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Grammars

(Part 1)

Lecture 21
Day 25/31

CS 154
Formal Languages and Computability
Spring 2019

Agenda of Day 25

- Summary of Lecture 20
- Quiz 8
- Lecture 21: Teaching …
  - Grammars (Part 1)

Summary of Lecture 20: We learned ...

REGEXs

- Two regular expressions r_1 and r_2 are equivalent iff ...
- ... both represent the same language.

$$r_1 \equiv r_2 \leftrightarrow L(r_1) = L(r_2)$$

Identities

- If r, s, and t are REGEXs, and a, b $\in$ Σ, then:

1. $r(s + t) = rs + rt$
2. $(s + t)r = sr + tr$
3. $(a^*)^* = a^*$
4. $(a \ldots a)^* a = a (a \ldots a)^*$
5. $a^* (a + b)^* = (a + b)^* a^* = (a + b)^*$

- A language is regular iff a REGEX represents it.
- The limitation of REGEXs ...
  - They just represent regular languages.
  - While more interesting languages are non-regular!
- Associated language to a REGEX is ...
  - ... the language that it represents.
- We don't have a standard REGEX.

Any Question?

NAME	Alan M. Turing		
SUBJECT	CS 154	TEST NO.	8
DATE	04/18/2019	PERIOD	1 / 2 / 3

TEST RECORD	
PART 1	123
PART 2	
TOTAL	

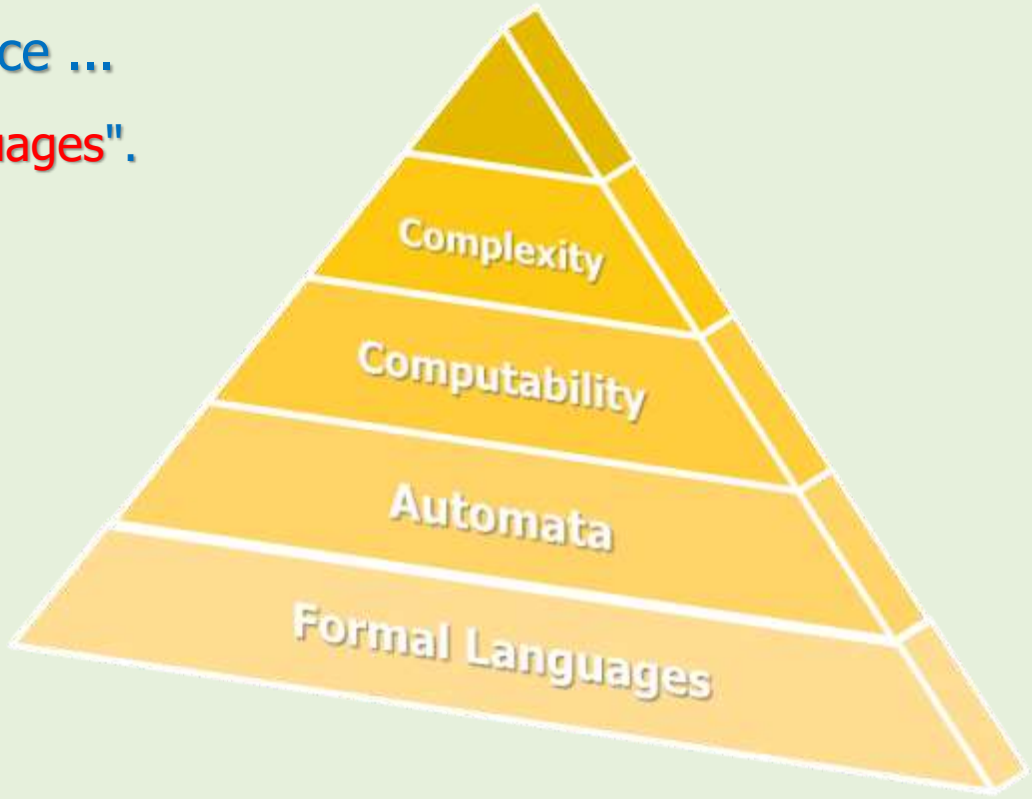
Quiz 8

Use Scantron

The Big Picture of the Course

Computer Science Foundation

- We started the semester with "Formal Languages" but we said:
  - we'd get back to it during the semester.
- So far, we've got back twice ...
  - Introduced "Regular Languages".
  - Introduced "REGEXs".
- ... and this is the 3rd time!



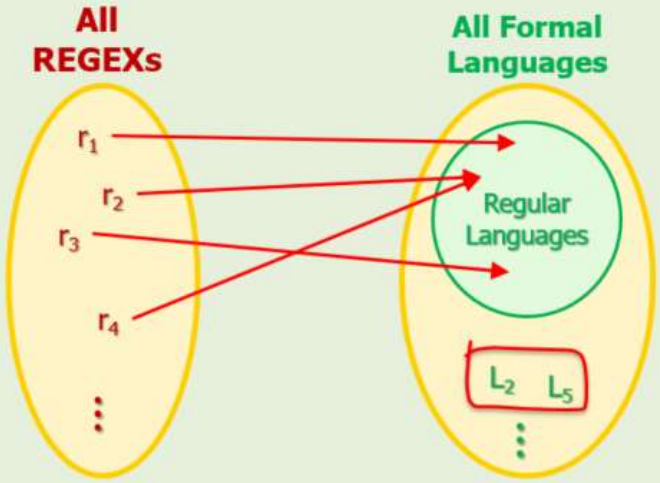
Motivation

- So far, we've represented formal languages by two mathematical tools:
  - Sets (set builder, roster method, Venn diagrams)
  - Regular Expressions (REGEXs)

What is the problem with them?

- Sets are NOT practical in computer science!

- REGEXs are limited to regular languages.



Objective of This Lecture

- We need a more powerful and practical tool to represent NONREGULAR languages!

That is Grammar!

- Our target is to represent all formal languages!
- But like any other tools, grammars have their own limitations.

- We'll be talking about grammars for 3 sessions!

Grammars

Introduction

- What would be your reaction if you encounter the following English sentences:
  - dog the runs.
  - dog runs the.
  - runs dog the.

- Even though all words are correct English words but the combinations are meaningless.
- The words' positions are not as they should be!

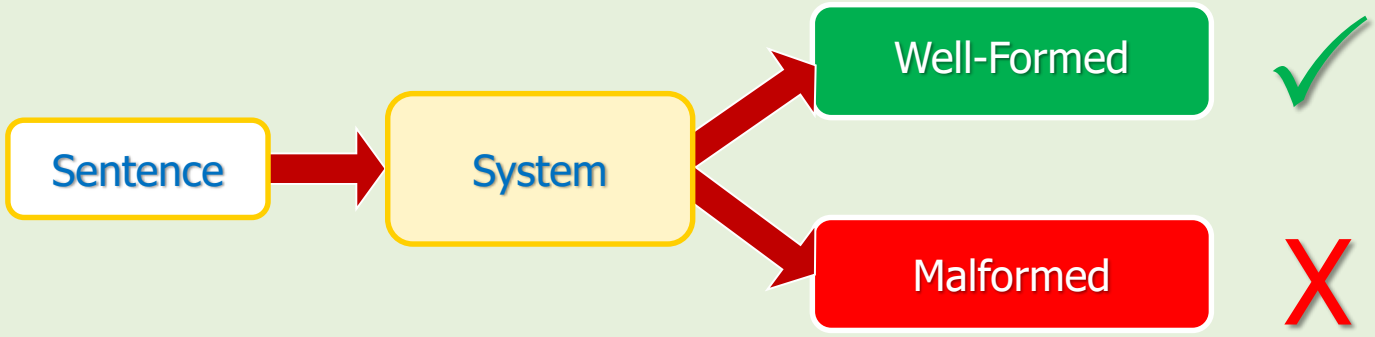
- In computer science and linguistic terminology:

  These sentences are not "well-formed".

What Are We Looking For?

- We need a system (or tool) to distinguish between:

  "well-formed" and "malformed" sentences



- Let's start with a simple example from a natural language like English.
- Then we'll generalize the idea to formal languages.

A Simple English Grammar

- A simple rule for constructing an English "sentence" is:

  <sentence> → <noun-phrase> <predicate>

  - Read "→" as: "is defined as"

- This rule is called "production rule".
  - Because we can produce an English sentence by this rule.

- The problem is ...
- We defined a sentence but we introduced two new "variables":

  <noun-phrase> and <predicate>!

- So, we need to define them before going further.

A Simple English Grammar

<sentence> → <noun-phrase> <predicate>

- We might define these new variables by the following production rules:

<noun-phrase> → <article> <noun>

<predicate> → <verb>

- Again, we introduced new variables <article>, <noun>, and <verb> that should be defined.

- So, we need to keep going ...

A Simple English Grammar

<sentence> → <noun-phrase> <predicate>

<noun-phrase> → <article> <noun>

<predicate> → <verb>

- We might define these new variables by the following production rules:

<article> → a

<article> → the

<noun> → dog

<noun> → boy

<verb> → runs

<verb> → walks

No variables left!

A Simple English Grammar: Notes & Definitions

- Variables are defined either by other variables,
- Or they are assigned "values" (aka terminals),
- Or mix of both (we'll see in the next slides)

Rough Definition of Grammar

- A set of production rules is called "grammar".
  - Later, we define it formally.
- Every grammar has a "starting variable".
  - In this example, <sentence> is the starting variable.

Simple English Grammar

1. <sentence> → <noun-phrase> <predicate>
2. <noun-phrase> → <article> <noun>
3. <predicate> → <verb>
4. <article> → a
5. <article> → the
6. <noun> → dog
7. <noun> → boy
8. <verb> → runs
9. <verb> → walks

Repeated

A Simple English Grammar: Examples

Example 1

- Is this a "well-formed" sentence?

"the dog runs"

Solution

- A sentence is well-formed if we can "derive" it from the production rules.
- We start from the "starting variable":

<sentence> ⇒ <noun-phrase> <predicate>

⇒ <article> <noun> <verb>

⇒ the <noun> <verb>

⇒ the dog <verb>

⇒ the dog runs

Simple English Grammar

1. <sentence> → <noun-phrase> <predicate>
2. <noun-phrase> → <article> <noun>
3. <predicate> → <verb>
4. <article> → a
5. <article> → the
6. <noun> → dog
7. <noun> → boy
8. <verb> → runs
9. <verb> → walks

Repeated

A Simple English Grammar: Examples

Example 1 (cont'd)

- We derived the sentence "the dog runs" from the set of production rules (grammar), so, IT IS WELL-FORMED.

- We used "⇒" notation for "derivation".
  - Read "⇒" as: "derives"
- We can also represent the whole process as:

$$\text{<sentence>} \overset{*}{\Rightarrow} \text{the dog runs}$$

to represent "multiple derivations" when we want to summarize the whole process.

- Now, let's take a failure example.

A Simple English Grammar: Examples

Example 2

- Is this a "well-formed" sentence?

"the runs dog"

Solution

- <sentence> ⇒ <noun-phrase> <predicate>

  ⇒ <article> <noun> <verb>

  ⇒ the <noun> <verb>

- "runs" is NOT a "noun"!
- It fails, so, the sentence IS MALFORMED.
- What else can we derive from this grammar?

Simple English Grammar

1. <sentence> → <noun-phrase> <predicate>
2. <noun-phrase> → <article> <noun>
3. <predicate> → <verb>
4. <article> → a
5. <article> → the
6. <noun> → dog
7. <noun> → boy
8. <verb> → runs
9. <verb> → walks

Repeated

A Simple English Grammar: Examples

Example 3

- The set of all sentences derivable from this grammar is:

{ "a dog runs",
"a dog walks",
"a boy runs",
"a boy walks",
"the dog runs",
"the dog walks",
"the boy runs",
"the boy walks" }

- What can we call this set of strings?

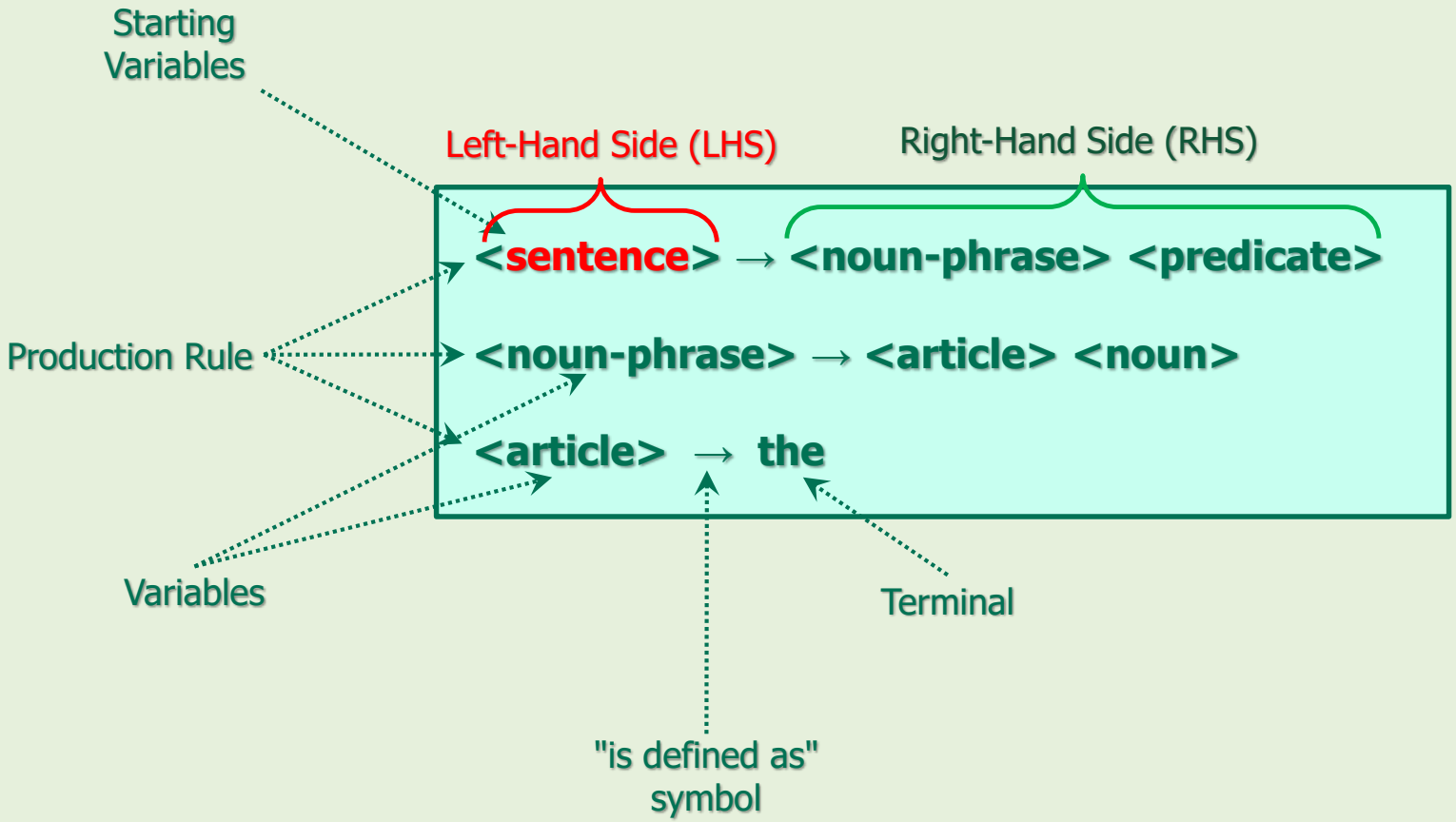
The language generated by the grammar

Simple English Grammar

1. <sentence> → <noun-phrase> <predicate>
2. <noun-phrase> → <article> <noun>
3. <predicate> → <verb>
4. <article> → a
5. <article> → the
6. <noun> → dog
7. <noun> → boy
8. <verb> → runs
9. <verb> → walks

Repeated

Grammar Terminologies



A Side Note About Natural Languages

- Unfortunately, natural languages were not developed by mathematicians.
- That's why, there are thousands of exceptions in their definition!
- So, learning them perfectly as second languages is very hard!

- The only successful language that was developed by a scientist is ...

  Esperanto!

- For more info, please look at the appendix A at the end of this lecture note.
- It is just for your information and is NOT part of this course.

Formal Grammars

Formal Grammars

- We can generalize the natural languages grammars to formal grammars.

Example 4

- Consider the following set of "production rules":

$$\begin{cases} S \rightarrow aB \\ B \rightarrow baB \\ B \rightarrow \lambda \end{cases}$$

- This set of production rules is an example of a "formal grammar".
- Let's see its ingredients in detail?

Ingredients of the Production Rules

- S and B are "variables" in this example.
  - Variables are represented by capital letters.

$$\begin{cases} S \rightarrow aB \\ B \rightarrow baB \\ B \rightarrow \lambda \end{cases}$$

- By default, the "starting variable" is 'S' unless we mention something else.

- 'a' and 'b' in this example are called "terminal symbols".
  - Terminals are represented by lower-case letters.
  - λ is our familiar empty string.
  - Terminals can be any sequence of terminal symbols or λ.

- "aB" and "baB" contain both variable and terminals and are called "sentential form".

How a string can be derived from a grammar?

Example 5

- Let grammar G be:
  1. $S \rightarrow a\ S\ b$
  2. $S \rightarrow \lambda$
- Derive string "ab"

Solution

$$S \overset{1}{\Rightarrow} a\ S\ b \overset{2}{\Rightarrow} a\ \lambda\ b = ab$$

- Could we derive this string if we had started with rule #2?

Derivation of Strings

Example 6

- Let grammar G be:
  1. $S \rightarrow a\ S\ b$
  2. $S \rightarrow \lambda$
- Derive string "aabb".

Solution

$$S \overset{1}{\Rightarrow} a\ S\ b \overset{1}{\Rightarrow} aa\ S\ bb \overset{2}{\Rightarrow} aa\ \lambda\ bb = aabb$$

- We can summarize the above derivation like this:

$$S \overset{*}{\Rightarrow} aabb$$

- As we said before, this notation is used when we just want to show that S drives the string.

A Convention

- When the left-hand sides of two or more production rules are the same, we can combine the right-hand sides by separating them with a vertical bar "|".

Example 7

- Let grammar G be:

  $S \rightarrow a\ S\ b$

  $S \rightarrow \lambda$

- We can represent it as:

  $S \rightarrow a\ S\ b \mid \lambda$

Associated Language to Grammars

- We can apply the production rules "recursively" in any arbitrary orders.
- Therefore, a grammar can generate zero, one, or more strings.

Definition

- The set of all strings generated (aka produced) by the grammar G is called the "associated language to G" and is denoted by L(G).

Grammar → Language Examples

Grammar → Language Examples

Example 8

- Let grammar G be:

  $S \rightarrow a\ S \mid \lambda$

- L(G) = ? // show it by a set-builder.

Solution

- How about this grammar?
  $S \rightarrow S\ a \mid \lambda$
- Is there any difference?

Grammar → Language Examples

Example 9

- Let grammar G be:

  $S \rightarrow a\ S\ b \mid \lambda$

- L(G) = ? // show it by a set-builder.

Solution

Conclusion

- After this example, we know that grammars can represents more languages than regular languages!
- So, they are more powerful tools!

Grammar → Language Examples

Example 10

- Let grammar G be:
  1. S → AB
  2. A → aA | λ
  3. B → bB | λ
- L(G) = ? // show it by a set-builder.

Solution

Language → Grammar Examples

Language → Grammar Examples

Example 11

- Find a grammar that generates the following language over $\Sigma = \{a, b\}$:

  $L = \{w : w \in \Sigma^*\}$

Solution

Language → Grammar Examples

Example 12

- Find a grammar that generates the following language over Σ = {a, b}:

  L = {w : w contains exactly one a}

Solution

Homework: Language → Grammar

- Find a grammar that generates the following languages over $\Sigma = \{a, b\}$:

1. $L = \{w : w \text{ contains at least one a}\}$
2. $L = \{w : w \text{ contains at least 2 a's}\}$
3. $L = \{w : w \text{ contains no more than 3 a's}\}$
4. $L = \{a^{2n} b^n : n \geq 0\}$
5. $L = \{a^{2n} b^m : n, m \geq 0\}$
6. $L = \{a^n b^m : n, m \geq 0, n \neq m\}$

References

1. Linz, Peter, "An Introduction to Formal Languages and Automata, 5th ed.," Jones & Bartlett Learning, LLC, Canada, 2012
2. Michael Sipser, "Introduction to the Theory of Computation, 3rd ed.," CENGAGE Learning, United States, 2013
   ISBN-13: 978-1133187790
3. The ELLCC Embedded Compiler Collection, available at: http://ellcc.org/

Appendix A

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Esperanto

A World Without War!

Esperanto Creator's Motivation

- Constructed in 1873 by Polish medical doctor, inventor, and writer, Ludwik L. Zamenhof (1859-1917).

- He had the dream of "a world without war".
- He believed this language can help international people to communicate easily.
- And the communications could prevent wars!

Alphabet (5 Vowels + 23 Consonants)

Letter	English Example
a	father
b	
c	cats
ĉ	chip
d	
e	bet
f	
g	go
ĝ	gem

Letter	English Example
h	
ĥ	kh in Persian
i	see
j	yes
ĵ	measure
k	
l	
m	
n	

Letter	English Example
o	so
p	
r	rolled "R"
s	
ŝ	share
t	
u	soon
ŭ	cow
v	
z	

Vowels & Consonants

- 5 Vowels: A, E, I, O, U
- 23 Consonants: the rest of alphabets

- Name the vowels by their pronunciation
- Name the consonants by: letter + o
- For example, we call "b" as "bo".

- The number of syllables of a word is the number of vowels.
- For example, "domo" (means house) has two syllables.

- The accented syllable is the second to the last syllable.

Rules

- All "nouns" are ended with 'o'.
- For example, "domo" means house.

- All "adjectives" are ended with 'a'.
- For example, "doma" means domestic.

- Adjectives can be placed before or after nouns.
- For example, "dolĉa pomo" or "pomo dolĉa" means "sweet apple".

- To make a negative adjective, just prefix it with "mal".
- For example, "bona" means "good" and "malbona" means "bad".

Rules

- All "adverbs" are ended with 'e'.
- For example, "rapide" means "quickly".

- Adverbs can be placed before or after the verb they modify.
- For example, "Ĝi flugas rapide" means "it flies quickly".
- Or we can say, "Ĝi rapide flugas".

- To make "plural", add 'j' at the end of the nouns and adjectives.
- For example, "seĝoj" means chairs.
- Example for plural, "rapidaj aŭtoj" means fast cars.
- Note that both adjective and noun should be plural.

Rules

- There is only one "definite article" "la" in Esperanto.
- For example, "la domo" means "the house".

- There is no indefinite article in Esperanto.
- So, if you don't use "la", it means the article is indefinite.

Rules

- To make "possessive personal pronoun", just add 'a' to the end of personal pronoun.
- For example, "mi" means "I", "mia" means "my".
- "vi" means "you", "via" means "your".

- To make a verb negative, just add "ne" before it.
- For example, "Ŝi estas alta." means "She is tall".
- "Ŝi ne estas alta." means "She is not tall".

Tenses

- Infinitives are ended with 'i'.
- For example, "flugi" means "to fly".

- Present tense is ended with "as".
- For example, "Mi flugas" means "I fly".

- Past tense is ended with "is".
- For example, "Mi flugis" means "I flied".

- Future tense is ended with "os".
- For example, "Mi flugos" means "I will fly".

Tenses

- Progressive Present is ended with "anta" but needs "estas" as well.
- For example, "Mi estas fluganta" means "I am flying".

Miscellaneous

- To make a question, add "Ĉu" at the beginning of the sentences.
- For example, "Mi flugas" means "I fly".
- "Ĉu mi flugas" means "Do I fly".
- It doesn't matter what tense the sentence has.

- Over all, Esperanto has 16 constants rules.
- These were some of them, I selected for your information.

General Info About Esperanto

- "Esperanto" means "one who hopes".
- The flag of Esperanto ...

- It is the most successful constructed language in the world.
- Almost two million people speak in Esperanto.
- The first World Congress of Esperanto was organized in 1905 in France.
- Esperanto was recognized as a language by UNESCO in 1954.
  - It was recommended as international non-governmental organizations language in 1985.
- In 2012, Google Translate added Esperanto into its list.
- Language codes in computer: ISO 639-1, ISO 639-2, ISO 639-3

Did You Know That ...

- Learning Esperanto as the second language will speed up learning 3rd and more languages.
- 8 Nobel laureates have been Esperantists
- Esperantists make up the largest non-political grouping in the British parliament.
- Leo Tolstoy helped found the Esperantist Vegetarian Association in 1908.
- In 1993, more than 4900 people (mainly non-Esperantists) visited the International Esperanto Museum in Vienna.
- Esperantists have continually suffered oppression from totalitarian governments!

Reference: http://esperanto.org/us/USEJ/world/index.html

References

1. Wikipedia, Esperanto available at: https://en.wikipedia.org/wiki/Esperanto
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3. Esperanto lessons: https://www.youtube.com/watch?v=bLx5hLag6WQ