Rhodes University

Submitted in partial fulfilment
of the requirements of the degree of

Bachelor of Science (Honours)

Creating and Optimizing a Sky Tessellation Algorithm for Direction-Dependent Effects

Antonio Bradley Peters

supervised by

Prof. Karen Bradshaw

Prof. Denis Pollney

project originated by

Dr. Cyril Tasse & Prof. Oleg Smirnov

October 1, 2016

Contents

Chapter 1

WIP

1.1 Cell Error

Once the Voronoi tessellation is determined, it is recentered based on the weighted average of the points in the cell. Sources are added to cells by determining the cell centre which is closest to it using the standard distance equation:

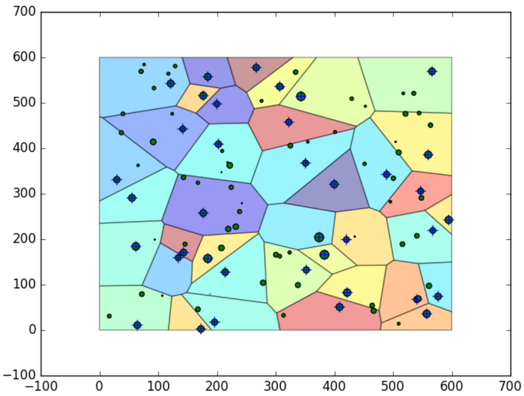
$$d = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}, \tag{1.1}$$

where (x_i, y_i) is the location of a source in the plane and (x_c, y_c) is the location of a centre. The closest centre is such that d is minimum. This is done to add the influence of weaker sources in overall correction. This is especially necessary when the cell is generated by a source slightly above the intensity threshold and contains a source slightly below the threshold. We seek a new weighted centre such that the error for a cell is minimum. The error for a cell containing N sources is defined as

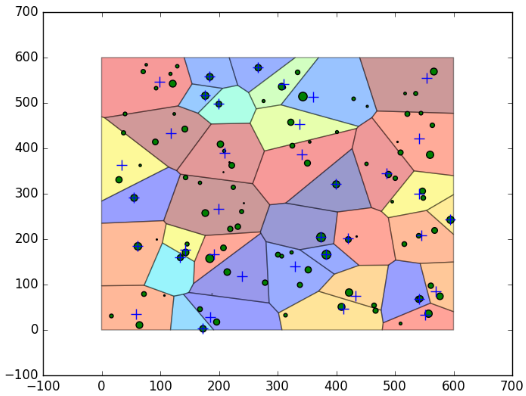
$$\epsilon = \sum_{i=0}^{N} z_i |\vec{r_i} - \vec{r_c}|^2, \tag{1.2}$$

with $\vec{r_i} = (x_i, y_i)$ is the location and z_i is the intensity of some source in the cell and $\vec{r_c}$ as the location of a new centre.

This error function will have a local minima at the point where its derivative with regards



(a) Highest intensity centre.



(b) Weighted average centre.

Figure 1.1: Changing the centre of a Voronoi cell.

to $\vec{r_c}$ is zero, or

$$\begin{aligned}\frac{d\epsilon}{d\vec{r_c}} &= \sum_{i=0}^{N} \frac{d}{d\vec{r_c}} z_i(|\vec{r_i}|^2 - 2\vec{r_i}\cdot\vec{r_c} + |\vec{r_c}|^2) \\ &= \sum_{i=0}^{N} z_i(2\vec{r_i} - 2\vec{r_c}) = 0\end{aligned}$$

or

$$2\sum_{i=0}^{N} z_i\vec{r_i} = 2\sum_{i=0}^{N} z_i\vec{r_c}$$

Since $\vec{r_c}$ is not dependant on the sum, it can be removed and the equation reordered to give

$$\vec{r_c} = \frac{\sum_{i=0}^{N} z_i\vec{r_i}}{\sum_{i=0}^{N} z_i} \tag{1.3}$$

From this, the new centre is determined, since it is required for the cell merge, the new centre retains the old centres intensity value i.e. the highest intensity from a source in the cell. Once the cell's new centre is obtained, its error is calculated using Equation 1.2. An example of a corrected centre can be seen in Figure 1.1.

1.2 Cell Merge

1.2.1 Obtaining the best Merge

References

Arthur, David, & Vassilvitskii, Sergei. 2007. k-means++: The advantages of careful seeding. *Pages 1027–1035 of: Proceedings of the Eighteenth Annual ACM-SIAM Symposium on Discrete Algorithms*. Society for Industrial and Applied Mathematics.

Aurenhammer, Franz. 1987. Power diagrams: properties, algorithms and applications. *SIAM Journal on Computing*, **16**(1), 78–96.

Cheng, Jingquan. 2009. Radio Telescope Design. *The Principles of Astronomical Telescope Design*.

Fortune, Steven. 1987. A sweepline algorithm for Voronoi diagrams. *Algorithmica*, **2**(1-4), 153–174.

Green, Peter J, & Sibson, Robin. 1978. Computing Dirichlet tessellations in the plane. *The Computer Journal*, **21**(2), 168–173.

Hamerly, Greg. Making k-means even faster. SIAM.

Moore, Gordon E. 2006. Cramming more components onto integrated circuits, Reprinted from Electronics, volume 38, number 8, April 19, 1965, pp. 114 ff. *IEEE Solid-State Circuits Newsletter*, **3**(20), 33–35.

NVIDIA. 2014. *GeForce GTX 750 Ti Whitepaper*. `http://international.download.nvidia.com/geforce-com/international/pdfs/GeForce-GTX-750-Ti-Whitepaper.pdf`. Accessed on: 26/04/2016.

NVIDIA. 2015. *CUDA C Programming Guide*. `http://docs.nvidia.com/cuda/`. Accessed on: 03/05/2016.

NVIDIA. 2016. *CUDA*. `http://www.nvidia.com/object/cuda_home_new.html`. Accessed on 26/05/2016.

NVIDIA. 2016a. *GeForce GTX 750 Ti*. `http://www.geforce.com/hardware/desktop-gpus/geforce-gtx-750-ti`. Accessed on: 26/04/2016.

NVIDIA. 2016b. *GPU-Accelerated Libraries*. `https://developer.nvidia.com/gpu-accelerated-libraries`. Accessed on: 22/05/2016.

Okabe, Atsuyuki, Boots, Barry, Sugihara, Kokichi, & Chiu, Sung Nok. 2009. *Spatial tessellations: concepts and applications of Voronoi diagrams*. Vol. 501. John Wiley & Sons.

Rajan, Krishna. 2013. *Informatics for materials science and engineering: data-driven discovery for accelerated experimentation and application*. Butterworth-Heinemann.

Rong, Guodong, & Tan, Tiow-Seng. 2006. Jump flooding in GPU with applications to Voronoi diagram and distance transform. *Pages 109–116 of: Proceedings of the 2006 Symposium on Interactive 3D Graphics and Games*. ACM.

Sault, RJ, & Wieringa, MH. 1994. Multi-frequency synthesis techniques in radio interferometric imaging. *Astronomy and Astrophysics Supplement Series*, **108**, 585–594.

Shamos, Michael Ian, & Hoey, Dan. 1975. Closest-point problems. *Pages 151–162 of: Foundations of Computer Science, 1975., 16th Annual Symposium on*. IEEE.

Smirnov, Oleg M. 2011. Revisiting the radio interferometer measurement equation-I. A full-sky Jones formalism. *Astronomy & Astrophysics*, **527**, A106.

Smirnov, OM, & Tasse, Cyril. 2015. Radio interferometric gain calibration as a complex optimization problem. *Monthly Notices of the Royal Astronomical Society*, **449**(3), 2668–2684.

Steinbach, Michael, Karypis, George, Kumar, Vipin, *et al.* 2000. A comparison of document clustering techniques. *Pages 525–526 of: KDD workshop on text mining*, vol. 400. Boston.

Subhlok, Jaspal, Stichnoth, James M, O'hallaron, David R, & Gross, Thomas. 1993. Exploiting task and data parallelism on a multicomputer. *Pages 13–22 of: ACM SIGPLAN Notices*, vol. 28. ACM.

Tasse, Cyril. 2014. Applying Wirtinger derivatives to the radio interferometry calibration problem. *arXiv preprint arXiv:1410.8706*.

Tasse, Cyril. 2016. *DDFacet imager*. TBD. Draft in preparation.

Thompson, A Richard, Moran, James M, & Swenson Jr, George W. 2008. *Interferometry and synthesis in radio astronomy*. John Wiley & Sons.

van Weeren, RJ, Williams, WL, Hardcastle, MJ, Shimwell, TW, Rafferty, DA, Sabater, J, Heald, G, Sridhar, SS, Dijkema, TJ, Brunetti, G, *et al.* 2016. LOFAR facet calibration. *arXiv preprint arXiv:1601.05422*.

Vuduc, Richard, & Choi, Jee. 2013. A brief history and introduction to GPGPU. *Pages 9–23 of: Modern Accelerator Technologies for Geographic Information Science*. Springer.

Way, Michael J, Scargle, Jeffrey D, Ali, Kamal M, & Srivastava, Ashok N. 2012. *Advances in machine learning and data mining for astronomy*. CRC Press.