Reti Bayesiane III

IALab A.A. 2018/2019

Inferenza Approssimata

Inference by stochastic simulation

Basic idea:

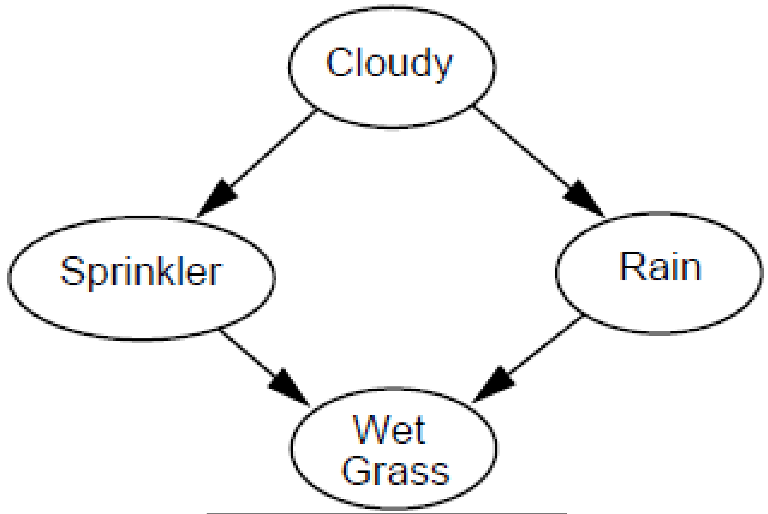
1) Draw N samples from a sampling distribution S
2) Compute an approximate posterior probability $\hat{P}$
3) Show this converges to the true probability P

Outline:

– Sampling from an empty network
– Rejection sampling: reject samples disagreeing with evidence
– Likelihood weighting: use evidence to weight samples

Example

P(C)
.50



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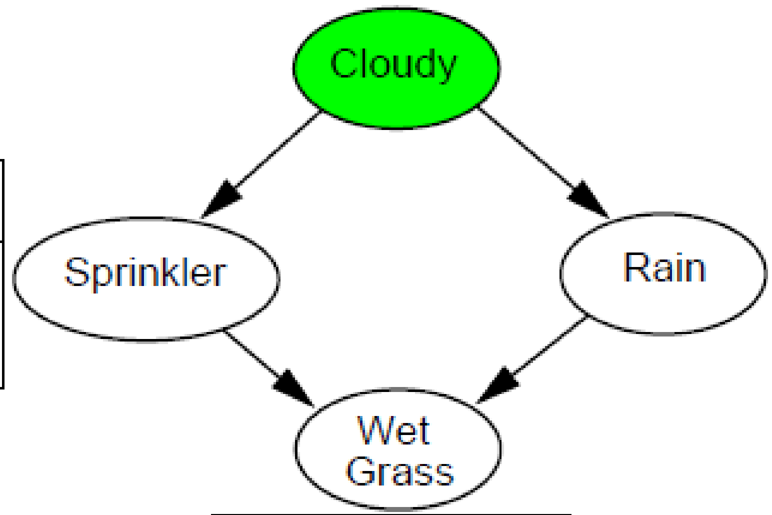

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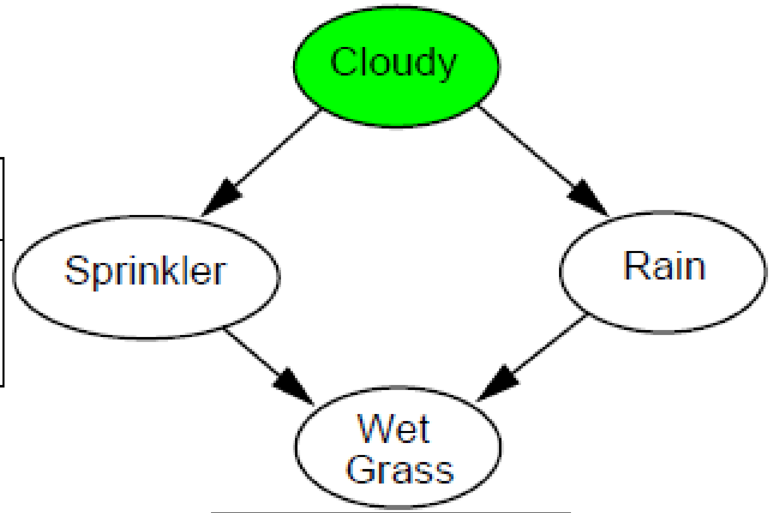

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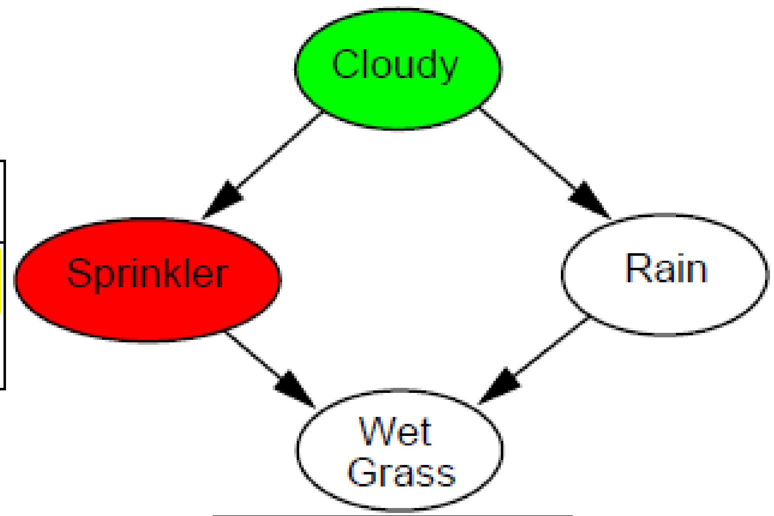

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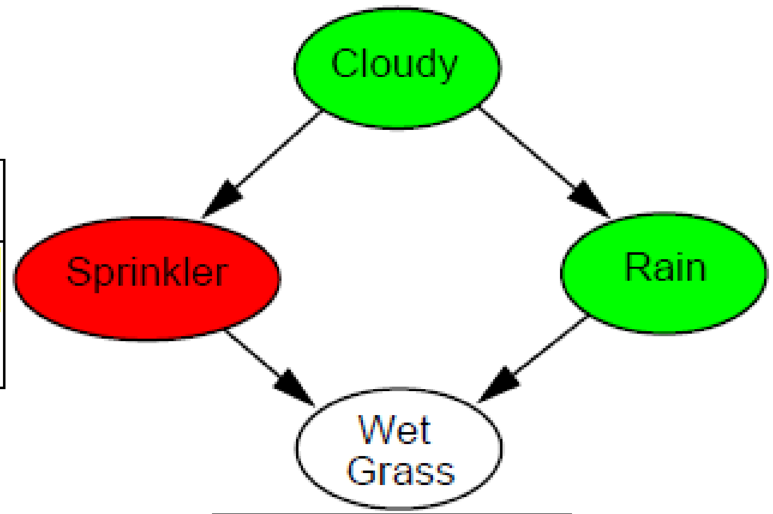

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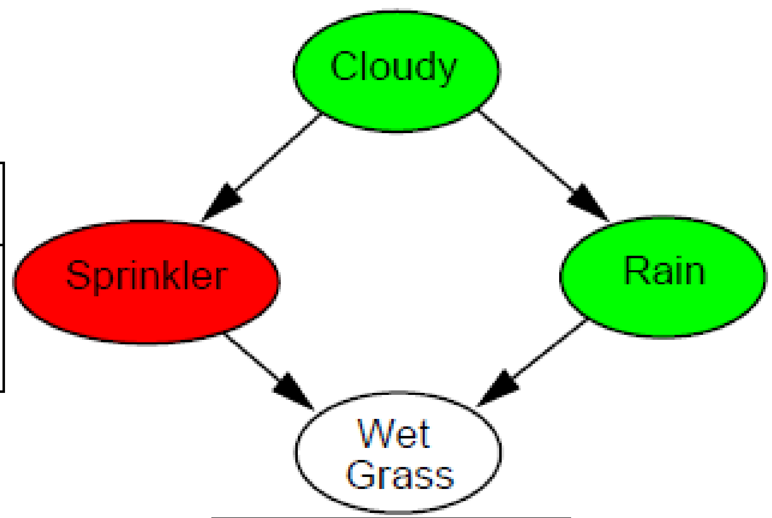

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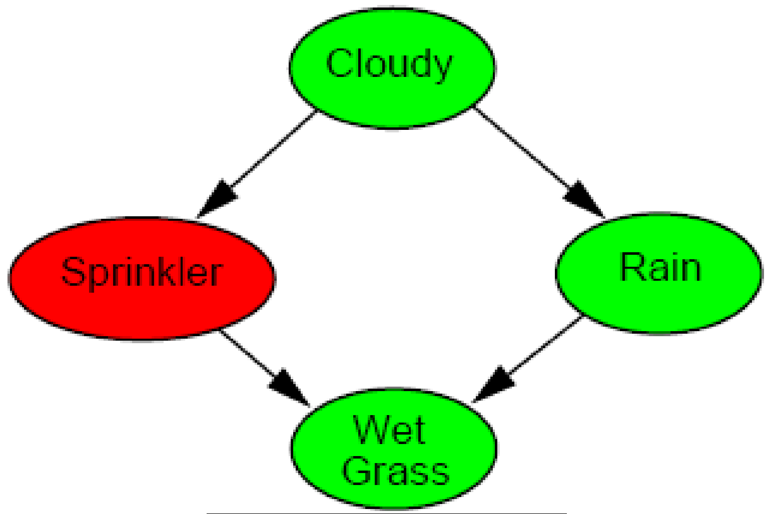

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Sampling from an empty network contd.

Probability that PRIORSAMPLE generates a particular event

$$S_{PS}(x_1 \ldots x_n) = \prod_{i=1}^{n} P(x_i|parents(X_i)) = P(x_1 \ldots x_n)$$

i.e., the true prior probability

E.g., $S_{PS}(t,f,t,t) = 0.5 \times 0.9 \times 0.8 \times 0.9 = 0.324 = P(t,f,t,t)$

Let $N_{PS}(x_1 \ldots x_n)$ be the number of samples generated for event $x_1, \ldots, x_n$

Then we have

stima

$$\begin{aligned} \lim_{N\to\infty} \hat{P}(x_1,\ldots,x_n) &= \lim_{N\to\infty} N_{PS}(x_1,\ldots,x_n)/N \\ &= S_{PS}(x_1,\ldots,x_n) \\ &= P(x_1 \ldots x_n) \end{aligned}$$

legge dei grandi numeri

That is, estimates derived from PRIORSAMPLE are consistent

Shorthand: $\hat{P}(x_1,\ldots,x_n) \approx P(x_1 \ldots x_n)$

Rejection sampling

$\hat{\mathbf{P}}(X|\mathbf{e})$ estimated from samples agreeing with $\mathbf{e}$

```
function REJECTION-SAMPLING(X, e, bn, N) returns an estimate of P(X|e)
   local variables: N, a vector of counts over X, initially zero

   for j = 1 to N do
      x ← PRIOR-SAMPLE(bn)
      if x is consistent with e then
         N[x] ← N[x]+1 where x is the value of X in x
   return NORMALIZE(N[X])
```

E.g., estimate $\mathbf{P}(Rain|Sprinkler = true)$ using 100 samples
  27 samples have $Sprinkler = true$
    Of these, 8 have $Rain = true$ and 19 have $Rain = false$.

$\hat{\mathbf{P}}(Rain|Sprinkler = true) = \text{NORMALIZE}(\langle 8, 19\rangle) = \langle 0.296, 0.704\rangle$

Similar to a basic real-world empirical estimation procedure

Analysis of rejection sampling

$\hat{\mathbf{P}}(X|\mathbf{e}) = \alpha \mathbf{N}_{PS}(X, \mathbf{e})$ (algorithm defn.)

$= \mathbf{N}_{PS}(X, \mathbf{e})/N_{PS}(\mathbf{e})$ (normalized by $N_{PS}(\mathbf{e})$)

$\approx \mathbf{P}(X, \mathbf{e})/P(\mathbf{e})$ (property of PriorSample)

$= \mathbf{P}(X|\mathbf{e})$ (defn. of conditional probability)

Hence rejection sampling returns consistent posterior estimates

Problem: hopelessly expensive if $P(\mathbf{e})$ is small

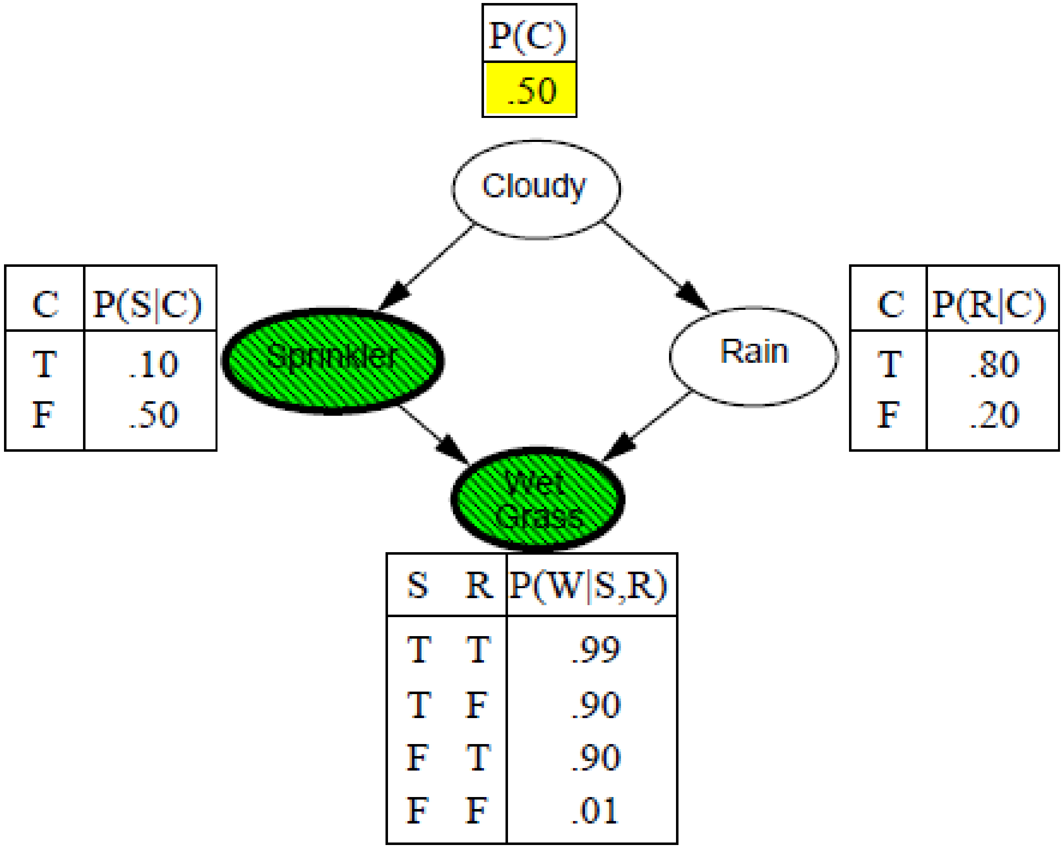
$P(\mathbf{e})$ drops off exponentially with number of evidence variables!

$N_{PS}(X,e)/N \approx P(X,e)$
$N_{PS}(e)/N \approx P(e)$

Likelihood weighting

Idea: fix evidence variables, sample only nonevidence variables,
and weight each sample by the likelihood it accords the evidence

Likelihood weighting example



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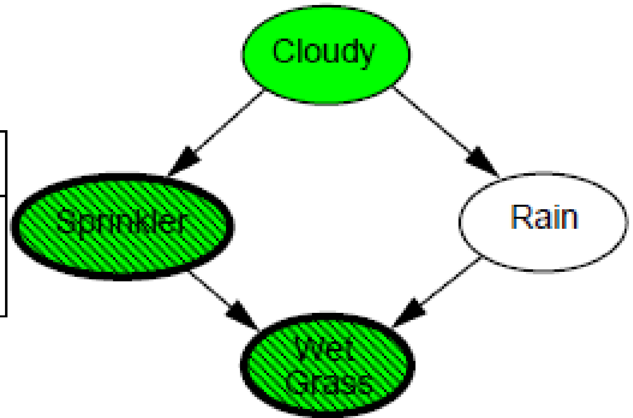

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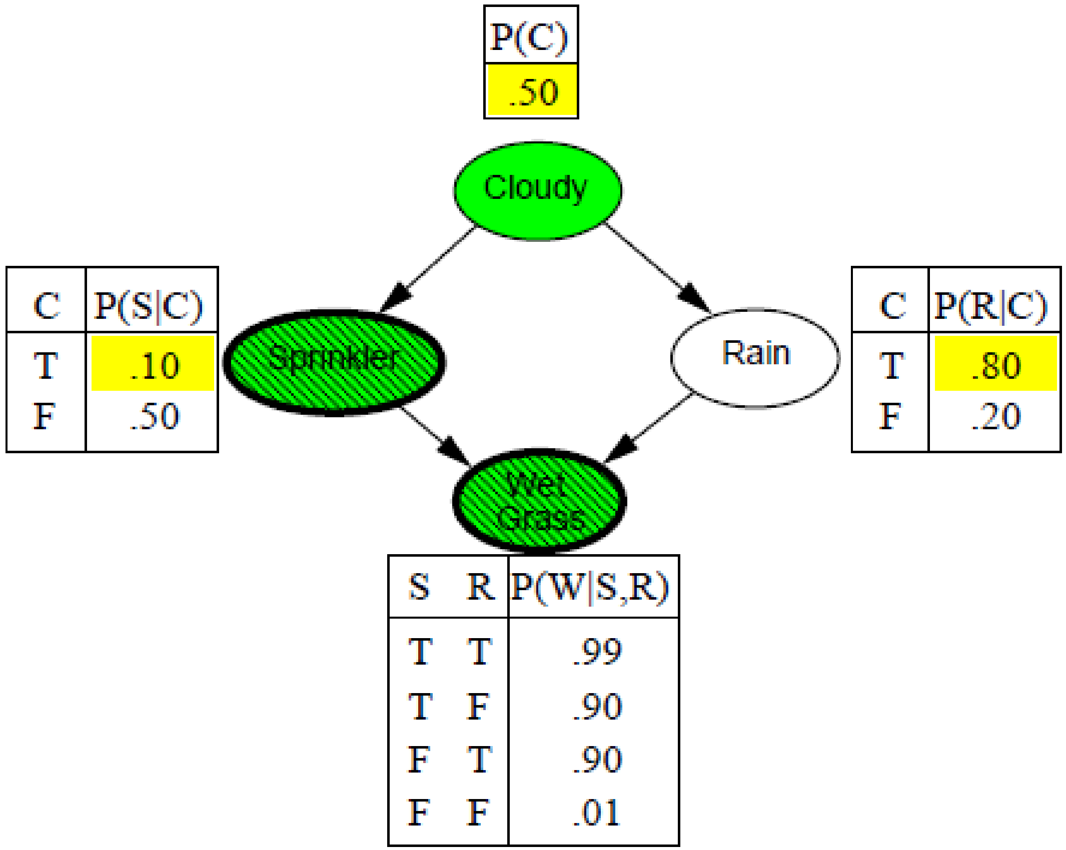

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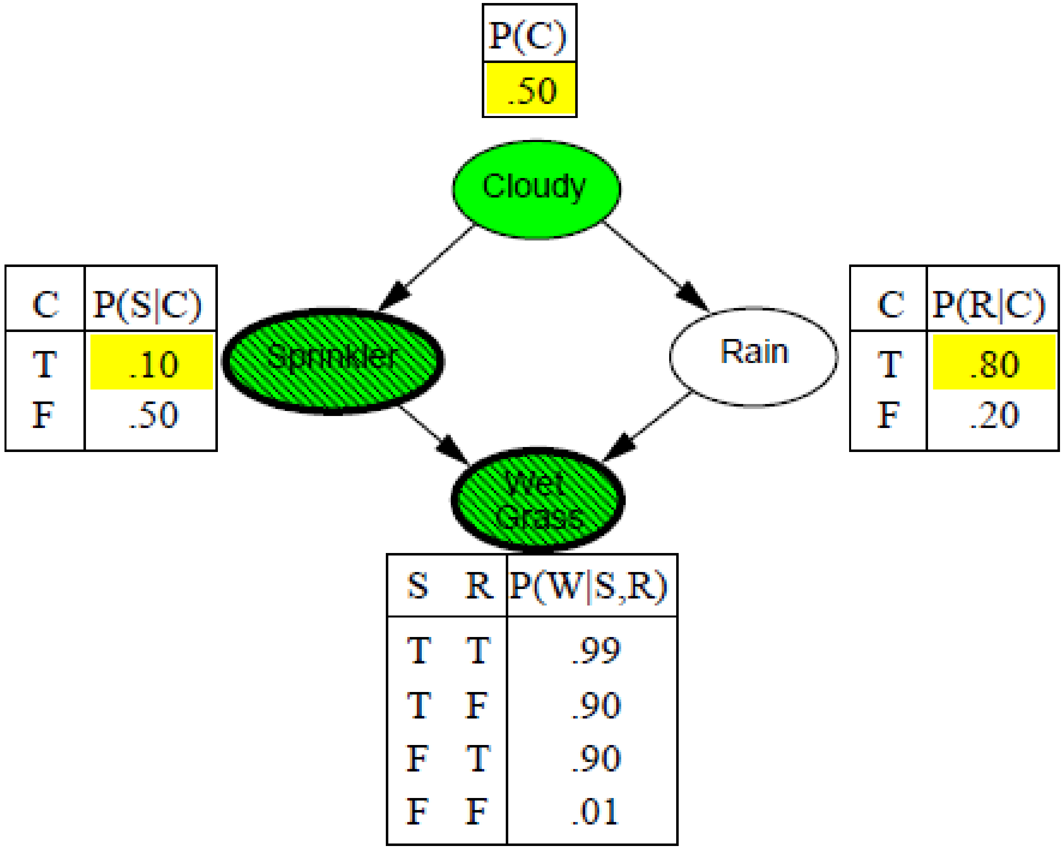

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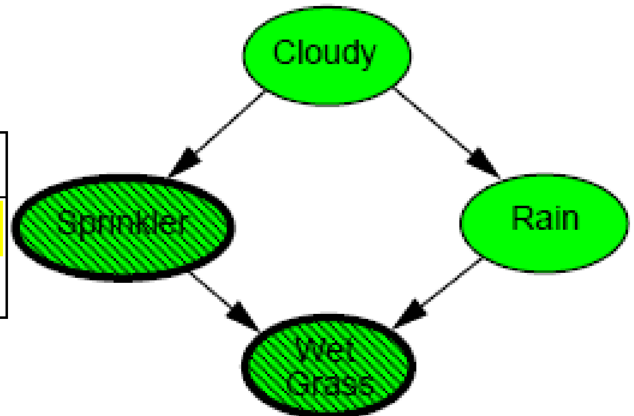

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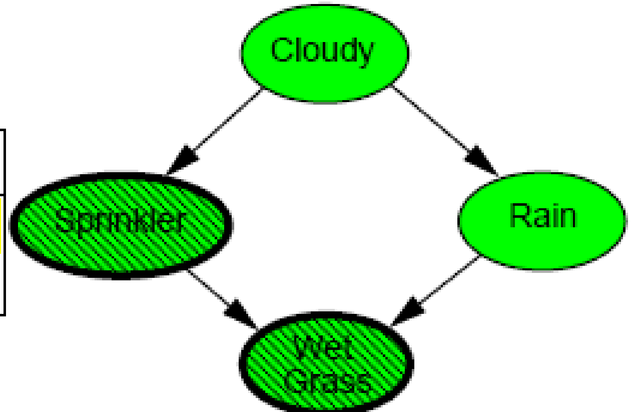

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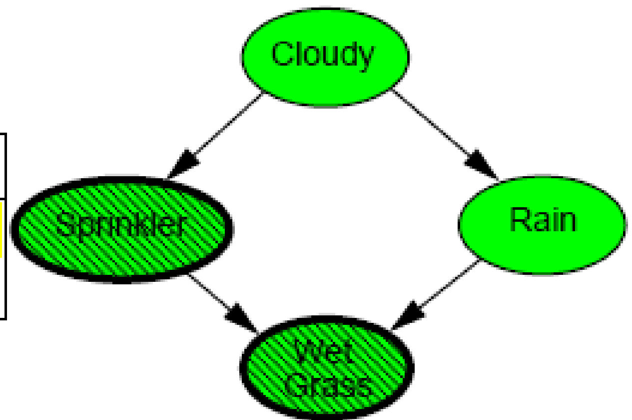

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$w = 1.0 \times 0.1 \times 0.99 = 0.099$

Likelihood weighting analysis

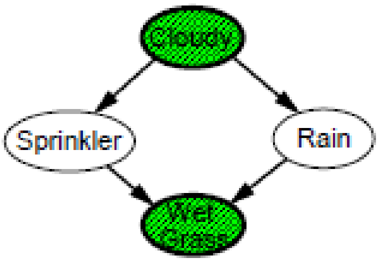
Sampling probability for WEIGHTEDSAMPLE is

$S_{WS}(\mathbf{z},\mathbf{e}) = \prod_{i=1}^{l} P(z_i|parents(Z_i))$

l variabili **z** di non-evidenza

Note: pays attention to evidence in ancestors only

$\Rightarrow$ somewhere "in between" prior and posterior distribution



prossima slide

Weight for a given sample $\mathbf{z},\mathbf{e}$ is

$w(\mathbf{z},\mathbf{e}) = \prod_{i=1}^{m} P(e_i|parents(E_i))$

m variabili **e** di evidenza

Weighted sampling probability is

$$S_{WS}(\mathbf{z},\mathbf{e})w(\mathbf{z},\mathbf{e})$$
$$= \prod_{i=1}^{l} P(z_i|parents(Z_i)) \ \prod_{i=1}^{m} P(e_i|parents(E_i))$$
$$= P(\mathbf{z},\mathbf{e})$$ (by standard global semantics of network)

Hence likelihood weighting returns consistent estimates but performance still degrades with many evidence variables because a few samples have nearly all the total weight

Likelihood Weighting

- che impatto ha il fatto che l'evidenza riguardi le "prime" o le "ultime" variabili (nell'ordinamento topologico)?

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- che impatto ha il fatto che l'evidenza riguardi le "prime" o le "ultime" variabili (nell'ordinamento topologico)?
- se l'evidenza è nelle "prime" variabili, i campioni generati saranno più probabili data l'evidenza stessa, quindi con peso maggiore (più significativi)
- se l'evidenza è nelle "ultime" variabili, i campioni generati saranno generati essenzialmente secondo le probabilità a priori, e gli verranno assegnati pesi potenzialmente molto bassi (poco significativi)

Esercizio Inferenza Approssimata

- sperimentate con le implementazioni AIMA code di Rejection Sampling (RS) e Likelihood Weighting (LW) per verificare:
  - la dipendenza dell'accuratezza dei due algoritmi dal numero di campioni generati
  - la miglior convergenza di LW rispetto a RS
  - la miglior convergenza di LW quando le osservazioni sono all'inizio dell'ordine topologico