Mutual Impedance of Cylindrical Coils at an Arbitrary Position and Orientation Above a Planar Conductor

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Mutual Impedance of Cylindrical Coils at an Arbitrary Position and Orientation Above a Planar Conductor

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We derive a closed-form expression for the mutual impedance due to eddy-current induction for a pair of cylindrical air-core coils with arbitrary position and orientation above a planar conductor. By extending a recently devised model for individual coils with an arbitrary tilt with respect to the surface, we obtain a remarkably simple result. We validated our model with measurements on a conductive plate. The results should be useful for designing new probe configurations and for evaluating the signals in eddy-current inspections when driver-pickup coil configurations are utilized.

***Index Terms*—Eddy-current testing, eddy currents, mutual impedance calculation, tilted coils.**

I. Introduction

EDDY-CURRENT testing probes work either in absolute, differential, or driver-pickup mode. The latter, also identified as send-receive or reflection mode, comprises a coil which generates the primary magnetic field and a coil or a set of coils which sense the response due to the inspected testpiece. The fact that the driver and pickup coils can be separately optimized for their intended purpose offers advantages compared to absolute and differential probes such as higher gain, wider frequency range, better signal-to-noise ratio, and immunity to thermal drift. Examples of coil designs working in the driver-pickup mode are the sliding probe for fastener inspections and the array probes for heat exchanger tubing inspections.

Since the induced voltage in the sensing coil largely depends on the mutual inductance M between the two coils, the calculation of M is very important for optimizing the design of such a probe and for evaluating its performance when used for flaw inspections [1]. Analytical expressions and accurate results were recently presented for the case of coils with parallel axes in air [2]–[4]. A model for the mutual impedance between a pair of coils above a conductive plate was recently presented in [5]. The coils were either parallel or perpendicular to the conductor surface and the results were verified by experimental measurements for the inductance change ΔM due to the conductor. In the present paper that model is further extended to cover also the case of tilted coils, thus allowing the calculation of the mutual impedance change with the coils in any relative position and orientation. The solution is based on an analytical expression for the field produced by a single tilted coil, which was also recently obtained [6].

With the proposed model, we have a fast and accurate analytical tool that can assist in understanding the acquired eddy-current test signals under conditions of tilt. Emphasis is placed on the validation of the model rather than on its use for parametric studies. For this reason, experimental measurements are acquired for various coil tilt angles by using an impedance analyzer.

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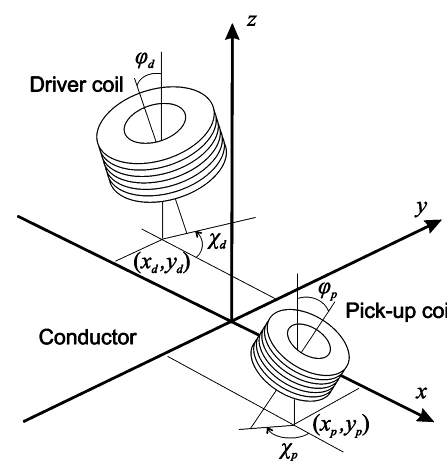

Fig. 1. General configuration for calculation of mutual impedance between a pair of tilted coils.

II. Analysis

A. Mutual Impedance Change

The 3-D configuration of a pair of tilted cylindrical coils above a planar conductor is shown in Fig. 1. The driver coil center is located at the point (x_d, y_d, d_d) and the coil axis is tilted through an angle φ_d with respect to the surface normal. Likewise, the pickup coil center is located at the point (x_p, y_p, d_p) and the coil axis is tilted through an angle φ_p with respect to the surface normal. Hereafter, the subscripts "d" and "p" refer to quantities of the driver and pickup coils, respectively. The configuration in Fig. 1 is constructed by taking coils such as in Fig. 2, where the tilt is in the direction of the x-axis, and then translating and skewing. The angle φ is taken positive for an anticlockwise rotation and negative for a clockwise rotation. The coils are wound with $N_{d,p}$ wire turns, hence they have a turns density $n_{d,p} = N_{d,p}/[(r_{2d,p} - r_{1d,p})l_{d,p}]$, and the excitation current in the driver coil is harmonically varying as the real part of $I\exp(i\omega t)$ where $\omega = 2\pi f$ and f denotes the frequency. Fig. 2 shows the rest of the coil dimensions. The

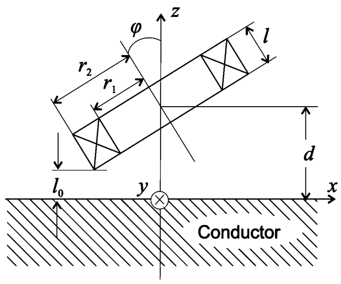

Fig. 2. Tilted cylindrical coil above a conductive half-space. The cross-sectional view shows the $y = 0$ plane. The coil will be subsequently translated and skewed to generate the final result for the driver and pickup coil source coefficients.

mutual impedance change between the two coils is given by the following general expression [5], derived from a reciprocity relation:

$$\Delta Z_{12} = 2i\omega\mu_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{a}\tilde{h}_p^{(s)}(-u,-v)\tilde{h}_d^{(s)}(u,v)R(a)\mathrm{d}u\mathrm{d}v \tag{1}$$

where $a = \sqrt{u^2+v^2}$ and for either driver or pickup coil the source term $\tilde{h}^{(s)}$ is the 2-D Fourier transform of the (per unit current) normal component of the free-space magnetic field incident on the surface $z = 0$

$$\tilde{h}^{(s)}(u,v) = \frac{1}{2\pi I}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_z^{(s)}(x,y,z=0)\mathrm{e}^{-iux}\mathrm{e}^{-ivy}\mathrm{d}x\mathrm{d}y. \tag{2}$$

The other term in the integrand of (1) is a reflection coefficient representing the effect of the planar conductor and is modified accordingly for the case of a conductive half-space, a plate, or the general case of a layered conductor.

B. Source Coefficient

Following [6], the expression for the source coefficient of either the driver or pickup coil in the initial configuration of Fig. 2, where the coil center coincides with the z-coordinate axis, is

$$\tilde{h}^{(s)}(u,v) = in\frac{\mathbf{I}(\psi r_1,\psi r_2)}{\psi^3}\mathrm{e}^{-ad}\sin\left(\frac{\psi l}{2}\right) \tag{3}$$

where n is the turns density, $d = l_0 + r_2\sin|\varphi| + (l/2)\cos\varphi, \psi = u\sin\varphi + ia\cos\varphi$ and

$$\mathbf{I}(x_1,x_2) = \int_{x_1}^{x_2} xI_1(x)\mathrm{d}x. \tag{4}$$

Having obtained the expression (3) for the source coefficient of the driver and pickup coils in the initial configuration, the general cylindrical coil arrangement can be modeled by applying 2-D Fourier identities for the skewed and shifted functions. Thus, if the coil is rotated around the z-axis by an angle χ, with $\chi \in (-\pi,\pi)$, and shifted so that its center lies above (x_0,y_0) the general source term is calculated from the initial configuration by

$$\tilde{h}^{(s)}(u\cos\chi + v\sin\chi, -u\sin\chi + v\cos\chi)\mathrm{e}^{iux_0}\mathrm{e}^{ivy_0}. \tag{5}$$

TABLE I
TEST PARAMETERS FOR THE NUMERICAL COMPUTATIONS IN FIG. 3

Coil data	Driver	Pickup	Testpiece	
r_1	7.04 mm	7.04 mm	σ_1	29.4 MS/m
r_2	12.20 mm	12.40 mm	μ_{r1}	1
l	5.04 mm	5.04 mm	c_1	2.47 mm
N	544	556		
L_0	5.55 mH	5.84 mH		

C. Layered Conductor

If the conductor has the form of a conductive half-space with conductivity σ_1 and relative magnetic permeability μ_{r1}, the reflection coefficient takes the following form:

$$R(a) = \frac{a-b_1}{a+b_1}. \tag{6}$$

In case the conductive half-space is replaced by a two-layered system comprising a top layer with conductivity σ_1, relative magnetic permeability μ_{r1} and thickness c_1 and a bottom layer with conductivity σ_2, relative magnetic permeability μ_{r2} the reflection coefficient becomes

$$R(a) = \frac{(b_1+b_2)(a-b_1) + e^{-2a_1c_1}(b_1-b_2)(a+b_1)}{(b_1+b_2)(a+b_1) + e^{-2a_1c_1}(b_1-b_2)(a-b_1)} \tag{7}$$

and $b_i = \sqrt{a^2 + i\omega\mu_{ri}\mu_0\sigma_i}/\mu_{ri}$ in (6) and (7). In case of a conductive plate, this reduces to the relevant expression used in [5] by setting $\sigma_2 = 0$. For multiple layers, a recursive approach can be followed for calculating $R(a)$ [7].

III. EXPERIMENTAL VALIDATION

A number of driver-pickup coil arrangements were studied both experimentally and theoretically and the mutual impedance change was measured and calculated as a function of frequency. The coils were located above a large aluminum alloy plate. The coil and plate parameters are shown in Table I. The impedance measurements were performed using a Agilent 4294A impedance analyzer operating in Gain-Phase mode, the general experimental details are given in [5]. The coils were wound on teflon formers, and perspex wedges were used for introducing coil tilt. The numerical aspects of calculating the infinite integrals in (1) as well as the finite integral in (4) are presented in [6].

Fig. 3 shows the variation of the mutual inductance change as a function of frequency for various pickup coil tilt angles when the base of the driver coil is parallel to the plate rsurface ($\varphi_d = 0$). The actual coil lift-off (vertical distance of the lower coil point from the upper slab surface) and coil center position were calculated using trigonometry from the coil and former dimensions and are given in Table II for all four arrangements after rounding to two decimal digits. The results are normalized with $i\omega$, so the quantity shown is actually the mutual in-

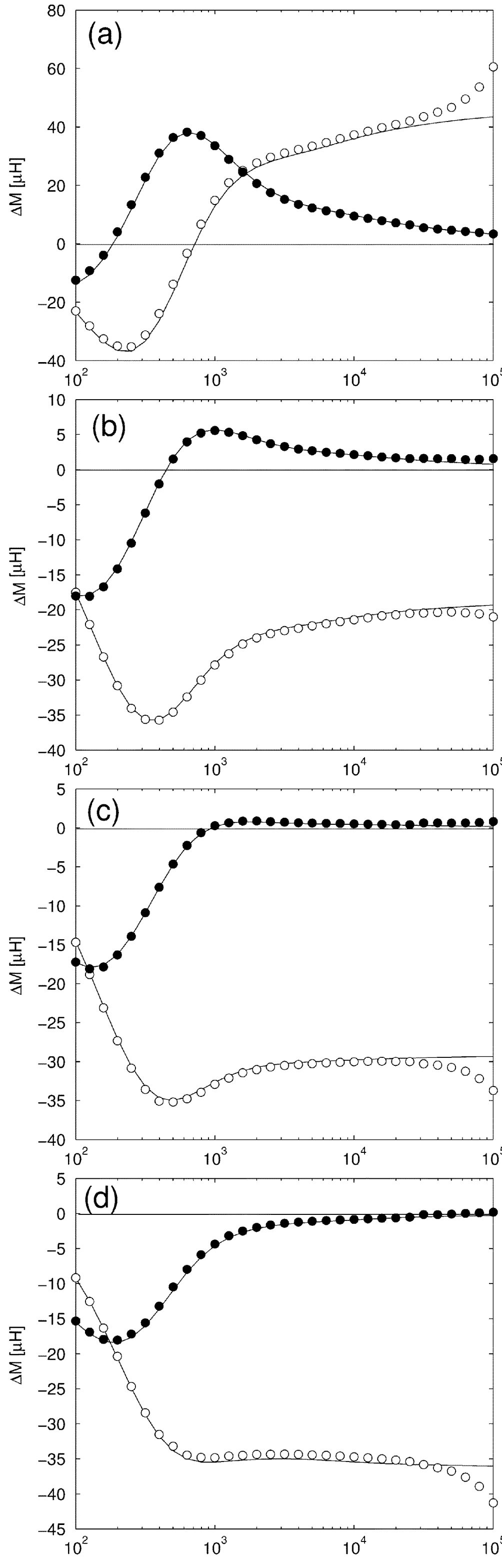

Fig. 3. Change in mutual inductance as a function of frequency for $\varphi_d = 0^o$ and (a) $\varphi_p = 0^\circ$ (b) $\varphi_p = 38^\circ$ (c) $\varphi_p = 56^\circ$ (d) $\varphi_p = 90^\circ$. The calculated variation is shown by the solid curves. The measured values are denoted by the symbols: real part ($\circ$), imaginary part ($\bullet$).

ductance change ΔM. In all cases, the experimental results are in very good agreement with the theoretical calculations. The small deviations observed at very high frequencies are due to nonideal coil behavior arising from stray capacitance across the coil wire-turns and within the leads. There are no free parameters in the calculations.

TABLE II
TEST PARAMETERS (IN DEG AND mm) FOR THE NUMERICAL COMPUTATIONS IN FIG. 3

Figure	3a	3b	3c	3d
φ_d	0	0	0	0
χ_d	0	0	0	0
x_d	0	0	0	0
y_d	0	0	0	0
l_{0d}	3.08	3.08	3.08	3.08
φ_p	0	38	56	90
χ_p	90	90	90	90
x_p	28	28	28	28
y_p	0	-5.75	-9.92	-18.52
l_{0p}	3.1	3.66	3.54	2.7

IV. CONCLUSION

A closed-form expression for the mutual impedance between a pair of cylindrical coils above a layered conductive half-space has been derived. The coils can be positioned and oriented in an arbitrary way.

The present work can be extended in various ways, for example for the computation of the mutual impedance change between a cylindrical coil and another probe coil. Furthermore, by utilizing a driver-pickup coil variation of an analytical crack model (thin skin or low frequency) it can be used for calculating crack signals for probes working in the send-receive mode. The result should be a realistic representation of an actual eddy-current inspection that is modeled accurately and with a fast computation scheme.

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