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Other Models of TMs

(Part 2)

Lecture 18

Day 19/31

CS 154

Formal Languages and Computability

Spring 2019

Agenda of Day 19

- Summary of Lecture 17
- Quiz 7
- Lecture 18: Teaching ...
  - Other Models of TMs (Part 2)

Summary of Lecture 17: We learned ...

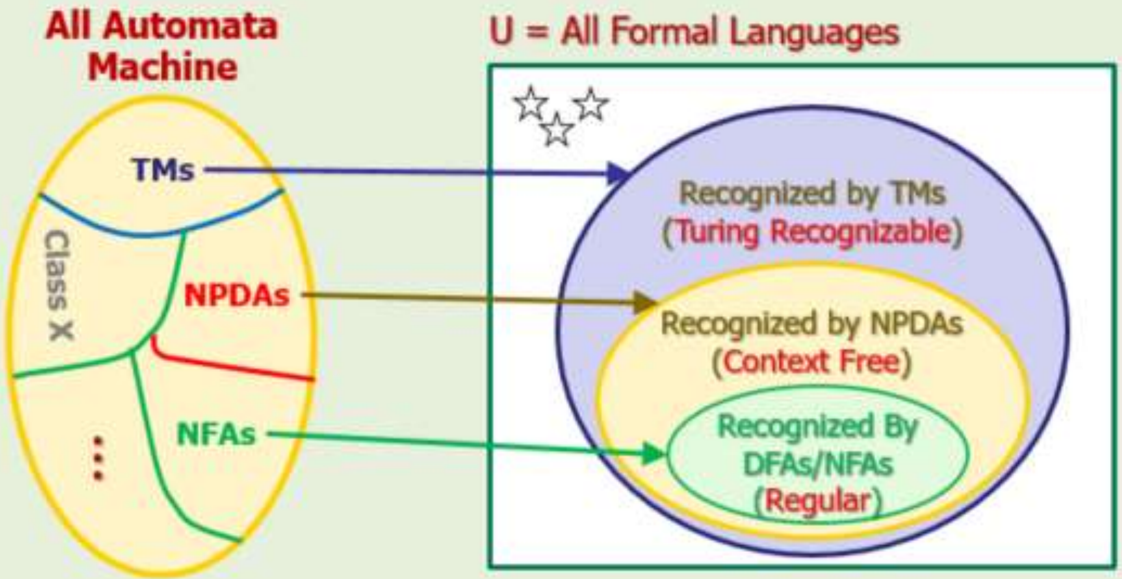
TMs vs NPDAs

- We use simulation to compare TMs and NPDAs.
- We can simulate whatever NPDAs do with TMs.
- But not vice versa!
  - At least we know some languages for which we could not construct NPDAs but could construct TMS
  - such as: $a^n b^n c^n$ and ww
- So, TMs are more powerful than NPDAs.

Any Question

Summary of Lecture 17: We learned ...

Machines and Languages Association



- TMs recognize some other non-regular languages called Turing recognizable.

Any Question

Summary of Lecture 17: We learned ...

TMs as Transducer

- Transducer is a device that converts an input to an output.
- We model a transducer by a ...
  - ... function.
- TMs can work in transducers mode.
  - Input is all or part of the nonblank symbols on the tape at the initial time.
  - Output is all or part of the tape's content when the machine halts.
- We learned how JFLAP shows the output.

- A function is called Turing-computable if ...
  - ... there exists a TM that implements it.
- We learned how to break a complex problem into smaller ones and how to combine TMs to make a bigger one.

Any Question

Summary of Lecture 17: We learned ...

Other Models of TMs

- We tried to figure out whether we can get more power by adding some capabilities to standard TMs.
- With any changes in standard TMs, we created a new class of automata.
- The changes we made:
  - TMs with stay-option ...
  - TMs with multidimensional-tape ...
- Were the new classes more powerful than the standard TM?

- We mentioned several theorems stating that the new classes were equivalent to standard TMs.

Any Question?

NAME	Alan M. Turing		
SUBJECT	CS 154	TEST NO.	7
DATE	03/28/2019	PERIOD	1 / 2 / 3

TEST RECORD	
PART 1	123
PART 2	
TOTAL	

Your **list #** goes here!

Take-Home Exam!

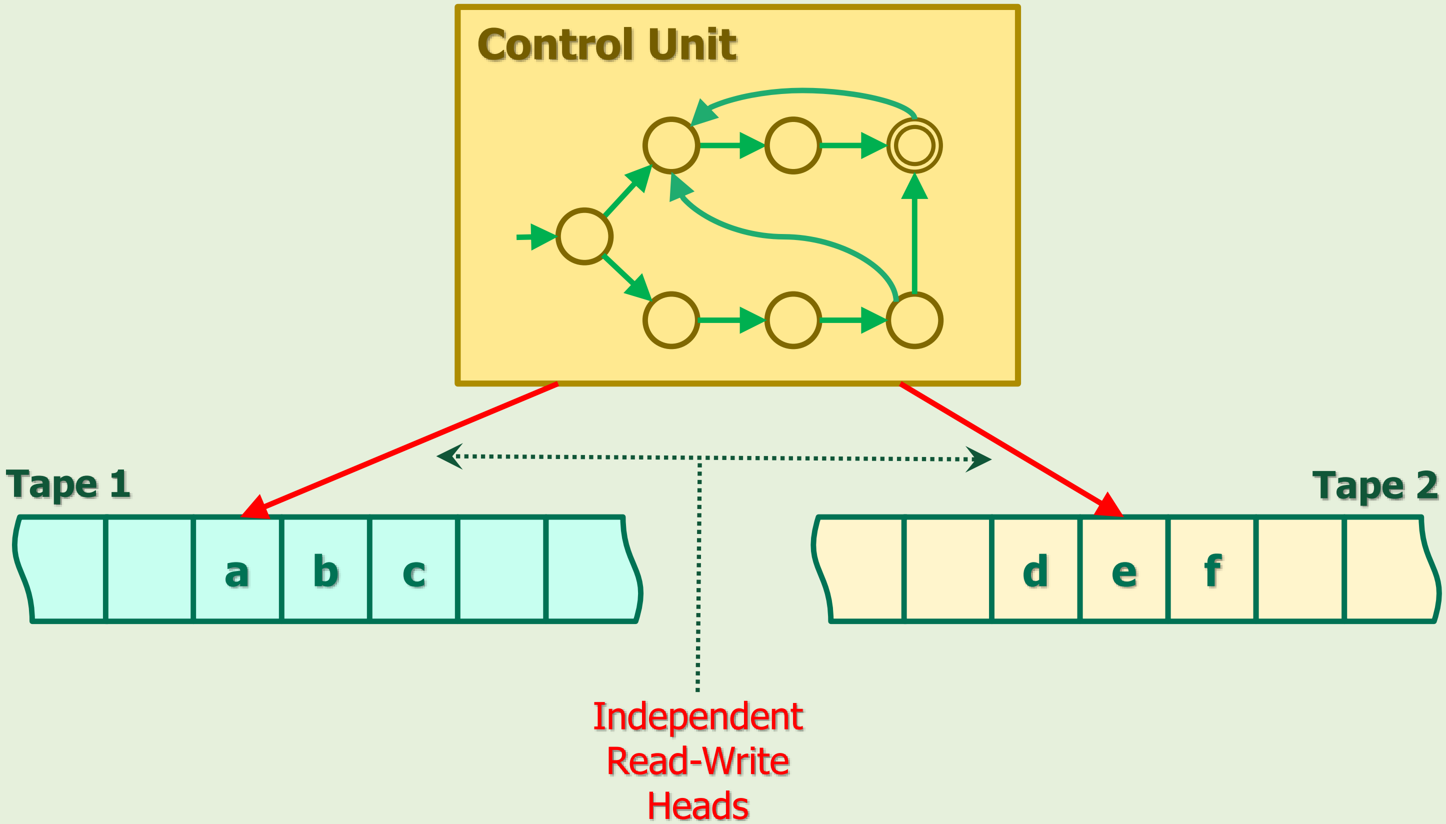
Quiz 7

Use Scantron

Multi-Tape TMs

Multi-Tape TMs: Building Block

- We can add additional tapes with independent read-write head to the standard TMs.
- For example, a double-tape TM looks like this:



Multi-Tape TMs: Transitions

- **Example 5**

- This is a transition of a double-tape TM.
  - We separate the labels of different tapes with "|".
- The transition condition is both inputs:

input symbol of tape 1 = 'a'

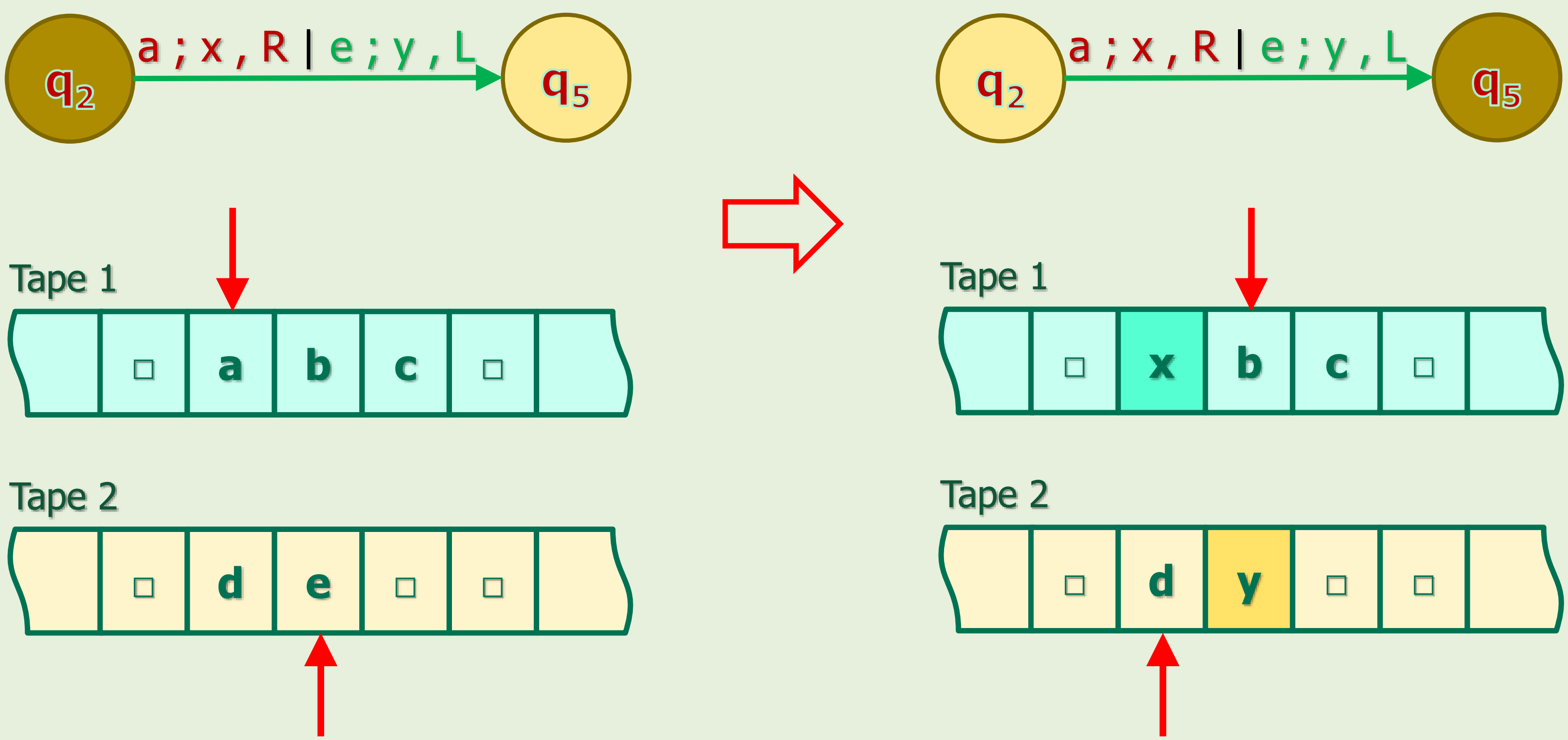
AND

input symbol of tape 2 = 'e'.

- The sub-rule looks like this: $\delta\ (q_2, a, e) = (q_5, x, y, R, L)$

Multi-Tape TMs: Transitions

- **Example 5 (cont'd)** $\delta\,(q_2, a, e) = (q_5, x, y, R, L)$



Before **After**

Multi-Tape TMs: Example

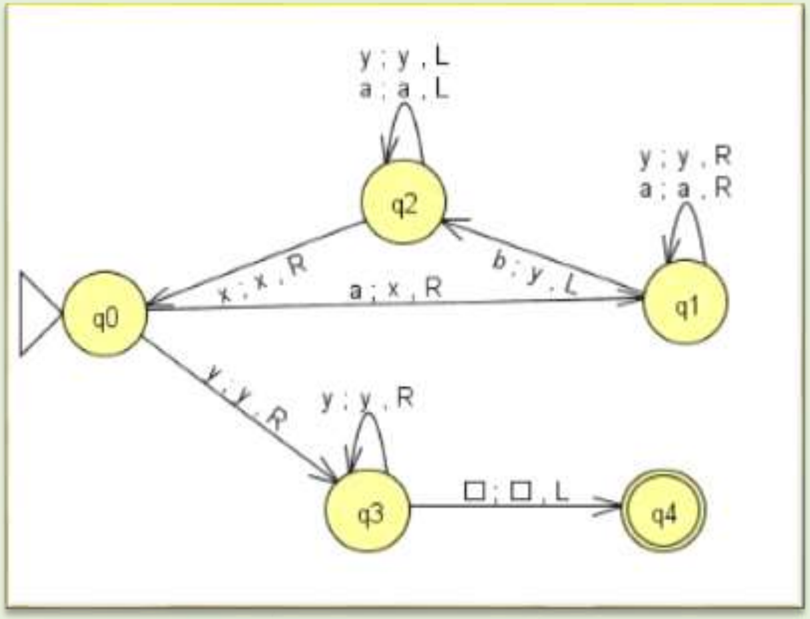
Example 6

- Design a double-tape TM for accepting the language $L = \{a^n b^n : n \geq 1\}$ over $\Sigma = \{a, b\}$.

Requirements:

1. The input is written on the tape 1.
2. You can use stay-option.

- Recall that we designed a standard TM for L before as the figure shows.



Multi-Tape TMs: Example

Example 6 (cont'd)

Strategy

- Read a's from tape 1 and write them on tape 2.
- When sensed the first 'b' on tape 1, match b's with the a's on tape 2.
- If all match, then accept,
- otherwise, reject.

- Do double-tape TMs facilitate our programming?

Homework

- Design a double-tape TM for accepting the following language:

  $L = \{a^n b^n c^n : n \geq 1\}$ over $\Sigma = \{a, b, c\}$

Requirements:

1. The input is written on the tape 1.
2. You can use stay-option.

Multi-Tape TMs: Formal Definition

- A TM with n-tape M is defined by the septuple:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$$

- Where:
  - ... (same as standard TM elements)

$$\delta: Q \times \Gamma^n \rightarrow Q \times \Gamma^n \times \{L, R\}^n$$

Where $\Gamma^n = \Gamma \times \Gamma \times \ldots \times \Gamma$ (Cartesian product)

Is this new class more powerful than standard TMs?

Theorem

- The TMs with multi-tape class is equivalent to the standard TMs class.
- We need to prove two things:
  1. Multi-tape TMs simulate standard TMs.
  2. Standard TMs simulate multi-tape TMs.

Proof

1. Multi-tape TMs simulate standard TMs.

- This step is trivial because if we just use one tape, then we have standard TM.

2. Standard TMs simulate multi-tape TMs.

Nondeterministic TMs (NTMs)

Nondeterministic TMs (NTMs)

Determinism:
During any timeframe, there is no more than one transition.

Any violation of this makes a machine nondeterministic.

Recap

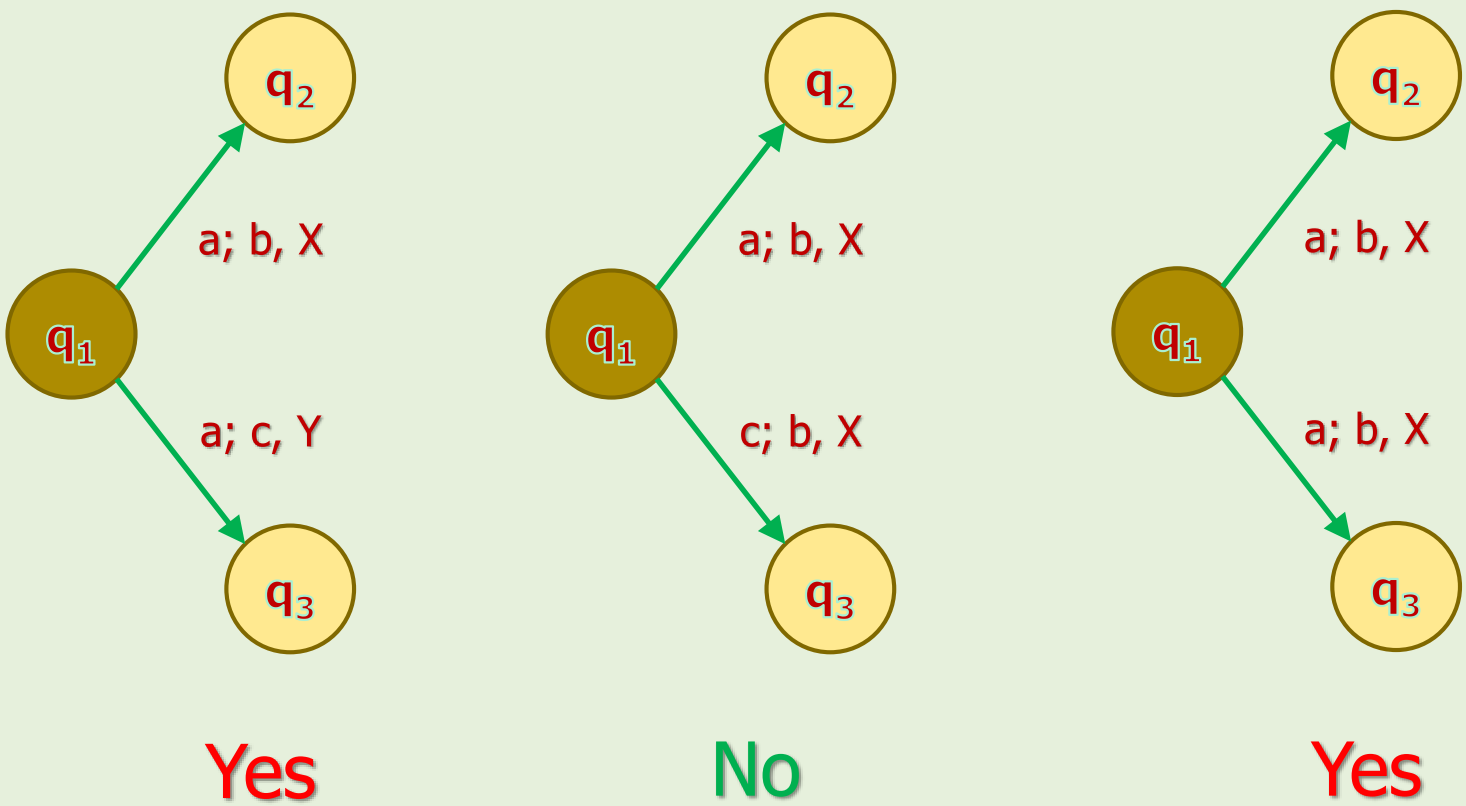
- What could be those violations in standard TMs?
  - ~~λ-transition~~
  - When δ is multifunction

- Theoretically, we can define λ-transition as usual.
- But historically it was not defined in TMs!

NTMs: Multifunction Examples

Example 7

- Are the following transitions violations for determinism? $X, Y \in \{L, R\}$



NTMs: Formal Definition

- A nondeterministic TM M is defined by the septuple:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$$

- Where:
  - ... (same as standard TM elements)

$$\delta: Q \times \Gamma \rightarrow 2^{Q \times \Gamma \times \{L, R\}}$$

δ is total function.

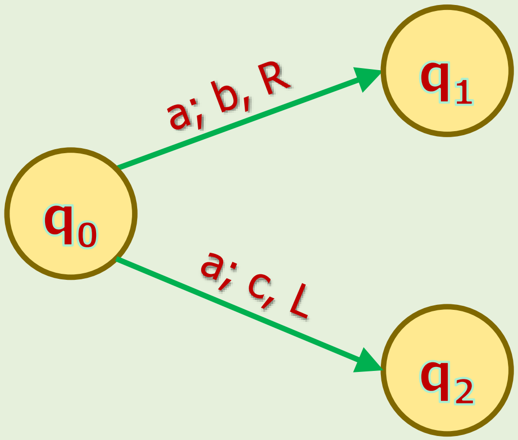
NTMs: Sub-Rules of Transition Function

Example 8

- Draw the transition graph of the following sub-rule:

$$\delta\ (q_0, a) = \{(q_1, b, R), (q_2, c, L)\}$$

Solution



How NTMs Behave If They Have Multiple Choices

- We already know that:

  All types of nondeterministic machines start parallel processing when they have multiple choices.

- In other words, for every possible choice, they create a new process and every process independently continues processing the string.

- The procedure of initiating a new process is exactly the same as NFAs.

How NTMs Behave If They Have Multiple Choices

Procedure of Initiating New Processes

1. It replicates its entire structure
   (transition graph + tape)
2. It initializes the new process with the current configuration.
3. The new process independently continues processing
   the rest of the input string.

- The only thing we need to know is:

What info do we need for the configuration?

NTMs' Configuration

1. Current state of the transition graph
2. Tape content + Position of the cursor

NTMs vs Standard TMs

NTMs vs Standard TMs

Theorem

- Nondeterministic TMs class is equivalent to standard TMs class.
- We need to prove two things:
  1. Nondeterministic TMs simulate standard TMs.
  2. Standard TMs simulate nondeterministic TMs.

Proof of 1

- Let's assume we've constructed a standard TM for an arbitrary language L.
- Can we always construct a NTM for L? How?
- Yes, just convert TMs definition to the NTMs',
  the same way we did for converting DFAs' to NFAs'.

NTMs vs Standard TMs

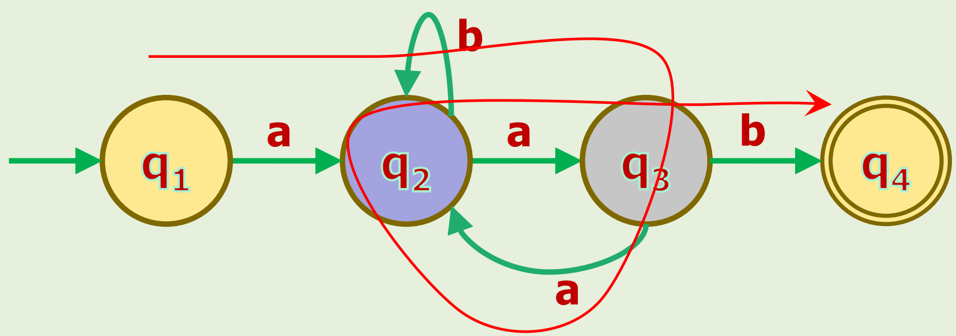
Proof of 2

- Mathematical proof of this part is not so easy but we can understand it intuitively.
- We'll explain it through an example.
- But first, we need some background.
- Next slide refreshes your knowledge about one-dimensional projection.

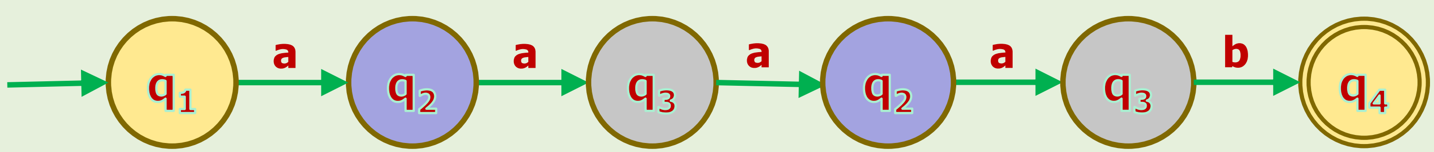
One-Dimensional Projection of a Walk

Recap

- As we learned before, we can represent a walk by one-dimensional projection.
- As an example, look at the string (walk) w = aaaab in the following NFA:

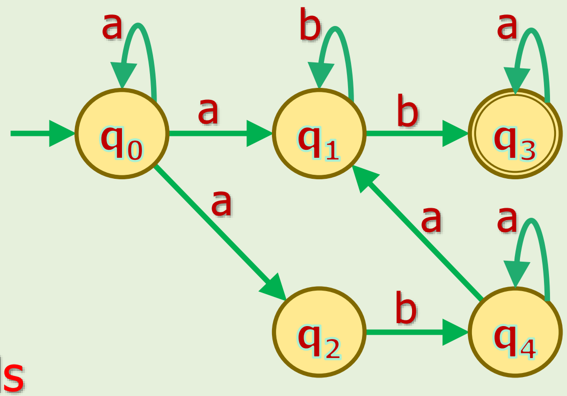


- This walk can be shown as:



NTMs vs Standard TMs

- **Proof of 2 (cont'd)**
- The following transition graph is an example of an NTM.

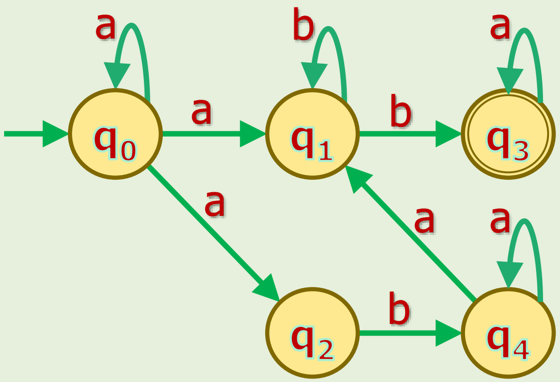


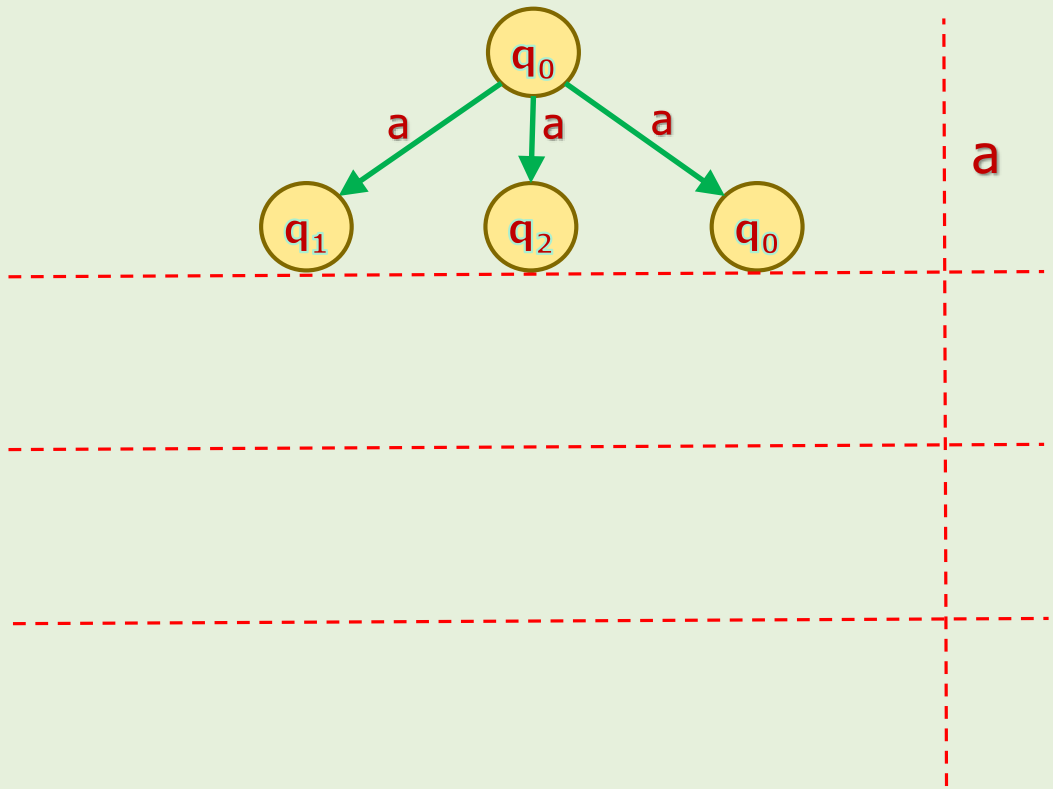
  - For simplicity, we showed only the input symbols of the labels.
  - It looks like an NFA, but we won't lose the generality of the point.

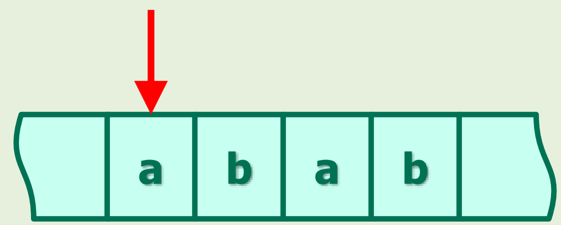
- If we input w = abab into this NTM, overall 6 processes will be initiated.
- We usually prefer to organize them as a tree.

Nondeterministic TMs vs Standard TMs

- **Proof of 2 (cont'd)**
- All processes for the string w = abab are organized in the following "processes tree":







Nondeterministic TMs vs Standard TMs

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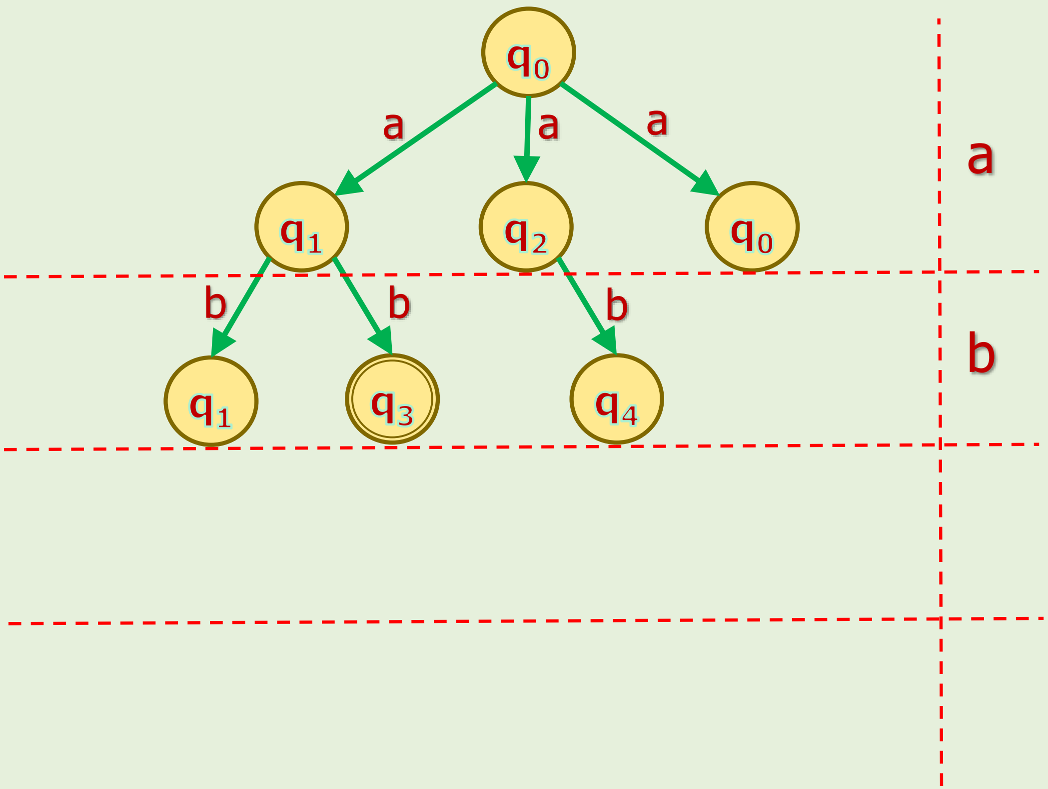

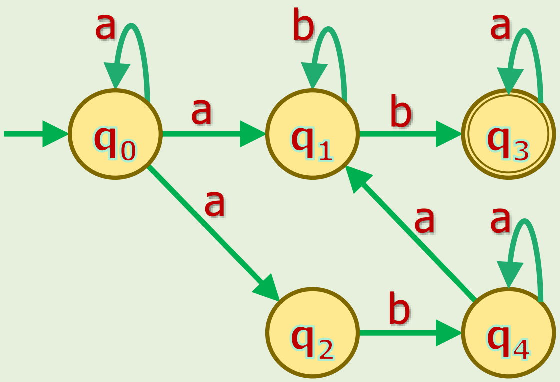

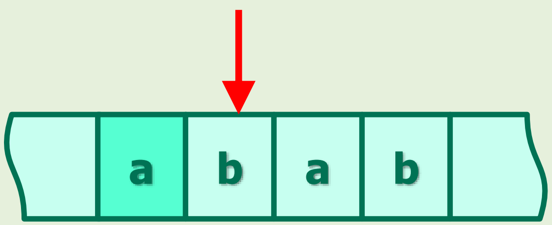

Nondeterministic TMs vs Standard TMs

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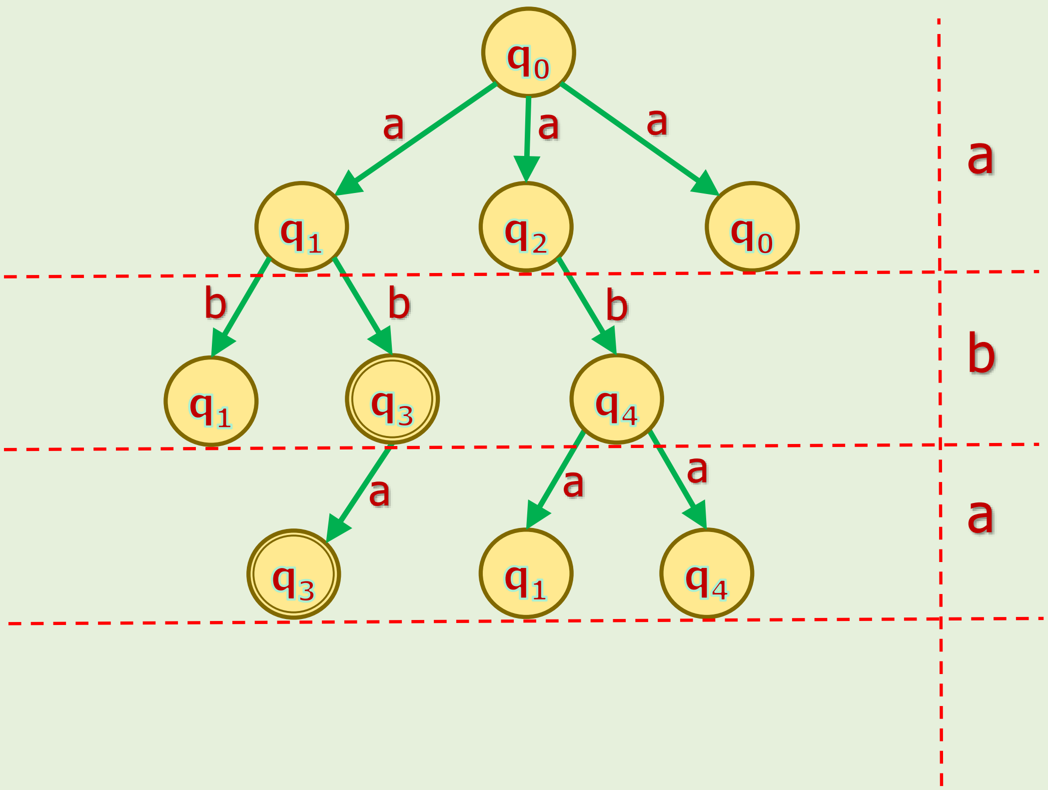

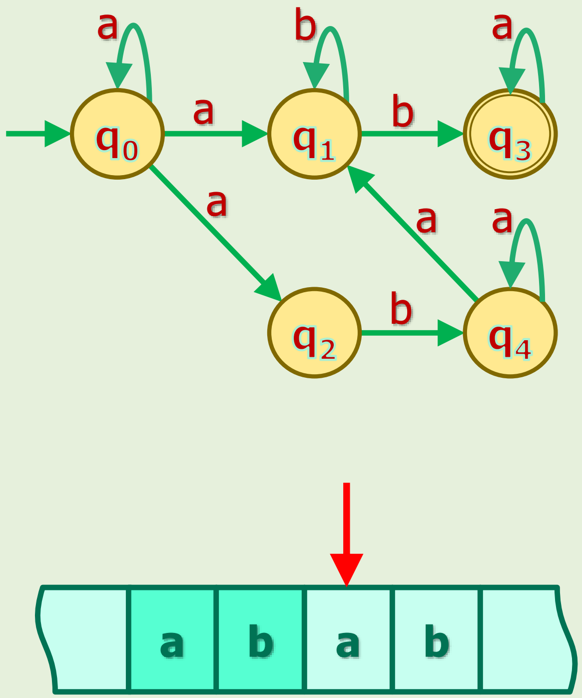

Nondeterministic TMs vs Standard TMs

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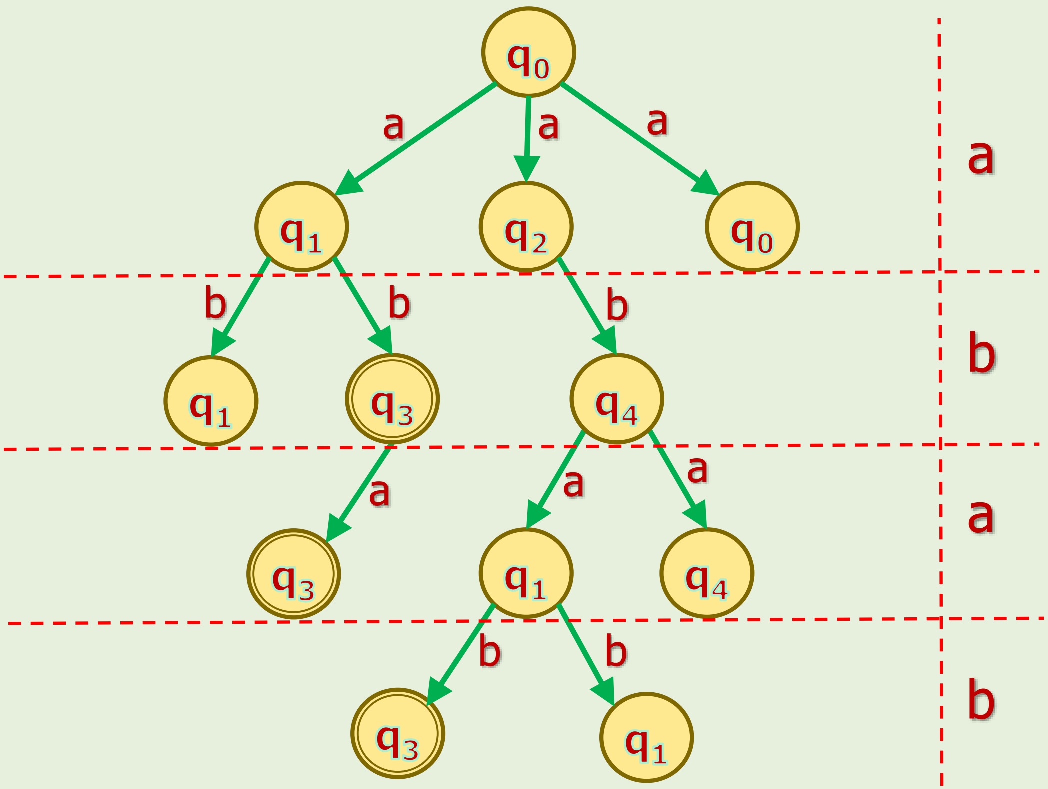

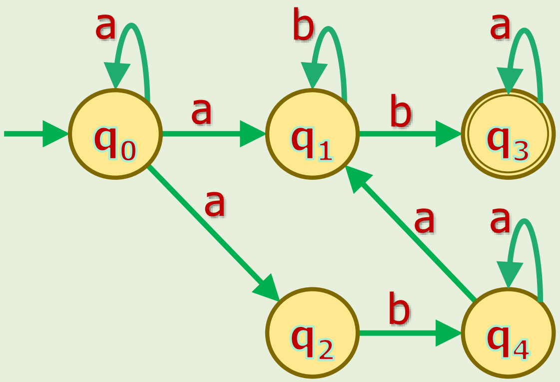

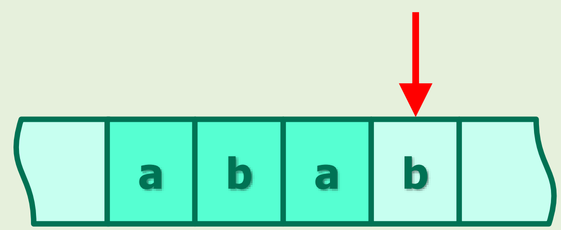

Nondeterministic TMs vs Standard TMs

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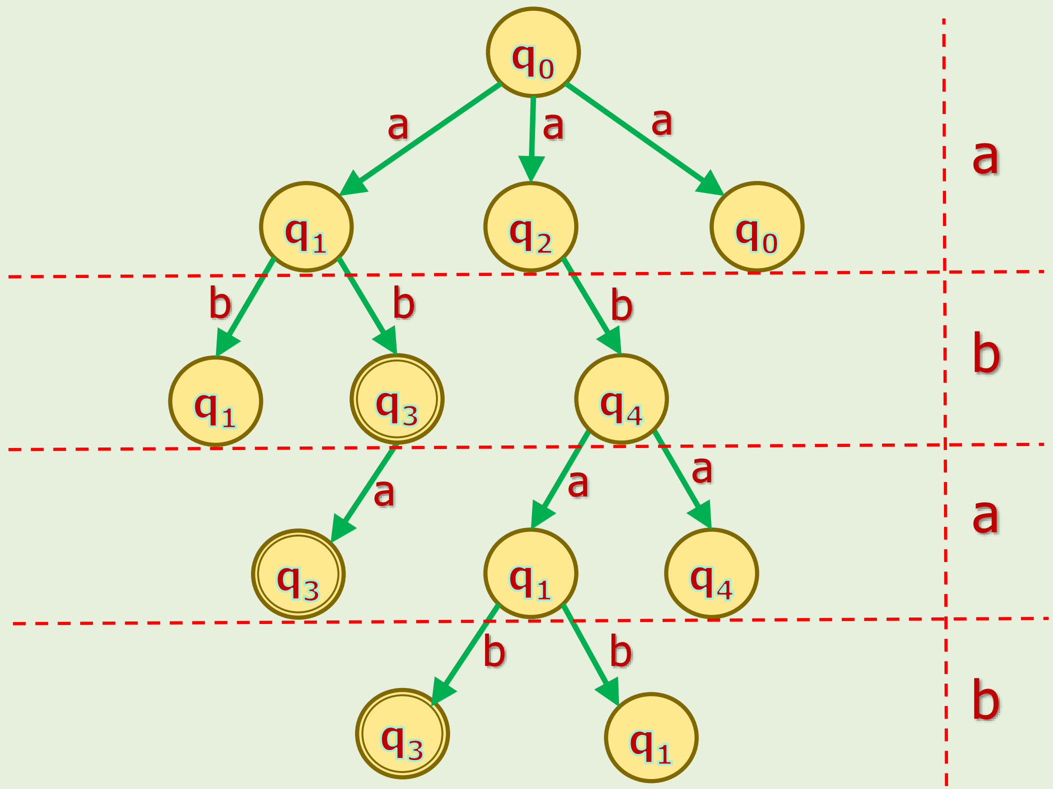
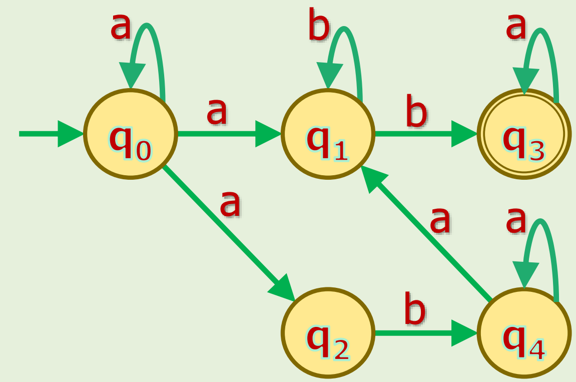
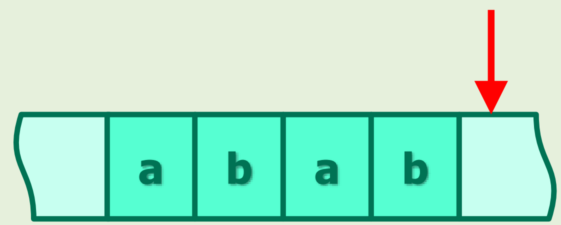


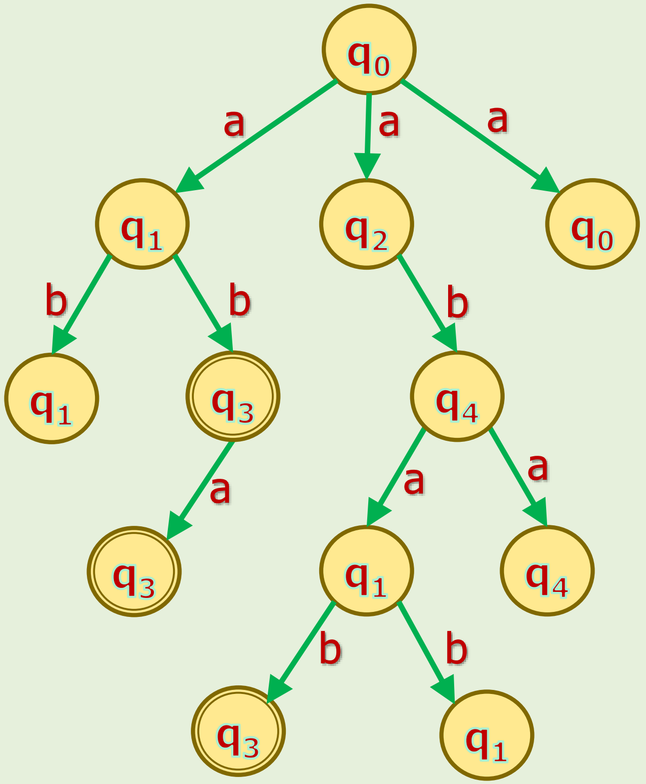




Nondeterministic TMs vs Standard TMs

- **Proof of 2 (cont'd)**
- Every walk from q_0 to a leaf is a process.
- Is every process a standard TM?
- Yes!

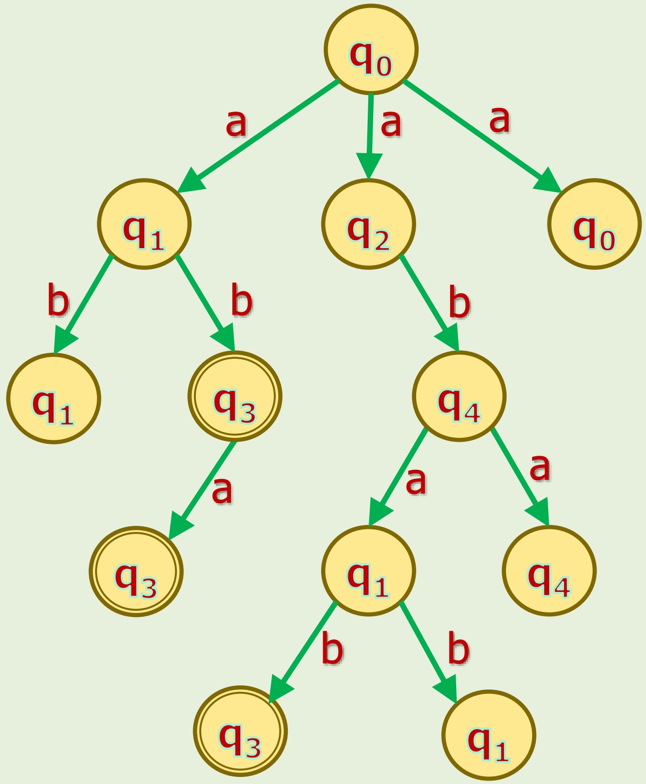
Therefore, an NTM
is a collection of standard TMs.



Nondeterministic TMs vs Standard TMs

Proof of 2 (cont'd)

- Can standard TM simulate NTMs?
- If it can handle the bookkeeping of the processes, then YES!
- Your term project and previous semesters term projects show that standard TMs can do this!



Nondeterministic TMs: Notes

1. Nondeterminism does NOT ADD any POWER to the automata theory.
   – It just speeds up the computation.

2. We are always looking for more power and speed is NOT our concern yet.
   – "Speed" will be a matter of concern when we will be talking about "complexity theory".

3. Quantum computing tries to implement nondeterminism!
   – So, it does NOT add any power to computing too!

Basic Concepts of Computation

Definition of Algorithm

Definition

- An algorithm for a problem L (= language) is equivalent to design a TM that solves L (= accept the language).

- Therefore, we define the TM structure as the "algorithm" for solving that problem.

Definition of Program

- A sub-rule defines how a machine acts in one transition for a specific state.
- The transition function defines all possible transitions of the machine for all possible situations.

- What is the "program" of a TM?

Definition

- The transition function of a TM is its "program".

References

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2. Michael Sipser, "Introduction to the Theory of Computation, 3rd ed.," CENGAGE Learning, United States, 2013
ISBN-13: 978-1133187790