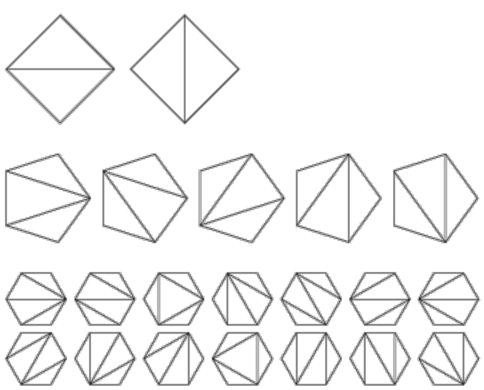
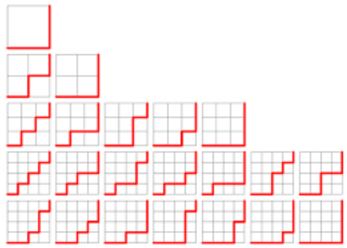
I saw a problem on project Rosalind about the Catalan numbers and counting RNA pairs which is really interesting to me (http://rosalind.info/problems/cat/). However, I think the entire problem is kind of difficult for me so for this session I did a research on Catalan numbers. The Catalan numbers are a list of infinite numbers named after 18th century mathematician Eugene Catalan. The Catalan numbers include 1,2,5,14,42,132,…. A formula for the Catalan numbers is $\frac{(2n)!}{(n+1)!n!}$[4]. Catalan numbers are actually solutions to approximately 200 combinatorics problems. Here I will elaborate on how the Catalan numbers are solution to the Euler's polygon problem and Dyck paths, and how to create a bijection between the Euler's polygon and Dyck paths.

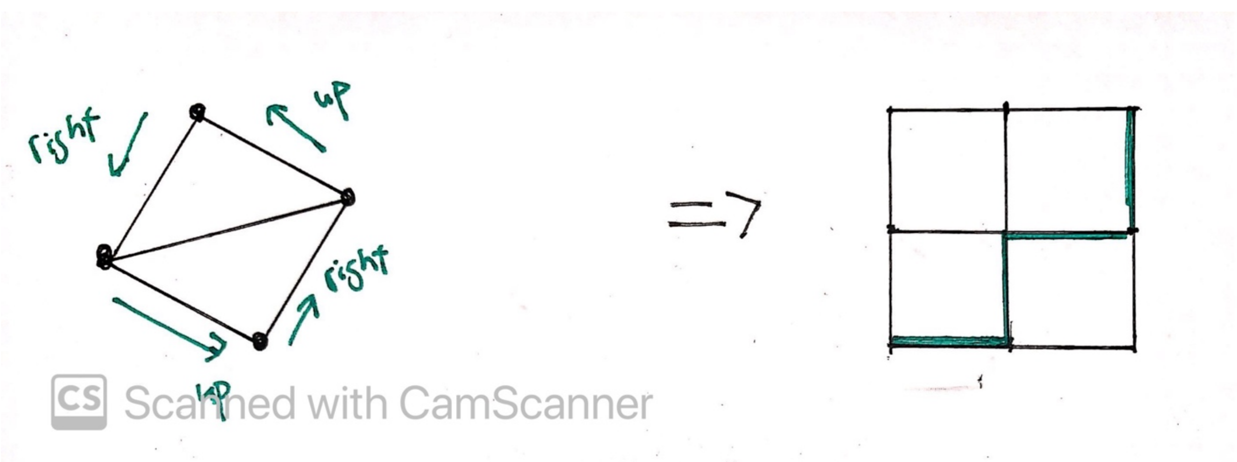
Euler's polygon problem states, "in how many ways can a regular n-side polygon be divided into n-2 triangles if different orientations are counted separately?" [3] The solution is the Catalan number C_{n-2}. For instance, in the diagram below, there are five ways we can divide a five-side polygon to triangles, which equals $\frac{(6)!}{(4)!3!} = 5$.


[3]

A Dyck path is a staircase walk from (0,0) to (n,n) that lies below or can touch the diagonal y=x. The number of Dyck paths of order n is given by the Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$ [2].

 [2]

Dyck paths and Euler's polygon could transform into one another using bijection. To start with the bottom vertex and travel counter clockwise, if the vertex is not connected to another vertex, take a step horizontally, if the vertex is connected to another vertex, take a step vertically. For instance, as illustrated in the diagram (drawn by myself):



The Catalan numbers actually have a recursive pattern. For the final session, I will create a program that calculates the nth Catalan number using recursion.

Reference:

[1] Richard P. Stanley, *Enumerative Combinatorics, Vol. I*, Cambridge University Press, 2011.

[2] https://mathworld.wolfram.com/DyckPath.html

[3] https://mathworld.wolfram.com/CatalanNumber.html

[4] https://www.youtube.com/watch?v=eoofvKI_Okg