Vagueness, Noise, and Signaling

Abstract

Signaling games have attracted a lot of attention as models for studying the evolution of meaning. One important aspect that is nevertheless typically missing from most models in the literature is the possibility for vagueness as a feature of successful signaling. Complementing recent models that have been proposed to explicitly address this limitation, we introduce the replicator diffusion dynamic. Provably a special case of the replicator mutator dynamic, it implements a versatile and natural integration of, on the one hand, stochastic noise in the form of probabilistic confusion of similar stimuli and, on the other, evolutionary pressure on optimal signal use. Applying the replicator diffusion dynamic to a generalization of David Lewis' signaling games, so-called similarity-maximizing games, we show that stochastic noise can not only lead to vague, yet communicative efficient signal use, but can also unify evolutionary outcomes and help avoid suboptimal categorization.

1 Introduction

Many of our concepts and words are vague. A vague category knows clear cases that fall under it, clear cases that do not, and also so-called borderline cases. Borderline cases neither clearly apply, nor clearly not apply, and there may be differences between borderline cases in terms of how well they represent the category in question. Vagueness does not seem to dramatically affect the success of everyday communication, but it is troublesome for some of our theories of language. This is especially the case for the logico-semantic tradition of Frege, Russell and the young Wittgenstein which is challenged by the paradoxes vagueness gives rise to. As can be expected, many proponents of this tradition have tried to address the problem, ranging from suggesting that there is nothing wrong with the classical approach since the problem is of a different nature, as in Williamson's epistemic view (Williamson, 1994), to proposing various kinds of modifications to either semantics, logic, or both, as in for example supervaluationism (e.g. Mehlberg, 1958; Fine, 1975), many-valued logic (e.g. Zadeh, 1975; Machina, 1976; Edgington, 1997), or paraconsistent logic (e.g. Cobreros et al., 2012).

Yet there are other intriguing aspects about vagueness. The puzzle that we are concerned with here is how vagueness could arise and be maintained in the first place. This may not seem a particularly deep issue at first glance, but on closer inspection it is a serious worry to functionalist accounts that maintain that our concepts and language use evolved in order to be efficient. Since the existence of unclear borderline cases seems to entail inefficiency of categorization or communication, the challenge, succinctly put forward by Lipman (2009), is to explain how vagueness can exist despite its obvious and demonstrable functional deficiency under evolutionary pressure to be optimally expressive.

This problem, call it Lipman's problem, has a conceptual and a technical side to it. Conceptually, it asks for reasons and general mechanisms by which we could plausibly conceive of vagueness as resisting the pressure of evolutionary selection for precision. Such reasons and general mechanisms are, arguably, not hard to imagine. But each putative explanation, no matter how plausible intuitively, must also be checked for internal consistency and its potential to account for the fact that we see vagueness as a part of a by-and-large efficient system of categorization and communication. In other words, not all first-shot rebuttals of Lipman's problem will do. Denying that there is any functional pressure on efficient communication, for instance, is a non-starter, because it leaves the relative efficiency of our communication with vague words entirely unexplained. In sum, the technical answer to Lipman's problem is to give, ideally, a conceptually sound model of evolution of categorization or language use that yields by-and-large efficient categories *and* vagueness.

A number of authors have recently tried to explain why vagueness evolved, based on considerations why a vague language might be useful (e.g. de Jaegher, 2003; Deemter, 2009; de Jaegher and van Rooij, 2010; Blume and Board, 2013). In contrast to these, others have argued that vagueness is a natural byproduct of limitations in information processing (e.g. Franke, Jäger, and van Rooij, 2011; O'Connor, 2014). This paper makes a contribution to the latter line of thought. Concretely, we introduce the replicator diffusion dynamics —a novel variant of the replicator mutator dynamic— that integrates stochastic noise on the differential confusability of similar stimuli. Our main contribution, therefore, is a technical answer to Lipman's problem in the sense introduced above. The dynamic proposed here generalizes and complements previous like-minded accounts. We show that stochastic noise can not only lead to vague, yet communicative efficient signal use, but can also unify evolutionary outcomes and help avoid suboptimal categorization.

The next section introduces the background against which the work presented here can be appreciated. Section 3 introduces the replicator diffusion dynamic and elaborates on its relation with the replicator mutator dynamic. Section 4 explores the replicator diffusion dynamic on the relevant class of generalized signaling games introduced in Section 2. Section 5 reflects on the results and compares what has been achieved to related accounts in more detail.

2 Background

The view that vagueness is a natural concomitant of cognitive limitations of language users has been formalized in a number of ways, using evolutionary game theory and certain generalizations of signaling games, so called *similarity-maximizing games*, or *sim-max games*, for short (Jäger, 2007; Jäger and van Rooij, 2007). Our contribution is best seen in relation to these accounts, as it also relies on sim-max games. Let's introduce these first, and then zoom in on the problem of vagueness.

2.1 Sim-max games & conceptual spaces

Signaling games, as introduced by Lewis (1969), have a sender and a receiver. The sender knows the true state of the world, but the receiver does not. The sender can select a signal, or message, to reveal to the receiver, who then chooses an act. In Lewis' games, states are maximally distinct from each other and the receiver's acts are related to them one-to-one. If the receiver chooses the act that corresponds to the actual state, the play is a success, otherwise a failure. Certain regular combinations of sender signaling and receiver reaction make messages meaningful, in the sense that their use is correlated systematically to certain states or acts. To investigate the conditions under which such meaning-generating behavior can evolve is a highly interesting topic that we are only beginning to fully understand (e.g. Wärneryd, 1993; Blume, Kim, and Sobel, 1993; Zollman, 2005; Huttegger, 2007; Pawlowitsch, 2008; Barrett, 2009; Wagner, 2009; Huttegger et al., 2010; Skyrms, 2010; Huttegger and Zollman, 2011).

Similarity-maximizing games are generalizations of Lewis' games where different states are allowed to be more or less similar to one another, and, roughly put, success of communication is a function of that similarity. Intuitively speaking, while Lewis' games treated communicative success as a matter of black and white, sim-max games allow for a gradient notion of communicative success: the more similar the receiver's interpretation is to the actual state, the more successful the play is taken to be. Signaling games with utility-relevant similarities in the state space are fairly standard in economics (e.g. Spence, 1973; Crawford and Sobel, 1982), but have received particular attention in a more philosophical and linguistic context for reasons that will become clear presently (Jäger, 2007; Jäger and van Rooij, 2007; Jäger, Metzger, and Riedel, 2011).

A sim-max game, in the relevant sense here, consists of a set of states T, a set of messages M with much fever messages than states, a prior probability distribution $\Pr \in \Delta(T)$ that gives the occurrence probabilities of states, a similarity metric on states $\mathrm{Sim} : T \times T \to \mathbb{R}$, and a utility function $\mathrm{U} : T \times T \to \mathbb{R}$. We identify the receiver's acts with the states of the world, so that the game is one of guessing the actual state, so to speak. We also assume that sender's and receiver's interests are alike, so we only have one utility function. We do not consider message costs, so utilities only depend on the actual state, and the receiver's response. The similarity function should satisfy the usual requirements for a metric:

$$\begin{aligned} &\mathrm{Sim}(t_1, t_2) \geq 0 && \mathrm{Sim}(t_1, t_2) = 0 \Leftrightarrow t_1 = t_2 \\ &\mathrm{Sim}(t_1, t_2) = \mathrm{Sim}(t_2, t_1) && \mathrm{Sim}(t_1, t_2) \leq \mathrm{Sim}(t_1, t_3) + \mathrm{Sim}(t_3, t_2)\,. \end{aligned}$$

The utility function should be a monotonically decreasing function of similarity:

$$\mathrm{Sim}(t_1, t_2) \geq \mathrm{Sim}(t_1, t_3) \quad \Rightarrow \quad \mathrm{U}(t_1, t_2) \geq \mathrm{U}(t_1, t_3)\,.$$

To keep matters simple, in this paper we mostly focus on cases where T contains finitely many points from the unit interval, all of which have equal probability.

Jäger, Metzger, and Riedel (2011) showed that the evolutionarily stable states of sim-max games are remarkably systematic. Their results were obtained for games with infinitely many states in n-dimensional Euclidean space $T \subseteq \mathbb{R}^n$ and a quadratic loss

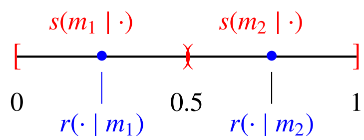


Figure 1: Example of a Voronoi language on $T = [0, 1]$. The (pure) sender strategy $s \in M^T$ uses one signal for the lower half, and another for the upper half of the unit interval. The (pure) receiver strategy $r \in T^M$ selects the central elements in the respective intervals.

function for utilities $\mathrm{U}(t_1, t_2) = -(t_1 - t_2)^2$. For these games, the evolutionarily stable states are demonstrably so-called Voronoi languages. Intuitively speaking, a Voronoi language is a pair of sender and receiver strategies, such that the sender strategy (quasi-)partitions the state space into convex categories, while the receiver's interpretations are the central spots in each category. Figure 1 gives an example of such a Voronoi language for a sim-max game with $T = [0, 1]$, two messages, and a flat prior distribution over states. The sender uses one message exclusively for all points in the lower half of the interval and another for all points in the upper half. The receiver's interpretations of messages are the central points in the respective intervals.

This result is interesting, because it demonstrates that signaling can impose orderly categories on a metric space, without that being the ulterior purpose of it all. Sender and receiver can be distinct entities, whose purpose is to communicate effectively about the actual state. In that case, evolving Voronoi languages would explain why linguistic categories are well-behaved and orderly in the way they appear to be. More abstractly, sender and receiver can also be thought of as distinct modules in a single system, where the first module must discretize the information by selecting a small sample of, suggestively, category labels. These are passed to a second module that tries to decode the original information. In this case, evolving Voronoi languages would explain why conceptual categories are well-behaved and orderly in the way that they appear to be (e.g. O'Connor, 2013, for more on this latter interpretation).

Seen in this light, sim-max games appear to have the potential of providing a foundation to approaches in cognitive semantics that rely on the notion of conceptual spaces. Gärdenfors (2000, pp. 70–77), for example, has prominently argued that natural categories are convex regions in conceptual space. If the conceptual space has a suitable metric, then it is possible to think of these convex categories as derived from a set of prototypes. Fixing a set of prototypes, we consider the category corresponding to each prototype p as the set of points that are more similar to p than to any other (e.g. Okabe et al., 2000). In this way, Gärdenfors (2000) argues, an efficient categorization system can be obtained: storing the prototypes lets us recover the categories without having to store each category's extension. However, what is left unexplained so far, is where the prototypes come from, and why we would not see just any distribution of prototypes as an equally efficient classification system. This is where sim-max games can contribute a principled approach to deriving, in an independent way, not only convex categories but also prototypical exemplars belonging to them.

2.2 Vague signaling in sim-max games

This brief outline of an approach to conceptual categorization using sim-max games leaves many problems unaddressed. One of them is that natural categories for continuously variable stimuli, like shades of color, pitch heights, spatial dimension and the like, do not have unique, point-valued prototypes and clear category boundaries. We would like to account for the possibility of such vagueness, and that means in particular for the following criteria (e.g. Sainsbury, 1991; Keefe and Smith, 1997; Smith, 2008): (i) clear positive examples of a vague category should show a gradient, perhaps smooth, transition to clear negative examples to accommodate also higher-order vagueness; (ii) prototypes should likewise be gradient regions, peaking at the center of the vague category they represent.

Douven et al. (2011) show that Gärdenfors's conceptual spaces approach can be extended to account for the existence of borderline cases. From the assumption that prototypes are not unitary points, but extended, yet convex regions in conceptual space, Douven et al. give a construction algorithm that yields "collated Voronoi diagrams" with thick, extended boundaries representing borderline regions. Building on this work, Decock and Douven (2012) show further how it is possible to arrive at higher-order vagueness and degrees of category membership, by, essentially, weighing in the distance of different borderline cases to various prototypical regions. This accounts for the first of the two desiderata mentioned above, but still assumes that crisp non-gradient prototype regions must be pre-given.

An alternative approach is taken by Franke, Jäger, and van Rooij (2011) and O'Connor (2014) who show how the above desiderata can be met by evolving strategies in sim-max games for various types of solution concepts. These can be divided into micro- and macro-level approaches. Micro-level approaches look at adaptive behavior of individual agents. Usually, changes in the behavioral dispositions of agents occur after every single interaction. In contrast, macro-level approaches outline more abstract, aggregate dynamics, happening in a population of agents, or otherwise abstracting from seemingly irrelevant detail. Usually, a macro-level dynamic captures changes of frequencies of behavioral types in the population over time.

As for micro-dynamics, Franke, Jäger, and van Rooij (2011) show how limited memory of past interactions can lead to vague signal use, when averaging over a single agent's behavior over time or over the momentary behavior of a population of several language users. O'Connor (2014) introduces a variant of reinforcement learning that entails a low-level form of stimulus generalization. Agents update their behavior after each round of play in such a way that states similar to the one that actually occurred are subject to behavioral adjustment as well.

Franke, Jäger, and van Rooij (2011) also consider a macro-level approach, using the notion of a *quantal response*. A quantal response function is a probabilistic choice rule that formalizes the idea that agents make small mistakes when calculating the expected utility of choice options. In aggregation, these probabilistic mistakes lead to systematic "trembles" that produce vague signal use.

The approach we take here is superficially similar, but there are crucial differences. For one, we adopt a dynamic perspective by looking at limited-time outcomes of a dynamic process. For another, we demonstrate in Section 5 that quantal responses can give rise to counterintuitive predictions. These counterintuitive examples suggest that vagueness in sim-max games is not convincingly explained by appeal to mistakes in calculating expected utility, but rather, as we assume here, as the result of confusing similar states. A more in-depth discussion of alternative approaches is deferred until Section 5. Let us first look into our own proposal in more detail.

3 Replicator diffusion dynamic

Against this background, we introduce a new macro-level approach to the evolution of vague signal use in sim-max games, where the source of vagueness is the agents' natural inability to sharply distinguish similar states. This applies to the sender as well as to the receiver role (albeit in slightly different manner). We call our dynamic *replicator diffusion dynamic* (RDD), because it is an extension of the replicator dynamic (RD) that adds diffusion of behavior. It is also a special case of the replicator mutator dynamic (RMD).

We begin by recapitulating the formulation of the RD in Section 3.1. Then we introduce the RDD in Section 3.2. Section 3.3 introduces the RMD, and Section 3.4 shows in what sense the RDD is a special case of the RMD.

3.1 Replicator dynamic in behavioral strategies

Fix a signaling game with finite states T, messages M and acts A. Let $\Pr(\cdot) \in \Delta T$ be the prior distribution over states and $\mathrm{U}_{S,R} : T \times M \times A \to \mathbb{R}$ the senders's and receiver's utility functions. The sender's behavioral strategies are functions $\sigma \in \Delta(M)^T$; the receiver's are functions $\rho \in \Delta(A)^M$. Define the expected utility of choices in each choice point as usual:

$$\begin{aligned}
\mathrm{EU}(m, t, \rho) &= \sum_{a \in A} \rho(a \mid m) \cdot U_S(t, m, a) \\
\mathrm{EU}(a, m, \sigma) &= \sum_{t \in T} \Pr(t) \cdot \sigma(m \mid t) \cdot U_R(t, m, a)\,.
\end{aligned}$$

The *fitness* of a behavioral strategy at a choice point is the frequency-weighted average of expected utilities of each choice, given the opponent's strategy:

$$\begin{aligned}
\Phi(t, \sigma, \rho) &= \sum_{m} \sigma(m \mid t) \cdot \mathrm{EU}(m, t, \rho) \\
\Phi(m, \sigma, \rho) &= \sum_{a} \rho(a \mid m) \cdot \mathrm{EU}(a, m, \sigma)\,.
\end{aligned}$$

The discrete-time replicator dynamic maps current strategies σ and ρ to future strategies $\mathrm{RD}(\sigma)$ and $\mathrm{RD}(\rho)$ in such a way that changes in frequency are proportional to expected utilities. For behavioral strategies, the changes take place locally at each of

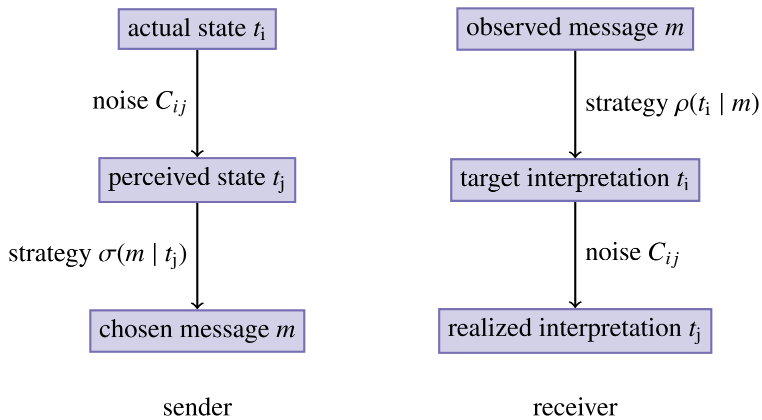


Figure 2: Effect of confusion of states on sender and receiver choices.

the agents' choice points:

$$\mathrm{RD}(\sigma)(m \mid t) = \frac{\sigma(m \mid t) \cdot \mathrm{EU}(m, t, \rho)}{\Phi(t, \sigma, \rho)}$$
$$\mathrm{RD}(\rho)(a \mid m) = \frac{\rho(a \mid m) \cdot \mathrm{EU}(a, m, \sigma)}{\Phi(m, \sigma, \rho)} \,.$$

3.2 Replicator diffusion dynamic in behavioral strategies

The replicator diffusion dynamic was first introduced by Correia (2013) as a noise-perturbed variant of the replicator dynamic (RD) in behavioral strategies. Fix a sim-max game with $T = A$ and a confusion matrix $C : T \times T$. C is a row-stochastic matrix whose elements C_{ij} give the probability that t_i is realized as t_j. The confusability of states affects senders and receivers alike, but in slightly different ways (see Figure 2). For the sender, C_{ij} is the probability that the actual state t_i is perceived as t_j. For the receiver, C_{ij} is the probability that t_j is the interpretation that is actually formed when t_i is the intended interpretation.

The aggregate effect of confusion of states on behavioral strategies can be captured in a function that maps behavioral strategies σ and ρ to their diffusions $D(\sigma)$ and $D(\rho)$. The idea is that σ and ρ are what, on average, the idealized, noise-free behavior would be, while $D(\sigma)$ and $D(\rho)$ are behavioral strategies that describe the agents' actual noise-perturbed probabilistic behavior. Since the effect of confusion of states is that behavior at one choice point percolates to behavior at similar choice points, we speak of *diffusion of behavior under confusion of states*. If we conceive of behavioral strategies σ and ρ as row-stochastic matrices, the diffusion effect of state confusability is easily captured by matrix multiplication as a diffusion function D_C that, based on confusion matrix C, maps behavioral strategies onto their diffused realizations:

$$\mathrm{D}_C(\sigma) = C\sigma \qquad\qquad \mathrm{D}_C(\rho) = \rho C \,. \tag{1}$$

The discrete-time replicator diffusion dynamic takes the replicator dynamic as basic, but factors in the confusion of states at each update step in a sequential update:

$$\mathrm{RDD}(\sigma) = \mathrm{D}_C(\mathrm{RD}(\sigma)) \qquad\qquad \mathrm{RDD}(\rho) = \mathrm{D}_C(\mathrm{RD}(\rho))\,.$$

This is equivalent to the following, perhaps more transparent formulation, given by Correia (2013):

$$\mathrm{RDD}(\sigma)(m \mid t_i) = \sum_j C_{ij} \cdot \frac{\sigma(m \mid t_j) \cdot \mathrm{EU}(m, t_j, \rho)}{\Phi(t_j, \sigma, \rho)}$$
$$\mathrm{RDD}(\rho)(t_i \mid m) = \sum_j C_{ji} \cdot \frac{\rho(t_j \mid m) \cdot \mathrm{EU}(t_j, m, \sigma)}{\Phi(m, \sigma, \rho)}\,.$$

The idea behind the sequential definition of the RDD is that, at each time step, strategies are gradually optimized along the current fitness landscape, as described by the RD, but the realization of optimized strategies is noisy, due to similarity of states. It is obvious that other (discrete-time) evolutionary dynamics can be subjected to state-confusability in an analogous fashion (see also Section 5.1). In the case of the RD, however, perturbation by state-confusion has a prominent close relative in the replicator mutator dynamic.

3.3 The replicator mutator dynamic

The RMD has been proposed first in the context of signaling game models for the evolution of grammar (e.g. Komarova, Niyogi, and Nowak, 2001; Nowak, Komarova, and Niyogi, 2001; Nowak, 2006). It adds stochastic mutation to the replicator dynamic in order to capture, in the context of grammar evolution, the differential learning success of different dispositions in first language acquisition. The relation of the RMD to other prominent evolutionary dynamics is well understood (Page and Nowak, 2002). The RMD has seen further fruitful applications in the context of signaling games (e.g. Huttegger et al., 2010; Deo, 2014). It is therefore desirable to relate the RDD that we propose here to the RMD.

A direct comparison between RDD and the RMD is not possible, because the usual formulation of the latter is in its continuous-time form, and based on mixed strategies, not behavioral strategies. We therefore give a discrete-time formulation of the RMD in behavioral strategies, which is independently useful. The relation between RDD and RMD will then be plain to see. We start with a delineation of mixed and behavioral strategies.

Mixed strategies. Pure sender (receiver) strategies are functions $s \in M^T$ ($r \in A^M$). Mixed sender (receiver) strategies are functions $\mathbf{s} \in \Delta(M^T)$ ($\mathbf{r} \in (A^M)$). The latter give the relative population frequencies of the former. We write $\mathbf{s}_i$ for the frequency $\mathbf{s}(s_i)$ of pure strategy s_i. Likewise for the receiver.

Every mixed strategy $\mathbf{s}$ converts to a unique behavioral strategy defined by:

$$\sigma(m \mid t) = \sum_{s(t)=m} \mathbf{s}(s)\,. \tag{2}$$

Let G be this mapping from mixed to behavioral strategies. Note that G is *not* an injection, as many mixed strategies map onto the same behavioral strategy. In this sense, mixed strategies hold more information than behavioral strategies about the distribution of pure strategies in a population.

In the context of evolutionary dynamics, we can think of behavioral strategies as treating each choice point as an independent update site (e.g. Cressman, 2003; Sandholm, 2013). Dynamics for mixed strategies conceive of agents as adjusting their whole pure strategy globally. In this way, dependencies between different choices at different choice points can matter to the evolutionary path. In contrast, dynamics for behavioral strategies conceive of agents as adjusting their strategies locally at each choice point, regardless of what they are disposed to do at other choice points. The latter perspective is then more coarse-grained, and not necessarily equivalent to the former. But an advantage of dynamics in behavioral strategies is that they reduce complexity, which is especially helpful for computational simulations of larger games, like the ones we are interested in here.

Replicator dynamic in mixed strategies. To understand the RMD in mixed strategies, we first introduce the RD in mixed strategies. Let $F_i^{\mathbf{r}}$ be s_i's fitness given $\mathbf{r}$ and $F_i^{\mathbf{s}}$ be r_i's fitness given $\mathbf{s}$. Then $\Phi(\mathbf{s}, \mathbf{r}) = \sum_k \mathbf{s}_k \cdot F_k^{\mathbf{r}}$ is the average fitness in the sender population and $\Phi(\mathbf{r}, \mathbf{s}) = \sum_k \mathbf{r}_k \cdot F_k^{\mathbf{s}}$ the average fitness in the receiver population.

The continuous-time replicator dynamic defines the change of frequency of mixed strategies. Its so-called two-population version, that takes sender and receiver roles as separate update sites:

$$\dot{\mathbf{s}}_i = \mathbf{s}_i \cdot \left(F_i^{\mathbf{r}} - \Phi(\mathbf{s}, \mathbf{r})\right) \qquad \dot{\mathbf{r}}_i = \mathbf{r}_i \cdot \left(F_i^{\mathbf{s}} - \Phi(\mathbf{r}, \mathbf{s})\right) .$$

Similar to the version for behavioral strategies, a discrete-time formulation of the RD for mixed strategies can be conceptualized as an update function that maps mixed strategies $\mathbf{s}$ and $\mathbf{r}$ onto mixed strategies $\mathrm{RD}(\mathbf{s})$ and $\mathrm{RD}(\mathbf{r})$ respectively, such that:

$$\mathrm{RD}(\mathbf{s})_i = \frac{\mathbf{s}_i \cdot F_i^{\mathbf{r}}}{\Phi(\mathbf{s}, \mathbf{r})} \qquad \mathrm{RD}(\mathbf{r})_i = \frac{\mathbf{r}_i \cdot F_i^{\mathbf{s}}}{\Phi(\mathbf{r}, \mathbf{s})} .$$

Replicator mutator dynamic. The replicator mutator dynamic extends the RD by adding probabilistic mutation. Let Q be a row-stochastic mutation matrix where Q_{ji} gives the probability that pure sender strategy s_j mutates into s_i. Similarly, let R be a row-stochastic mutation matrix where R_{ji} gives the probability that pure receiver strategy r_j mutates into r_i.

The RMD is usually given only in its continuous-time form. A two-population (non-payoff adjusted) formulation is then:

$$\dot{\mathbf{s}}_i = \sum_j Q_{ji} \cdot \mathbf{s}_j \cdot F_j^{\mathbf{r}} - \mathbf{s}_i \cdot \Phi(\mathbf{s}, \mathbf{r}) \qquad \dot{\mathbf{r}}_i = \sum_j R_{ji} \cdot \mathbf{r}_j \cdot F_j^{\mathbf{s}} - \mathbf{r}_i \cdot \Phi(\mathbf{r}, \mathbf{s}) .$$

A discrete-time, two-population formulation of the RMD can be defined as follows (c.f. Page and Nowak, 2002, p. 97):

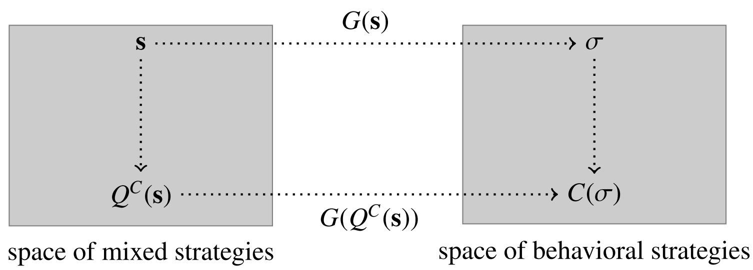


Figure 3: Correspondence between state-confusion and mutation.

$$\mathrm{RMD}(\mathbf{s})_i = \sum_j Q_{ji} \frac{\mathbf{s}_j \cdot F^{\mathbf{r}}_j}{\Phi(\mathbf{s}, \mathbf{r})} \qquad \mathrm{RDD}(\mathbf{r})_i = \sum_j R_{ji} \frac{\mathbf{r}_j \cdot F^{\mathbf{s}}_j}{\Phi(\mathbf{r}, \mathbf{s})} \,.$$

The discrete-time RMD has the same sequential nature as the RDD: first we compute the fitness-driven change according to the standard replicator dynamic; then we compute the perturbation from mutation. This becomes clear if we define independent mutation functions $\mathrm{M}_{Q,R}$ that map mixed strategies onto the outcome of (one round of) mutation:

$$\mathrm{M}_Q(\mathbf{s})_i = \sum_j \mathbf{s}_j \cdot Q_{ji} \qquad \mathrm{M}_R(\mathbf{r})_i = \sum_j \mathbf{r}_j \cdot R_{ji} \,. \tag{3}$$

The discrete-time RMD given above can then be rewritten as:

$$\mathrm{RDD}(\mathbf{s}) = \mathrm{M}_Q(\mathrm{RD}(\mathbf{s})) \qquad \mathrm{RDD}(\mathbf{r}) = \mathrm{M}_R(\mathrm{RD}(\mathbf{r}))\,.$$

3.4 Diffusion as a special kind of mutation

The sequential reformulation of the RMD in terms of a sequence of updates already closely resembles our initial formulation of the RDD. However, there is a major difference between these. While the RDD is a dynamic for behavioral strategies, the RMD, as taken from the literature, applies to mixed strategies. Similarly, the confusability of states is an operation that is straightforwardly definable for behavioral strategies (see Equations (1)), while mutation in the RMD is defined as mutation from one pure strategy to another. Nonetheless, it is possible to map each confusion matrix to a mutation matrix in a natural way so that the correspondence between mixed and behavioral strategies are conserved. This is the main result presented in this section, and it is our justification for regarding the RDD as the behavioral-strategy analogue of the replicator-mutator dynamic when the only source of mutation is confusability of states.

More concretely, since there is a non-injective mapping G from behavioral to mixed strategies, we would like to show that there is a systematic translation from a confusion matrix C to a mutation matrices Q^C such that for each mixed sender strategy $\mathbf{s}$, its unique corresponding behavioral strategy $\sigma = G(\mathbf{s})$ has a confusion-perturbation $C(\sigma)$ that corresponds, via G, to the mutation-perturbation $Q^C(\mathbf{s})$ (see Figure 3). In other words, we hypothesize that a confusion matrix C should give rise to a unique mutation matrix Q^C so that whenever $G(\mathbf{s}) = \sigma$ we also have $G(Q^C(\mathbf{s})) = C(\sigma)$. Similarly, for the receiver.

Confusion-based mutations. There are natural conversions of C into Q^C and R^C. The case for the receiver is easier, so we start with that.

The probability that r_i is realized as r_j under diffusion with C is the product of the probabilities, for each m, that the state $r_i(m)$ is perceived as state $r_j(m)$. Abusing notation by referring to the indices of states $r_i(m)$ and $r_j(m)$ with $r_i(m)$ and $r_j(m)$ directly, we define:

$$R^C_{ij} = \prod_m C_{r_i(m) r_j(m)} \,. \tag{4}$$

Now look at the sender. The probability that s_j is realized as s_i under diffusion with C is the product of the probabilities, over all states t_k, that the message $s_i(t_\mathrm{k})$, that s_i would produce at state t_k in the absence of noise, is produced by a noisy realization of s_j, which is the probability $\sum_{t_l \in s_j^{-1}(s_i(t_k))} C_{kl}$ that the state t_k is realized as a state t_l which s_j would map unto $s_i(t_\mathrm{k})$. So, define:

$$Q^C_{ji} = \prod_{t_k} \sum_{t_l \in s_j^{-1}(s_i(t_k))} C_{kl} \,. \tag{5}$$

For example, consider a signaling game with two states and two messages. Let the confusion matrix be:

$$C = \begin{pmatrix} .8 & .2 \\ .2 & .8 \end{pmatrix} .$$

The resulting mutation matrices are:

$$Q^C = \begin{array}{c} \\ 11 \\ 12 \\ 21 \\ 22 \end{array} \begin{array}{c} \begin{array}{cccc} 11 & 12 & 21 & 22 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ .16 & .64 & .04 & .16 \\ .16 & .04 & .64 & .16 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \qquad R^C = \begin{array}{c} \\ 11 \\ 12 \\ 21 \\ 22 \end{array} \begin{array}{c} \begin{array}{cccc} 11 & 12 & 21 & 22 \end{array} \\ \begin{pmatrix} .64 & .16 & .16 & .04 \\ .16 & .64 & .04 & .16 \\ .16 & .04 & .64 & .16 \\ .04 & .12 & .12 & .64 \end{pmatrix} \end{array} .$$

Here, a pair like 21, for example, refers to a pure sender strategy with $s(t_1) = m_2$ and $s(t_2) = m_1$. Similarly for the receiver.

Based on the translations from confusion matrices C to mutation matrices Q^C and R^C in Equations (4) and (5), we can finally formulate the desired correspondence result, that reveals the RDD as a special case of the RMD in behavioral strategies where the only source of mutation is imperfect discriminability of states.

Theorem 1. (i) If $G(\mathbf{s}) = \sigma$, then $G(\mathrm{M}_{Q^C}(\mathbf{s})) = \mathrm{D}_C(\sigma)$. And, (ii) if $G(\mathbf{r}) = \rho$, then $G(\mathrm{M}_{R^C}(\mathbf{r})) = \mathrm{D}_C(\rho)$.

A proof is given in Appendix A.

4 Exploring the RDD

The previous section showed that the RDD can be conceptualized as a two-step procedure: we first calculate where the replicator dynamic would take us, then apply diffusion. To understand the RDD better, in Section 4.1 we look closer at the novel diffusion part, before exploring in Section 4.2 how diffusion and fitness-based selection interact.

4.1 (Iterated) Diffusion

Confusion of states should intuitively be a function of their perceptual similarity. To make this concrete, let us assume that the state space of the sim-max game consists of $n \geq 2$ states that are equally spaced across the unit interval, including 0 and 1. The distance $\left| t_\mathrm{i} - t_\mathrm{j} \right|$ is the objective, physical similarity between two states t_i and t_j. Distance in physical space feeds into a perceptual similarity function, as described by Nosofsky (1986):

$$\mathrm{Sim}(t_\mathrm{i}, t_\mathrm{j}\,;\,\alpha) = \begin{cases} 1 & \text{if } \alpha = 0 \text{ and } t_\mathrm{i} = t_\mathrm{j} \\ 0 & \text{if } \alpha = 0 \text{ and } t_\mathrm{i} \neq t_\mathrm{j} \\ \exp\left(-\frac{\left| t_\mathrm{i} - t_\mathrm{j} \right|^s}{\alpha^2}\right), & \text{otherwise} \end{cases} \tag{6}$$

where $\alpha >= 0$ is an imprecision or indiscriminability parameter. When $\alpha = 0$ agents perfectly discriminate between states; when $\alpha \to \infty$ agents cannot discriminate states at all. The top part of Figure 4 gives an impression of Nosofsky-similarity for different parameter values. Other formalizations of perceptual similarity are possible, including ones that allow for different discriminability in different areas of the state space, but we stick with Nosofsky's similarity function for the time being, because it is mathematically simple, yet an established notion in mathematical psychology.

We further assume that the probability of confusing any two states t_i and t_j is proportional to their perceived similarity and therefore obtained by normalization:

$$C_{ij} = \frac{\mathrm{Sim}(t_\mathrm{i}, t_\mathrm{j}\,;\,\alpha)}{\sum_j \mathrm{Sim}(t_\mathrm{i}, t_\mathrm{j}\,;\,\alpha)}\,.$$

Confusion of states is then also a function of imprecision parameter α (see Figure 4, bottom part). For $\alpha = 0$ the confusion matrix has $C_{ii} = 1$; for $\alpha > 0$, C is positive, i.e., $C_{ij} > 0$ for all i and j; for $\alpha \to \infty$ we find $C_{ij} = {}^{1}/_{|T|}$ for all i and j.

Diffusion of behavior was defined in Equation (1), repeated here:

$$\mathrm{D}_C(\sigma) = C\sigma \qquad\qquad \mathrm{D}_C(\rho) = \rho C\,.$$

If states are confused on average with a probability proportional to their similarity, the effect on noise-perturbed behavior is that behavioral dispositions gradually diffuse along a gradient of similarity of states as well. Consequently, iterated diffusion leads to a smoothing out and an eventual equalization of behavioral strategies, for both the sender and the receiver. Figure 5 shows an arbitrary sender and receiver strategy after one or several rounds of iterated application of diffusion, for different values of perceptual imprecision α.

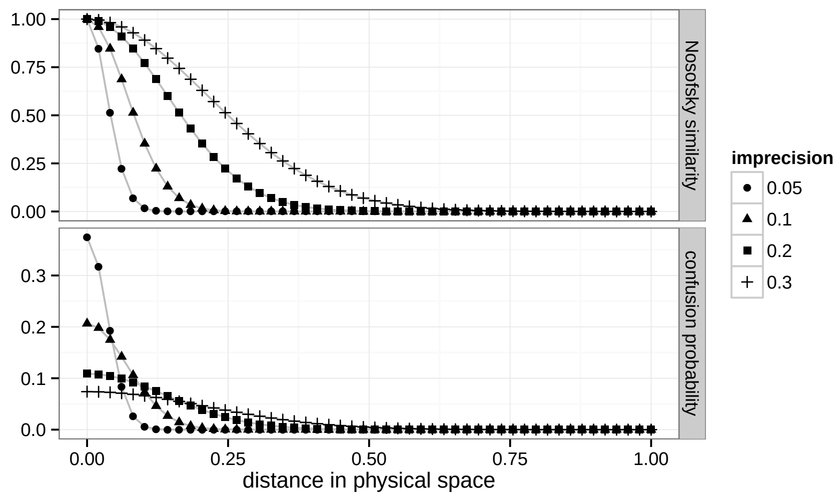


Figure 4: Examples of Nosofsky-similarity (non-normalized) and derived probability of state confusion (normalized) for $|T| = 50$ and different values of imprecision α.

The example in Figure 5 suggests that iterated diffusion leads to sender and receiver behavior that is the same at all choice points. This holds in general. After n steps of diffusion an initial sender strategy σ will be $C^n\sigma$, and an initial receiver strategy ρ will be ρC^n. If we assume that $\alpha > 0$, C is a positive row-stochastic matrix (each state could in principle be confused as any other state, albeit with possibly a very low probability). It then follows from the Perron-Frobenius theorem that the limit $C^\infty = \lim_{n\to\infty} C^n$ exists (it is the Perron-projection) and all of its rows are identical. That is why, in the limit of iterated diffusion, $C^\infty S$ has identical rows (messages are sent with the same probability in each state), and RC^∞ likewise has identical rows (every message is interpreted equally). All of this is in line with the intuition that diffusion of behavior under confusion of states iteratively equalizes behavior for similar states; if all states can be confused for one another in principle, behavior smoothes out entirely in the limit.

4.2 Exploring the replicator diffusion dynamics

Now that we understand better what diffusion does, let us look at how diffusion interacts with optimization of behavior, as described by the replicator dynamics. The main question we should address is whether diffusion and fitness-based replication reasonably interact, and if so, whether the inclusion of diffusion has any noteworthy effects on the evolving meaning of signals. Clearly, if $\alpha = 0$, the RDD reduces to the RD. If $\alpha \to \infty$, the diffusion component takes over and the RDD reduces to the trivial iterated diffusion process that we looked at in the previous section. We will show presently that for reasonable in-between levels of imprecision α, the RDD leads to communicatively

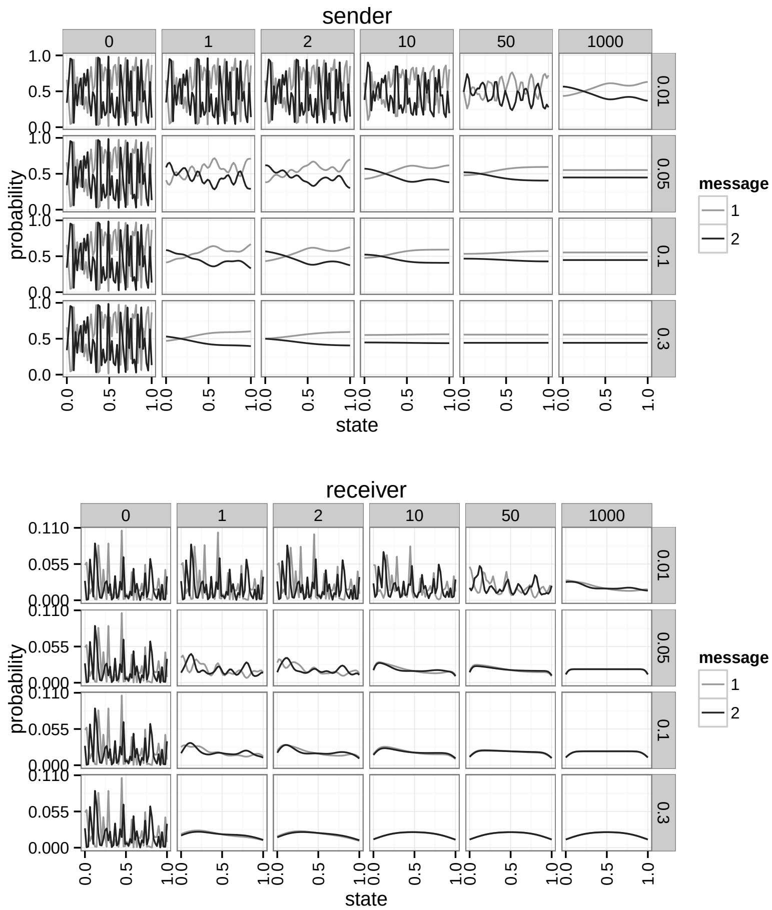
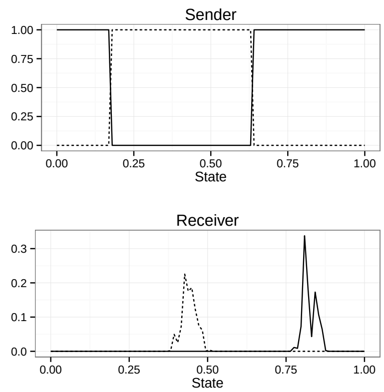
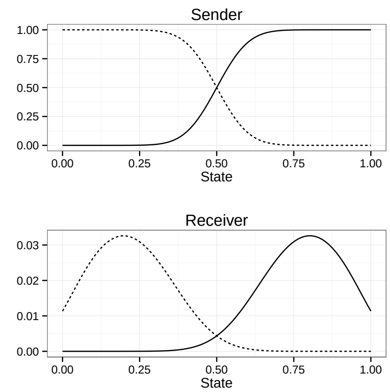


Figure 5: Iterated diffusion for one arbitrary sender (top) and one receiver strategy (bottom), for a game with 50 states and 2 messages. Each row corresponds to a different confusion level α, each colum to a different number of applications of iterated diffusion.



(a) $n_s = 90, \beta = 0.1, \alpha = 0$



(b) $n_s = 90, \beta = 0.1, \alpha = 0.1$

Figure 6: Example strategies under RDD at stopping time of our simulations.

successful, yet vague signal meaning. Suitable levels of imprecision can have further accelerating and unifying effects on meaning evolution.

4.2.1 Experimental set-up

To explore the RDD, we turn to numerical simulations. Let states be evenly spaced elements of the unit interval that always include 0 and 1, and let priors be flat: $\Pr(t) = \Pr(t')$ for all t, t'. As for utility functions, we take another Nosofsky-style function:

$$\mathrm{U}(t_\mathrm{i}, t_\mathrm{j}\,; \beta) = \begin{cases} 1 & \text{if } \alpha = 0 \text{ and } t_\mathrm{i} = t_\mathrm{j} \\ 0 & \text{if } \alpha = 0 \text{ and } t_\mathrm{i} \neq t_\mathrm{j} \\ \exp\left(-\frac{|t_\mathrm{i} - t_\mathrm{j}|^s}{\beta^2}\right), & \text{otherwise} \end{cases} \tag{7}$$

The tolerance parameter $\beta \geq 0$ models the amount of tolerable pragmatic slack or communicative imprecision. This choice of utility function is governed partly by convenience in parallel to the similarity function, but also because it has, as we believe, the right general properties for a communicative payoff function. Unlike utilities that are, say, linearly or quadratically decreasing in physical distance (c.f. Jäger, Metzger, and Riedel, 2011; Franke, Jäger, and van Rooij, 2011), utilities that are exponentially decreasing in negative quadratic distance can model situations where a small amount of imprecision in communication is tolerable, whereas similarly small differences in intolerably far away interpretations matter very little, with a smooth transition between these regimes (c.f. O'Connor, 2014).

We ran 50 trials of the RDD, starting with randomly sampled sender and receiver strategies, for each triplet of independent parameter values: $n_s = |T| \in \{6, 10, 50, 90\}$, $\alpha \in \{0, 0.05, 0.1, 0.2, 0.3\}$, $\beta \in \{0.05, 0.1, 0.2, 0.3\}$. Each trial ran for a maximum of 200 update steps of the RDD. This may not be enough to guarantee convergence to the eventual attracting state, but we were interested especially in whether the dynamics could lead to reasonable outcomes in reasonable time. A trial was considered converged and stopped before the maximum of 200 rounds if the total amount of change between strategies before and after the current RDD step was smaller than 0.001, i.e., if:

$$\sum_{t \in T} \sum_{m \in M} |\textsc{rdd}(\sigma)(t,m) - \sigma(t,m)| < 0.001\,, \text{ and}$$
$$\sum_{m \in M} \sum_{t \in T} |\textsc{rdd}(\rho)(m,t) - \rho(m,t)| < 0.001\,.$$

Representative examples for resulting strategy pairs at stopping time are given in Figure 6. Figure 6a shows a strategy pair at stopping time with 90 states, tolerance $\beta = 0.1$ and imprecision $\alpha = 0$. Zero imprecision means that the trial was effectively an application of the plain RD. Noteworthily, the given sender strategy approximates a pure sender strategy that crisply partitions the state space into non-convex sets. The irregular shape of the receiver strategy shows clearly that the pictured strategy pair has not yet reached a dynamically stable state under the RD. Indeed, the trial did not converge in our technical sense, but was stopped after the maximum of 200 rounds. In contrast, the outcome of a trial with identical parameters, except with imprecision $\alpha = 0.1$, which is shown in Figure 6b, had converged (in our technical sense) after 99 rounds of iterated RDD. The sender strategy shows a smooth blending from one "category" to the other, and the receiver's interpretations are rather extended curves, peaking at a central point in the relevant "categories".

These examples already show two interesting things. Firstly, inclusion of imprecision can lead to seemingly well-behaved, yet vague strategies in the sense that we are after. The sender strategy in Figure 6b identifies clear positive and clear negative cases for each signal, with a smooth transition in-between. The receiver's interpretations of signals can be seen as smoothed out prototype regions.

Secondly, in contrast to results concerning evolutionary stability in sim-max games (Jäger, 2007; Jäger, Metzger, and Riedel, 2011), (sender) strategies can approach non-convex pure strategies under the replicator dynamic. We see this in our numerical simulations for the non-limiting cases, but this also holds, for some types of utility functions, for the limiting case. This was first observed by Elliott Wagner in unpublished work, and is further discussed by O'Connor (2013). Our simulation results do not speak directly to limiting results, but we will see shortly that diffusion from confusability of states clearly prevents evolutionary paths that meander for a long time in the vicinity of non-convex strategies.

4.2.2 Dependent measures

To further explore what happened in all of our 50 trials for each parameter triple, we defined a small number of metrics that aim to capture numerically how vague, communicatively efficient and generally well-structured the resulting strategy pairs were. *Entropy* and *informativity* capture (slightly different aspects of) the amount of systematicity or regularity in signal use. *Voronoiness* and *convexity* are, respectively, gradient and categorical measures for the coherence of signal-induced categories. *Expected utility* measures the communicative efficiency of evolved strategy pairs.

Entropy. This classic information-theoretic notion captures the amount of uncertainty in a probability distribution. Roughly put, entropy of a signaling strategy captures inverse distance from a pure strategy. The usual definition of entropy applies directly to mixed strategies:

$$E(\mathbf{s}) = \sum_{s \in M^T} \mathbf{s}(s) \cdot \log(\mathbf{s}(s)) \qquad\qquad E(\mathbf{r}) = \sum_{r \in T^M} \mathbf{r}(r) \cdot \log(\mathbf{r}(r))\,.$$

These measures are computationally expensive to calculate, since the size of the domain over which the sum is computed grows exponentially with the number of choice points. But provably equivalent metrics for behavioral strategies are ready to hand:

$$E(\sigma) = -\sum_{t \in T} \sum_{m \in M} \sigma(m \mid t) \cdot \log(\sigma(m \mid t))$$

$$E(\rho) = -\sum_{m \in M} \sum_{t \in T} \rho(t \mid m) \cdot \log(\rho(t \mid m))\,.$$

Values obtained by these definitions are lower bounded by 0 and upper bounded by, respectively, $\log(|M^T|) = |T| \cdot \log(|M|)$ and $\log(|T^M|) = |M| \cdot \log(|T|)$. We work with normalized values.

The sender strategies in Figures 6a and 6b have entropy $1.19e^{-5}$ and 0.21, respectively. The receiver strategies have respective entropies 0.43 and 0.84. In general, we expect that vague languages will have higher entropy than crisp ones and that increasing imprecision will also increase entropy, all else being equal.

Informativity. The quantity of information in a signal is an old notion that also goes back to the start of information theory. Skyrms (2010, ch. 3) discusses its use in the context of signaling games. Intuitively speaking, informativity in Skyrms' sense measures how revealing the sender's signal use is about which state is actual. It therefore measures, in our given set-up with flat priors, a sense of well-behavedness or regularity of a sender strategy.

The main idea behind Skyrms' notion is that we can quantify the amount of information about a state t in a message m via the relation between the probability of the state given the message $\Pr(t \mid m)$ and the unconditional probability of the state $\Pr(t)$. Following Bayes' theorem, we can express $\Pr(t \mid m)$ as $\frac{\Pr(t) \cdot \Pr(m \mid t)}{\Pr(m)}$. We then have $\Pr(m \mid t) = \sigma(t, m)$ and $\Pr(m) = \sum_{t' \in T} \Pr(t') \cdot \sigma(t', m)$. Finally, we equate sender infomativity with the average overall information about states in each signal. Following Skyrms (2010, p. 36), and the considerations above, the informativity about states conveyed by a sender strategy is:

$$I(\sigma) = \frac{1}{|M|} \sum_{m \in M} \sum_{t \in T} \frac{\Pr(t) \cdot \sigma(t, m)}{\sum_{t' \in T} \Pr(t') \cdot \sigma(t', m)} \cdot \log\left(\frac{\sigma(t, m)}{\sum_{t' \in T} \Pr(t') \cdot \sigma(t', m)}\right).$$

This measure is bounded between 0 and 1. The sender strategies in Figures 6a and 6b have informativity 0.69 and 0.54, respectively. In general, we expect that vague languages will have lower informativity than crisp ones and that increasing imprecision will decrease informativity.

Skyrms (2010, p. 39) also offers a notion of informativity about the acts performed by the receiver, but it turns out that this is perfectly correlated with information about the states in the results of our simulations. We therefore only look at informativity about states.

Voronoiness. This metric aims to quantify how close a strategy is to being a part of a Voronoi tessallation of the state space. Bearing in mind that this metric is specifically geared towards the sim-max games in our set-up, we define Voronoiness as follows. Let $\pi(\rho, m) = \arg\max_{t \in T} \rho(m, t)$ be the prototype of a message $m \in M$, then

$$V(\sigma, \rho) = \sum_{t \in T} \sum_{m_1 \in M} \sum_{m_2 \in M} F(\sigma, \rho, t, m_1, m_2) \quad \text{with}$$

$$F(\sigma, \rho, t, m_1, m_2) = \begin{cases} 1 & \text{if } \mathrm{Sim}(t, \pi(\rho, m_1)) > \mathrm{Sim}(t, \pi(\rho, m_2)) \Leftrightarrow \\ & \quad \sigma(t, m_1) > \sigma(t, m_2) \\ 0 & \text{otherwise.} \end{cases}$$

is the Voronoiness of the sender strategy σ given the receiver strategy ρ and

$$V(\rho) = \sum_{m \in M} \sum_{t_1 \in T} \sum_{t_2 \in T} F(\rho, m, t_1, t_2) \quad \text{with}$$

$$F(\rho, m, t_1, t_2) = \begin{cases} 1 & \text{if } \mathrm{Sim}(t_1, \pi(\rho, m)) > \mathrm{Sim}(t_2, \pi(\rho, m)) \\ & \quad \Leftrightarrow \rho(m, t_1) > \rho(m, t_2) \\ 0 & \text{otherwise.} \end{cases}$$

is the Voronoiness of the receiver strategy ρ. The metrics are lower bound by 0 and upper bound by, respectively, $|T| \times |M|^2$ and $|M| \times |T|^2$, thus we will normalize them by dividing by these values.

Despite being vague, a language can have maximal Voronoiness according to this gradient measure. For instance, the sender strategies in Figures 6a and 6b have Voronoiness 0.91 and 1, respectively. The receiver strategies have Voronoiness 0.78 and 1, respectively. We do not *prima facie* expect a conceptual connection between vagueness and Voronoiness, and so do not predict a clear effect of imprecision on Voronoiness either.

Convexity. At least for sender strategies, which converge faster, it also makes sense to define a categorical measure of convexity, that compensates for potential vagueness. To determine whether a sender strategy σ is convex despite possibly being vague, we look at the derived pure strategy s for which $s(t) = \arg\max_{m' \in M} \sigma(t, m')$. If that s is convex, we also count σ as convex. The sender strategy in Figure 6a is not convex, while the one in Figure 6b is. Again, we do not expect a conceptual relation between vagueness and imprecision, on the one hand, and convexity on the other.

Expected utility. We also record the expected utility of a strategy pair:

$$\mathrm{EU}(\sigma, \rho\,; \beta) = \sum_{t \in T} \sum_{m \in M} \sum_{t' \in T} \Pr(t) \cdot \sigma(t, m) \cdot \rho(m, t') \cdot \mathrm{U}(t, t'\,; \beta)\,.$$

To make direct comparisons across different parameter settings, we normalize expected utility by the maximal amount of expected utility obtainable in the relevant game.

The strategy pair in Figure 6a has a normalized expected utility of 0.992, the pair in Figure 6b has 0.901. Generally, vagueness and imprecision can be expected to decrease expected utility (c.f. Lipman, 2009). The crucial question is whether communicative success drops unacceptably fast with moderate levels of vagueness and imprecision.

4.2.3 Results

Figure 7 shows the means of the recorded metrics, together with their estimated 95% confidence intervals, for each triple of independent variables. Most trends are quite clear, and not unexpected. Entropy of sender and receiver strategies are increasing with imprecision. Entropy of sender strategies seems independent of tolerance levels and the number of states, while there seems to be an effect of both parameters on the entropy of the receiver strategy. Sender informativity and expected utility are decreasing under growing imprecision. There seem to be mild effects of the number of states in the game, but hardly any in terms of pragmatic tolerance. What was not expected *a priori* was that Voronoiness of both sender and receiver strategies appear to be non-decreasing under increasing impairment, with the only exception being the 6 states games and the transition from $\alpha = 0$ to $\alpha = 0.05$. Expected utility of evolved languages mostly decreases in increasing imprecision, with the sole exception of a rise for $\beta = 0.2$ and $n_s = 6$ at $\alpha = 0.1$. Still, normalized expected utility of evolved strategy pairs under mild values of imprecision are not necessarily much worse than their respective perfect precision RD-analogues. This is especially so for lower numbers of states (which increase the chance of non-convergence and non-convexity; see Figure 8 and the discussion below).

Taken together, these findings suggest the following effect of confusability of states and diffusion of behavior. Mild forms of perceptual imprecision lead to slightly less communicative efficiency, more vagueness and more regular, well-behaved languages. This impression is also supported by the proportion of converged and convex trials, pictured in Figure 8. Non-converged and non-convex sender strategies showed for several values of parameters in the absence of impairment. Increasing impairment always assured convergence and convexity: diffusion speeds up convergence to a convex and regular language.

Diffusion also has another interesting regularizing effect on the evolution of signaling. If we look at the confidence intervals in Figure 7, we see that there is very little variation in the recorded metrics for evolved strategies, at least for higher values of impairment. On closer inspection, it turns out that variability in low-impairment conditions is not only due to non-convergence or non-convexity. Figure 9 gives two more examples of strategy pairs at stopping time. Both are obtained for the same triple of parameters, both converged and are convex. However, they are not equally efficient. In

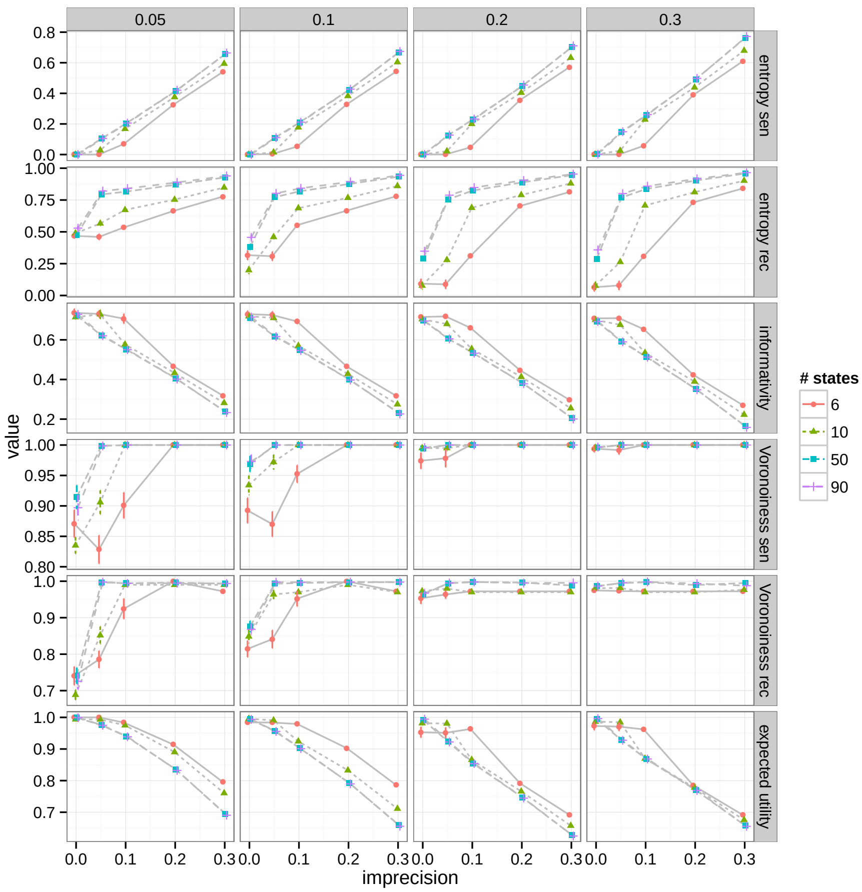


Figure 7: Means of recorded metrics for each triple of independent variables. Notice that the plots also show 95% confidence intervals, but these are most often invisible, owing to the uniformity of outcomes across trials.

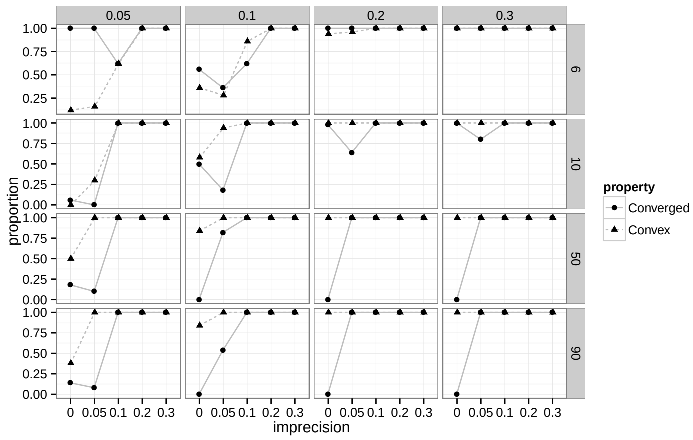


Figure 8: Proportion of converged and convex trials.

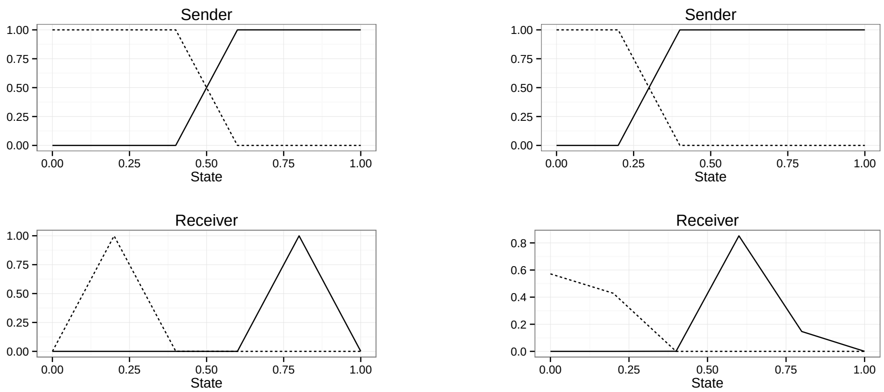
fact, the pair in Figure 9a has a normalized expected utility of 0.99 while the one in Figure 9b only has 0.89.

Interestingly, this type of variability in evolutionary outcomes is also weeded out by mild increases in impairment. To see this, we calculated the average distance between evolved sender strategies within each group of trials that had identical independent parameter values. We determined the distance between sender strategies σ and σ' as the average Hellinger distance between probability distributions $\sigma(t)$ and $\sigma'(t)$ over messages at each choice point t:

$$\mathrm{HD}(\sigma, \sigma') = \frac{1}{|T| \cdot \sqrt{2}} \cdot \sum_{t \in T} \sqrt{\sum_{m \in M} \left(\sqrt{\sigma(t,m)} - \sqrt{\sigma'(t,m)} \right)^2} .$$

To compensate for the arbitrariness of message use, we set the distance between strategies σ and σ' to be the maximum of $\mathrm{HD}(\sigma, \sigma')$ and $\mathrm{HD}(\sigma^*, \sigma')$ where σ^* is σ with reversed message indices. The *within group distances*, i.e., the average distances between all sender strategies in each parameter group, are plotted in Figure 10. This clearly shows that with increasing imprecision, the sender strategies evolving under RDD were exactly alike, modulo which message was used for which part of the unit interval. In other words, perceptual imprecision clearly unifies evolutionary outcomes, and in fact guarantees sender strategies that are not only convex, but also maximally efficient in that they induce a vague category split exactly in the middle of the unit interval.

This unifying property of perceptual imprecision could be considered an evolutionary beneficial side-effect. Higher imprecision can lead to higher *within group expected utility*, i.e., the average expected utility that each evolved language (i.e., pair of sender and receiver strategy at stopping time) scored when playing against an arbitrary other language of the same parameter group (see Figure 10). What this means is that, while



(a) $n_s = 6, \beta = 0.2, \alpha = 0$ (b) $n_s = 6, \beta = 0.2, \alpha = 0$

Figure 9: More example strategies under RDD at stopping time of our simulations.

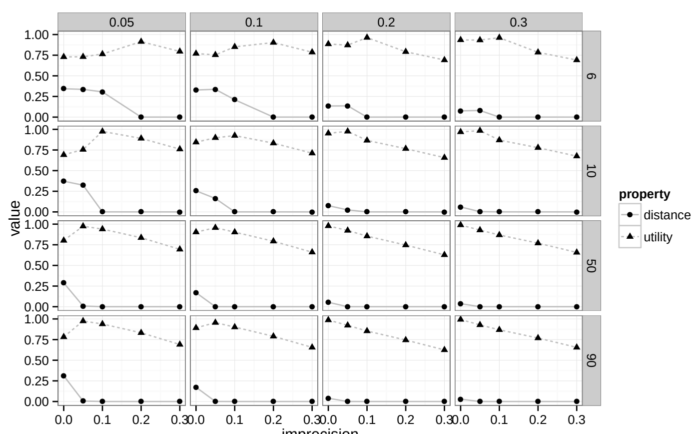


Figure 10: For each group, i.e., triplet of independent variables, the plots show (i) within group distances and (ii) within group expected utilities, as defined in the main text.

imprecision might decrease the communicative efficiency of individual languages, it increases the conceptual coherence and communicative success between independently evolving strategies. It is almost as if mere confusability of states imposes a regularity constraint on evolving linguistic meanings.

5 Discussion

We have introduced the RDD as a new variant of the RMD that implements stochastic noise in the form of confusability of similar states. The probabilities of confusing similar states is easily implemented for behavioral strategies, but, as seen in Section 3, can also be translated into a mutation matrix for pure strategies. This allows, in principle, several conceptual interpretations of the RDD, on which we will briefly elaborate below in Section 5.1.

Next to this technical contribution, the results of our numerical simulations also advance our understanding of the possibility of evolving regular and efficient categories despite their (higher-order) vagueness. Section 5.2 zooms in on the relation with closely related accounts once more, to delineate the present approach more precisely.

5.1 Model interpretation

We mentioned in passing in Section 3.2 that adding diffusion to other discrete-time evolutionary dynamics that operate on behavioral strategies is entirely straightforward. We could, for instance, easily diffuse the outcome of a best-response dynamic at each time step. That would make sense if we thought of agents as prone to confusing similar states, but otherwise rationally optimizing behavior. The reason that we chose the RD to combine with diffusion is twofold. A minor reason is that it makes for a conceptually interesting link with the RMD. A more important reason is that the RD is especially versatile and non-committal about what the exact process of adaption is that is being modelled.

Originally the RD was introduced as mathematical model of evolution under asexual reproduction, motivated by concepts from the theory of natural selection (Taylor and Jonker, 1978). The most conservative interpretation of the RDD in our present context is thus a biological one: we can imagine signaling strategies as innate or fixed behaviorial tendencies of organisms, steps in the evolutionary process as successive generations, and selection as capturing the reproductive advantage of fitter individuals. This interpretation, strictly speaking, requires formalization in terms of mixed strategies via the RMD, and possibly also a symmetrizing of the game, so that every agent is assumed to have a unique sender and receiver role at the same time, the pair of which is bequeathed onto the next generation. Diffusion, in this context, could be either of two things. It could be differential mutation probabilities in line with other interpretations of mutation in the RMD (e.g. Nowak, Komarova, and Niyogi, 2001; Komarova, Niyogi, and Nowak, 2001): pure strategies are not faithfully reproduced, e.g. by imperfections of learning or underdetermination of input, and mistakes in inheriting pure strategies are more likely if they result from the confusion of similar states. Another possible biological interpretation of the RDD, is that inheritance is faithful, but strategies are noisily realized. Strategies are not necessarily selected for what they are, but rather for how they are realized, once noise is factored in.

But the replicator dynamic is not only a model of biological evolution. It can also be interpreted as a high-level description of the likely development of other behavioral adaptation processes, like differential imitation (see Sandholm, 2013, for various derivations of the RD). Under this interpretation, the RD is consistent with the idea that what is subject to evolutionary optimization are not organisms but behavior: agents or systems can adapt their behavior to the perceived environment or agents can adopt strategies from others. Fitness captures the success of behavioral patterns, which in the case of language can be thought of in terms of communicative success. Differential reproduction represents the tendency of more successful strategies to be more likely realized, be it through imitation or some solitary learning or optimization mechanism.

Under this non-biological interpretation, we have again two options of picturing what diffusion is, similar to the biological cases before. One possibility is that diffusion is noise in the adoption of strategies, e.g. by conditional imitation, of strategies by other signalers. Another possibility is that diffusion is again noise on the realization of strategies: while behavior is optimized to be efficient (be it due to learning, introspection or imitation), realization of strategies is bound to be noisy due to confusability of similar states.

We believe that all of the four mentioned interpretations are, on first approximations, feasible conceptualizations of the RDD, and that it is a good thing to know of a working account of vague signaling that sketches where fitness-based selection under state-confusability will take is, abstracting away from the details of actually playing the game, inheriting, imitating or otherwise optimizing behavior in whatever particular way. It is a good thing to know this on the macro-level, especially since there are also micro-level accounts that nicely complement the picture. We turn to one such next.

5.2 Relation with previous accounts

Generalized reinforcement. Under *Herrnstein reinforcement learning*, sender and receiver play the game repeatedly and adjust their dispositions to act after each round of play, in such a way that the actual (non-negative) payoff gained in the current interaction is added to the non-normalized propensities for repeating the same behavior. For sim-max games, this means that when the sender chose m in state t, and this resulted in some non-negative payoff (which is guaranteed by our choice of utility function), the probability that the sender chooses m again in t is increased, but nothing else changes. In particular, the sender's behavior in other choice points does not change. *Generalized reinforcement learning* is different here. When the use of m in t gave positive payoff, then not only will its future use probability be promoted at t but also at other states, proportional to how similar these are to t. Similar amendments take care of the way that the receiver updates his choice dispositions.

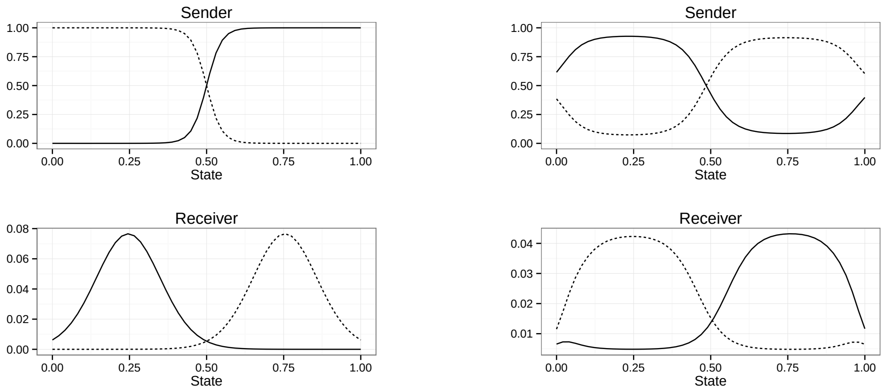
O'Connor (2014) shows that this extension not only leads to vague signaling of the appropriate kind, but also speeds up learning in such a way that, especially for sim-max games with higher numbers of states, higher levels of communicative success are reached in shorter learning periods than is possible without stimulus generalization. Technically, this result is partly due to the fact that signalers make bigger changes to their behavioral strategies after each round of play under generalized reinforcement than under the Herrnstein variety. But that only explains the speed of adaptation, not necessarily that generalization also leads to regularity and communicative efficiency.

Diffusion of strategies in the RDD can also be conceived of as a form of generalization, and works in large part quite analogous to stimulus generalization in O'Connor's approach. But there are still differences. O'Connor showed that generalized reinforcement learning can speed up the development of efficient signaling, especially for higher numbers of states. Complementing this, we showed that diffusion regularizes evolutionary outcomes and prevents meandering around sub-optimal and non-convex strategies, also for low numbers of states.

Conceptually, the RDD is a more abstract framework than generalized reinforcement learning. The latter is foremost motivated as a learning dynamic that has two players adapt their individual strategies after each concrete round of play. In contrast, the RDD describes a more abstract, average dynamical change in behavioral dispositions. Although the behavior of (some forms of) reinforcement learning mirror those of the RD (at some stage in time) (Börgers and Sarin, 1997; Hopkins and Posch, 2005; Beggs, 2005), this does not mean that *generalized* reinforcement learning is also necessarily a plausible high-level description of, say, generalized learning in a population of agents. Seen in this light, generalized reinforcement learning and the RDD nicely complement each other, as similarly-minded accounts operating at different levels of abstraction.

Quantal response equilibria. Franke, Jäger, and van Rooij (2011) suggested a number of ways in which information processing limitations of signaling agents could lead to vague strategies. The model that is most clearly related to the present approach uses the notion of a *quantal response*, also known as a *logit response* or a *soft-max function* (e.g. Luce, 1959; McFadden, 1976; McKelvey and Palfrey, 1995, 1998; Goeree, Holt, and Palfrey, 2008). A quantal response function is a paramterized generalization of the classic best response function. For example, if $U : A \to \mathbb{R}$ is the measure of expected utility over choices A of an agent, then a best response function would have the agent choose a only if $U(a) = \max_{a' \in A} U(a)$. A quantal response function rather assumes that agents would choose a with a probability proportional to $\exp(\lambda \cdot U(a))$, where λ is a rationality parameter. If $\lambda \to \infty$ we retrieve the behavior of the best response function, but if it is positive but finite, any choice a will receive a positive probability, but acts with higher expected utility will be more likely. The underlying motivation for this choice rule is the assumption that there is noise in the computation of expected utilities and/or in maximization of expected utilities. Consequently, choices with almost equal expected utilities will be chosen with almost equal probability (for moderate values of λ).

Franke, Jäger, and van Rooij (2011) show that quantal response equilibria of sim-max games, i.e., pairs of sender and receiver strategies such that the sender strategy is the quantal response to the expected utilities under the receiver strategy and vice versa, can show the desired marks of vague signaling. Figure 11a gives an example of a quantal response equilibrium for a sim-max game, as used in our set-up but with higher tolerance $\beta = 0.5$. Sender and receiver strategies look very much like what evolves under RDD with modest values of perceptual imprecision.



(a) $n_s = 90, \lambda = 15, \beta = 0.5$

(b) $n_s = 90, \lambda = 15, \beta = 0.05$

Figure 11: Examples of vague quantal response equilibria.

But not all quantal response equilibria are equally plausible, and the ones that are not suggest that it is less natural to think of vagueness as arising from of errors in the computation or maximization of expected utility than that it arises from confusion of similar states. To see what the problem is, we can look at cases like given in Figure 11b, which is the quantal response equilibrium for a game with lower tolerance $\beta = 0.05$. Unlike what evolves under RDD in this case, sender strategies have vague boundaries also towards the end of the unit intervals. Technically, this is because quantal responses equalize message use far away from the "prototypical" interpretation, not just in-between categories, so to speak. This, in turn, is because quantal responses introduce noise into the decision making at the level of computing or maximizing expected utility of choices.

That this is conceptually odd shows even more clearly in a case where the state space is intuitively unbounded, as for instance for the property "tall". If the usual interpretation of a "tall man" peaks at around, say, 195cm then when meeting a giant of n meters, senders would, according to the quantal response approach, be ever more inclined to describe the giant indifferently as either "tall" or "short" the larger n gets. This is because, as the distance from the prototype increases for larger n, the difference between the expected utilities of saying "short" or "tall" will converge to zero.

Admittedly, this argument hinges on the choice of utility function. Still, to the extent that the chosen lower bounded utility functions are reasonable—and we think they are very reasonable— the case suggests that quantal responses are not a good model, intuitively speaking, for why linguistic categories are vague. Vagueness is more plausibly an effect of perceptual confusion of similar states than of computational errors in maximizing expected utility.

6 Conclusions

Vagueness is a pervasive but seemingly non-disruptive feature of natural communication and classification systems. From an evolutionary point of view, the challenge arises to explain how vagueness can persist under selective pressure for precision. This is foremost a technical challenge, probing for the possibility of integrating into a consistent model forces that lead to vagueness with forces that lead to efficient information transfer. This paper proposed one such model in the context of sim-max games and explored some of its consequences.

We introduced the replicator diffusion dynamic as a special case of the established replicator mutator dynamic. It is a special case in the sense that the RDD incorporates a particular kind of mutation that is based on the similarity of states (as defined in the sim-max games). More concretely, we started from the idea that similar states are more easily confused than less similar states. To formalize this idea, we assumed that there is a confusion matrix C, where C_{ij} is the probability that a state t_i is realized as a possibly different state t_j. Based on this, we showed two things. For one, we showed how the discrete-time formulation of the replicator dynamic can be easily adapted to integrate diffusion of behavior due to confusion of states. This yielded the RDD in its basic discrete-time formulation for behavioral strategies. For another, we also showed that there exists a translation of the confusion matrix C into suitable mutation matrices Q_C and R_C such that the effect of former and latter are effectively equivalent, although C applies to behavioral strategies, while Q_C and R_C apply to mixed strategies. This shows that the RDD may be thought of as a special case of the RMD.

Based on data from numerical simulations, we demonstrated that the inclusion of mild levels of stochastic confusability of states does not undermine the possibility of evolving communicatively successful signaling strategies at all. On the contrary, strategies that evolved under mild diffusion induce a highly regular and systematic category structure that shows the signs of vagueness as desired. There might even be a higher-order benefit to the presence of imprecision, in that it can accelerate convergence to optimal categorization, by preventing evolutionary paths to stay near inefficient non-convex strategies for a long time. Diffusion thus also unifies and regularizes evolutionary outcomes in such a way that sub-optimal categorization is avoided. These results complement similar findings by O'Connor (2014), obtained for a micro-dynamic extension of reinforcement learning. The RDD adds to this a more general, abstract framework that is not tied to the specific assumptions of turn-based reinforcement.

Many interesting issues arise in this context that we have not yet explored. Firstly, we concentrated on the RDD in behavioral strategies, because numerical simulations of these are much less complex and unwieldy than of mixed strategies or analyses of the symmetrized game. But we also noted that for especially biological interpretations of the dynamic, symmetrizing would be conceptually more plausible. It remains to be seen whether there are substantial differences between the symmetrized version of the RDD and the behavioral strategy version that we have explored here.

Secondly, we have only explored the effect of a single confusion matrix that affected both sender and receiver roles. Obviously, we could allow different roles to be affected differently by confusability of states. We might hypothesize that for the unifying and regularizing effects of diffusion, it is less important that the receiver's strategies are diffused, as long as the sender's are. But, again, these issues must wait for further exploration. Connected to that is the obvious extension that also allows for stochastic confusability of signals.

Further promising extensions of the work presented here include investigating higher-dimensional state spaces, possibly with less uniform, more psychologically real similarity metrics, such as the three-dimensional color space. Also, to compare the work presented here better to other work on efficient categorization (e.g. Mohlin, 2014), it would be worthwhile exploring the possibility of signal innovation (e.g. McKenzie Alexander, Skyrms, and Zabell, 2012) also for sim-max games. This way, the game dynamics would ideally lead to an optimal number of categories, not only an optimal shape of a predefined number of categories.

A Proof of Theorem

Part (i). Fix $\mathbf{s}$ and $\sigma = G(\mathbf{s})$. Look first at the rhs of the consequent:

$$\begin{aligned}
\mathrm{D}_C(\sigma)(m_y \mid t_x) &= \sum_{t_l} C_{xl} \cdot \sigma(m_y \mid t_l) && \text{(by Equation (1))} \\
&= \sum_{t_l} C_{xl} \cdot \sum_{s_i(t_l)=m_y} \mathbf{s}_i && \text{(by Equation (2))} \\
&= \sum_{t_l} \sum_{s_i(t_l)=m_y} \mathbf{s}_i \cdot C_{xl} \,.
\end{aligned}$$

Next consider the lhs of the consequent:

$$\begin{aligned}
G(\mathrm{M}_{Q^C}(\mathbf{s}))(m_y \mid t_x) &= \sum_{s_i(t_x)=m_y} \mathrm{M}_{Q^C}(\mathbf{s}_i) && \text{(by Equation (2))} \\
&= \sum_{s_i(t_x)=m_y} \sum_{s_j} \mathbf{s}_j \cdot Q^C_{ji} && \text{(by Equation (3))} \\
&= \sum_{s_i(t_x)=m_y} \sum_{s_j} \mathbf{s}_j \cdot \prod_{t_l} \sum_{t_m \in s_j^{-1}(s_i(t_l))} C_{lm} && \text{(by Equation (5))} \\
&= \sum_{s_j} \mathbf{s}_j \cdot \sum_{s_i(t_x)=m_y} \prod_{t_l} \sum_{t_m \in s_j^{-1}(s_i(t_l))} C_{lm}
\end{aligned}$$

To simplify this further we look at a fixed s_j and consider the term:

$$\sum_{s_i(t_x)=m_y} \prod_{t_l} \sum_{t_m \in s_j^{-1}(s_i(t_l))} C_{lm} \,. \tag{8}$$

Let Y be the row-stochastic matrix with $Y_{kl} = \sum_{t_m \in s_j^{-1}(m_l)} C_{km}$. Every pure sender strategy maps each state t_k onto exactly one Y_{kl}. If we quantify over all pure strategies, we essentially look at each such mapping. Term (8) above sums over all pure strategies that map t_k onto m_y. The above term then sums over all products whose factors are tuples in $\times_{k>2}\{y \mid \exists l : y = Y_{kl}\}$. So term (8) expands to (where $e = |T|$ and $d = |M|$):

$$(Y_{11} \cdot Y_{21} \cdot Y_{31} \cdot \ldots \cdot Y_{e1}) + (Y_{11} \cdot Y_{21} \cdot Y_{31} \cdot \ldots \cdot Y_{e2}) + \ldots$$
$$+ (Y_{11} \cdot Y_{2d} \cdot Y_{3d} \cdot \ldots \cdot Y_{ed})$$

But since Y is a row-stochastic matrix, this simplifies to Y_{xy}. Continuing the derivation with this:

$$\begin{aligned} G(\mathrm{M}_{Q^C}(\mathbf{s}))(m_y \mid t_x) &= \sum_{s_j} \mathbf{s}_j \cdot \sum_{t_l \in s_j^{-1}(m_y)} C_{xl} \\ &= \sum_{s_j} \sum_{t_l \in s_j^{-1}(m_y)} \mathbf{s}_j \cdot C_{xl} \\ &= \sum_{t_l} \sum_{s_i(t_l)=m_y} \mathbf{s}_i \cdot C_{xl} \,. \end{aligned}$$

□

Part (ii). Fix $\mathbf{r}$ and $\rho = G(\mathbf{r})$. The rhs of the consequent expands to:

$$\begin{aligned} \mathrm{D}_C(\rho)(t_x \mid m_y) &= \sum_{t_j} \sum_{i \in \{k \mid r_k(m_y)=t_x\}} \mathbf{r}_i \cdot C_{jx} && \text{(by Equation (1))} \\ &= \sum_{r_i} \sum_{j \in \{k \mid r_i(m_y)=t_j\}} \mathbf{r}_i \cdot C_{jx} \\ &= \sum_{r_i} \mathbf{r}_i \cdot C_{r_i(m_y)x} \,. \end{aligned}$$

The rhs expands to (by Equations (2), (3) and (4)):

$$G(\mathrm{M}_{R^C}(\mathbf{r}))(t_x \mid m_y) = \sum_{r_i} \mathbf{r}_i \cdot \sum_{j \in \{j \mid r_k(m_y)=t_x\}} \prod_m C_{r_i(m)r_j(m)} \,.$$

Without loss of generality, assume that $x = y = 1$, and fix $|M| = d$ and let e be the number of pure receiver strategies. Then the last term can be rewritten as:

$$\begin{aligned} = \sum_i \mathbf{r}_i \cdot (&C_{r_i(m_1)1} \cdot C_{r_i(m_2)r_1(m_2)} \cdot \ldots \cdot C_{r_i(m_d)r_1(m_d)} + \ldots \\ &+ C_{r_i(m_1)1} \cdot C_{r_i(m_2)r_2(m_2)} \cdot \ldots \cdot C_{r_i(m_d)r_2(m_d)} + \ldots \\ &+ C_{r_i(m_1)1} \cdot C_{r_i(m_2)r_e(m_2)} \cdot \ldots \cdot C_{r_i(m_d)r_e(m_d)})) \,. \end{aligned}$$

For every messages m_l, $C_{r_i(m_l)}$ is a stochastic vector. For $l > 1$, all elements of these vectors appear equally often. But that means that these cancel out. What remains is:

$$= \sum_{r_i} \mathbf{r}_i \cdot C_{r_i(m_y)x} \,.$$

□

References

Barrett, Jeffrey A. (2009). “The Evolution of Coding in Signaling Games”. In: *Theory and Decision* 67, pp. 223–237.

Beggs, A.W. (2005). “On the Convergence of Reinforcement Learning”. In: *Journal of Economic Theory* 122.1, pp. 1–36.

Blume, Andread and Oliver Board (2013). “Intentional Vagueness”. In: *Erkenntnis*.

Blume, Andreas, Yong-Gwan Kim, and Joel Sobel (1993). “Evolutionary Stability in Games of Communication”. In: *Games and Economic Behavior* 5.5, pp. 547–575.

Börgers, Tilman and Rajiv Sarin (1997). “Learning Through Reinforcement and Replicator Dynamics”. In: *Journal of Economic Theory* 77.1, pp. 1–14.

Cobreros, Pablo et al. (2012). “Tolerant, Classical, Strict”. In: *Journal of Philosophical Logic* 41.2, pp. 347–385.

Correia, José Pedro (2013). “The Bivalent Trap: Vagueness, Theories of Meaning, and Identity”. MA thesis. Universiteit van Amsterdam.

Crawford, Vincent P. and Joel Sobel (1982). “Strategic Information Transmission”. In: *Econometrica* 50, pp. 1431–1451.

Cressman, Ross (2003). *Evolutionary Dynamics and Extensive Form Games*. Cambridge, MA: MIT Press.

Decock, Lieven and Igor Douven (2012). “What is Graded Membership?” In: *Noûs*, pp. 1–30.

Deemter, Kees van (2009). “Utility and Language Generation: The Case of Vagueness”. In: *Journal of Philosophical Logic* 38.6, pp. 607–632.

Deo, Ashwini (2014). “The Semantic and Pragmatic Underpinnings of Grammaticalization Paths: The Progressive to Imperfective Shift”. manuscript, Yale University.

Douven, Igor et al. (2011). “Vagueness: A Conceptual Spaces Approach”. In: *Journal of Philosophical Logic* 42.1, pp. 137–160.

Edgington, Dorothy (1997). “Vagueness by Degrees”. In: *Vagueness: A Reader*. Ed. by Rosanna Keefe and Peter Smith. MIT Press.

Fine, Kit (1975). “Vagueness, Truth and Logic”. In: *Synthese* 30.3-4, pp. 265–300.

Franke, Michael, Gerhard Jäger, and Robert van Rooij (2011). “Vagueness, Signaling & Bounded Rationality”. In: *JSAI-isAI 2010*. Ed. by T. Onoda, D. Bekki, and E. McCready. Springer, pp. 45–59.

Gärdenfors, Peter (2000). *Conceptual Spaces: The Geometry of Thought*. MIT Press.

Goeree, Jacob K., Charles A. Holt, and Thomas R. Palfrey (2008). “Quantal Response Equilibrium”. In: *The New Palgrave Dictionary of Economics*. Ed. by Steven N. Durlauf and Lawrence E. Blume. Basingstoke: Palgrave Macmillan.

Hopkins, Ed and Martin Posch (2005). “Attainability of boundary points under reinforcement learning”. In: *Games and Economic Behavior* 53, pp. 110–125.

Huttegger, Simon M. (2007). “Evolution and the Explanation of Meaning”. In: *Philosophy of Science* 74, pp. 1–27.

Huttegger, Simon M. and Kevin J. S. Zollman (2011). “Signaling Games: Dynamics of Evolution and Learning”. In: *Language, Games, and Evolution*. Ed. by Anton Benz et al. LNAI 6207. Springer, pp. 160–176.

Huttegger, Simon M. et al. (2010). “Evolutionary Dynamics of Lewis Signaling Games: Signaling Systems vs. Partial Pooling”. In: *Synthese* 172.1, pp. 177–191.

de Jaegher, Kris (2003). “A Game-Theoretic Rationale for Vagueness”. In: *Linguistics and Philosophy* 26.5, pp. 637–659.

de Jaegher, Kris and Robert van Rooij (2010). “Strategic Vagueness, and Appropriate Contexts”. Unpublished manuscript.

Jäger, Gerhard (2007). “The Evolution of Convex Categories”. In: *Linguistics and Philosophy* 30.5, pp. 551–564.

Jäger, Gerhard, Lars P. Metzger, and Frank Riedel (2011). “Voronoi Languages: Equilibria in Cheap-Talk Games with High-Dimensional Types and Few Signals”. In: *Games and Economic Behavior* 73.2, pp. 517–537.

Jäger, Gerhard and Robert van Rooij (2007). “Language Structure: Psychological and Social Constraints”. In: *Synthese* 159.1, pp. 99–130.

Keefe, Rosanna and Peter Smith, eds. (1997). *Vagueness: A Reader*. MIT Press.

Komarova, Natalia L., Partha Niyogi, and Martin A. Nowak (2001). “The Evolutionary Dynamics of Grammar Acquisition”. In: *Journal of Theoretical Biology* 209, pp. 43–59.

Lewis, David (1969). *Convention. A Philosophical Study*. Cambridge, MA: Harvard University Press.

Lipman, Barton L. (2009). “Why is Language Vague?” Manuscript, Boston University.

Luce, Duncan R. (1959). *Individual Choice Behavior: A Theoretical Analysis*. New York: Wiley.

Machina, Kenton F. (1976). “Truth, Belief, and Vagueness”. In: *Journal of Philosophical Logic* 5.1, pp. 47–78.

McFadden, Daniel (1976). “Quantal Choice Analysis: A Survey”. In: *Annals of Economic and Social Measurement* 5.4, pp. 363–390.

McKelvey, Richard D. and Thomas R. Palfrey (1995). “Quantal Response Equilibrium for Normal Form Games”. In: *Games and Economic Behavior* 10.1, pp. 6–8.

— (1998). “Quantal Response Equilibrium for Extensive Form Games”. In: *Experimental Economics* 1, pp. 9–41.

McKenzie Alexander, Jason, Brian Skyrms, and Sandy L. Zabell (2012). “Inventing New Signals”. In: *Dynamic Games and Applications* 2, pp. 129–145.

Mehlberg, Henryk (1958). “Truth and vagueness”. In: *The Reach of Science*. University of Toronto Press, pp. 256–259.

Mohlin, Erik (2014). “Optimal Categorization”. In: *Journal of Economic Theory* 152, pp. 356–381.

Nosofsky, Robert M. (1986). “Attention, Similarity, and the Identification-Categorization Relationship”. In: *Journal of Experimental Psychology: General* 115.1, pp. 39–57.

Nowak, Martin A. (2006). *Evolutionary Dynamics: Exploring the Equations of Life*. Harvard University Press.

Nowak, Martin A., Natalia L. Komarova, and Partha Niyogi (2001). “Evolution of Universal Grammar”. In: *Science* 291.5501, pp. 114–118.

O’Connor, Cailin (2013). “Evolving Perceptual Categories”. Manuscript, UCI.

— (2014). “The Evolution of Vagueness”. In: *Erkenntnis* 79.4, pp. 707–727.

Okabe, Atsuyuki et al. (2000). *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*. New York: Wiley.

Page, Karen M. and Martin A. Nowak (2002). “Unifying Evolutionary Dynamics”. In: *Journal of Theoretical Biology* 219, pp. 93–98.

Pawlowitsch, Christina (2008). “Why Evolution does not Always Lead to an Optimal Signaling System”. In: *Games and Economic Behavior* 63.1, pp. 203–226.
Sainsbury, Mark (1991). “Is There Higher-Order Vagueness?” In: *The Philosophical Quarterly* 41.163, pp. 167–182.
Sandholm, William H. (2013). “Population Games and Deterministic Evolutionary Dynamics”. Prepared for *Handbook of Game Theory*.
Skyrms, Brian (2010). *Signals: Evolution, Learning, and Information*. Oxford: Oxford University Press.
Smith, Nicholas J. J. (2008). *Vagueness and Degrees of Truth*. Oxford: Oxford University Press.
Spence, Andrew Michael (1973). “Job market signaling”. In: *Quarterly Journal of Economics* 87, pp. 355–374.
Taylor, Peter D. and Leo B. Jonker (1978). “Evolutionary Stable Strategies and Game Dynamics”. In: *Mathematical Bioscience* 40.1–2, pp. 145–156.
Wagner, Elliott (2009). “Communication and Structured Correlation”. In: *Erkenntnis* 71.3, pp. 377–393.
Wärneryd, Karl (1993). “Cheap Talk, Coordination, and Evolutionary Stability”. In: *Games and Economic Behavior* 5.4, pp. 532–546.
Williamson, Timothy (1994). *Vagueness*. London and New York: Routledge.
Zadeh, L. A. (1975). “Fuzzy Logic and Approximate Reasoning”. In: *Synthese* 30.3-4, pp. 407–428.
Zollman, Kevin J. S. (2005). “Talking to Neighbors: The Evolution of Regional Meaning”. In: *Philosophy of Science* 72, pp. 69–85.