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Regular Expressions

(Part 2)

Lecture 20

Day 24/31

CS 154

Formal Languages and Computability

Spring 2019

Agenda of Day 24

- Solution and Feedback of Quiz 7 and Quiz ++
- Summary of Lecture 19
- Lecture 20: Teaching ...
  - Regular Expressions (Part 2)

Solution and Feedback of Quiz 7 (Out of 20)

Section	Average	High Score	Low Score
01 (TR 3:00 PM)	18.94	20	14
02 (TR 4:30 PM)	17.27	20	11
03 (TR 6:00 PM)	18.37	20	15

Solution and Feedback of Quiz ++ (Out of 45)

Section	Average	High Score	Low Score
01 (TR 3:00 PM)	39.32	45	30
02 (TR 4:30 PM)	37.38	45	14
03 (TR 6:00 PM)	39.55	45	31

Summary of Lecture 19: We learned ...

Regular Expressions (REGEXs)

- REGEXs are another way to represent formal languages.
- We like REGEXs because ...
  - ... they represent formal languages in a more compact way.
  - They are shorthand for some formal languages.
  - They have practical applications in OS's and programming languages.
- This course introduces the mathematical base of them.

- The elements of REGEXs are:
  - ϕ, λ, Σ
  - ()
  - Operators:
    + (union)
    . (dot or concatenation)
    * (star-closure)

Any Question?

Summary of Lecture 19: We learned ...

REGEXs

- We defined REGEXs formally as:

1. ϕ, λ, and $a \in \Sigma$ are all REGEXs.
2. If r_1 and r_2 are REGEXs, then the following expressions are REGEXs too:

   $r_1 + r_2$

   $r_1 . r_2$

   r_1^*

   (r_1)
3. A string is REGEX if it can be derived recursively from the primitive REGEXs by a finite number of applications of the rule #2.

- Between REGEXs and languages, there are the following correspondence:

  1. $L(\phi) = \{ \}$
  2. $L(\lambda) = \{\lambda\}$
  3. $L(a) = \{a\}$ for all $a \in \Sigma$
  4. $L(r_1 + r_2) = L(r_1) \cup L(r_2)$
  5. $L(r_1 . r_2) = L(r_1) L(r_2)$
  6. $L((r_1)) = L(r_1)$
  7. $L(r_1^*) = (L(r_1))^*$

- We learned how to calculate the language represented by a REGEX by using the above correspondences.

Any Question?

REGEX → Language Examples

Example 18

- Given $r = (aa)^*$
- $L(r) = ?$

Solution

REGEX → Language Examples

Example 19

- Given $r = (bb)^* b$
- $L(r) = ?$

Solution

REGEX → Language Examples

Example 20

- Given r = $(aa)^* \, b \, (bb)^*$
- $L(r)$ = ?

Solution

REGEX → Language Examples

Example 21

- Given $r = (a + b)^* (a + bb)$
- $L(r) = ?$

Solution

Associated Languages to REGEXs

Definition

- If REGEX r represents language L, then L is called the "associated language" to r and is denoted by L(r).

- As we saw in the previous slides ...

  If $r = (aa)^*$, then

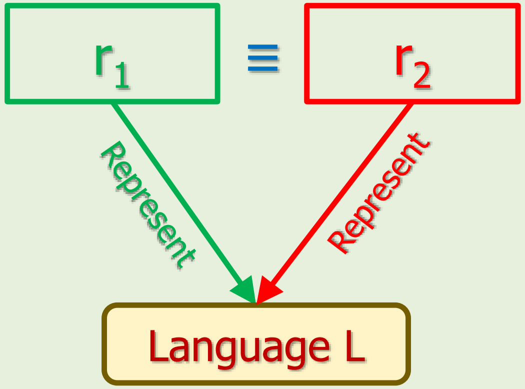
  $L(r) = \{a^{2n} : n \geq 0\}$

Equivalency of REGEXs

Definition

- Two regular expressions r_1 and r_2 are equivalent iff both has the same associated language.

$$r_1 \equiv r_2 \;\leftrightarrow\; L(r_1) = L(r_2)$$



Equivalency of REGEXs Example

Example 22

- Given r_1 and r_2 as:
- $r_1 = (a + b)^* a$
- $r_2 = (a + b)^* (a + b)^* a$
- Are r_1 and r_2 equivalent?

- Both of these REGEXs are expressing a language containing any string of 'a' and 'b' terminated by an 'a'.

- For a given language L, how many REGEX we can make?
  - Infinite

REGEXs Identities

REGEXs Identities

- If r, s, and t are REGEXs, and $a, b \in \Sigma$, then:
  1. $r(s + t) = rs + rt$
  2. $(s + t)r = sr + tr$
  3. $(a^*)^* = a^*$
  4. $(a \dots a)^* a = a (a \dots a)^*$
  5. $a^* (a + b)^* = (a + b)^* a^* = (a + b)^*$
- We can use the seven mathematical rules mentioned before to prove the above identities.
- Obviously, we should show both sides represent the same language.
- For example, for the first one, we should show:

$$L(r(s + t)) = L(rs + rt)$$

REGEXs Identities Examples

Example 23

$a\,b^* + b\,b^*$

$= (a + b)\,b^*$

Example 24

$b^* + b^*\,a$

$= b^*\,(\lambda + a)$

Example 25

$aaa^*\,bb^* + aa^*\,b\,bb^*$

$= (aaa^* + aa^*b)\,bb^*$

$= (aa^*\,a + aa^*b)\,bb^*$

$= aa^*\,(a + b)\,bb^*$

Homework: Identities

- Given r = (aa)* (λ + ab) (bb)*
- L(r) = ?

Language → REGEX Examples

Language → REGEX

Example 26

- Given $L(r) = \{w \in \Sigma^* : w$ has exactly one a$\}$ over $\Sigma = \{a, b\}$

  $r = ?$

Solution

Language → REGEX

Example 27

- Given $L(r) = \{w \in \Sigma^* : w$ has at least one pair of consecutive a's$\}$ over $\Sigma = \{a, b\}$

  $r = ?$

Solution

Language → REGEX

Example 28

- Given $L(r) = \{a^n b^m : n \geq 3, m \text{ is even}\}$ over $\Sigma = \{a, b\}$

  $r = ?$

Solution

Language → REGEX

Example 29

- Given $L(r) = \{w : |w| \geq 3, 3^{rd}$ symbol of w is 'a'$\}$ over $\Sigma = \{a, b\}$

  $r = ?$

Solution

Language → REGEX

Example 30

- Given $L(r) = \{a^n b^m : n + m \text{ is even}\}$ over $\Sigma = \{a, b\}$

  $r = ?$

Solution

Homework

- Find a REGEX for the following languages.
  1. $L(r) = \{w \in \{a, b\}^* : \text{w contains no a}\}$
  2. $L(r) = \{w \in \{a, b\}^* : \text{w contains exactly two a's}\}$
  3. $L(r) = \{a^{2n} : n \geq 0\}$ over $\Sigma = \{a\}$
  4. $L(r) = \{a^{2n+1} : n \geq 0\}$ over $\Sigma = \{a\}$
  5. $L(r) = \{w \in \{a, b\}^* : \text{w contains at least two a's}\}$
  6. $L(r) = \{w \in \{a, b\}^* : \text{w begins with an 'a' and ends with a 'b'}\}$
  7. $L(r) = \{w \in \{a, b\}^* : \text{w begins and ends with the same symbol}\}$
  8. $L(r) = \{w \in \{a, b\}^* : \text{w contains exactly one occurrence of aa}\}$

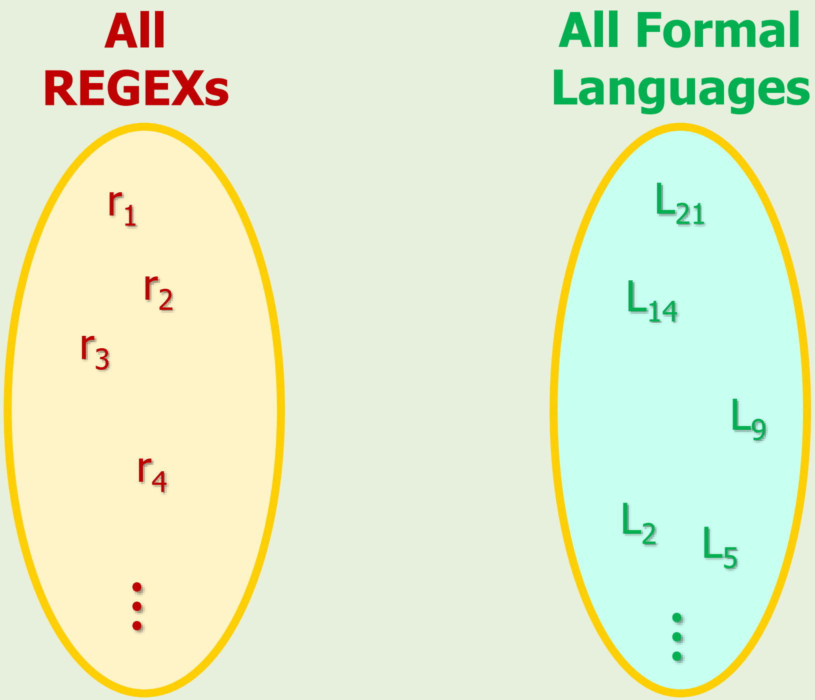
REGEXs and Languages Association

REGEXs and Languages Association

- What is the relationship between:

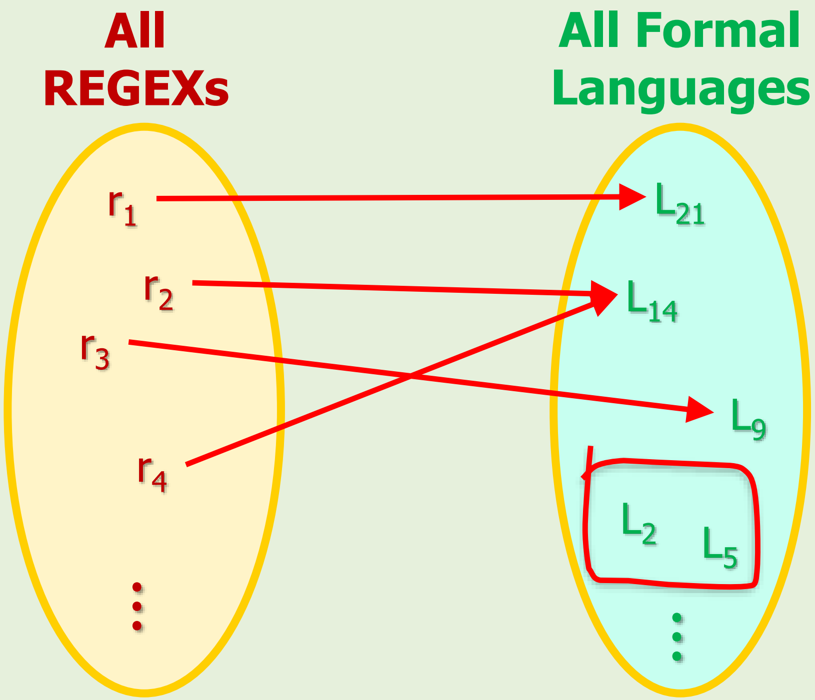
  the set of REGEXs, and

  the set of all formal languages?



REGEXs and Languages Association

- You agree that "every REGEX represents a language".
- BUT we don't know yet whether we can represent every language by a REGEX or not!
  - Our knowledge is not enough yet.



REGEX for More Complex Languages

- Find a REGEX for each of the following languages:

1. $L = \{a^n b^n : n \geq 0\}$ over $\Sigma = \{a, b\}$
2. $L = \{ww^R : w \in \Sigma^*\}$ over $\Sigma = \{a, b\}$

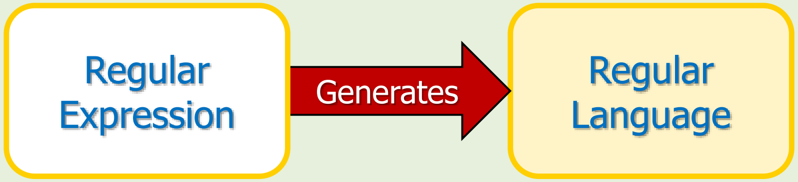
Solution

- …
- Struggling?!
- After some struggling, you realize that you cannot find any REGEX for these languages! Why?
- Look at the theorems in the next slide!

REGEXs and Regular Languages

Theorem

- If r is a REGEX, then L(r) is a regular language over Σ.



Theorem

- Let L be a regular language over Σ.
  Then there exists a REGEX r such that L = L(r).

REGEXs and Regular Languages

- The following definition shows that REGEXs are another way to represent regular languages.

Definition

- A language is regular iff a REGEX represents it.

Set of Languages Represented by REGEXs	=	Set of Regular Languages

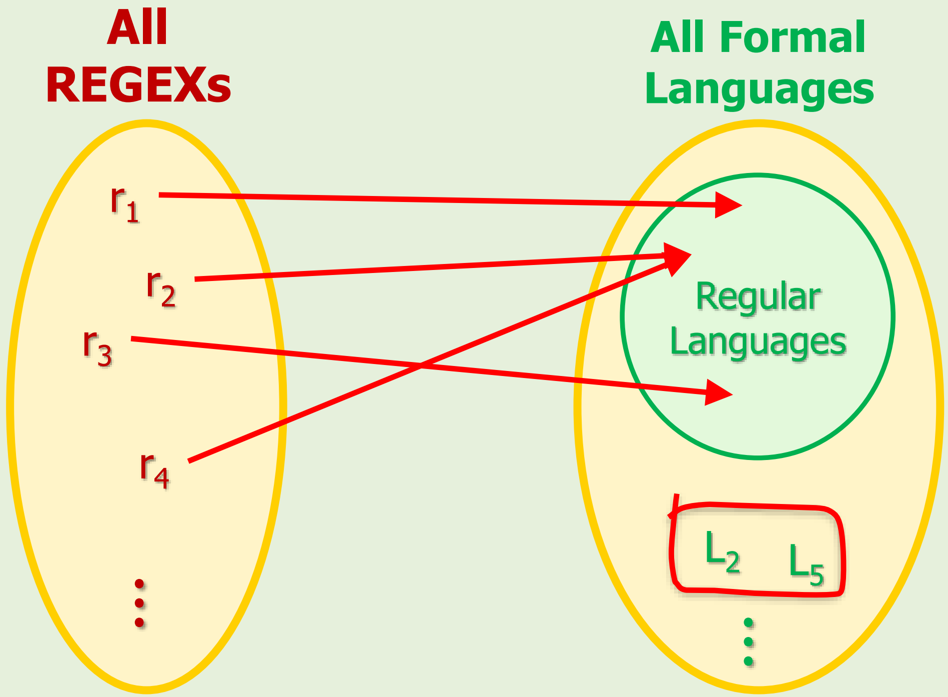
REGEXs and Languages Association

Revisited

- We've already agreed that "every REGEX represents a language".
- Now we know that:

Those languages are regular.

And there is no association between non-regular Languages and REGEXs.



What is the Next Step?

- We started this topic to look for a compact way to represent formal languages.
- We introduced REGEXs and experienced their usefulness.
- But the theorems showed their limitations.
  - REGEXs represent only regular languages.
- So, the next step would be looking for ...
  a practical compact way to represent non-regular languages.

Last Question: Do We Have a Standard REGEX?

- In computer science, we do NOT have a standard REGEX!
- Every OS and every programming language has its own REGEX.
- Of course, there are some common elements and rules between all of them.
  - So, you should learn each one based on their elements and rules.
- But the basic idea is the same.
  - In fact, they have implemented their REGEXs based on the REGEX we introduced here.

Homework

- Fill out the following tables.
- For example, ϕ + a = ϕ + a = a or a . a = aa
  - Note that '+' and '.' are binary operators and need two operands but '*' is unary operator and needs one operand.

+	ϕ	λ	a
ϕ			a
λ			
a	a		a

.	ϕ	λ	a
ϕ			
λ			
a			aa

ϕ*	
λ*	
a*	a*

References

1. Linz, Peter, "An Introduction to Formal Languages and Automata, 5th ed.," Jones & Bartlett Learning, LLC, Canada, 2012
2. Michael Sipser, "Introduction to the Theory of Computation, 3rd ed.," CENGAGE Learning, United States, 2013 ISBN-13: 978-1133187790