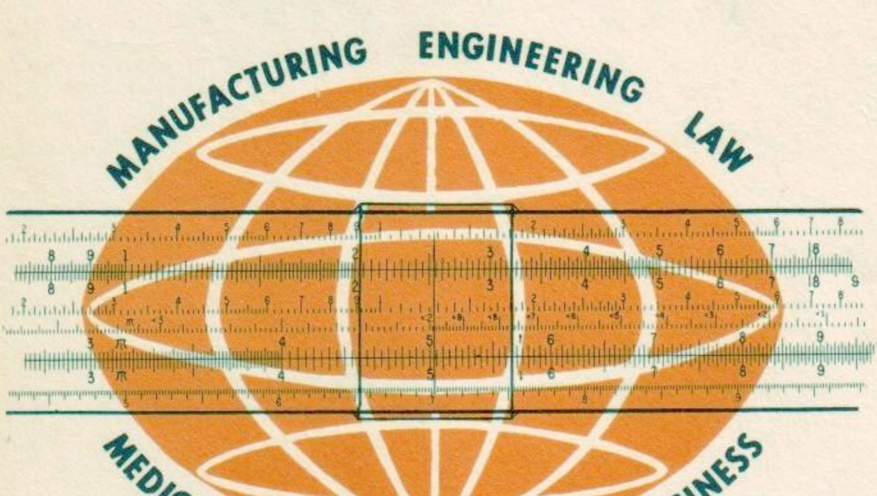
University Interscholastic League

BEGINNERS' SLIDE RULE MANUAL



By Otto G. Brown and H. Grady Rylander

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University Interscholastic League

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

SAM HOUSTON

while guided and controlled by virtue, the noblest attribute of man. It is the only dictator that freemen acknowledge, and the only security which freemen desire.

MIRABEAU B. LAMAR

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Acknowledgments

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EUGENE DIETZGEN COMPANY.
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PICKETT & ECKEL, INC.
FREDERICK POST COMPANY.

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Preface

This revision of the University Interscholastic League Slide Rule Manual has been written for the express purpose of helping the beginner to understand more clearly the basic slide rule operations. In particular, the fundamental operations and the location of the decimal point in these operations have been completely rewritten while other topics are partially revised. Also, the expanded illustrations should be more helpful in reading the settings in the examples. These illustrations are shown for the most generally used slide rule, the ten inch straight rule. Also, at the beginning of the answer section in this manual, some methods of answer presentation are given which are the basis for the judges' scoring in formal competition.

You will discover that advanced slide rule operations are not discussed in this manual. These operations, as well as some of the more complicated but fundamental operations, can best be mastered by consulting your coach or instructor.

The specific rules for University Interscholastic League Slide Rule Competition are not included in this manual. However, for this information, the reader is referred to "Constitution and Contest Rules of the University Interscholastic League."

With the ever-growing popularity of the slide rule in a great many fields, not only are you preparing for interesting individual or club competition, but you are also paving the way for the future by learning its proficient use.

RODNEY J. KIDD, *Director*
University Interscholastic League

I. Introduction

The slide rule is a very handy instrument that serves as a medium for performing rapid calculations. Due to its relatively small size, it is readily portable and may be easily carried about in one's pocket or stored in a desk. There are countless numbers and types of problems that it can aid in solving. It can multiply, divide, give ratios and proportions, raise numbers to powers, take roots of numbers, give the trigonometric and hyperbolic functions of angles, give the logarithms of numbers to any base, indicate the reciprocals of numbers, give the areas of circles, solve certain types of equations directly, and perform many other useful operations at a speed many times greater than that of longhand calculations. It can even be induced to add and subtract.

The first slide rule appeared in 1620 as a result of Gunter's invention of the straight logarithmic scale based upon Napier's invention of logarithms. Even though it did not look like our present-day rules, its principle of operation was the same as that employed by the most modern, complicated slide rules now in existence. This early ancestor of our slide rule consisted of a single scale on which the settings were made by employing dividers to indicate line segments the lengths of which were logarithmically proportional to the magnitudes of numbers. It was rather simple and crude, but it performed its job well. In the centuries that have followed, many different types of slide rules have appeared. Some called rectilinear rules are straight; others in the form of a disk are called circular rules; a few have appeared in the form of a cylinder consisting of a number of rotating drums. Other special forms of rules, particularly useful for performing specialized tasks required by engineering, science, industry, or the military forces, have been developed. A few of the many names that have been given to these different types of rules are: polyphase, log log duplex decitrig, log log vector, binary, deci log log, vector hyperbolic, flight calculator, power computer, versalog, and other equally queer sounding names to the beginning slide rule operator.

Probably the most commonly used rule is the rectilinear rule, although the circular rules are used by a large number of individuals.

For this reason, this manual will employ the straight slide rule for its instructional examples.

This publication is designed to assist any interested individual to become an efficient operator or a slide rule. With adequate practice he should become a proficient operator. The beginner should keep his rule before him while he is reading and studying these instructions. He should also make all settings indicated in the illustrative examples, and should compute the answers for all the exercises a number of times until he has assured himself that he has mastered that particular phase of calculations. The principles involved are rather simple and should be understood with a small amount of study. However, a great amount of practice is absolutely necessary before one gains complete confidence in his ability to operate the rule proficiently. Learning to use a slide rule is like learning algebra; it cannot be done overnight or in a week's time. Consequently, the beginner, justifiably, should not be too disappointed over his seemingly slow progress in learning the efficient use of the rule.

Practice exercises are given in this manual with each particular phase of instructions pertaining to that particular type of slide rule operation. Also, an additional number of practice problems of a more complex nature are listed in section XVI of this bulletin. Finally, the answers to all practice exercises and to the practice problems are given at the end (section XVII) to enable the operator to check his calculations. With the answers thus shown separately from the problems, it permits one to solve the particular problem in its entirety before checking the answer.

One must remember that, although the slide rule is capable of doing a great many useful things when in the hand of a proficient operator, it cannot think. It can never be more accurate than the readings or settings made on it by its user. This is the operator's particular task: **think for the rule, and let it perform the mechanics of the operation;** only then will you begin to reap the benefits of its use.

II. Selecting a Slide Rule

Any reputable manufacturer or dealer will furnish a price list of the various type of rules. At the end of this section some of the more reputable firms that produce slide rules are listed for information. The prices for the various rules will vary, depending upon the manufacturer and the type of rule. Also, if a case is purchased, of course, the total cost will be more. Generally, however, it might be said that a first class ten inch rule might be purchased for a price in the range of twenty dollars. It is worthwhile to consider the purchase of a case for the rule, as it affords an excellent means of protection against most types of damage. Section V gives useful information on adjusting a slide rule.

In calculations where extreme accuracy is required, the twenty inch straight rule is probably the best choice for use, as it is capable of giving an accuracy of 1 part in 2000. However, for almost all calculations necessary in engineering, science, and related fields, the ten inch rule gives sufficient accuracy, 1 part in 1000. Another factor to be considered here is that, although the twenty inch straight rule is twice as accurate as the ten inch straight rule, it is not as handy to carry or store.

In comparing straight and circular slide rules, it must be borne in mind that each has favorable factors. An eight to ten inch diameter circular rule gives about the same accuracy as a twenty inch straight rule. However, the time required for a particular operation is generally greater for the circular rule. On the other hand, the operator does not have to decide which index to use in order to prevent running out of range of the scale when using a circular rule.

Slide rules are constructed of wood, metal, or plastic. Generally speaking, a plastic rule is cheaper in cost than either a metallic or a wooden rule. However, the plastic rule also has a greater tendency to warp with age and to chip when dropped. Wooden rules are affected by high humidity conditions in that the slide sometimes has a tendency to stick. Nevertheless, with care this condition can usually be avoided. Metallic rules generally remain in adjustment the best. However, many operators prefer the ease of reading the scales of the wooden rules rather than the lithographed or printed scales generally found on metallic rules.

Probably the most satisfactory slide rule for all around use is the ten inch wooden or metallic slide rule. An accompanying case also serves as insurance against damage when not in use.

The following manufacturers are among those who produce slide rules of excellent quality:

Eugene Dietzgen Company, 318 Camp St., New Orleans, Louisiana 70130

Keuffel & Esser Company, 1701 Walker Ave., Houston, Texas 77001

Gilson Slide Rule Company, P. O. Box 111, Stuart, Florida

John Henschel Company, 195 Marine St., Farmingdale, New York 11735

Pickett Inc., 436 Gutierrez St., Santa Barbara, California 93102

Frederick Post Company, Box 138, Houston, Texas 77002

F. Weber Company, 2000 Windrim, Philadelphia, Pennsylvania 19144

III. Parts of a Slide Rule

The straight slide rule consists essentially of three parts; the body, the slide, and the cursor. These are as indicated in Figure 1.

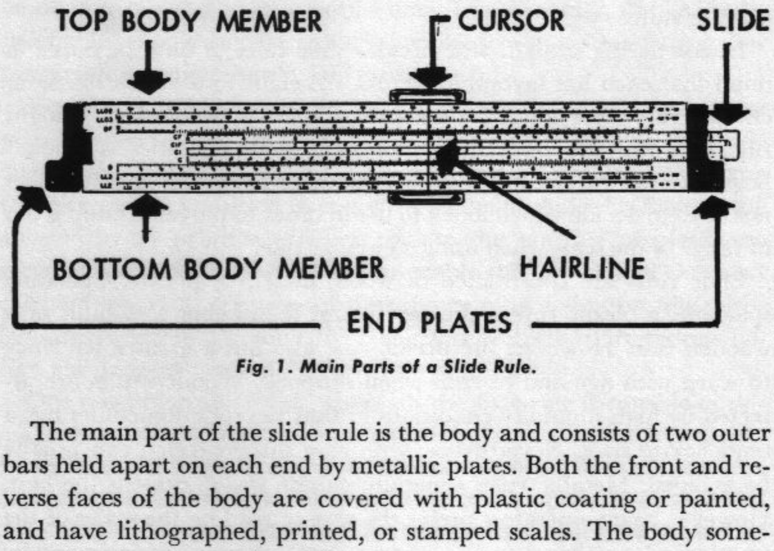

Fig. 1. Main Parts of a Slide Rule.

The main part of the slide rule is the body and consists of two outer bars held apart on each end by metallic plates. Both the front and reverse faces of the body are covered with plastic coating or painted, and have lithographed, printed, or stamped scales. The body sometimes is referred to as the stock.

The second part of the rule is the slide, a long sliding member which moves between the two bars of the body. The slide, like the body, has scales on both its faces. This part is also called the slider.

The cursor is the glass or plastic runner, generally framed in a spring loaded metallic holder, with a black or red hairline placed perpendicular to the direction of sliding of the cursor along the body. Generally, a glass or plastic runner is provided for both the front and rear faces of the rule. The accompanying hairlines move together since they are carried by the same cursor. Sometimes the cursor is referred to as the indicator.

The horizontal rows of calibration marks on both the body and the slide are called scales. Each scale is named by a letter or letters generally placed at both ends. It is interesting to note that, even though the most commonly used straight slide rule is referred to as a ten inch rule, its scales generally are twenty-five centimeters in length, approximately 0.16 inch less than ten inches. A few of the recent slide rules have scales that actually measure ten inches in length.

IV. Care of the Rule

One must remember that the slide rule is a precision instrument and measures should be taken to prevent damage to it which might affect its use.

A slide rule case can do much to help prevent damage to the rule. The cases are usually made of leather but some are also made of plastic. A slide rule case can help prevent such damage as scratched scales, broken indicators, and warped slides or body members.

The rule should not be stored where it is subjected to conditions of high temperature or high humidity, such as by an open window, in direct sunlight, or near a source of heat. This practice can, of course, cause warping and discoloring.

Although, in some instances, chemicals are used in an effort to ease the motion of the slide, this practice should be avoided. The best solution for a sticky rule is proper adjustment. However, a slight application of powder will do much to alleviate the friction between slide and body.

The use of an artgum eraser or carbon tetrachloride cleaner is recommended for use on discolored and dirty scales. However, care should be exerted in order to prevent damage to the figures on the scales. The use of soap and water is not recommended with wooden rules.

The best way to clean the indicator is to disassemble it, clean all parts, reassemble it, and finally adjust it. The indicator can be cleaned somewhat by placing a strip of soft paper between the indicator and the body and, while applying a moderate force on the indicator, moving it to and fro across the paper.

The use of a bleaching agent, ammonia, alcohol, dye solvent, or naptha is useful in removing stains from the rule. However, care should be exerted in order to prevent damage to the calibration marks on the scale. Frequently these solutions will dissolve the dye or ink used to mark the scales or will even dissolve the material from which the scales are made.

It is alarming how many slide rules are damaged by cigarettes. Careless smoking and slide rule operating are not winning combinations.

Above all things, handle the rule carefully. Exert caution not to drop it nor to step on it accidentally.

After each cleaning of the rule, it should be checked and corrective adjustments should be made, if necessary.

V. Adjusting a Slide Rule

When purchasing a slide rule, one should check the serial number (Fig. 2) listed on the slide and the body of the rule to be sure that

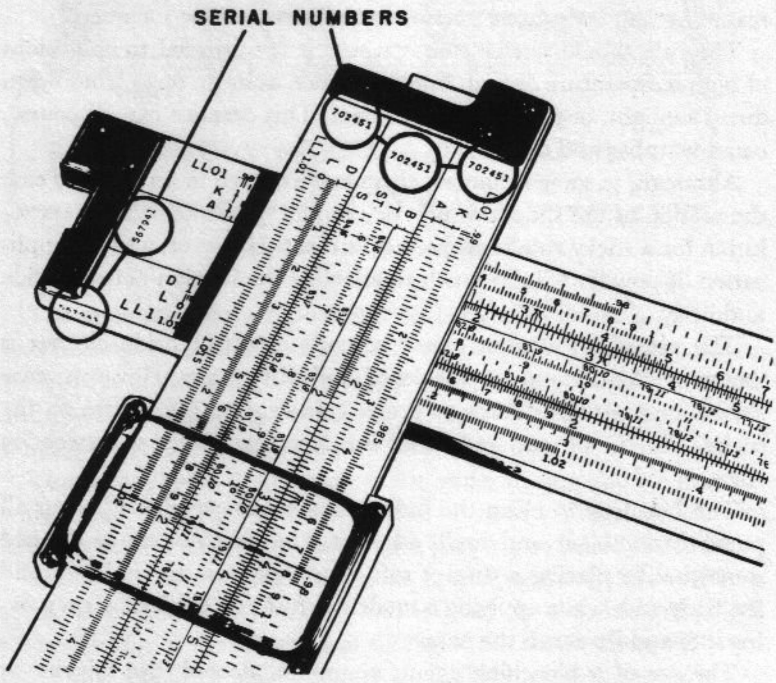

Fig. 2. Slide Rule Serial Numbers.

indexes on the reverse side are not in proper alignment, the rule cannot be corrected and, thus, this alignment also should be checked at time of purchase.

With the slide set and the rule adjusted so the eight pairs of indexes coincide (four on front side and four on reverse side), the indicator hairline now must be adjusted. To do this, loosen the indicator adjustment screws (*C, D, E,* and *F*, in Fig. 3) on on both sides of the rule, adjust the indicator until the hairline coincides with the vertical pairs of indexes on the left end of the front face of the rule, and then tighten the four adjusting screws. This same procedure should then be repeated on the reverse side of the rule in order to insure proper hairline alignment there also. Extreme care should be taken when performing these adjustments in order to prevent damage to the heads and threads of the adjusting screws. Figure 3 illustrates a slide rule in proper adjustment on the front side.

The rule should also be tested for tightness. To do this, move the slide back and forth between the two extreme positions of sliding. If the slide appears to move with great difficulty, or, on the other hand, moves too freely, the tightness of the rule should be adjusted. If the slide moves too stubbornly, the two body adjusting screws *A* and *B* (Fig. 3) should be loosened and the top body member should be moved away from the slide. The screws should be tightened and the slide can then be tested for freedom of movement. A trial and error method of adjustment should finally permit satisfactory operation. On the other hand, if the slide is adjusted too loosely (the slide falls under the effects of gravity only when the rule is held in a vertical position), correct tightness can be effected in the following manner: loosen the two body adjusting screws (*A* and *B* in Fig. 3), insert two pieces of thin paper (tracing paper or tissue paper) between the slide and the top body member as indicated in Figure 4, press each end of the top body member tightly to the slide, and tighten the body adjusting screws. The slide should then be removed from the body, the traces of paper extracted, and the slide reinserted in the body. When making these adjustments on slide tightness, one must be careful not to disturb the alignment of the eight pairs of coincident indexes. Best results are usually obtained by loosening the screws at only one end of the rule at a time.

The slide rule is now in correct adjustment and ready for efficient operation.

In some cases, rules have warped slides or body members. The above adjustments cannot completely correct this condition, and if the movement of the slide is extremely jerky, the rule should be rejected.

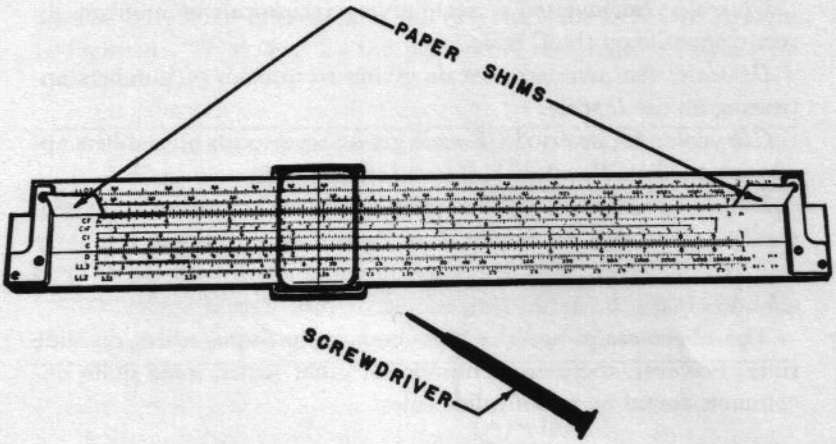

Fig. 4. Adjusting a Slide Rule.

VI. Common Scales of the Slide Rule

Except for the same model rule made by the same manufacturer, it is very uncommon that two slide rules will happen to have the same number and kind of scales on them. Moreover, the placement of the various scales will vary from slide rule to slide rule. Therefore, the listing of the scales that follow is a more general listing according to their most generally known names. Also, the most important uses of these scales are given.

C and D scales: Used for multiplication and division, or a combination of multiplication and division, and in conjunction with *A, B,* and *K* scales for square and cube roots.

CF and DF scales: Folded at π; otherwise identical to the *C* and *D* scales. The *CF* and *DF* scales, when used in conjunction with the *C* and *D* scales, permit direct calculations involving multiplication and division by π. In most cases where the wrong index has been used when multiplying on the *C* and *D* scales, the answer may be found on either the *CF* or *DF* scale.

A and B scales: Used in conjunction with the *D* and *C* scales to obtain the squares and square roots of numbers. Also used for multiplication by setting left index of *B* in first range of *A* or by setting right index of *B* in second range of *A*.

K scale: Used in conjunction with the *C* or *D* scale to obtain the cubes and cube roots of numbers.

CI scale: An inverted *C* scale giving reciprocals of numbers directly opposite on the *C* scale.

DI scale: An inverted *D* scale giving reciprocals of numbers appearing on the *D* scale.

CIF scale: An inverted *CF* scale giving reciprocals of numbers appearing on the *CF* scale.

L scale: A uniformly divided scale that is used in conjunction with the *D* scale to give the mantissas of logarithms to the base ten.

TRIG scales (T, ST, S): Used in conjunction with the *C* or *D* scale to obtain the natural trigonometric functions of angles.

The above scales are the most commonly found scales on slide rules; however, there are a number of other scales, some quite uncommon, found on various slide rules.

VII. Reading a Scale

Nearly everyone at one time or another has had experience in measuring distances by use of a rule, yardstick, or tape calibrated in inches. A slide rule is similar to a straight twelve inch rule in that the numbers appearing on its scales are a measure of the magnitude of numbers rather than the magnitude of distances. On a straight 12-inch rule, the numbers appearing on the calibration indicate inches; however, between these there are several unnumbered lines indicating fractions of an inch. Thus, on the 12-inch straight rule shown (Fig. 5), the point at A designates a reading of $1\frac{5}{8}''$ or $1.625''$ on a decimal

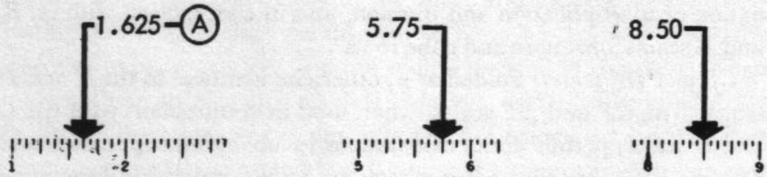

Fig. 5. Reading a Linear Scale.

basis. Like a straight rule, the figures on a slide rule scale indicate numbers, and the division lines between figures indicate magnitude of numbers between the printed figures on the scales. On the slide rule, the scale divisions are not uniform. This is due to the fact that the slide rule operates on a logarithmic background. Nevertheless, the method of reading intermediate values is the same as for the straight 12-inch rule. Thus (Fig. 6) the point at A designates a reading on

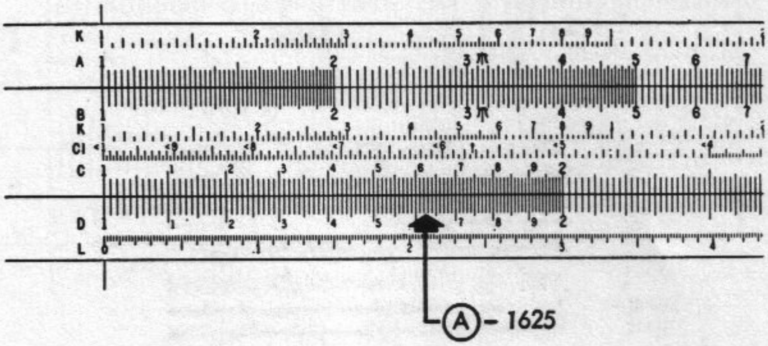

Fig. 6. Reading a Slide Rule Scale.

the *D* slide rule scale of 1.625. However, as we shall see, this point could also represent a reading of 0.01625, 0.1625, 1.625, 16.25, 162.5, 16,250, or the same ordered combination of numbers (1625) with any number of zeros immediately following the combination 1625 and before the decimal point or any number of zeros immediately preceding the combination 1625 and after the decimal point. As odd as it may appear to the beginner, with few exceptions, the decimal point has nothing to do with the location of a number on the scales of a slide rule; it is the sequence of digits that determines this location. On either the *C* or *D* scales, the most generally used scales, the location of 0.002, 0.020, 0.200, 2.00, 20.0, 200.0, 2000.0, or 2 with any number of zeros either before or after it will fall at exactly the same position on the scale. Thus (Fig. 7) point A represents a scale reading of 167, point B represents a reading of 255, and point C represents a reading of 670. Note that decimal points have not been imposed on the above readings since "a slide rule setting is independent of the decimal point."

You will notice that, in many cases, interpolation will be necessary to set a particular number on the *C* or *D* scale. Suppose, for example,

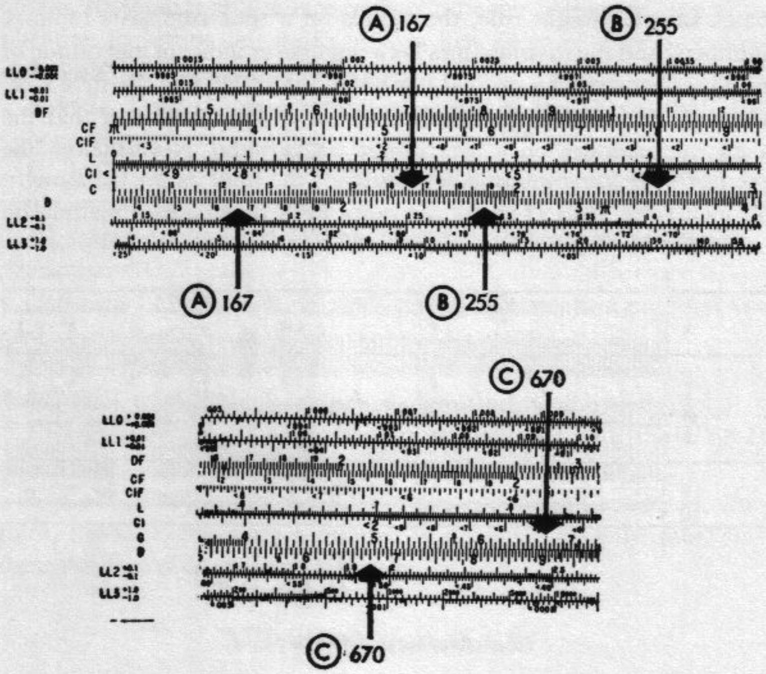

Fig. 7. Reading the C and D Scales.

the number sequence 1543 is to be set on the *D* scale. In Figure 8 we see that this setting will fall between points *D* and *E* (between 1500 and 1600). Further, the setting will fall between *F* (1540) and *G* (1550). Finally, the actual setting 1543 will fall, roughly, 3/10 of the distance between *F* and *G* away from *F*, or 7/10 of the distance between *F* and *G* away from *G*. Thus point *H* represents a setting of 1543. It should be noted that, in the range 1 to 2 on the *C* and *D* scales, a sequence of three significant figures can be set in the rule. Thus, if the sequence of numbers is greater than three, interpolation is necessary. Furthermore, in the range 2 to 10, only a sequence of two significant digits can be set. Thus, in this range, if the number of significant digits is greater than two, you must again resort to interpolation.

However, the fundamental rule to remember here is: "The position of a number on the *C* and *D* scales is independent of the decimal point."

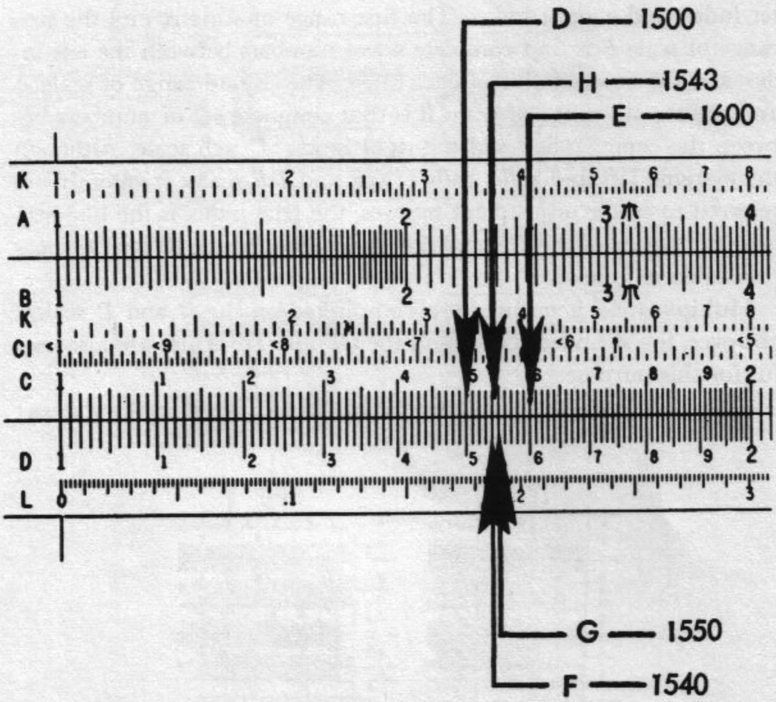

Fig. 8. Interpolation on a Slide Rule Scale.

Exercise 1

1. Make a sketch of the *C* or *D* scale indicating the general method of dividing the scale.
2. On the sketch of problem 1, indicate the main numbers of the scale (1, 2, 3, etc.).
3. In Figure 9 give the slide rule setting for the positions *A* through *L*, inclusive. Give as many significant figures as possible with one digit of interpolation and, of course, neglect decimal points.

VIII. Multiplication

As the indexes of the scales play an important part in the multiplication operation, it is worthwhile to recall at this point the location of the indexes. The left index of the *C* or *D* scale is the line at 1 on the extreme left end of the scale. The right index of the *C* or *D* scale is the line at 1 (or 10) on the extreme right end of the scale. The *A* and *B* scales actually have three indexes each, namely, a left index, a center index and a right index. The first range of scale *A* and the first range of scale *B* is that complete set of numbers between the left index and the center index of each scale. The second range of scale *A* and the second range of scale *B* is that complete set of numbers between the center index and the right index of each scale. Although the extreme left and right ends of *CF* and *DF* scales frequently are referred to as the adjustment indexes, the true index is the line at 1 near the center of the *CF* and *DF* scales. See Figure 10 for the location of these indexes.

Multiplication is ordinarily accomplished on the *C* and *D* scales; however, the *A* and *B* scales, and the *CF* and *DF* scales are also useful for this purpose.

The process of multiplication can be accomplished in three steps:

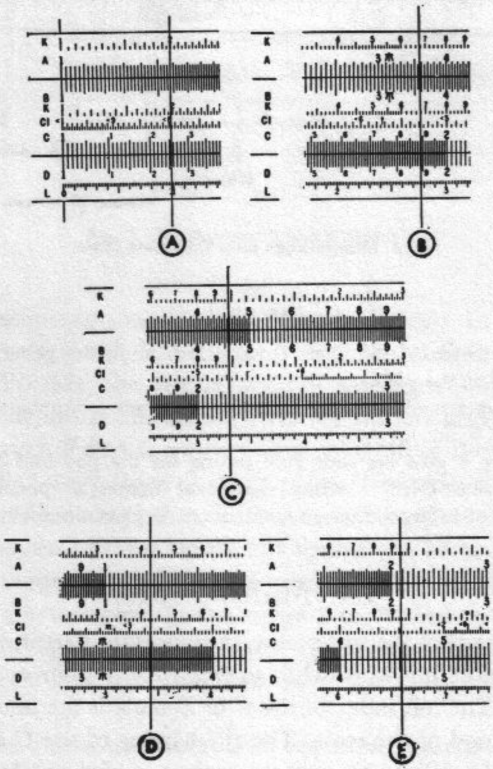

Fig. 9. Problem No. 3. (Part A—E)

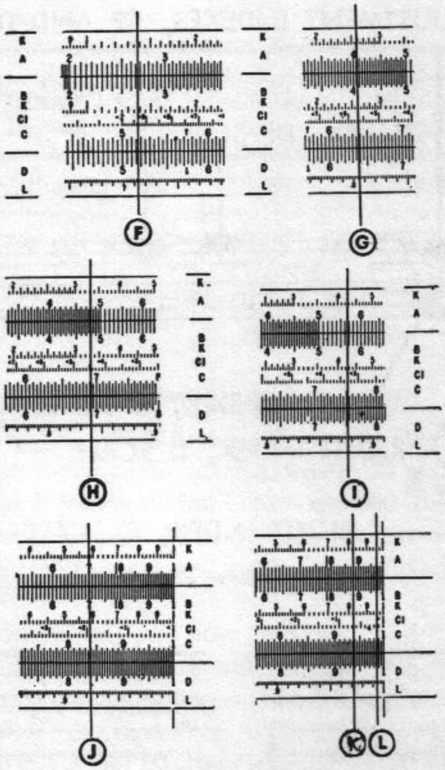

Fig. 9. Problem No. 3 (Part F–L)

1) the first number is set on the *D* scale by adjusting the slide so that one index of the *C* scale coincides with the number, 2) the cursor is then moved until the hairline coincides with the second number on the *C* scale, 3) the answer is read on the *D* scale under the hairline. These three steps are illustrated in Figure 11 where the operation of multiplying 22×3 is shown. The first step is to move the slide until the *left* index of the *C* scale coincides with 22 on the *D* scale; the second step is to move the cursor until the hairline coincides with 3 on the *C* scale; the third step is to read the answer (66) under the hairline on the *D* scale.

It should be noted that, in some cases, the *left* index of the *C* scale is moved to coincide with the first number on the *D* scale while, in other cases, the *right* index is moved to coincide with the first number on the *D* scale; thus, in multiplying 52×3 (Fig. 12), the *right*

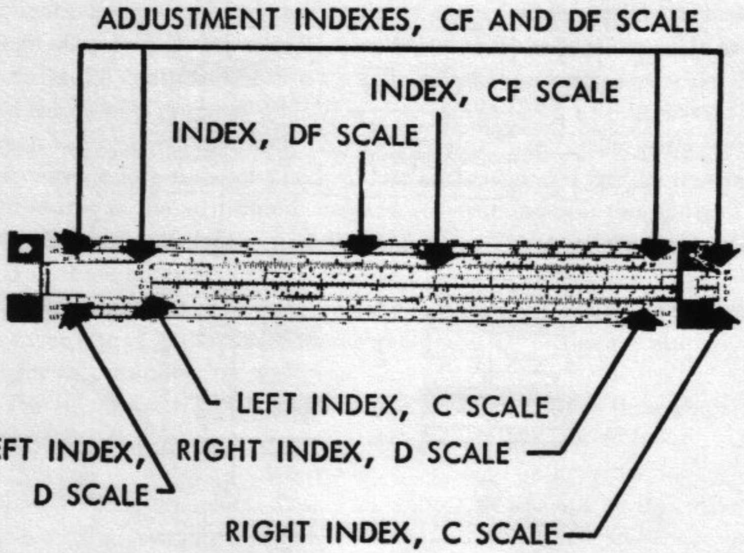

Fig. 10. Location of Indexes.

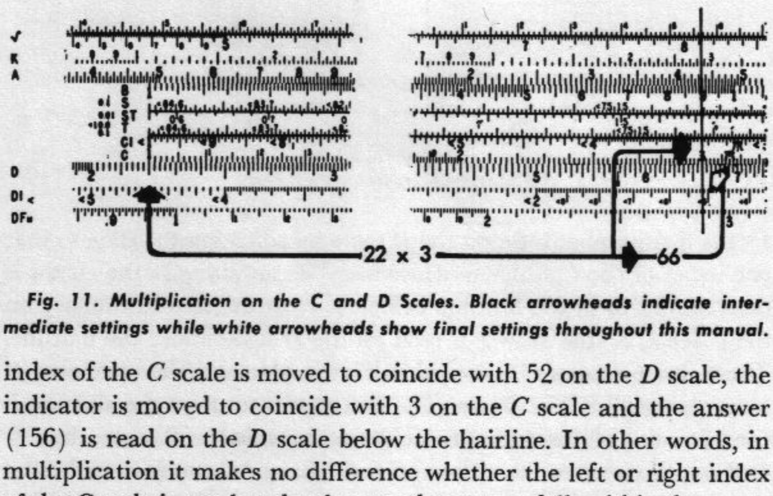

Fig. 11. Multiplication on the C and D Scales. Black arrowheads indicate intermediate settings while white arrowheads show final settings throughout this manual.

index of the *C* scale is moved to coincide with 52 on the *D* scale, the indicator is moved to coincide with 3 on the *C* scale and the answer (156) is read on the *D* scale below the hairline. In other words, in multiplication it makes no difference whether the left or right index of the *C* scale is employed as long as the answer falls within the range of the *D* scale. This means that the operator should use a little "mental forethought" to decide which index to use. Of course, with practice, the selection of the correct index becomes almost automatic.

Note that in the above operations the decimal point location was neglected. In Figure 11, we multiplied 22×3; however, if we had

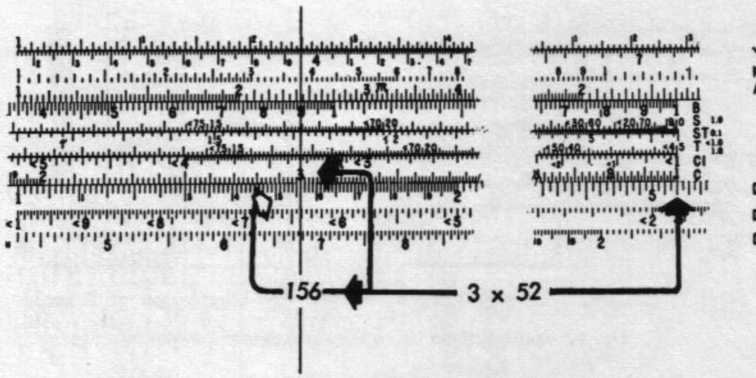

Fig. 12. Multiplication on the C and D Scales.

multiplied 0.22 × 0.3, 0.22 × 3, 2.2 × .3, or any other arrangement with the same combination of numbers, namely 22 and 3, the answer we read from the *D* scale would still have been 660. In short, we have to determine the placement of the decimal point in every operation. How to find the correct position of the decimal point will be discussed later.

Had we so desired, we could have used the *A* and *B* scales to multiply. In this case, the operation would be exactly the same as before with the *B* scale (on the slide) corresponding to the *C* scale and the *A* scale (on the body) corresponding to the *D* scale. At one glance, it will be seen that the *A* and *B* scales are similar in nature to the *C* and *D* scales except that the range of the *A* and *B* scales is twice that of the *C* and *D* scales.

This double range for the *A* and *B* scales has certain advantages and, also, certain disadvantages. The "mental forethought" necessary to determine which index to use when employing the *C* and *D* scales is not necessary here.

Suppose, for example, we wanted to multiply 135 × 8; Figure 13 shows the left index of the *C* scale set on 135 on the *D* scale. We can see immediately that when we move the hairline to coincide with 8 on the *C* scale, the answer is out of range of the *D* scale. Thus, we have chosen the wrong index and in order to solve the problem by using the *C* and *D* scales we must move the slide until the right index of the *C* scale coincides with 135 on the *D* scale. Then we move the hairline to coincide with 8 on the *C* scale and we read the answer (1080) on the *D* scale. Thus, in order to multiply these two factors on the *C* and *D* scales, only the right index of the *C* scale can be used.

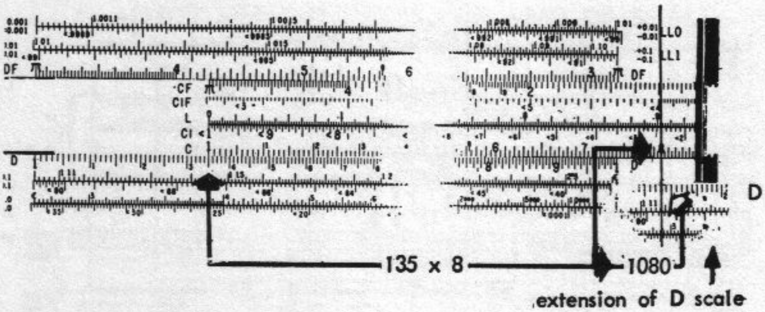

Fig. 13. Multiplication Using Wrong Index of C Scale.

This problem can be avoided by using the A and B scales, although multiplication is not the primary use of the A and B scales. The only rule to remember in order to avoid running out of range of scale A when multiplying by use of the A and B scales, is to set the left index of scale B so that it coincides with the first number in the first range of scale A, or conversely, to set the right index of scale B so that it coincides with the first number in the second range of scale A. Thus Figure 14 shows the multiplication of 135×8 using the left index of

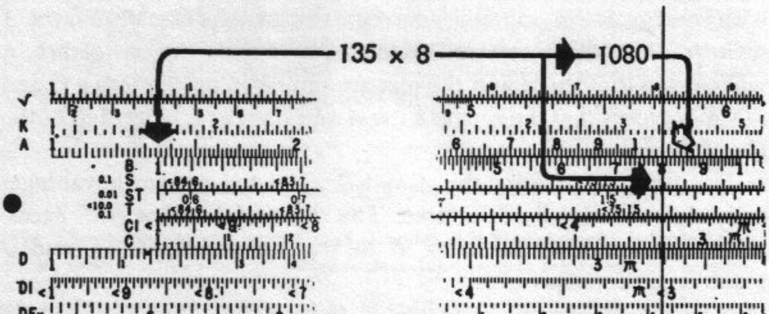

Fig. 14. Multiplication on the A and B Scales.

scale B while Figure 15 shows the same operation using the right index of scale B. Of course, it can be seen that the accuracy of multiplication when using the A and B scales is less than that when using the C and D scales since the length of one range of the C and D scales is twice the length of one range of the A and B scales. However, the center index of scale B can be set in either range of scale A.

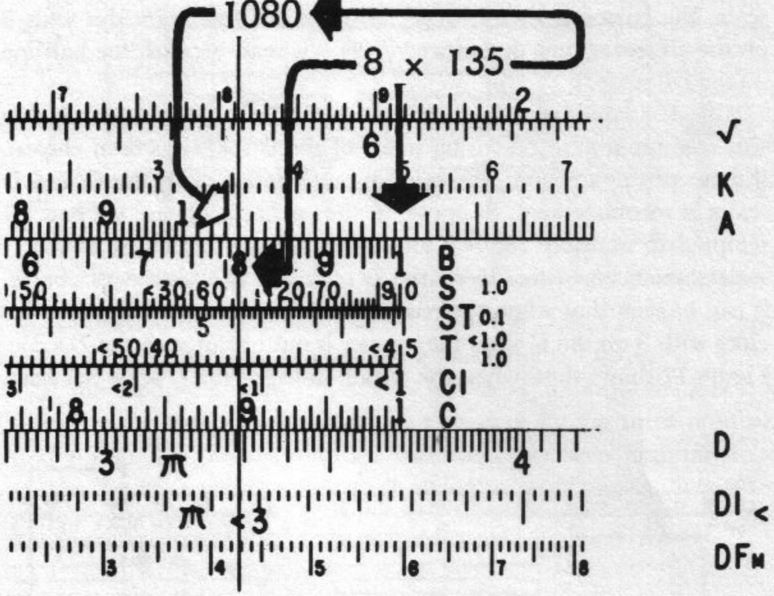

Fig. 15. Multiplication on the A and B Scales.

Also, the *CF* and *DF* scales (sometimes called the folded scales since the range is folded at π) can be used to multiply, the *CF* scale corresponding to the *C* scale and the *DF* scale corresponding to the *D* scale. Figure 16 indicates the use of the *CF* and *DF* scales to multiply 135×8. This operation is, again, accomplished in three steps.

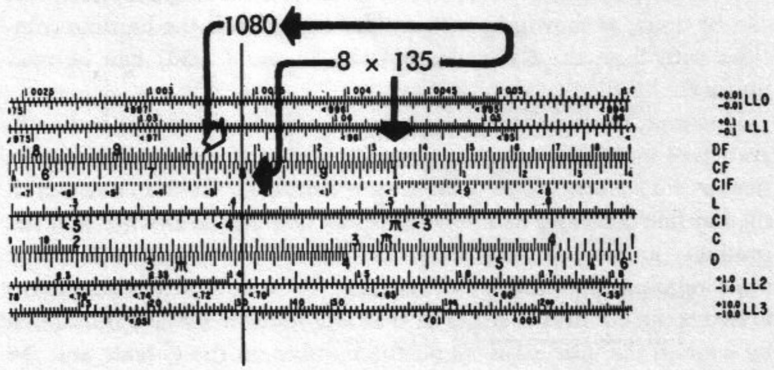

Fig. 16. Multiplication on the CF and DF Scales.

The index of the *CF* scale is placed to coincide with 135 on the *DF* scale, the cursor is then moved until the hairline coincides with 8 on the *CF* scale, and the answer (1080) is read beneath the hairline on the *DF* scale.

Also, for rapidity of calculations, particularly when the *C* and *D* scales are used and the wrong index of the *C* scale has been chosen, the use of the *CF* and *DF* scales in conjunction with the *C* and *D* scales is recommended. Suppose, in the earlier problem, we had attempted to multiply 135×8 and had used the left index of the *C* scale to coincide with 135 on the *D* scale. At one glance, of course, it can be seen that when the cursor is moved until the hairline coincides with 8 on the *C* scale the answer is out of range of the *D* scale. Figure 17 shows that, when the wrong index of the *C* scale has been

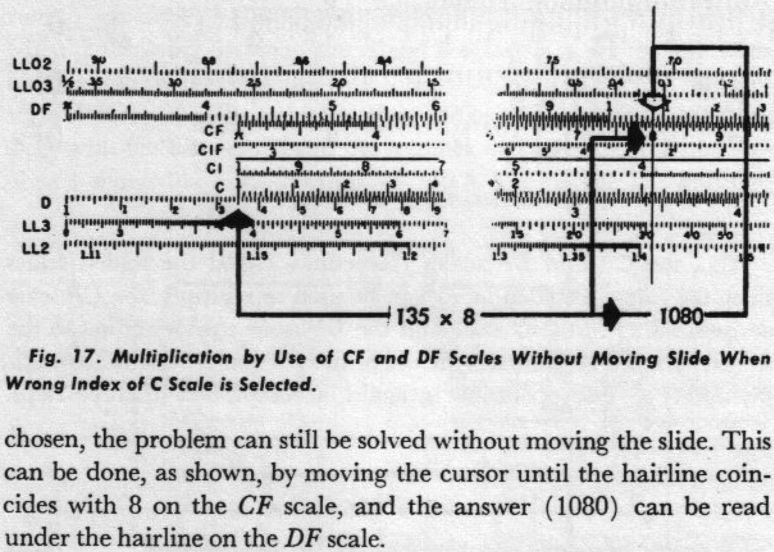

Fig. 17. Multiplication by Use of CF and DF Scales Without Moving Slide When Wrong Index of C Scale is Selected.

chosen, the problem can still be solved without moving the slide. This can be done, as shown, by moving the cursor until the hairline coincides with 8 on the *CF* scale, and the answer (1080) can be read under the hairline on the *DF* scale.

However, the primary function of the *CF* and *DF* scales is the operation of multiplication by π. Actually, this can be done with no particular slide setting. The procedure is simply to set the cursor until the hairline coincides with the multiplier of π on the *D* scale, and the product (answer) is read under the hairline on the *DF* scale. The same relationship existing between the *DF* and *D* scales also exists between the *CF* and *C* scales in that any number may be multiplied by π when the hairline is set on the number on the *C* scale and the answer is read directly above on the *CF* scale. Figure 18 shows the

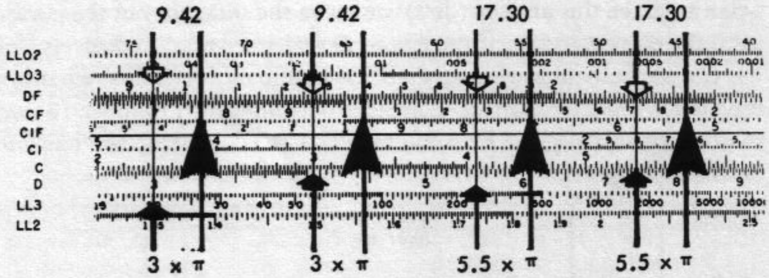

Fig. 18. Multiplication by π Using the DF and D Scales and also the CF and C Scales.

multiplication of $\pi \times 3$ and $\pi \times 5.5$ using the *D* and *DF* scales, and also the same operation by employing the *C* and *CF* scales.

If the product of several numbers is desired, multiply the first two numbers together, then multiply this product by the third number, and repeat this process until all numbers have been multiplied together. In other words, when multiplying three or more numbers together such as $12 \times 25 \times 8$ (Fig. 19), we first multiply 12×25 and

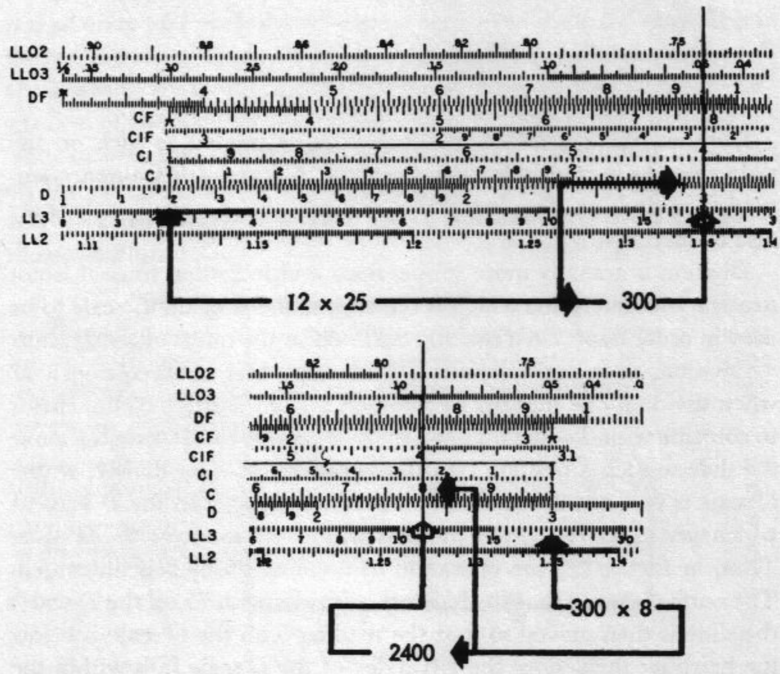

Fig. 19. Multiplication of Three Numbers Using the C and D Scales.

when we have this answer (300) we move the slide but not the cursor until the correct index (if the *C* and *D* scales are used exclusively) of the *C* scale coincides with the hairline on the cursor. Then, we move the cursor until the hairline coincides with the third number (8) on the *C* scale. Finally, we read the answer (2400) beneath the hairline on the *D* scale.

Exercise 2

a. By use of the *C* and *D* scales only, perform the following multiplications:

1. 16×5	*6.* 0.16×0.05
2. 17×41	*7.* $1{,}080 \times 0.003$
3. 9.5×2	*8.* 63.5×18.12
4. 3.75×5	*9.* $2 \times 6 \times 9$
5. 64×400	*10.* $0.2 \times 0.7 \times 3.5$

b. Perform the same calculations as in part a, but use the *A* and *B* scales only.

c. Perform the same calculations as in part a, but by using the *CF* and *DF* scales in conjunction with the *C* and *D* scales.

IX. Division

Division, the reverse of multiplication, is treated as such on the slide rule. As in multiplication, the *C, D, CF* and *DF* scales are employed in division. For less accurate work, the *A* and *B* scales may also be used.

Division is actually more simple than multiplication since it is not necessary to determine which is the correct index of the *C* scale to be used in order to obtain a reading that falls in the range of the *D* scale.

Division, as is multiplication, is accomplished in three steps: 1) when dividing one number by another, set the hairline of the cursor to coincide with the first number, or dividend, on the *D* scale, 2) move the slide to such a position that the second number, or divisor, on the *C* scale is beneath the hairline, 3) read the answer on the *D* scale at whichever index of the *C* scale falls within the range of the *D* scale. Thus, in Figure 20, the operation of dividing 75 by 5 is illustrated. The cursor is set so that the hairline coincides with 75 on the *D* scale; the slide is then moved so that the reading 5 on the *C* scale is below the hairline; then, since the left index of the *C* scale falls within the range of the *D* scale, the answer (15) is read on the *D* scale directly

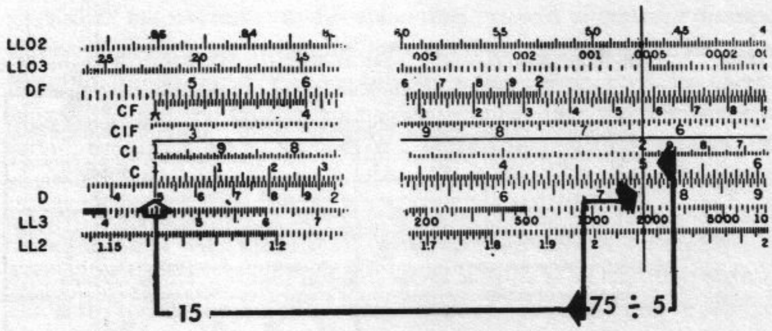

Fig. 20. Division by Use of the C and D Scales.

below the left index of the *C* scale. The division of 378 by 6 is shown, also, in Figure 21.

Figure 22 illustrates the use of the *CF* and *DF* scales in the process of division. In this case, 91 is divided by 13 to give 7. The indicator hairline is set to coincide with 91 on the *DF* scale, the slide is moved until 13 on the *CF* scale coincides with the hairline, and the answer 7 is read on the *DF* scale directly above the index of the *CF* scale. If it is desired merely to divide a number by π, the *DF* and *D* scales or the *CF* and *C* scales are useful since no setting of the slide need be made. Regardless of the slide setting, wherever the hairline crosses the *D* scale is the answer that will be obtained by dividing the number indicated on the *DF* scale under the hairline by π. Thus, 4 divided by π is 1.273, 12 divided by π is 3.82, and 8 divided by π is 2.54. This is shown in Figure 23.

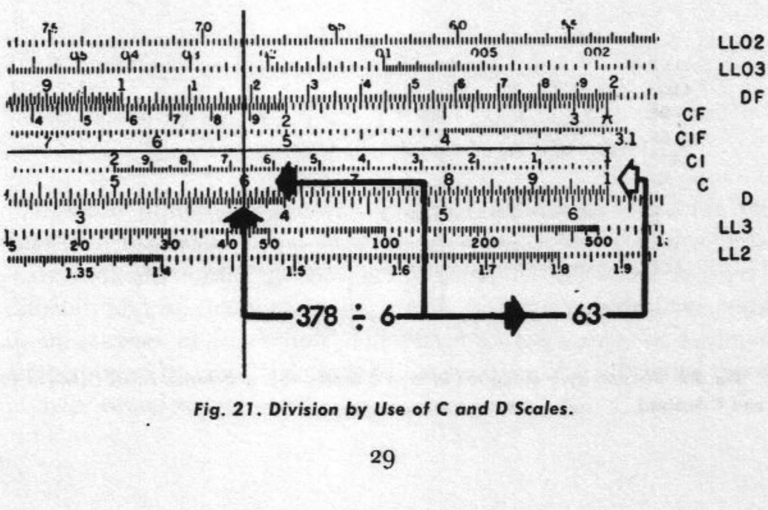

Fig. 21. Division by Use of C and D Scales.

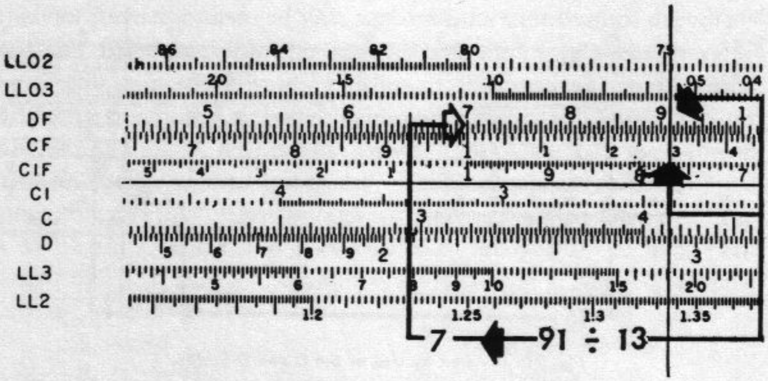

Fig. 22. Division by Use of CF and DF Scales.

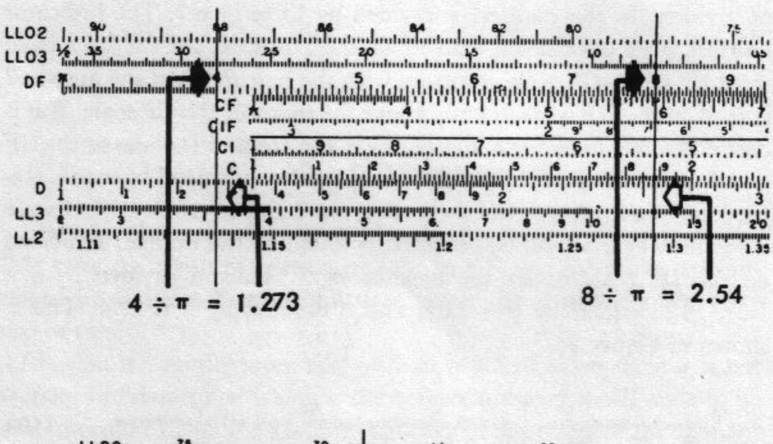

Fig. 23. Division by π Using the DF and D Scales. (Same Relation Exists Between CF and C Scales.)

Also, of course, the *A* and *B* scales may be used to perform division although they are not intended to be used for this purpose. The procedure here again can be accomplished in three steps. First, the cursor is moved until the hairline coincides with the first number, or dividend, on the *A* scale; second, the slide is moved until the second number, or divisor, is beneath the hairline on the *B* scale; third, the answer is read on the *A* scale at the point where any index of the *B* scale coincides with the *A* scale. Also, when dividing by use of the *A* and *B* scales, any combination of ranges may be used. In other words, the first number, or dividend, may be set on either the first or second range of scale *A*, and the slide may be moved until the second number, or divisor, is on either range of scale *B* beneath the hairline. Again, here, the answer is read on the *A* scale at the point where any index of the B scale coincides with the *A* scale.

Exercise 3

a. By use of the *C* and *D* scales, perform the following divisions:

1. $\frac{25}{5}$ *5.* $\frac{125}{17}$

2. $\frac{76}{5.85}$ *6.* $\frac{18}{6.67}$

3. $\frac{102}{0.010{,}9}$ *7.* $\frac{34}{53.8}$

4. $\frac{99.6}{22.3}$ *8.* $\frac{1{,}785}{47.4}$

b. Perform the above divisions by use of the *CF* and *DF* scales.

c. Perform the above divisions by use of the *A* and *B* scales.

X. Placement of the Decimal Point

In many simple problems the decimal point can be located in the answer by the method of inspection. However, in more complex problems involving several divisions and several multiplications, it is very difficult, and, in many cases, impossible to locate the decimal point in the answer by inspection. Therefore, a rapid, sure, and simple method must be used to determine the location of the decimal point in more complex problems.

A survey of the methods of decimal placement employed by the University Interscholastic League Slide Rule Contestants conducted over a period of several years revealed that the majority of contestants used the "significant digit" method of location. Also, in the opinion of the authors, this method is the most reliable for all types of problems. Therefore, for the above reasons, the "significant digit" method will be followed in this simplified manual.

The "significant digits" in a number greater than 1 is a positive number and is numerically equal to the number of digits to the left of the decimal point; on the other hand, the "significant digits" in a number less than 1 is a negative number and is numerically equal to the number of zeros immediately following the decimal point. The following table is given to help clarify the meaning of "significant digits."

Table A

RANGE OF NUMBER N	SIGNIFICANT DIGITS OF NUMBER N	EXAMPLE OF NUMBER N
$10{,}000 \leq N < 100{,}000$	+5	83,145.32
$1{,}000 \leq N < 10{,}000$	+4	6,220.19
$100 \leq N < 1{,}000$	+3	457.23
$10 \leq N < 1000$	+2	33.84
$1.00 \leq N < 10$	+1	2.75
$0.10 \leq N < 1.00$	0	0.68
$0.01 \leq N < 0.10$	—1	0.035
$0.001 \leq N < 0.010$	—2	0.004,9
$0.0001 \leq N < 0.0010$	—3	0.000,52
$0.00001 \leq N < 0.00010$	—4	0.000,019

The "significant digit" method is employed in the following manner:

For Multiplication: The significant digits in the answer is *numerically equal* to the *sum* of the significant digits in the numbers to be multiplied *minus one* for each time the slide is extended to the *right* of the body during multiplication operation. Any *movements* of the slide to the *left* of the body during the process of multiplication are *ignored*.

Thus, for the following examples:

a.	$58 \times 300 = 17{,}400$	Problem
	$(+2) + (+3) = (+5)$	Decimal Placement
b.	$45 \times 34 = 1{,}530$	Problem
	$(+2)+(+2) = (+4)$	Decimal Placement

c.	$920 \times 0.05 = 46$	Problem
	$(+3)+(-1) = (+2)$	Decimal Placement

In each of the above problems, the slide extends to the *left*; thus, the significant digits in the answer is numerically *equal* to the *sum* of the significant digits in the numbers to be multiplied.

d.	$35 \times 25 = 875$	Problem
	$(+2)+(+2)-1 = (+3)$	Decimal Placement
e.	$420 \times 1{,}600 = 672{,}000$	Problem
	$(+3)+(+4)-1 = (+6)$	Decimal Placement
f.	$80 \times 0.12 = 9.6$	Problem
	$(+2)+(0)-1 = (+1)$	Decimal Placement

In each of the above problems, the slide extends to the *right*; thus, the significant digits in the answer is numerically equal to the *sum* of the significant digits in the numbers to be multiplied *minus one*.

For Division: The significant digits in the answer is *numerically equal* to the significant digits in the dividend *minus* the significant digits in the divisor *plus one* for each time the slide is extended to the *right* of the body during the division operation. Any movements of the slide to the *left* of the body during the process of division are *ignored*.

Thus, for the following examples:

a.	$\dfrac{250}{5} = 50$	Problem
	$(+3)-(+1) = (+2)$	Decimal Placement
b.	$\dfrac{0.003{,}5}{70} = 0.000{,}05$	Problem
	$(-2)-(+2)=(-4)$	Decimal Placement
c.	$\dfrac{3.60}{9} = 0.40$	Problem
	$(+1)-(+1)=(0)$	Decimal Placement

In each of the above problems, the slide extends to the *left*; thus, the significant digits in the answer is numerically *equal* to the significant digits in the dividend minus the significant digits in the divisor.

d.	$\dfrac{465}{3} = 155$	**Problem**
	$(+3)-(+1)+1 = (+3)$	**Decimal Placement**

e. $\dfrac{824}{0.04} = 20{,}600$ Problem

$(+3)-(-1)+1=(+5)$ Decimal Placement

f. $\dfrac{606}{303} = 2$ Problem

$(+3)-(+3)+1=(+1)$ Decimal Placement

In each of the above problems, the slide extends to the *right;* thus, the significant digits in the answer is one *more* than the significant digits in the dividend minus the significant digits in the divisor.

Exercise 4

Determine the correct answers to the following problems; include the decimal point in your answer.

1. $17.34 \times 0.003{,}57$

2. 192×46.8

3. 37×12

4. $0.004{,}32 \times 0.333$

5. $109.5 \times 0.000{,}018{,}4$

6. $\dfrac{37}{12}$

7. $\dfrac{0.354}{0.098}$

8. $\dfrac{0.004{,}32}{0.333}$

9. $\dfrac{123.4}{567{,}890}$

10. $\dfrac{125}{52}$

XI. Complex Multiplication and Division

Multplication of Three or More Numbers. This operation is merely an expansion of the multiplication of two numbers which was discussed in section VIII. The entire process can be completed without recording any intermediate results. Thus, let us examine the problem of multiplying 52.7 × 350 × 0.173. The operation is broken down in this manner. First, multiply 52.7 × 350, and then multiply this product by 0.173. As before, to multiply 52.7 × 350 the slide is moved until the correct index (right, in this case) of the *C* scale coincides with 527 on the *D* scale; the cursor is then moved until the hairline coincides with 350 on the *C* scale, and the answer (18,450) is read under the hairline on the *D* scale. But why record this intermediate result? The ultimate answer desired is this product (18,450) multiplied by 0.173. So, without ever recording this value, the cursor is left in position and the slide is moved until the correct (left, in this case) index of the *C* scale falls under the hairline; the cursor is then moved until the hairline coincides with 173 on the *C* scale and the answer (3190) is read under the hairline on the *D scale*. Where is the decimal point located? In order to locate the decimal point in the product of a series of numbers the method for simple multiplication is extended. The number of significant digits in the answer is equal to the sum of the significant digits in the numbers to be multiplied minus one for each time the slide extends to the right of the body during the operation. Thus, the significant digits in the numbers to be multiplied are 52.7 (+2), 350 (+3), and 0.173 (0). The sum of the significant digits in the three multipliers is +5. During the operation the slide extended to the right of the body one time (when multiplying the product of [52.7 × 350] and [0.173]). Therefore, 1 is subtracted from 5 and the significant digits in the answer is 4. Consequently, the answer is 3,190.

If more than three numbers are to be multiplied together, the process would be simply an expansion of the above method, with the multiplications performed in pairs on a cumulative basis.

Division of One Number by a Product of Two or More Numbers. This operation is simply a series of divisions. The entire process, again, can be completed without tabulating any intermediate value. Thus, take the case of dividing (725) by (17 × 55). The solution is to divide 725 by 17 and then, in turn, divide this quotient by 55. In order to divide 725 by 17 (a simple division process), the cursor is moved until the hairline coincides with 725 on the *D* scale; the slide is then moved until 17 falls under the hairline on the *C* scale and the answer (42.6) is read on the *D* scale at the point where either index of the *C* scale falls in the range of the *D* scale. But, again, why record this intermediate reading? The answer desired is this quotient (42.6) divided by 55. Thus, without recording this intermediate value the cursor is left in position and the slide is moved until the second divisor (55) falls under the hairline on the *C* scale. The answer (0.775) is read on the *D* scale at whichever index of the *C* scale falls in range of the *D* scale. Where is the decimal point located? In order to determine its location in this case, the rule of decimal point location for simple division is expanded. The significant digits in the answer is equal to the significant digits of the dividend minus the sum of the significant digits of the divisors plus one for each time the slide extends to the right of the body during the entire operation. Thus, in the above example, the significant digits in the dividend (725) is +3. The sum of the significant digits in the divisors, 17 (+2) and 55(+2), is +4. During the operations, the slide extended to the right of the body one time (when dividing 725 by 17). Therefore, the significant digits in the answer is $3 - 4 + 1 = 0$, and thus the answer is 0.775.

If the original problem had three or more divisors, the process above would be continued in more or less cyclical operations until all divisions had been completed and the final answer obtained.

Extended Operations. Example: Find the result of $\dfrac{82 \times .037 \times 432}{.54 \times 8.63 \times 127}$

This problem is broken down into simple operations as shown below. The method of alternating multiplication and division is recommended, where possible, rather than a series of multiplications followed by a series of divisions. The reason is that a multiplication following a division often does not require a movement of the slide and, thus, this procedure will consume a minimum amount of time. Therefore, the above problem would be performed in the following steps:

1) Divide 82 by 0.54.
2) Multiply the quotient obtained in (1) by .037.
3) Divide the product obtained in (2) by 8.63.
4) Multiply the quotient obtained in (3) by 432.
5) Divide the product obtained in (4) by 127.
6) Read the final sequence of numbers in the answer on the *D* scale.

Each of the above operations is a simple multiplication or division of two factors.

*Thus,

1. $\dfrac{82}{54} = 1{,}518$

2. $1{,}518 \times 37 = 561$

3. $\dfrac{561}{863} = 651$

4. $651 \times 432 = 2{,}815$

5. $\dfrac{2{,}815}{127} = 2{,}215$ (the sequence of numbers in the answer)

* Note: Only sequence of numbers is determined here; decimal point is ignored.

To determine the decimal point location in the answer, the rules for simple multiplication and division are extended. Operations (1), (3), and (5) are simple divisions. In two of these operations, (1) and (5), the slide extends to the right of the body. Operations (2) and (4) are simple multiplications. In one of these operations, (2), the slide extends to the right of the body. The sum of the significant digits of the numbers in the original problem above the division line is $+ 2 - 1 + 3$ or $+ 4$. The sum of the significant digits of the numbers below the division line in the original problem is $0 + 1 + 3$ or $+ 4$. Therefore, the significant digits in the answer is equal to $4 - 4 + 2$ (two extensions of the slide to the right in division) $- 1$ (one extension of the slide to the right in multiplication). Consequently, the significant digits in the answer is $+ 1$ and the value is 2.215.

Actually, when performing an operation such as the above example, each time the slide extends to the right during multiplication a $- 1$ should be recorded and when the slide extends to the right during division a $+ 1$ should be recorded. These can then be mentally summed to find the total correction for slide movement to be applied to the significant digits of the original numbers in order to determine the significant digit value for the final answer.

There are many complex combinations of numbers similar to the above that can be solved by simply an extension or a slight variation of the above methods.

Investigate the problems given in the following exercise to test your ability to solve a few such complex arrangements.

Exercise 5

Determine the answers to the following problems by use of the *C* and *D* scales. Be sure to indicate the decimal point in your answers.

1. $\dfrac{36 \times 51 \times 17}{25 \times 16 \times 37}$

2. $\dfrac{96.1 \times 48.5 \times 0.033}{21.7 \times 16.4 \times 45}$

3. $\dfrac{0.002{,}77 \times 0.046{,}2}{0.003{,}66}$

4. $\dfrac{484 \times 5.99 \times 63.3}{41.2 \times 0.067{,}7}$

5. $\dfrac{0.098{,}8 \times 0.136 \times 44.4}{100.4 \times 0.007{,}7 \times 39.5}$

6. $\dfrac{48.3 \times 17.7 \times 39.3}{0.000{,}733 \times 0.000{,}044{,}1 \times 96{,}400{,}000}$

7. $\dfrac{0.012 \times 0.033 \times 41.4 \times 76.3}{36.2 \times 0.000{,}339 \times 0.442}$

8. $\dfrac{414 \times 2.41 \times 0.017{,}8}{0.013 \times 0.016 \times 9.5 \times 48.2}$

9. $\dfrac{40{,}000 \times 58{,}100 \times 0.061{,}3}{0.002{,}4 \times 78{,}000 \times 209}$

10. $\dfrac{200{,}000 \times 48{,}100 \times 0.000{,}072}{48.2 \times 0.000{,}393 \times 750{,}000}$

XII. Squares and Square Roots

The determination of squares and square roots on a slide rule varies depending upon the scales appearing on the rule. In general there are two different types of square root or square scales used, namely, the A and B scales, and the R_1 and R_2 scales or two $\sqrt{\ }$ scales.

For Rules Having A and B Scales. The A and B scales are identical in calibration, just as the C and D scales are identical and also the CF and DF scales are identical. The A and B scales each consist of two D scales that have been reduced to half length and placed end on end, with the A and B scales each having approximately one-half as many calibration lines as the D scale.

If one desires, he may multiply and divide by use of the A and B scales; however, as mentioned earlier, the accuracy of his answer will be less than that obtained by using the C and D scales. This is not the intended use of the A and B scales.

In order to determine the square of a number, slide the indicator hairline until is coincides with the number on the D scale and read the square of the number directly under the hairline on the A scale. See Figure 24. If the number to be squared (set on the D scale) is

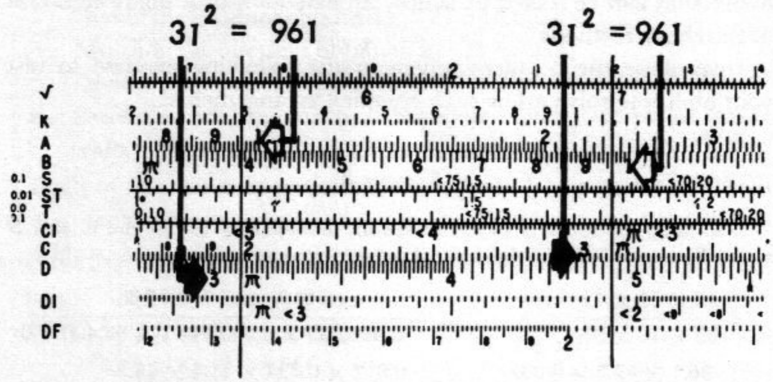

Fig. 24. Squares and Square Roots Using the A and D Scales and also the B and C Scales.

greater than one and the answer (read on A scale) falls in the first range of scale A, there will be an odd positive number of significant digits in the answer. For example, (Fig. 24) when we set 31 on the D scale we read 961 under the hairline in the first range of scale A. Therefore, we know the answer will either be 9.61, 961, 96100, etc.

Logical reasoning tells us the answer for 31^2 is 961. Conversely, if the number (55) to be squared is greater than one, and the answer (3025) falls in the second range of scale *A*, there will be an even positive number (+ 4) of significant digits in the answer. If the number (0.02) to be squared is less than one, and the answer (0.0004) falls in the first range of scale *A*, there will be an odd negative number (− 3) of significant digits in the answer. If the number (0.06) to be squared is less than one, and the answer (0.0036) falls in the second range of scale *A*, there will be either zero significant digits or an even negative number (− 2) of significant digits in the answer.

Determining the square root of a number is the reverse of the above procedure; however care must be taken to locate the number on the correct half of the *A* scale. If it is desired to determine the square root of a number greater than one and having an odd number of digits before the decimal, such as 9 or 125, slide the indicator hairline to this number on the left half of the *A* scale and read the square root directly under the hairline on the *D scale*. If it is desired to determine the square root of a number greater than one and having an even number of digits before the decimal, such as 25 or 4900, slide the indicator hairline to the number on the right half of the *A* scale and read the square root directly under the hairline on the *D* scale. If it is desired to obtain the square root of a number less than one and having an odd number of zeros between the decimal and the first significant digit, such as 0.04 or 0.0009, slide the indicator hairline to the number on the left half of the *A* scale and read the square root of the number directly under the hairline on the *D* scale. To obtain the square root of a number less than one and having either no zeros or an even number of zeros between the decimal and the first significant digit, such as 0.25 or 0.009, slide the indicator hairline to the number on the right half of the *A* scale and read the square root directly under the hairline on the *D* scale. The decimal point is determined by reversing the procedure for squaring a number.

The above procedures may be applied to the *B* and *C* scales exactly as described above. The number is set on the *C* scale and the square is read on the *B* scale. This permits square and square root determination on the slide independent of its position relative to the body.

For Rules Having R_1 and R_2 or Two $\sqrt{}$ Scales. Note that the R_1 and R_2 scales (or the two $\sqrt{}$ scales) are similar to one-half of the *D* scale; the R_1 and R_2 scales (or two $\sqrt{}$ scales) placed with their ends together would be similar to a complete *D* scale but would be twice as long as a *D* scale.

To determine the square of a number, move the indicator hairline to the number on whichever of the R_1 or R_2 scales (or the top or bottom $\sqrt{}$ scale) on which the number appears and read the square directly under the hairline on the D scale. Note that the slide is not used and could have been removed from the rule if the operator so desired. The decimal point is determined either by reasoning or by approximation.

Determining the square root of a number is the reverse of the above procedure; however care must be taken to locate the square root on the correct scale (either the R_1 or R_2 scale or the first or second $\sqrt{}$ scale). If it is desired to determine the square root of a num-

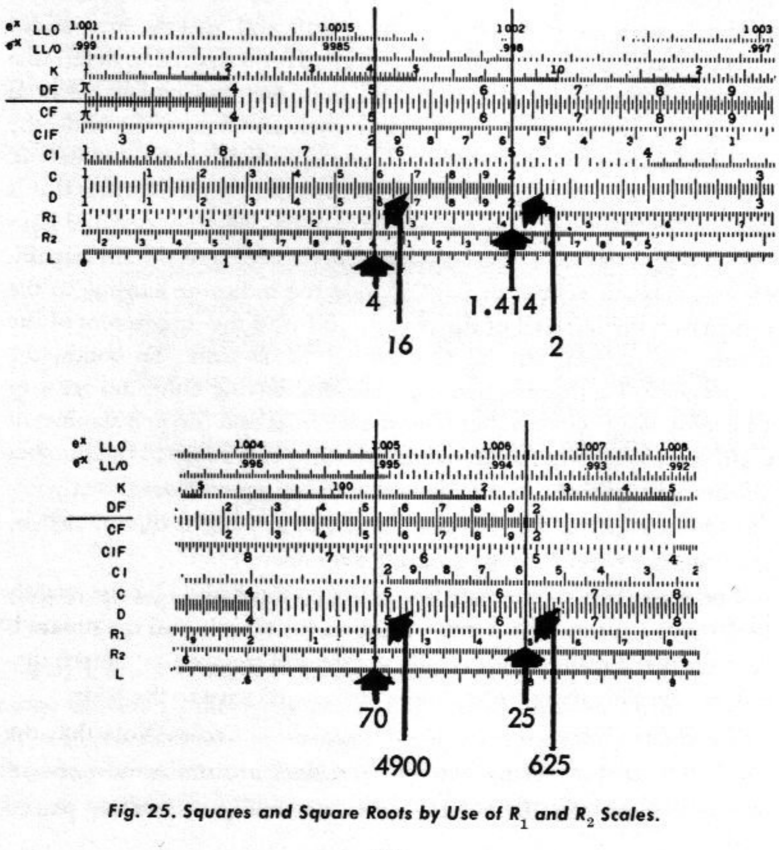

Fig. 25. Squares and Square Roots by Use of R_1 and R_2 Scales.

ber greater than one and having an odd number of digits before the decimal (such as 2 or 625), set the indicator hairline on the number on the D scale and read the square root under the hairline on the R_1 scale (or first $\sqrt{}$ scale). To determine the square root of a number greater than one and having an even number of digits before the decimal (such as 16 or 4,900), set the indicator hairline on the number on the D scale and read the square root under the hairline on the R_2 scale (or second $\sqrt{}$ scale). To determine the square root of a number less than one and having an odd number of zeros between the decimal and the first significant digit (such as 0.04 or 0.0009), set the hairline on the number on the D scale and read the square root under the hairline on the R_1 scale (or first $\sqrt{}$ scale). To find the square root of a number less than one and having either no zeros or an even number of zeros between the decimal and the first significant digit (such as 0.25 or 0.007), set the hairline on the number on the D scale and read the answer under the hairline on the R_2 scale (or second $\sqrt{}$ scale). The decimal point is determined either by reasoning or by approximation.

Figure 25 indicates a number of squares and square roots that may be obtained by use of the R_1 and R_2 scales. Figure 26 indicates similar operations by use of the two $\sqrt{}$ scales.

Exercise 6

Obtain the squares and square roots indicated in the following problems. Be sure to locate the decimal in the answer.

1. $\sqrt{4}$	*6.* $(19.6)^2$
2. $\sqrt{16}$	*7.* $(0.031)^2$
3. $\sqrt{144}$	*8.* $(17{,}241)^2$
4. $\sqrt{0.915}$	*9.* $(5.01)^2$
5. $\sqrt{0.000{,}25}$	*10.* $(0.002{,}32)^2$

XIII. Cubes and Cube Roots

Determining the cubes and cube roots of numbers is very similar to determining the squares and square roots of numbers except the K or the $\sqrt[3]{}$ scales are used in place of the A, B, the R_1, R_2 or the $\sqrt{}$ scales. These scales are used in conjunction with the D scale.

For Rules Having a K Scale. The K scale is similar to three D scales that have been shrunk to one-third their regular size and placed end on end. This is shown in Figure 27.

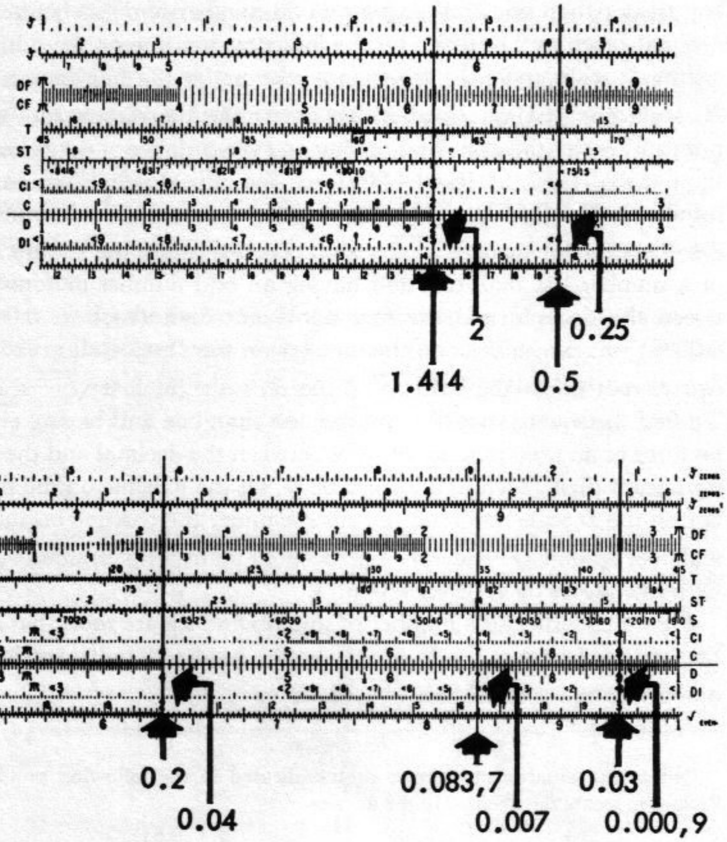

Fig. 26. Squares and Square Roots by Use of the Two $\sqrt{\ }$ Scales.

In order to determine the cube of a number, set the indicator hairline on the number on the *D* scale and read the cube on the *K* scale directly under the hairline. If the answer (read on the *K* scale) falls in the first third (left section) of scale *K* the significant digits in the answer is equal to three times the significant digits in the number to be cubed minus two; thus, 2^3 is 8. If the answer falls in the middle third (center section) of the *K* scale, the significant digits in the answer is equal to three times the significant digits in the number to be cubed minus one; thus, 3^3 is 27. If the answer falls in the last third (right section) of scale *K* the significant digits in the answer is equal to exactly three times the significant digits in the number; thus, 7^3 is 343. This procedure is shown in Figure 27.

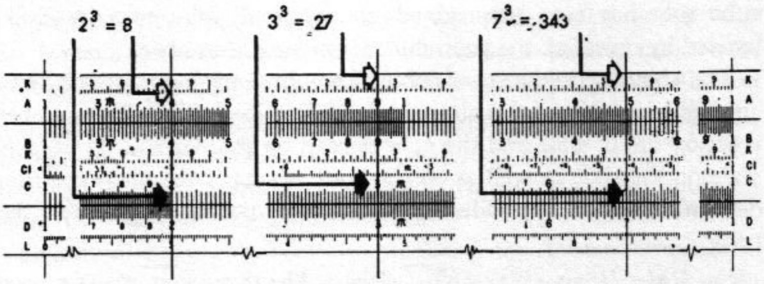

Fig. 27. Cubes and Cube Roots by Use of the K Scale.

Determining the cube root of a number is the reverse of the above procedure; however care must be taken to locate the number on the correct third of the *K* scale. If it is desired to determine the cube root of a number greater than one and having 1, 4, 7, 10, 13, etc., digits before the decimal (such as 8 or 1,728), set the indicator hairline on the number on the first third (the left section) or the *K* scale and read the cube root on the *D* scale under the hairline. To determine the cube foot of a number greater than one and having 2, 5, 8, 11, 14, etc., digits before the decimal (such as 27 or 25,000), set the hairline on the number on the middle third (center section) of the *K* scale and read the cube root on the *D* scale under the hairline. The cube root of a number greater than one and having 3, 6, 9, 12, 15, etc., digits before the decimal (such as 125 or 175,000) may be found by setting the hairline on the number on the last third (right section) of the *K* scale and reading the answer on the *D* scale under the hairline. To determine the cube root of a number less than one and having 2, 5, 8, 11, 14, etc., zeros between the decimal and the first significant digit (such as 0.008 or 0.000009), set the hairline on the number on the first third (left section) of the *K* scale and read the cube root on the *D* scale under the hairline. To find the cube root of a number less than one and having 1, 4, 7, 10, 13, etc., zeros between the decimal and the first significant digit (such as 0.06 or 0.00008), set the hairline on the number on the middle third (center section) of the *K* scale and read the cube root on the *D* scale under the hairline. To obtain the cube root of a number less than one and having 0, 3, 6, 9, 12, 15, etc., zeros between the decimal and the first significant digit (such as 0.85 or 0.00072) set the hairline on the number on the last third (right section) of the *K* scale and read the cube root on the *D* scale under the hairline. Once the number sequence of the cube root has been determined, the decimal point may be easily located by reversing the procedure for cubing a number.

The above procedures for cubing and obtaining cube roots may be applied to the *K* and *C* scales exactly as described above. The number is set on the *C* scale and the cube is read on the *K* scale. Thus, a *K* scale on the slide as well as on the body permits cube and cube root determination on the slide regardless of its position relative to the body.

For Rules Having Three $\sqrt[3]{\ }$ *Scales.* The three $\sqrt[3]{\ }$ scales placed end on end would equal one *D* scale; however the total length would be three times as great as the length of the *D* scale. This is shown in

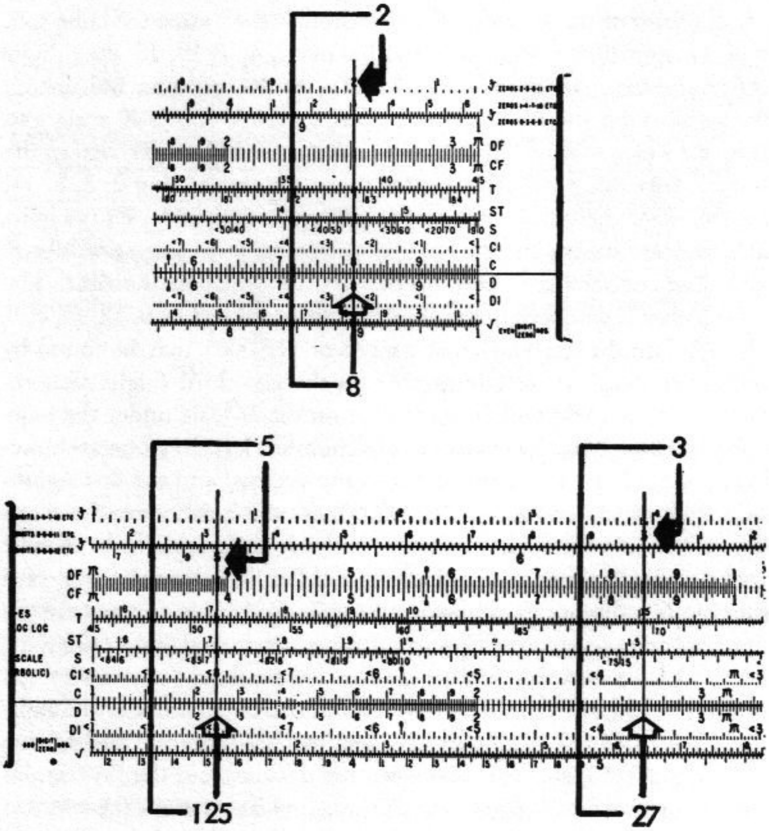

Fig. 28. Cubes and Cube Roots by Use of the Three $\sqrt[3]{\ }$ Scales.

Figure 28. Since the three $\sqrt[3]{\ }$ scales are used in conjunction with the *D* scale, the slide is not used in determining cubes and cube roots.

In order to determine the cube of a number, set the hairline on the number on whichever $\sqrt[3]{\ }$ scale it appears and read the cube directly on the *D* scale under the hairline. Thus 2^3 is 8, 3^3 is 27, and 5^3 is 125. This is shown in Figure 28. The decimal point is located either by reasoning or by approximation.

Determining the cube root of a number is the reverse of the above procedure; however care must be exerted to locate the cube root on the correct one of the three $\sqrt[3]{\ }$ scales as determined by the relative position of the decimal point and the first significant digit in the original number. If it is desired to determine the cube root of a number greater than one and having 1, 4, 7, 10, 13, etc., digits before the decimal (such as 8 or 1,728), set the hairline on the number on the *D* scale and read the cube root on the first (top) $\sqrt[3]{\ }$ scale under the hairline. The cube root of a number greater than one and having 2, 5, 8, 11, 14, etc., digits before the decimal point (such as 27 or 25,000) may be found by setting the hairline on the number on the *D* scale and reading the cube root of the number on the second (or middle) $\sqrt[3]{\ }$ scale under the hairline. To determine the cube root of a number greater than one and having 3, 6, 9, 12, 15, etc., digits before the decimal (such as 125 or 175,000) set the hairline on the number on the *D* scale and read the cube root on the third (or bottom) $\sqrt[3]{\ }$ scale under the hairline. Cube roots of numbers less than one and having 2, 5, 8, 11, 14, etc., zeros between the decimal and the first significant digit (such as 0.008 or 0.000009) may be determined by setting the indicator hairline on the number on the *D* scale and reading the cube root on the first (or top) $\sqrt[3]{\ }$ scale under the hairline. Cube roots of numbers less than one and having 1, 4, 7, 10, 13, etc., zeros between the decimal and the first significant digit (such as 0.06 or 0.00008) are determined by setting the hairline on the number on the *D* scale and reading the cube root on the second (or center) $\sqrt[3]{\ }$ scale under the hairline. To find the cube root of a number less than one and having 0, 3, 6, 9, 12, 15, etc., zeros between the decimal and the first significant digit (such as 0.85 or 0.00072) set the indicator hairline on the number on the *D* scale and read the cube root on the third (or bottom) $\sqrt[3]{\ }$ scale under the hairline. The decimal point is determined either by reasoning or by approximation.

Exercise 7

Perform the following operations:

1. $(5)^3$	*6.* $\sqrt[3]{27}$
2. $(12.6)^3$	*7.* $\sqrt[3]{0.486}$
3. $(436)^3$	*8.* $\sqrt[3]{12{,}144}$
4. $(0.058)^3$	*9.* $\sqrt[3]{0.006{,}8}$
5. $(0.33)^3$	*10.* $\sqrt[3]{12{,}460{,}149}$

XIV. Inverted Scales

The *CI, CIF,* and *DI* scales yield further simplification for solving problems involving multiplication and division. They are of particular assistance in obtaining the reciprocals of numbers.

By inspecting the *CI* and *C* scales, one will find that the *CI* scale is exactly the reverse of the *C* scale; these two scales are equal in length. The *CI* and *C* scales have the property that, regardless of the indicator setting, the number on one of the scales under the hairline is the reciprocal of the number on the other scale under the hairline. This is shown in Figure 29.

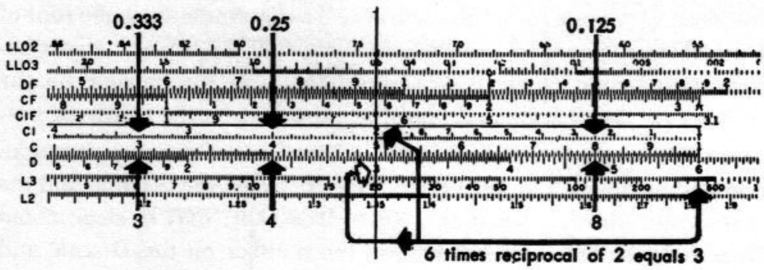

Fig. 29. Reciprocals by Use of the CI and C Scales.

The *DI* and *D* scales and the *CIF* and *CF* scales have the same properties of finding reciprocals as have the *CI* and *C* scales.

To illustrate the use of these scales, consider the problem of 6 divided by 2. This may be rewritten as 6 times the reciprocal of 2; this is accomplished on the slide rule by the following operations. Set the right index of the *C* scale to coincide with 6 on the *D* scale, slide the indicator until the hairline coincides with 2 on the *CI* scale, and read the answer 3 on the *D* scale under the hairline. This is illustrated in Figure 29.

The problem $6 \div 2\pi$ may be rewritten as 6 times the reciprocal of

2π. This may be accomplished on the slide rule by setting the left index of the *C* scale to coincide with 6 on the *D* scale, sliding the indicator until the hairline coincides with the 2 on the *CIF* scale, and reading the answer 0.955 under the hairline on the *D* scale.

The proficient slide rule operator soon determines for himself short-cut methods of employing the *CF, DF, CI, DI,* and *CIF* scales along with the *C* and *D* scales to reduce the number of slide rule operations to a minimum and, consequently, to minimize the time required to perform the calculations. This, however, is something that can be accomplished only with practice.

Exercise 8

To practice the use of the inverted (or reciprocal) scales, determine the answers to the following problems by use of several different combinations of scales.

1. $\frac{625}{25}$
2. $\frac{2}{18}$
3. $\frac{100}{2\pi}$
4. $\frac{1}{750}$
5. $\frac{1{,}920}{62.4}$
6. $\frac{756}{25\pi}$
7. $\frac{9.04}{39.1}$
8. $\frac{1}{0.042}$
9. $\frac{0.006{,}91}{89.75}$
10. $\frac{1{,}230{,}000}{1{,}920\pi}$

XV. Conclusion

It is hoped that the instructions given in this manual will aid you in learning to use the slide rule. When you have learned how to perform the operations discussed previously you should be able to classify yourself as an efficient slide rule operator. In order to become a proficient operator, one should practice using the rule at every opportunity.

Of course, in Interscholastic League competition, accuracy and speed are of primary importance. Efficiency yields accuracy and speed. Therefore, after you have mastered the basic operations described herein, accuracy and speed will be a natural consequence. Moreover, both of these attributes are fundamental qualities for winning competition.

There are many other scales appearing on the modern slide rule which have not been investigated in these elementary discussions, for example, the log log scales and the trigonometric scales. However, since the ultimate objective of most readers of this manual is to prepare for district, regional, and state slide rule competition, the scales not discussed are of secondary importance at this time; the reason for this is the fact that the slide rule competitive tests contain only problems that involve the use of the fundamental scales. As one prepares for slide rule competition by learning the use of the fundamental scales, his knowledge of the slide rule will increase so rapidly that learning the use of the advanced scales will be a trivial procedure.

The following set of practice problems are given with the objective of helping you to perform many types of operations in order to survey your "cumulative" use of the rule. These problems are considerably more difficult than those following each individual exercise and when you can solve all of them without reference to the answers, you are well on the road to "slide rule mastery."

XVI. Practice Problems

1. 326.419×63.4

2. $0.005{,}23 \times 391.4$

3. $418.7 \times 0.001{,}22 \times \pi \times 63{,}402 \times 0.009{,}18$

4. $26.8 \times 0.007{,}87 \times 5{,}210$

5. $\dfrac{913 \times 7.68}{89.5}$

6. $\dfrac{5.92 \times 0.008{,}74}{20.45}$

7. $\dfrac{1}{0.007{,}465 \times \pi \times 92.83}$

8. $\dfrac{2{,}860}{0.003{,}42 \times 19.65 \times 0.041{,}6 \times 0.048{,}6}$

9. $\dfrac{1}{1{,}600 \times \pi \times 0.040{,}03}$

10. $\dfrac{8{,}460 \times 7{,}230}{6{,}241 \times 92.20}$

11. $\left(\frac{3}{0.078{,}2}\right) \times \left(\frac{\pi}{19.2}\right) \times 2{,}983$

12. $\frac{0.412 \times 27.6 \times 1{,}305}{66.7 \times 0.96 \times 27{,}500}$

13. $\frac{6.67 \times 67.4 \times 11.34}{32.8}$

14. $\frac{\left(\frac{\pi}{14.92}\right)}{0.020{,}8 \times (17.3)^3}$

15. $\frac{286}{19{,}650 \times 0.041{,}6 \times 0.048{,}6 \times 0.003{,}42}$

16. $\frac{0.046 \times 0.000{,}713 \times 68.1}{234 \times 9.68 \times 5.1 \times \pi}$

17. $(\pi)^2 \times 0.000{,}032{,}9 \times 18{,}650{,}000$

18. $0.000{,}000{,}912 \times 71{,}432{,}710 \times \frac{1}{\frac{1}{(\pi)^2}} \times \left(\frac{16.1}{49.8}\right)^2$

19. $\frac{(891)^3 \times \pi \times (406)^2}{(10)^3 \times (406)^3 \times 8{,}910}$

20. $\frac{2 \times 32.2 \times \pi \times 0.005{,}9 \times 0.98 \times (2.916)^2}{4}$

21. $\frac{0.119{,}23 \times (17.8)^2 \times 0.628}{2.607 \times (\pi)^2}$

22. $\frac{6.298{,}4 \times 0.000{,}047}{\pi \times (316)^3 \times 2}$

23. $\frac{(\pi)^3 \times (0.004{,}3 \times 7)^2}{0.028 \times 4{,}917}$

24. $\frac{(2.14)^2 \times (6.28)^3}{\pi \times (0.146)^2 \times 987}$

25. $\frac{1{,}004 \times (2.160)^2 \times (4.87)^3}{(4.87)^2 \times (2.17)^3 \times 1{,}004}$

26. $\frac{1}{6.09 \times 72.1 \times (0.094 \times 4)^3 \times 3.02}$

27. $\dfrac{0.034 \times \pi \times (4.27)^3}{0.008{,}3 \times (7.19)^2}$

28. $\left(\dfrac{0.063 \times \pi}{92}\right)^3$

29. $\dfrac{0.001{,}64 \times \left(\dfrac{\pi}{2.75}\right)^3 \times 7{,}240 \times 3}{(1.732)^2 \times 0.002{,}93}$

30. $\pi \times 2 \times \sqrt{\dfrac{12 \times 18.1}{32.2}}$

31. $\dfrac{666 \times 0.022 \times 44.4}{1{,}004 \times \sqrt{\pi}}$

32. $64.3 \times \sqrt{\dfrac{1}{17.2}}$

33. $\dfrac{\sqrt[3]{4{,}750} \times 3.141{,}6}{(\pi)^3 \times \sqrt{71.8}}$

34. $\dfrac{\left(\dfrac{1}{62.4}\right) \times \sqrt[3]{31.006}}{9.869{,}7 \times \left(\dfrac{\pi}{429}\right)}$

35. $\dfrac{12 \times (12)^2 \times (12)^3}{\sqrt{902{,}500}}$

36. $\dfrac{(72)^2 \times (72)^3}{\sqrt{7{,}123{,}360}}$

37. $\dfrac{1}{\sqrt{40.9} \times 0.018{,}4 \times 0.043{,}1} \times \dfrac{\pi}{16{,}218}$

38. $\dfrac{(0.038)^3 \times \pi \times (1{,}000)^3 \times \sqrt{7{,}000}}{(94)^3 \times (94)^2}$

39. $\sqrt[3]{(\pi)^2}$

40 $\dfrac{100}{\sqrt{17.8} \times 29.17}$

41. $\dfrac{(24)^3 \times (785)^2}{\sqrt{962}}$

42. $\dfrac{1{,}634.278 \times \sqrt[3]{\pi}}{0.008{,}85}$

43. $\dfrac{\left(\dfrac{17.9}{2.08}\right)^2}{(0.039)^3 \times \sqrt[3]{11.543}}$

44. $\sqrt[3]{\sqrt{8{,}303{,}766{,}000}}$

45. $\sqrt{\dfrac{0.021{,}8 \times \sqrt{83.9}}{(31.6)^3 \times 0.000{,}000{,}067}}$

46. $\sqrt[3]{\sqrt{\dfrac{49.7 \times 0.032}{(\pi)^3}}}$

47. $\sqrt{\dfrac{\pi \times 98.6}{1{,}492 \times (29)^2}}$

48. $\dfrac{\left(\dfrac{6.28}{84.9}\right) \times \left(\dfrac{0.043{,}7}{91.4}\right)}{4.17 \times \sqrt{870.25}}$

49. $\sqrt{\dfrac{\pi \times 934}{\sqrt[3]{93{,}400}}}$

50. $\sqrt[3]{\left(\dfrac{8.496 \times \pi}{4.248}\right)^2}$

Methods of Answer Presentation

As stated in the Constitution and Contest Rules of the University Interscholastic League, the answers to the problems may be written in full or expressed in powers of ten. This is a matter of choice for a particular operator.

Thus, suppose the answer to a particular problem were 12,300. This answer could be expressed as 0.123×10^5, 1.23×10^4, 12.3×10^3, 123×10^2, etc. However, when using this method of recording answers, care must be taken to insure that at least three significant figures appear in the answer to obtain full credit for the problem. Again, suppose the answer to another problem were 0.000210. This value could be indicated as 2.10×10^{-4}, 21.0×10^{-5}, 210×10^{-6}, etc. for full credit, but an answer of 2.1×10^{-4}, 21×10^{-5}, etc. would not be completely correct according to Interscholastic League

Rules and full credit could not be allowed. The reason that $21. \times 10^{-4}$ or 21×10^{-5} would not be worth full credit in this case is due to the fact that only two significant figures are indicated. Regardless of whether the significant figures are zeros or not they must be recorded.

The following answers to particular problems would all be worth full credit:

a) 5,340, 0.534×10^4, 5.34×10^3, 53.4×10^2, 534×10, etc.

b) 0.200, 2.00×10^{-1}, 20.0×10^{-2}, 200×10^{-3}, etc.

c) 0.0820, 0.820×10^{-1}, 8.20×10^{-2}, 82.0×10^{-3}, etc.

The point to remember is that, regardless of the method of answer presentation followed, at least three significant figures must be shown.

XVII. Answers

Exercise 1.

3. A) 1,253
B) 1,892
C) 214
D) 349
E) 456
F) 517
G) 638
H) 691
I) 752
J) 839
K) 938
L) 986

Exercise 2.

1. 80.0
2. 697
3. 19.00
4. 18.75
5. 25,600
6. 0.008,00
7. 3.24
8. 1,150
9. 108.0
10. 0.490

Exercise 3.

1. 5.00
2. 13.00
3. 9,360
4. 4.47
5. 7.35
6. 2.70
7. 0.632
8. 37.7

Exercise 4.

1. 0.061,9
2. 8,990
3. 444
4. 0.001,440
5. 0.002,01
6. 3.08
7. 3.61
8. 0.013,00
9. 0.000,217
10. 2.40

Exercise 5.

1. 2.11	*5*. 0.019,5	*8*. 1.86
2. 0.009,61	*6*. 10,800	*9*. 3,640
3. 0.035,0	*7*. 231	*10*. 48.8
4. 65,800		

Exercise 6.

1. 2.00	*5*. 0.015,8	*8*. 297,000,000
2. 4.00	*6*. 384	*9*. 25.1
3. 12.00	*7*. 0.000,961	*10*. 0.000,005,38
4. 0.956		

Exercise 7.

1. 125	*5*. 0.035,9	*8*. 23.0
2. 2,000	*6*. 3.00	*9*. 0.189
3. 82,900,000	*7*. 0.786	*10*. 232
4. 0.000,195		

Exercise 8.

1. 25.0	*5*. 30.8	*8*. 23.8
2. 0.111	*6*. 9.63	*9*. 0.000,077,0
3. 15.9	*7*. 0.231	*10*. 204
4. 0.001,33		

Practice Problems

1. 20,700	*22*. 0.000,001,49
2. 2.05	*23*. 0.000,204
3. 934	*24*. 17.2
4. 1,100	*25*. 2.25
5. 78.3	*26*. 0.014,2
6. 0.002,53	*27*. 19.4
7. 0.459	*28*. 0.000,000,010,0
8. 21,000,000	*29*. 6,010
9. 0.004,97	*30*. 16.3
10. 106	*31*. 0.366
11. 18,700	*32*. 15.5
12. 0.008,43	*33*. 0.201
13. 155	*34*. 0.697
14. 0.001,96	*35*. 3,140
15. 2,100	*36*. 725,000
16. 0.000,000,061,5	*37*. 0.038,2
17. 6,060	*38*. 0.001,97
18. 67.2	*39*. 2.14
19. 0.614	*40*. 0.810
20. 2.49	*41*. 275,000,000
21. 0.922	*42*. 270,000

43. 552,000
44. 45.0
45. 9.72
46. 0.610
47. 0.015,7
48. 0.000,000,287
49. 8.04
50. 3.41

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