Carissa Ying Geok Teng (A0205190R/E0425113)

1a)

$\pi_1(A) = L$

$\pi_2(A) = S$

$U^{\pi}(A) = -0.1 + \gamma P(A|A, a)U^{\pi}(A) + \gamma P(B|A, a)U^{\pi}(B)$

$(1 - \gamma P(A|A, a))U^{\pi}(A) = \gamma P(B|A, a) - 0.1$

$U^{\pi}(A) = \frac{\gamma P(B|A,a)-0.1}{1-\gamma P(A|A,a)} = \frac{\gamma(1-P(A|A,a))-0.1}{1-\gamma P(A|A,a)} = \frac{\gamma-\gamma P(A|A,a))-0.1}{1-\gamma P(A|A,a)} = \frac{\gamma-\gamma P(A|A,a))}{1-\gamma P(A|A,a)} - \frac{0.1}{1-\gamma P(A|A,a)}$

As $P(A|A,a) \to 1$, $\frac{\gamma-\gamma P(A|A,a))}{1-\gamma P(A|A,a)} \to 0$, $\frac{0.1}{1-\gamma P(A|A,a)} \to \frac{0.1}{1-\gamma}$

As $P(A|A,a) \to 0$, $\frac{\gamma-\gamma P(A|A,a))}{1-\gamma P(A|A,a)} \to \gamma$, $\frac{0.1}{1-\gamma P(A|A,a)} \to 0.1$

Hence, as $P(A|A, a)$ decreases, $U^{\pi}(A)$ increases

Since $P(A|A, L) < P(A|A, S)$,

$U^{\pi 1}(A) > U^{\pi 2}(A)$

Hence,

$\pi^*(A) = L$

1b)

$P(A|A, L) = \frac{N(A|A,L)}{N(A,L)} = \frac{3}{7}$

$P(B|A, L) = \frac{N(B|A,L)}{N(A,L)} = \frac{4}{7}$

$U^{\pi^*}(A) = \frac{\gamma P(B|A,a)-0.1}{1-\gamma P(A|A,a)} = \frac{0.5*\frac{4}{7}-0.1}{1-0.5*\frac{3}{7}} = 0.23636$

2)

$V(s) = R(s) + \gamma \, max \sum_{s'} P(s'|s, a)V(s')$

V(s1) = 1 + 0.5*(0.1*2+0.9*5) = 3.35

V(s2) = 2 + 0.5*(0.6*2+0.4*5) = 3.6