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Abstract	A vexing puzzle about vagueness, rationality, and evolution runs, in crude abbreviation, as follows: vague language use is demonstrably suboptimal if the goal is efficient, precise and cooperative information transmission; hence rational deliberation or evolutionary selection should, under this assumed goal, eradicate vagueness from language use. Since vagueness is pervasive and entrenched in all human languages, something has to give. In this paper, we focus on this problem in the context of signaling games. We provide an overview of a number of proposed ways in which vagueness may come into the picture in formal models of rational and evolutionary signaling. Most argue that vague signal use is simply the best we can get, given certain factors. Despite the plausibility of the proposals, we argue that a deeper understanding of the benefits of vagueness needs a more ecological perspective, namely one that goes beyond the local optimization of signaling strategies in a homogeneous population. As an example of one possible way to expand upon our current models, we propose two variants of a novel multi-population dynamic of imprecise imitation where, under certain conditions, populations with vague language use dominate over populations with precise language use.

Towards an Ecology of Vagueness

José Pedro Correia and Michael Franke

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1 Vagueness and Rationality

18 The classical philosophical problem of vagueness is most starkly embodied by the
19 sorites paradox. The original formulation is attributed to Eubulides, an ancient Megar-
20 ian philosopher (Sorensen 2009), and uses the example of a heap of sand: if no

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removal of one grain of sand can make a heap into a non-heap, one can repeatedly remove all but one grain of sand from something that is clearly a heap and be forced to acknowledge that the remaining single grain of sand is still a heap; otherwise, it seems, one would have to accept that there is a determinate number of grains that forms a heap, and anything under it is not a heap. Neither choice is, however, intuitively satisfying. The paradox is interesting because it can be made general and re-applied to many other words besides 'heap'. Predicates for which one can find a suitable instance of the general formulation of the sorites paradox are called *vague*. Paradigmatic examples besides 'heap' include 'tall', 'red', 'bald', 'tadpole', and 'child' (Keefe and Smith 1997). How widespread is the problem? It is easy to find more examples of predicates based on more finely grained properties—as 'tall' is intuitively based on height—for which constructing a sorites paradox would be easy. Mereological nihilists argue that instances of the sorites paradox can be designed for any material object that can be decomposed into small enough parts. If one subscribes to the scientific picture of matter as composed of molecules and atoms, this applies to tables and chairs, cats and mats, and any other ordinary thing (Unger 1979). Bertrand Russell famously argued (Russell 1923) that all words, including "the words of pure logic," are vague when used by human beings.

If one thinks of language as governed by logical rules, and of rationality as including the ability to follow those rules, the sorites paradox seems *prima facie* to demonstrate that vagueness and rationality are incompatible. But one can think of language in different terms. One possibility is to think of signs as tools agents use to coordinate actions. An example of a way to formally study language along those lines is the game-theoretic framework of signaling games (Lewis 1969/2011). Within such an approach, rationality can be seen as the ability to choose the use of signs that is optimal to achieve some form of coordination. The existence of vagueness in such models would not be at odds with rationality as long as vague languages turn out to be optimal for the purposes at hand. However, as Lipman (2009) argues in detail, this is typically not the case. The problem can be put very succinctly as follows. In standard game-theoretic models of communication, vague signal use yields a lower expected utility than crisp use. Therefore, given that the dynamics (be it natural selection, cultural evolution, or rational choice) maximize utility, vagueness should be weeded out by these forces, giving rise to only precise languages. Thus it would be irrational to stick to vague language use when one could (theoretically) switch to a better system. But vagueness is pervasive in natural language, and there is no reason to believe it is going away. The problem seems to be a theoretically serious one, but it does not obviously undermine our everyday linguistic practices. Most of the time we seem to communicate just fine. Therefore, the issue must lie with the conceptions we have of the forces or mechanisms that underlie those practices. Lipman concludes that "we cannot explain the prevalence of vague terms in natural language without a model of bounded rationality which is significantly different from anything in the existing literature" (Lipman 2009, p. 1).

Here we survey proposals for addressing vagueness in the context of signaling games, but before we start we want to establish some vocabulary to better frame the upcoming discussion. Rationality is an elusive notion that is frequently debated

in areas like philosophy, psychology, and economics. Discussions around it usually touch on different aspects of the concept without always clearly demarcating them; this is what we want to do before going further. The aforementioned logical picture of language and meaning is focused on dichotomies like true and false, meaningful and meaningless, correct and incorrect. A sentence can be true or false only if it is meaningful, and it is meaningful if it is constructed according to correct rules. Rationality is intimately connected with the ability to follow certain procedures, not only of sentence production, but ultimately of sentence combination and reasoning (think of what is required for making a logically valid deductive inference). This is an example of what we will call a *procedural* account of rationality, one which focuses more on the means rather than the ends. One can also do the opposite and focus on the consequences instead. *Instrumental* rationality, as it is typically called, characterizes an agent's choice as rational if it maximizes the possibility of achieving a desired goal, regardless of the means. The notion is linked to Hume and Levine (1738/2005), epitomized in the following assertion: "Reason is, and ought to only be the slave of the passions." Instrumental rationality is close to a notion of rational choice that is used in economics and game theory[1]: agents are rational if and only if they make decisions that maximize their expected utility. A more in-depth discussion of the opposition in the context of theoretical economics can be found, for example, in the work of Simon (e.g. 1986).[2]

When developing models where rationality is relevant, be it constructing a logical system or setting up a formal game, the assumed epistemic relation between agents and their environment can come to bear on considerations of rationality. The verdict over how rational certain choices are can vary depending on how accurate and complete an agent's knowledge is of its environment, the goals to be achieved, the choices or rules available, the relation between those choices and the objectives, and so forth. In this respect, we will call an agent *omniscient* if it is in possession of the same information as the modeler, whereas of an agent with less than that we will say that it only has *limited* awareness of the relevant aspects of the model. Models working within the logical picture typically do not make a distinction between modeler and agent, and thus lack room to express these epistemic gaps. By abstracting away from language users, these models also typically do not represent potential interactions between them, let alone allow for repeated interaction and language change. In other types of models, however, a further aspect of rational choice needs to be considered, namely the ability of agents to make accurate predictions about how other agents behave. We can say that an agent is more or less *strategic* depending on the extent to which she is able to anticipate the actions and beliefs of other agents, and to predict medium/long term gains from repeated play. Lack of perfect knowledge of the situation or lack of ability to choose strategically can be caused by many possible factors. These include, among other things, limitations in handling informa-

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[1] In fact, it has been argued (Vanderschraaf 1998) that Hume's whole account of convention is very closely in line with modern game theory.

[2] Simon uses the term substantive instead of instrumental rationality, but the characterization is basically the same (Simon 1986, pp. 210–212).

tion (receiving, storing, retrieving, transmitting) and limited computational resources to solve complex problems. We can talk about *bounded* rationality to characterize the choices of such agents. While the above definition of instrumental rationality is usually understood as requiring a single choice to maximize the expected utility in a single concrete decision situation, one might also be interested in more general choice mechanisms (e.g. Zollman and Smead 2010; Hagen et al. 2012; Fawcett et al. 2013; Galeazzi and Franke 2017). A choice mechanism is a general way of behaving for an agent involved in a variety of decision situations. When considering only a single situation, rationality can only be *local*. If we take into account the possibility of the agent's choice or choice mechanism hinging on multiple situations, we can also talk about *global* rationality. This can be important because, hypothetically, there could be choice mechanisms that are sub-optimal at a local level (for each situation) but are actually perfectly rational at a global level.

Note that we consider that, in theory, most combinations of these aspects are possible. Although we have been pinning procedural rationality to the logical picture of language, this is only with the most traditional logical systems in mind. We are not denying that advances in dynamic, epistemic, fuzzy, paraconsistent, and other types of logic could potentially enable one to capture procedural rationality with different characteristics. Game-theoretical models of language can, on the other hand, also combine local instrumental rationality with omniscient highly strategic agents. Our objective here is not to survey all the possibilities. We want to focus on signaling games as a framework for the study of language use and meaning. The vocabulary just introduced will, we hope, help inform the discussion that follows. We proceed by introducing the framework of signaling games in Sect. 2. In Sect. 3 we look into explanations of vagueness in a particular kind of signaling game. Section 4 tries to generalize the considerations of these proposals to argue for an approach to vagueness anchored in a more global notion of rationality. We propose and analyse the results of a novel multi-population model of imprecise imitation in Sect. 5, and summarize our conclusions in Sect. 6.

2 Signaling Games

Signaling games were first introduced as models of communication by Lewis (1969/2011). In order to support the idea that linguistic conventions can arise without any prior conventional activity, Lewis considers situations where agents' choices involve sending and receiving signals or messages.[3] One can think of two players with different roles. The first player, the sender, has knowledge about which of a number of possible states of affairs obtains and, depending on this information, chooses a signal to send. The second player, the receiver, has knowledge about which signal the sender chose and, based on this information, chooses one of several possible responses. A preference relation exists between responses and states of affairs, and

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[3]These terms will be used interchangeably.

¹⁴⁵ a payoff is attributed to each player based on the choices of both. Note that Lewis
¹⁴⁶ assumes that no player has any preference regarding the particular signal that is used
¹⁴⁷ for a given state, provided that it enables advantageous coordination with responses.
¹⁴⁸ Formally, in order to describe the setup all we need is to specify a set of possible states
¹⁴⁹ of affairs T, a probability measure P such that $P(t)$ is the probability or frequency
¹⁵⁰ with which $t \in T$ occurs, a set of available signals or messages M, a set of responses
¹⁵¹ or actions A, and a pair of utility functions $U_{S,R} : T \times A \to \mathbb{R}$, one for the sender
¹⁵² and one for the receiver, each of which yields a payoff value for each possible pairing
¹⁵³ of state and action. These so-called signaling problems can be seen as particular cases
¹⁵⁴ of coordination problems if we consider the players' choices to be of contingency
¹⁵⁵ plans or strategies. A sender strategy is a specification of a choice of message for
¹⁵⁶ each possible state of affairs. It thus describes the sender's behavior conditional on
¹⁵⁷ the state of affairs that obtains. A receiver strategy analogously specifies a choice of
¹⁵⁸ action for each possible message. Formally, what the sender chooses is a function
¹⁵⁹ $\sigma : T \to M$ and the receiver a function $\rho : M \to A$. The expected utility EU of a
¹⁶⁰ strategy can be calculated by using the utility function and aggregating payoffs for all
¹⁶¹ pairings of states of affairs and actions, weighted by the probability of each state. Con-
¹⁶² cretely, the expected utility of σ given ρ is $\mathrm{EU}_S(\sigma \mid \rho) = \sum_{t \in T} P(t) U_S(t, \rho(\sigma(t)))$,
¹⁶³ and the expected utility of ρ given σ is $\mathrm{EU}_R(\rho \mid \sigma) = \sum_{t \in T} P(t) U_R(t, \rho(\sigma(t)))$.
¹⁶⁴ As an example, consider a game with $T = \{t_1, t_2\}$, $M = \{m_1, m_2\}$, $A = \{a_1, a_2\}$,
¹⁶⁵ $P(t_1) = P(t_2) = 0.5$ and the following utility matrix:

¹⁶⁶
$$
\begin{array}{c|c|c}
 & a_1 & a_2 \\
\hline
t_1 & 1, 1 & 0, 0 \\
\hline
t_2 & 0, 0 & 1, 1 \\
\end{array}
$$

¹⁶⁷ Consider sender strategy $\sigma = \{t_1 \mapsto m_2, t_2 \mapsto m_1\}$ and receiver strategy $\rho = \{m_1 \mapsto$
¹⁶⁸ $a_2, m_2 \mapsto a_1\}$. These would have an expected utility of 1 for both sender and
¹⁶⁹ receiver, since when t_1 obtains with probability 0.5 the sender will use m_2, and
¹⁷⁰ to this message the receiver will respond with a_1, which achieves a payoff of 1,
¹⁷¹ and when t_2 obtains with probability 0.5 the sender will use m_1, and to this mes-
¹⁷² sage the receiver will respond with a_2, which also achieves a payoff of 1. They
¹⁷³ also represent one of the two stable conventions in this game, the other being the
¹⁷⁴ pair of strategies $\sigma = \{t_1 \mapsto m_1, t_2 \mapsto m_2\}$ and $\rho = \{m_1 \mapsto a_1, m_2 \mapsto a_2\}$. Con-
¹⁷⁵ ventions of this kind in a signaling problem are what Lewis calls *signaling sys-*
¹⁷⁶ *tems*. An example of complete miscoordination would be $\sigma = \{t_1 \mapsto m_1, t_2 \mapsto m_2\}$
¹⁷⁷ and $\rho = \{m_1 \mapsto a_2, m_2 \mapsto a_1\}$. Partial coordination is achieved, for example, by
¹⁷⁸ $\sigma = \{t_1 \mapsto m_1, t_2 \mapsto m_1\}$ and $\rho = \{m_1 \mapsto a_1, m_2 \mapsto a_2\}$.

¹⁷⁹ Lewis' account of the stability of conventions rests on what could be consid-
¹⁸⁰ ered strong demands. Namely, there needs to be a state of affairs that indicates to
¹⁸¹ everyone involved that a certain regularity will hold, as well as "mutual ascription of
¹⁸² some common inductive standards and background information, strategic rationality,
¹⁸³ mutual ascription of strategic rationality, and so on" (Lewis 1969/2011, pp. 56–57).
¹⁸⁴ Agents are thus envisioned as omniscient and highly strategic. These requirements
¹⁸⁵ can seem excessive, and even more so if we consider how simple signaling sys-
¹⁸⁶ tems are when compared to human languages. The models were introduced to help

187 explain how language could have gotten off the ground as a conventional system
188 without any sort of prior agreement. However, if one considers the circumstances
189 of the origins of human language, it seems implausible that the agents that started
190 making use of primordial signaling systems which (hypothetically) evolved into lan-
191 guages possessed such advanced rationality. Furthermore, communication through
192 simple message exchange is something that almost all animals do: monkeys use calls,
193 birds use singing, bees use dances, ants use pheromone trails, and so on. A plausible
194 account of the origin of language should first explain how signaling systems could
195 get started, without requiring high standards of rationality from the agents involved.

196   In order to address this problem, Skyrms (1996) proposes studying signaling
197 problems in evolutionary terms. Rather than imagining, as Lewis does, rational agents
198 making conscious decisions in possession of knowledge of the game and expectations
199 of the behavior of other agents, one can imagine a simpler scenario inspired by
200 biological evolution: there is a population of agents with biologically hardwired
201 behaviors for engaging in interactions characteristic of a signaling problem; utility
202 does not represent preference, but rather fitness for survival and reproduction; the
203 make-up of the population evolves based on the relative fitness of the strategies
204 represented in the population. Such a setup attempts to capture the main features
205 of natural selection: in a diverse population, agents with more successful strategies
206 thrive, while agents with less fit strategies die off. Although the inspiration for this
207 scenario is biological evolution, similar things could be said about how ideas spread in
208 a population of agents who can adopt or abandon them depending on how successful
209 they prove to be (e.g. Benz et al. 2006; Pagel 2009; Thompson et al. 2016), i.e. we
210 can interpret these notions in terms of cultural evolution (Dawkins 1978; Boyd and
211 Richerson 1985). The principles can be captured in formal models that abstract away
212 from details of single interactions and behavior of individual agents, for example in
213 the replicator dynamics (Taylor and Jonker 1978). The only things relevant to this
214 equation are the relative proportions of strategies in a given population and the utility
215 function. Using it, one can compute which strategies evolve under which conditions.

216   Skyrms' evolutionary game theory approach to signaling games not only gives
217 more plausible grounds to support Lewis' discussion of convention, it also accom-
218 plishes an important conceptual change: it moves most of the theory and mathe-
219 matical formalism to the descriptive side of the investigation. Utility represents how
220 the modeler views the signaling problem and understands the relative advantages or
221 disadvantages of different possible strategy combinations. Dynamics describe how
222 strategies can evolve when driven by mechanisms of utility maximization. The shift
223 in perspective allows interpretations that accommodate limited non-strategic agents.
224 While the general framework manages to abstract quite some details away from the
225 formalization, it nevertheless leaves room for them, especially when it comes to the
226 dynamics. We have already mentioned the replicator equation that can be seen as
227 representing biological or cultural evolution, but one can also use dynamics inspired
228 by learning mechanisms (e.g. Roth and Erev 1995), or even ones assuming a high
229 degree of knowledge of the game and other players (e.g. Gilboa and Matsui 1991;
230 Mühlenbernd 2011; Spike et al. 2017). This range of options goes hand in hand with
231 a range of pictures of rationality, from nothing more than survival of the fittest in a

biologically-inspired setting, to a certain degree of instrumental but limited and non-strategic rationality in the case of learning dynamics, to higher levels of rationality and even recursive strategic reasoning about the co-players' beliefs and choices. Each of these can be utilized depending on the problem that one is interested in characterizing. Thus, although Skyrms shows that high requirements of rationality are not necessary for signaling conventions to evolve, the framework does leave room for the study of linguistic interactions between highly strategic agents.

The characterization of signaling problems in terms of evolutionary game theory allows us to explain why certain equilibria come to be and how. A core notion in this context is that of an *evolutionary stable state* (Maynard Smith 1982): an equilibrium situation that a population tends to under standard evolutionary pressures, and to which it returns if slightly disturbed. With these tools, one can better understand why signaling systems are stable even without any strong assumptions of rationality. One can also map out which initial conditions drive the system towards which equilibria and which do not. In a simple case like the example discussed above, an evolutionary process of the kind described always drives the population into a state where one signaling system takes over completely. More complex signaling problems may have different evolutionary outcomes, sometimes unexpected ones. Skyrms (2010) gives an overview of different topics studied using signaling games, including expansions of the framework itself (for example, considering other dynamics beyond the replicator equation), exploration of other factors that impact the evolution of signaling (for example, how agents are interconnected), or variations on the signaling problem and its basic assumptions (for example, loosening the alignment of interests in order to provide accounts of deceptive signal use). Other uses of signaling games include discussions of categorization (e.g. Jäger and Rooij 2007), compositionality (e.g. Barrett 2009), incommensurability (e.g. Barrett 2010), to name a few. More recent overviews are given by Huttegger (2014), Huttegger et al. (2014), and Franke and Wagner (2014). In the following section, we discuss how a particular type of signaling game has been used to address the problem of vagueness.

3 Vagueness in Sim-Max Games

The sorites paradox requires us to assume a relation between the vague terms and a more precise underlying dimension (height for tallness, number of hairs for baldness, number of grains of sand for "cheapness", and so on). Not only does this property need to be much more fine-grained than the vague term, but it also needs to have some structure: there is at least an order between the elements in it (thinking of height in centimeters, $180 > 179 > \cdots > 120$), and usually even a degree of how far apart these elements are from each other. In terms of signaling games, one can model this using a state space constituted of values of the underlying dimension, and a message space constituted by the terms in question. Because of the difference in granularity, we will typically be interested in cases where the state space is much larger than the message space. We can model the structure of the state space by defining a

273 distance or similarity function between every value, effectively making it a metric
274 space. Another important ingredient of the sorites paradox is the acknowledgment
275 of a certain degree of tolerance with respect to whether a certain term applies or not.
276 This tolerance decreases with distance in state space: assuming a 180 cm person is
277 tall, one would easily tolerate the use of the term for a person measuring 179 cm, less
278 so for someone who is 170 cm, and much less so for 160 cm. This can be modeled
279 using a utility function that is continuous rather than discrete and that monotonously
280 decreases with distance, i.e. success is not a matter of black and white, right or wrong,
281 but a matter of degree, of how close the receiver got to the optimal response to the
282 sender's perceived state.

283   The simplest type of game to study in this scenario is one where the state space
284 and the action space are the same. We can imagine this as a game of guessing states
285 of affairs: the sender has knowledge of a particular state, sends a message to the
286 receiver, who in turn has to guess it; their payoff, as discussed above, is proportional
287 to how close the guess got to the original state. These games, called similarity-
288 maximization or sim-max games for short, were first introduced by Gerhard Jäger
289 and Robert van Rooij (Jäger and Rooij 2007; Jäger 2007) and further studied by Jäger
290 et al. (2011). What these authors find about this setup is that the evolutionary stable
291 states are what they call Voronoi languages. Roughly, these are situations where the
292 sender uses messages in a way that can be seen as partitioning the state space into
293 convex regions, and the receiver responds with the central element of those regions.

294   In an abstract setup, using 50 states uniformly distributed over the unit interval and
295 two possible messages, such an optimal language looks like what we see in Fig. 1a:
296 at a specific point, the probability that the sender uses one message decays sharply
297 from 1 to 0, and increases sharply from 0 to 1 for the other message; the response
298 of the receiver for each message is a degenerate distribution over the state space
299 which assigns all probability mass to a single state. These strategies give mutually
300 optimal behavior for a case where the prior probability is the same for each state
301 and utility is a linear or quadratic function of the distance between the actual state
302 and the receiver's guess. In general, at which point the sender switches the use of

(a) Crisp   (b) Vague
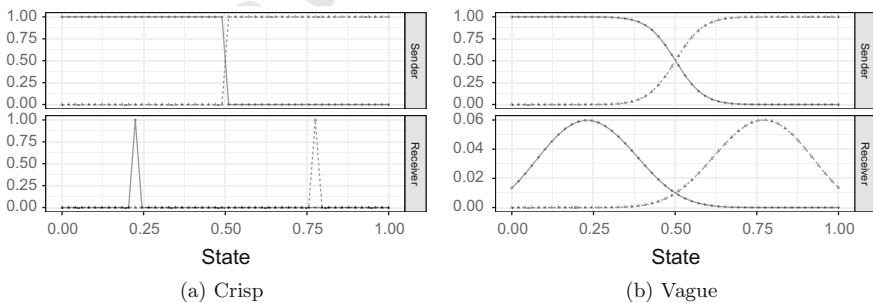
Fig. 1 Example strategies for a state space with 50 states. Each line corresponds to a message. For the sender, it plots for each state the probability that the message is used. For the receiver, it plots for each state the probability that the response is that state, given the message

303 messages and which guesses of the receiver are optimal critically hinges on the priors
304 over states and the utility function. Still, confirming Lipman's argument, there is no
305 vagueness in such optimal languages.

306 The sender/receiver strategy pairs that we are looking for look more like Fig. 1b,
307 where the sender's probability of choosing a certain message gradually changes,
308 and the receiver strategy assigns a positive probability to more than one state for
309 each message. What characterizes vagueness in these models is thus a smooth and
310 monotonous transition between parts of the state space where a message is clearly
311 used and where it is clearly not. This means that, for some states in the middle of
312 the state space, there is uncertainty as to which message will be used by the sender,
313 whereas for states in the extremes this is as good as certain. For the receiver part,
314 there is uncertainty as to which state will be picked for each message.

315 The interpretation of this uncertainty will be different depending on the interpre-
316 tation of the model. If we see it as an explanatory model of how two agents play the
317 game, we can see it as randomization. If we interpret the model as descriptive, it sim-
318 ply represents expected behavior in a manner agnostic to the underlying mechanisms.
319 A third option is to see probabilities as capturing relative numbers in a population of
320 agents. For example, if the sender strategy assigns a probability of 0.4 to the event
321 of message m being sent for state t, this would mean that 40% of the population uses
322 that message for that state. This latter option leaves open the possible interpretation
323 that each agent commands a crisp language and vagueness is only a population-level
324 effect. However, given the level of abstraction of the description so far, none of this
325 is necessarily implied by the model. Our preferred stance is to see the model as
326 descriptive, but we do not wish to focus on that debate here. The question we want to
327 address is: what additional modifications to sim-max games are sufficient for optimal
328 languages to be more like Fig. 1b, rather than like Fig. 1a?

329 Franke et al. (2011) make two suggestions as to how vagueness in a signaling
330 game can be boundedly rational, i.e. how vagueness could arise as a consequence of
331 cost-saving limitations in the cognitive capacities of instrumentally rational agents.
332 The first proposal is called limited memory fictitious play (LMF) and models agents
333 playing a sim-max game where their ability to recall past interactions with others
334 is limited to a certain number. For a given interaction, each agent uses her limited
335 memory of the other agent's past behavior to estimate the other's strategy, and plays
336 an instrumentally rational best response to that strategy. In order to study the evolution
337 of strategies in repeated interaction, the authors model several individual agents in
338 actual play. What they observe is the emergence of vague signaling at the level of the
339 population, i.e. population averages of individual strategies exhibit the hallmarks of
340 a vague language. This is because each agent recalls a different history of previous
341 play and so holds slightly different beliefs about language use. However, each agent
342 still commands a crisp language, which is an inadequate feature of the model if the
343 intention is to capture how vagueness presents itself in human languages.

344 In order to overcome this limitation, Franke, Jäger, and van Rooij make another
345 proposal using the notion of a quantal response equilibrium (QRE). The idea, inspired
346 by experimental psychology, is to model the choice of best response as stochastic

347 rather than deterministic.[4] A prominent explanation for such soft-max or quantal
348 choice behavior is that agents make small mistakes in the calculation of their expected
349 utilities (Train 2009). They still choose the option with the highest expected utility,
350 but each assessment of the expected utilities is noise perturbed. This, in turn, may
351 actually be boundedly rational since the calculation of expected utilities relies on
352 assessing stochastic uncertainty, which in turn may be costly to calculate precisely.
353 Choice based on a few samples from a distribution can be optimal if taking more sam-
354 ples or other means of better approximating probabilistic beliefs is resource costly
355 (e.g. Vul et al. 2014; Sanborn and Chater 2016). The degree to which agents tremble
356 in the calculation of expected utilities and therefore deviate from the instrumen-
357 tally rational behavior can be characterized by a parameter. Franke, Jäger, and van
358 Rooij find that for low values of this parameter, only babbling equilibria are possible,
359 where sender and receiver simply randomize, respectively, message and interpreta-
360 tion choice uniformly. Above a certain value of the parameter, other equilibria of
361 the kind described in the beginning of this section arise, where agents communicate
362 successfully, though not perfectly, using fuzzy strategies similar to those depicted in
363 Fig. 1b. However, it is not clear whether soft-max choices capture the right stochastic
364 trembles in decision making as they would arise under natural sources of uncertainty
365 about the context (see Franke and Correia 2017).

366   O'Connor (2014) proposes a way in which vagueness could be expected to evolve
367 as a side-effect of a particular type of learning process. She studies sim-max games
368 driven not by rational choice dynamics, but by generalized reinforcement learning
369 (GRL), a variant of Herrnstein reinforcement learning (HRL) (Roth and Erev 1995).
370 In HRL, agents learn to play a signaling game by strengthening particular choices
371 (of messages for the sender, of responses for the receiver) proportionally to how
372 successful those choices prove to be in an interaction. O'Connor's proposal is to
373 model generalization as the propagation of reinforcement to nearby states, where
374 "nearby" is defined in terms of distance in state space. For example, if a sender was
375 successful in using message m for state t, she will not only positively reinforce that
376 choice of message for t, but also for states similar to t. This is done to a degree that
377 is proportional to the similarity between t and other states. The dynamics gives rise
378 to vague signaling of the kind we are looking for.

379   Although there is a close relationship between reinforcement learning and
380 population-level dynamics (Börgers and Sarin 1997; Beggs 2005), O'Connor's GRL
381 is, on the face of it, an account of learning between individual players. Also, we
382 need further justification for linking generalization in reinforcement so closely to
383 the underlying payoff function of the sim-max game. Why should agents evolve
384 to generalize in exactly the right way? O'Connor suggests that, despite a language
385 with vagueness having lower expected utility than a language without it, the learning
386 mechanism that induces vagueness does have evolutionary advantages: it achieves
387 higher payoffs in a shorter period of time. From a global point of view, learning speed
388 can be an advantage. Imagine an initial population of agents with random strategies,

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[4]Probabilistic choice rules are also the source of vagueness in recent accounts by Lassiter and
Goodman (2017) and Qing and Franke (2014).

some using GRL and others using classical HRL. Although the latter type of agent can hypothetically develop a precise and more efficient signaling system, agents using GRL could coordinate on vague signaling strategies with high (though not optimal) expected utility sooner than agents using HRL. In such a scenario, they could drive the other agents to extinction before the latter had time to achieve coordination and reap the benefits of a more precise signaling system.

A similar finding is made by Franke and Correia (2017) when studying a variant of the replicator dynamics in which individual agents do not have the ability to generalize but simply make perceptual mistakes. In a scenario where agents learn by imitation, if one assumes that they do not have perfect perception, there will always be the possibility for senders to confuse states and thus learn associations with messages that are different than the ones observed, and for receivers to similarly mix up responses. Furthermore, it seems reasonable to assume that this confusability is proportional to state similarity, i.e. that the more similar two states are, the more likely it is that they will be mistaken for each other. Incorporating these considerations into a derivation of the replicator dynamics based on imitation processes, Franke and Correia develop a variant of the dynamic that also induces vague signal use of the kind we expect here. The consequence is very similar to that of the GRL model discussed above, in that the way the behavior for a given state is updated takes into account the behavior of similar states, proportionally to their similarity. Given the known relation between reinforcement learning and the replicator dynamics (Beggs 2005), it is actually quite plausible that the two are tightly related (although this would need to be formally demonstrated). The account is, furthermore, interpretable at a lower level of rationality.

The motivation underlying this model of vague signaling is still one of inevitability. A vague strategy is not claimed to have higher expected utility than a crisp one. However, the authors observe an effect similar to that pointed out by O'connor: signaling converges faster and more often in scenarios where there is some degree of state confusability. Furthermore, they observe one additional potentially beneficial property. Running several rounds of simulation for each parameter set, they measure for each group of results how close resulting strategies are to each other, and how they would fare playing against one another. The results show that the within group distance between strategies becomes smaller with growing confusability, and the within group expected utility is actually higher for strategies evolving under a certain degree of state confusion. Thus, some amount of uncertainty seems to promote more homogeneous populations of signalers that are better at achieving cooperation within a group. What is left to show, as it was with O'Connor's GRL approach, is whether the potential payoff advantages that were observed in simulations actually suffice to promote vague language use in an encompassing model of multi-level selection. The following section motivates the need for taking a more ecological approach to the evolution of vagueness, before Sect. 5 gives a concrete model.

4 The Ecology of Vagueness

Arguments of the kind presented by Lipman (2009), that vague signal use is sub-optimal when compared to crisp use, work under various assumptions. Part of the picture formed by these assumptions is a highly idealized conception of the agents involved and of the context in which they develop and use signals. These idealizations probably originate, via game theory, from the conception of rationality of traditional theoretical economics. Herbert A. Simon describes this picture as follows:

> Traditional economic theory postulates an "economic man," who, in the course of being "economic" is also "rational." This man is assumed to have knowledge of the relevant aspects of his environment which, if not absolutely complete, is at least impressively clear and voluminous. He is assumed also to have a well-organized and stable system of preferences, and a skill in computation that enables him to calculate, for the alternative courses of action that are available to him, which of these will permit him to reach the highest attainable point on his preference scale (Simon 1955, pp. 99).

Both Simon and Lipman call for this picture to be revised, and this is what the proposals surveyed here all do. In order to account for vagueness in natural language in the context of these models, they peel away from this idealized picture and bring some of these assumptions down to earth. In the process, they point to ways in which we, as language learners and language users, are finite beings finding ways to cope with a highly complex and dynamic environment:

1. Our existence is temporally finite; language does not have an infinite amount of time to evolve, nor can it take an infinite time to be learned. The faster a language can start being useful, the better;
2. Language learning through experience has to rely on a limited number of observations. Not only is the state space typically much larger than one can survey in sufficient time, it is even potentially infinite and constantly changing;
3. A corollary of the former is that there will always be heterogeneity in a population of language learners, at the very least in their prior experience, since each agent will have relied on a different set of observations. Furthermore, this information is not directly or fully accessible to others.

All of these observations support the weakening of modeling assumptions. The research surveyed here shows us some examples of assumptions which, when weakened, make vague signal use a natural outcome of certain evolutionary dynamics. But it gives us even more. It suggests ways in which the mechanisms that lead to vagueness can have positive effects that are extremely important in the context of the points just enumerated. We learned from O'Connor and Franke and Correia that vague languages are quicker to converge and adapt, which is valuable given the finite and dynamic character of our experience (point 1). O'Connor also showed how generalization, an invaluable feature of any procedure for learning from a limited number of observations (point 2), leads to vagueness. We also learned from Franke and Correia that state confusability, a mechanism that leads to vague signal use, can have a homogenizing effect on vocabularies, potentially compensating for the heterogeneity of agents' experiences (point 3).

What do these observations tell us about rationality? GRL (O'Connor 2014) and the work of Franke and Correia (2017) both assume a picture of agents with a basic level of instrumental rationality, possibly limited awareness of the game, and a lack of strategic capabilities, adapting their behavior with only short-term gains in sight. These approaches introduce constraints on agent behavior or information processing that prevent the evolution of crisp signal use. But a crisp language would still have a higher expected utility than the evolved strategies. Agents in those models seem to be only as rational as the modelers allow them to be. Despite the plausibility of the mechanisms proposed (limited memory, imprecise calculation of expected utilities, generalization, state confusability), the results of these models feel somewhat bitter-sweet because of the hypothetical possibility of an ideal strategy, seemingly barred from the agents in an artificial manner. Couldn't a more rational agent evolve and drive the system into crisp signal use? Aren't we, human beings, that kind of agent?

Perhaps a deeper understanding of vagueness and the reasons for its pervasiveness in natural language are to be found only when we broaden the scope of the models employed. All the models discussed so far explore evolutionary dynamics for one homogeneous population playing one game. Different types of agents and different game setups are considered, but each of these different possible scenarios is tested separately. We see at least two ways in which one could embrace a more ecological perspective. The first is to think about meaning and vagueness from a more Wittgen-steinian perspective. One can see each signaling game as embodying a particular language game. In Wittgenstein's picture of language, however, we do not play only one language game over the course of our existence; there is a plurality of them and which one an agent is engaged in at a particular moment is never clearly identified, neither are the exact benefits one might gain by choosing a certain behavior over another. These are furthermore not fixed in time; old language games fall out of fashion or stop being useful, and new ones emerge all the time (see Wittgenstein 1953/2009, and in particular §23).

We can look for rationality at several levels in this pluralistic picture. First, as before, there is the actual behavior of a single agent in each actualized language game. As mentioned above in connection the soft-max choice function used by Franke et al. (2011), behavior that strictly maximizes expected utility under uncertainty may be resource heavy, so it might be compatible with local strategic rationality that agents' production choices are stochastic. Second, if we look at behavior across many game types and contexts, there is also the level of an agent's internal theory of how words and phrases are likely to be used (or even normatively: how messages should be used), conditional on a given context. Notice that a single agent's rational beliefs about linguistic practices or linguistic meaning may well have to reflect the actual stochasticity: under natural assumptions about information loss, the best belief for prediction matches the actual distribution in the real world (e.g. Vehtari and Ojanen 2012). In sum, both at the level of behavior and at the level of beliefs about use or meaning, we should expect to find vagueness. Still, despite the natural vagueness, there does not seem to be anything fundamentally missing or conceptually incoherent in a naturalistic, rationality- or optimality-driven explanation of what each agent is doing or what each agent believes about language, use and meaning.

518   Another way to go beyond locality is to work with more heterogeneous popu-
519 lation models. The mechanisms that lead to vague signal use, as O'Connor (2014)
520 and Franke and Correia (2017) stress, have the aforementioned important advantages
521 of faster speed of convergence, higher flexibility, and homogenization. The argument
522 goes that these side-effects, by temporarily enabling a higher expected utility, could
523 allow a population using some generalization (or affected by some imprecision) to
524 take over. However, despite its intuitiveness, the argument is based on comparing
525 isolated runs of different dynamics. The models do not allow the hypothesis to be
526 tested, because they do not accommodate different populations evolving together. In
527 the remainder of the paper we propose a way to do this based on the model of Franke
528 and Correia (2017). We introduce two variants of a multi-population model of the
529 imprecise imitation dynamics, where populations characterized by different impre-
530 cision values interact and evolve together. Using this model, we can better see under
531 which conditions the hypothesized advantages of some imprecision can lead to the
532 evolution of vagueness.

5 Two Multi-population Models of Imprecise Imitation

534   We build upon the model of Franke and Correia (2017), adding to the imprecise imi-
535 tation dynamics support for multiple populations with different imprecision values
536 evolving together. The resulting evolutionary dynamic has two layers: first, signaling
537 strategies change by imprecise imitation following the model of Franke and Correia
538 (2017); second, there is evolutionary selection at the level of the imprecision values
539 themselves. More concretely, imagine agents who are born with varying perceptual
540 abilities, subsequently learn a signaling strategy by imitation of other agents (either
541 within or across populations), and depending on the success of the strategies they
542 develop, are more or less likely to survive and reproduce. In such a setup, evolution-
543 ary processes will not only change the distribution of strategies within a population,
544 where a population is identified by a shared level of imprecision in imitation, they
545 will also promote those levels of imprecision which result in the development of
546 more successful signal use.
547   This is closely related to the ideas behind theories of kin selection (Hamilton 1964)
548 and multi-level (or group) selection (Wilson 1975). These theories build upon the
549 hypothesis that selection acts not only to directly favor genes that result in behaviors
550 that benefit individuals, but also to indirectly favor genes that lead to behaviors that
551 benefit either genotypically (kin) or socially (group) related individuals (see Okasha
552 2006, for more details). In our model there is a similar structure: there is a process of
553 selection of behavior within each population, and levels of imprecision are selected
554 across populations based on the strategies they give rise to. We believe there is an
555 important difference with group and kin selection models in that the two selection
556 processes in our model act on different entities: the inner shaping signaling behavior,
557 and the outer selecting levels of imprecision. In any case, we intend our model to
558 be descriptive rather than causal or explanatory. That means that we do not want

to commit to seeing populations either as kin or as social groups, and use them as merely descriptive abstractions that allow us to capture the hypothetical impact of indirect selection processes.[5]

In the following, we lay down the formal details of our model. Let's start by defining A to be the set of imprecision values considered. For each value $\alpha \in A$, the proportion of its population is given by $P(\alpha)$, such that $\sum_{\alpha \in A} P(\alpha) = 1$. Each population has its own sender and receiver strategies, represented as σ^α and ρ^α. The probability that a given agent with imprecision α (or of type α) observes t_o when the actual state is t_a is given by $P_o^\alpha(t_o|t_a)$. If the same agent intends to realize t_i, the probability that she actually realizes t_r instead is given by $P_r^\alpha(t_r|t_i)$. Following Franke and Correia (2017, p.26), we can then define the following values.

Probability that t_a is actual if t_o is observed by an agent of type α:

$$P_{\hat{o}}^\alpha(t_a|t_o) \propto P_a(t_a) P_o^\alpha(t_o|t_a)$$

Probability that a random sender of type α produces m when the actual state is t_a:

$$P_\sigma^\alpha(m|t_a) = \sum_{t_o} P_o^\alpha(t_o|t_a)\sigma^\alpha(m|t_o)$$

Probability that the actual state is t_a if a random sender of type α produced m:

$$P_{\hat{\sigma}}^\alpha(t_a|m) \propto P_a(t_a) P_\sigma^\alpha(m|t_a)$$

Probability that t_r is realized by a random receiver of type α in response to message m:

$$P_\rho^\alpha(t_r|m) = \sum_{t_i} P_r^\alpha(t_r|t_i)\rho^\alpha(t_i|m)$$

These formulations are merely parameterized versions of the single-population model. They encapsulate calculations that one can use to compute expected utilities and strategy update steps for each type. The latter, however, depend on the types of interaction that we imagine occurring between populations. In the following sections, we consider two different possibilities.

5.1 Tight Population Interaction

In a multi-population model with tight interaction between populations, each agent plays with, observes, and potentially imitates any other agent, regardless of their type. This has an impact on the expected utilities of sender and receiver strategies of

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[5]We make this note because of the heated debate between the two theories. See Kohn (2008), and Kramer and Meunier (2016) for more details.

each type, and on the update steps for those strategies. Let's start with the expected utilities. For a sender of type α, the expected utility of its strategy σ^α against all other receiver strategies $\rho^\star$, is given by:

$$\mathrm{EU}^\alpha_\sigma(m, t_o, \rho^\star) = \sum_{t_a} P^\alpha_{\check{o}}(t_a|t_o) \sum_{\alpha' \in A} P(\alpha') \sum_{t_r} P^{\alpha'}_\rho(t_r|m) U(t_a, t_r)$$

and, for a receiver of type α, the expected utility of its strategy ρ^α against all other sender strategies $\sigma^\star$, is given by:

$$\mathrm{EU}^\alpha_\rho(t_i, m, \sigma^\star) = \sum_{\alpha' \in A} P(\alpha') \sum_{t_a} P^{\alpha'}_{\check{\sigma}}(t_a|m) \sum_{t_r} P^\alpha_r(t_r|t_i) U(t_a, t_r)$$

576 Expected utilities thus take into account the existence of strategies of other types,
577 and weigh the relevance of each type α' according to its relative proportion $P(\alpha')$.

Another important value to calculate has to do with the types that agents observe and imitate. In a model with tight interaction, we imagine this occurring across populations. Therefore, we can define the probability that a sender of type α observes a randomly sampled agent play message m for observed state t_o as:

$$P^\alpha_o(m|t_o) = \sum_{t_a} P^\alpha_{\check{o}}(t_a|t_o) \sum_{\alpha' \in A} P(\alpha') P^{\alpha'}_\sigma(m|t_a)$$

and the probability that a receiver of type α observes a randomly sampled agent choose interpretation t_o given message m as:

$$P^\alpha_o(t_o|m) = \sum_{t_r} P^\alpha_o(t_o|t_r) \sum_{\alpha' \in A} P(\alpha') P^{\alpha'}_\rho(t_r|m)$$

Again, these calculations incorporate the probabilities that the imitating agent might observe the behavior of an agent of another type α', weighed by its relative proportion. Finally, the update step for a sender strategy of type α at time instant $i + 1$ is given by:

$$\check{\sigma}^\alpha_{i+1}(m|t) \propto P^\alpha_o(m|t) \mathrm{EU}^\alpha_{\sigma_i}(m, t, \rho^\star_i)$$

and similarly for a receiver strategy of type α by:

$$\check{\rho}^\alpha_{i+1}(t|m) \propto P^\alpha_o(t|m) \mathrm{EU}^\alpha_{\rho_i}(t, m, \sigma^\star_i)$$

578 We here use $\check{\sigma}$ and $\check{\rho}$ since there is still an additional adjustment to these values to
579 be calculated before we get the final strategies σ and ρ.
580   These formulations cover the evolution of the particular strategies of each type.
581 We can think of this as the level of cultural evolution: agents are born with a certain
582 level of imprecision and adopt strategies based on the behavior of others. Alongside
583 this process, we can imagine another level of selection, where agents die and new

584 agents are born. More successful agents have a higher likelihood of surviving and
585 reproducing, giving rise to more agents with their level of imprecision. Levels of
586 imprecision are thus subject to an evolutionary dynamic that is indirectly influenced
587 by the cultural dynamic. Importantly, only the level of imprecision is passed on to
588 new generations under this dynamic, not the actual strategies developed by the agents
589 at the cultural level.

We model this process by changing the proportion of each type $P(\alpha)$ according to
the replicator dynamic. A population of type α consists of agents employing both a
sender and a receiver strategy, thus the overall fitness of the population must include
the expected utilities of both. We could imagine this process happening at a different
speed to the cultural process, in which case we could have different time scales. For
the sake of simplicity, we choose to have them both happen at each time step, but
calculate the changes occurring at this level of selection after the calculation for the
other level. We define the proportion of type α at time step $i + 1$ as:

$$P_{i+1}(\alpha) \propto P_i(\alpha)(\mathrm{EU}^\alpha_{\sigma_{i+1}}(m, t, \rho_i^\star) + \mathrm{EU}^\alpha_{\rho_{i+1}}(t, m, \sigma_i^\star))$$

In order to additionally account for the fact that strategies are not passed on to new
generations, we mix the evolved strategies of a certain type σ^α and ρ^α with new
random strategies $\tilde{\sigma}^\alpha$ and $\tilde{\rho}^\alpha$ (generated at each time step). The idea is that, at each
time step, a certain percentage of each population will consist of "newborn" agents,
i.e. agents that haven't yet had time to evolve their strategies. We define a parameter
γ that quantifies this percentage, or as we can also call it the birth rate, which we
consider to be the same for every population. This mixing finally defines the strategies
for time step $i + 1$ and can be described in the following formulas:

$$\sigma^\alpha_{i+1}(m|t) = (1 - \gamma)\check{\sigma}^\alpha_{i+1}(m|t) + \gamma\tilde{\sigma}^\alpha_{i+1}(m|t)$$

$$\rho^\alpha_{i+1}(t|m) = (1 - \gamma)\check{\rho}^\alpha_{i+1}(t|m) + \gamma\tilde{\rho}^\alpha_{i+1}(t|m)$$

590 **Simulation results** We performed 25 simulation runs of this model for each of
591 three population scenarios: only one population with $\alpha = 0$ (for reference), two
592 populations with $\alpha \in \{0, 0.05\}$, and three populations with $\alpha \in \{0, 0.05, 0.1\}$. For
593 each scenario, starting proportions $P(\alpha)$ were equal for each value of α. Given
594 the observations by Franke and Correia (2017) that state space size and tolerance
595 parameter β do not result in important qualitative difference, we fixed these values at
596 $n_S = 30$ and $\beta = 0.1$. We also used a fixed uniform distribution for the priors and a
597 message space with 2 messages. Each type started with its own randomly generated
598 strategy. Regarding the duration of the simulations, due to the mixing in of new
599 individuals into the population (birth rate was fixed at $\gamma = 0.05$), the convergence
600 criteria is no longer applicable because each strategy is randomly perturbed at each
601 time step. Therefore, all simulation runs were stopped after 200 iterations.

602 The first thing to observe from the simulation results is that the population with
603 no imprecision ($\alpha = 0$) dominated the other populations in every run. In Fig. 2 we
604 plot the evolution of the proportion of each population in the two-population and
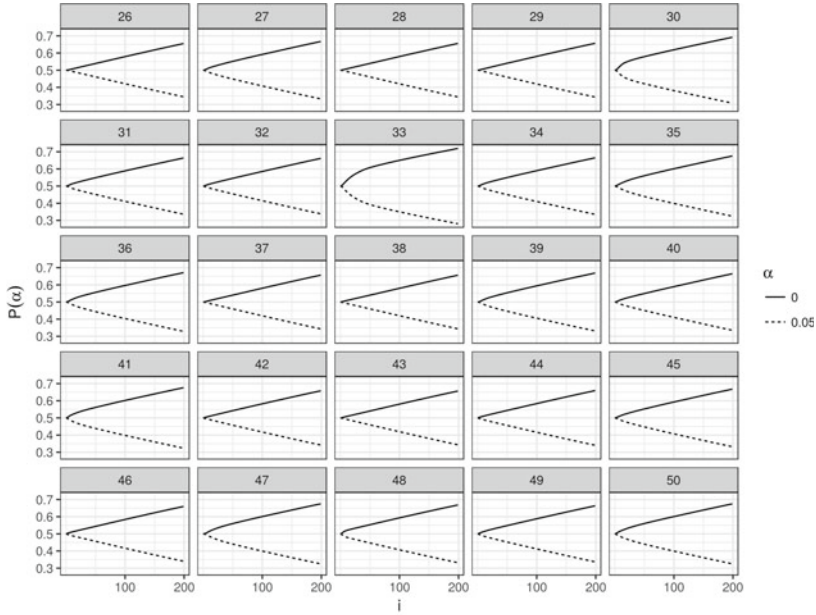
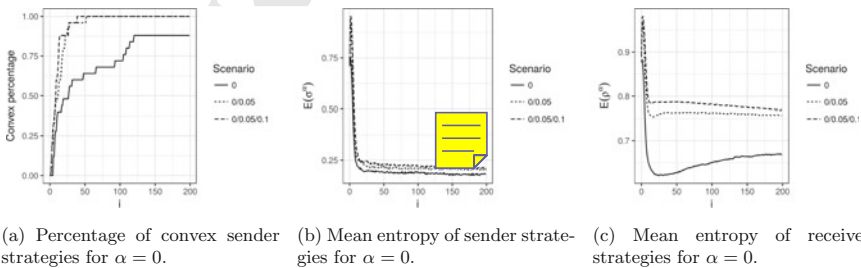(a) Two-population scenario.
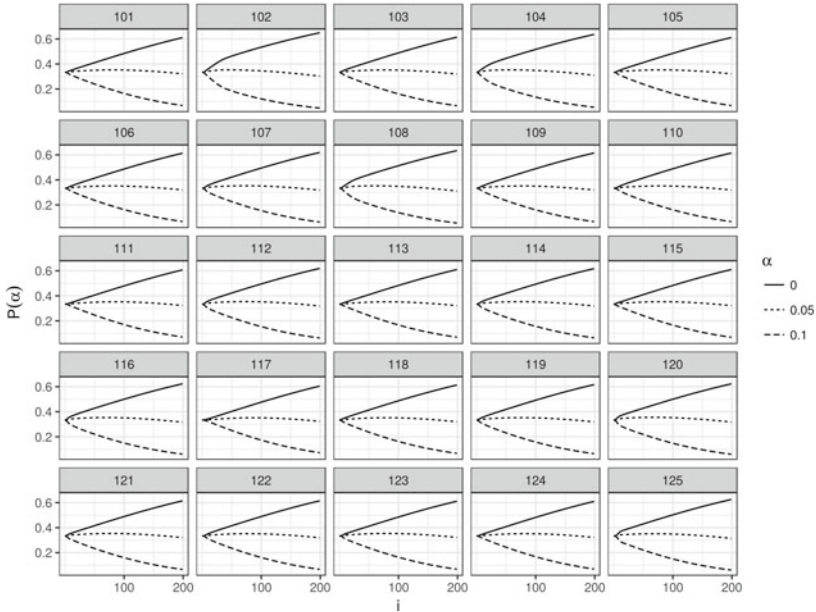
(b) Three-population scenario.

Fig. 2 Evolution of population proportions through time for each simulation trial of the tight interaction model. Numbers on top of each plot identify each trial

three-population scenarios for all trials. As the plot shows, the population with no imprecision steadily increased its proportion against the others in every trial. In the three-population scenario, the proportion of type $\alpha = 0.05$ sees a slight increase in the beginning of the simulation, but inexorably starts a downward trend. These observations go against the expectation of Franke and Correia (2017) that faster convergence to a convex strategy by populations with a certain level of imprecision could give them a temporary advantage to take over and eliminate other types. The reason for this is interesting in itself. What happens is that, because of the tight interaction between populations, the strategies of each type evolve in close tandem with each other. One of the consequences of this is that the population with no imprecision reaches convexity faster than it would on its own because of the interaction with the populations with imprecision. We can see this effect by looking into the percentage of trials with convex sender strategies at a given iteration, for each scenario, and comparing the three scenarios: populations with no imprecision evolving alone, two populations ($\alpha \in \{0, 0.05\}$), and three populations ($\alpha \in \{0, 0.05, 0.1\}$). This is plotted in Fig. 3a. What we see is more trials reaching convexity earlier for the multi-population scenarios when compared with the single-population scenario. This effect precludes the hypothesized temporary advantage of imprecision manifesting itself, but it can be seen as a positive influence of the population(s) with imprecision on the population with no imprecision.

The flip side of this tight connection between populations is that strategies evolved by populations with no imprecision are also more vague (in the sense defined in Sect. 3). This is visible by looking at mean entropy values, namely sender strategy entropy, plotted in Fig. 3b, and receiver strategy entropy, plotted in Fig. 3c. The values for the population with no imprecision are clearly higher in the scenarios where it evolves together with populations with imprecision than in the scenario where it evolves on its own. Given the trends in population proportions, one expects this to eventually be eliminated when the population with no imprecision finally takes over the others, but it is interesting to observe that while populations with vague strategies persist, the population with no imprecision takes much longer to evolve a fully crisp strategy.

(a) Percentage of convex sender strategies for $\alpha = 0$.

(b) Mean entropy of sender strategies for $\alpha = 0$.

(c) Mean entropy of receiver strategies for $\alpha = 0$.

Fig. 3 Development of some metrics through time for the $\alpha = 0$ population in each of three scenarios: evolving alone ('0'), with an $\alpha = 0.05$ population ('0/0.05'), and with an additional $\alpha = 0.1$ population ('0/0.05/0.1')

636 ## *5.2 Loose Population Interaction*

We can also imagine a scenario where populations interact more loosely. Namely, the dynamic we want to model here is one where agents of a certain type α imitate and learn only from other agents of the same type, but nevertheless use their signaling strategies with agents from all types. In order to capture this, we need to make changes to some calculations. Any formula that is not redefined in this section should be assumed to remain the same. First, if agents learn only within their population, the imitation dynamic needs to consider only the expected utility against agents of that population. We thus define the following expected utilities for a sender of type α:

$$\text{EU}^\alpha_\sigma(m, t_o, \rho^\alpha) = \sum_{t_a} P^\alpha_{\check{o}}(t_a | t_o) \sum_{t_r} P^\alpha_\rho(t_r | m) U(t_a, t_r)$$

and for a receiver of type α:

$$\text{EU}^\alpha_\rho(t_i, m, \sigma^\alpha) = \sum_{t_a} P^\alpha_{\check{\sigma}}(t_a | m) \sum_{t_r} P^\alpha_r(t_r | t_i) U(t_a, t_r)$$

637 The main difference from the previous model is that these expected utilities are not
638 calculated against all populations ($\sigma^\star$ and $\rho^\star$) but only against the agent's own type
639 (σ^α and ρ^α). This also implies that population proportions do not play a role at this
640 level of selection.

Regarding the imitation process, we can redefine the probability that a sender of type α observes a randomly sampled agent play message m for observed state t_o as:

$$P^\alpha_o(m | t_o) = \sum_{t_a} P^\alpha_{\check{o}}(t_a | t_o) P^\alpha_\sigma(m | t_a)$$

and the probability that a receiver of type α observes a randomly sampled agent choose interpretation t_o given message m as:

$$P^\alpha_o(t_o | m) = \sum_{t_r} P^\alpha_o(t_o | t_r) P^\alpha_\rho(t_r | m)$$

Again, the main difference is that agents make observations within their own population, so to model a randomly sampled agent one needs only to take into account agents of the same type. Population proportions again do not play a role in these calculations. Based on these formulas, we can define the update step for a sender strategy of type α at time instant $i + 1$ as:

$$\check{\sigma}^\alpha_{i+1}(m | t) \propto P^\alpha_o(m | t) \text{EU}^\alpha_{\sigma_i}(m, t, \rho^\alpha_i)$$

and similarly for a receiver strategy of type α as:

$$\check{\rho}^{\alpha}_{i+1}(t|m) \propto P^{\alpha}_o(t|m)\mathrm{EU}^{\alpha}_{\rho_i}(t, m, \sigma^{\alpha}_i)$$

Note that, because imitation and learning occur only within populations, these formulations are essentially the same as for the single-population model of Franke and Correia (2017), only parameterized by type α.

The selection process between different populations still follows the same motivation as before: agents of a certain type that evolve successful strategies (with respect to other types) will be more likely to survive and reproduce, benefiting the proportion of their population. The formulation of the dynamic for the proportion of each type $P(\alpha)$ thus stays the same. In order to avoid confusion, we want to stress that this means that these calculations rely on the definitions of expected utility across populations (i.e. $\mathrm{EU}^{\alpha}_{\sigma}(m, t_o, \rho^{\star})$ and $\mathrm{EU}^{\alpha}_{\rho}(t_i, m, \sigma^{\star})$) and not the newly introduced $\mathrm{EU}^{\alpha}_{\sigma}(m, t_o, \rho^{\alpha})$ and $\mathrm{EU}^{\alpha}_{\rho}(t_i, m, \sigma^{\alpha})$.

Simulation results We ran the same number of simulation trials under the same conditions as for the model with tight population interaction. In Fig. 4 we plot the evolution of population proportions for all trials of the multi-population scenarios. The first thing to observe is that proportions evolve faster than in the model with tight interaction. Whereas in the latter no given population ever reached much more than 70% proportion, in this model we see that many trials resulted in one population fully dominating the others. In the two-population scenario, some population reached at least 99% in 24 out of 25 trials. In the three-population scenario, this happened in 18 out of 25 trials. More interestingly for our investigation, some trials actually resulted in the population with $\alpha = 0$ being dominated. For the two-population scenario, $\alpha = 0.05$ fully reached 100% in 8 trials (79, 86, 88, 89, 91, 93, 96, 98), a point from which it is technically impossible for the other population to recover. In most cases the dominating population gains its ground from the start, but in 5 cases we see a temporary advantage of $\alpha = 0.05$ that is then lost to the other population. In one interesting trial (85), $\alpha = 0.05$ reached 99.87% only to then steadily start losing ground to $\alpha = 0$. For the three-population scenario, $\alpha = 0$ was reduced to 0% in 4 trials (152, 157, 159, and 175). In all of those cases, $\alpha = 0.05$ clearly has the upper hand over $\alpha = 0.1$, despite a temporary advantage of the latter in some trials.

Because of the loose interaction between populations, different types can now evolve separately. One consequence of this is that populations with a certain level of imprecision again reach convexity faster than those without imprecision. In Fig. 5 we plot, for each trial, the first iteration when each type reached convexity. What we see is that, even though $\alpha = 0$ usually reaches convexity later, this is not always the case. This has certainly to do with the initial conditions of each trial, since the randomly generated strategies can simply by chance be more favourable to reaching convexity. More importantly, we also see that reaching convexity sooner is not a sufficient condition for achieving population dominance. There were many trials where another type reached convexity much sooner than $\alpha = 0$ but its population was completely dominated nevertheless (e.g. 76, 84, 92, and 99 in the two-population
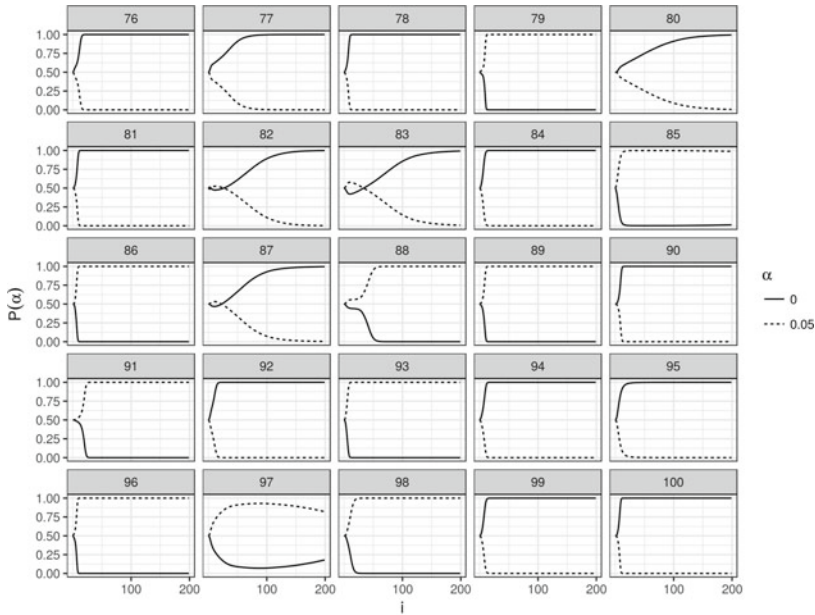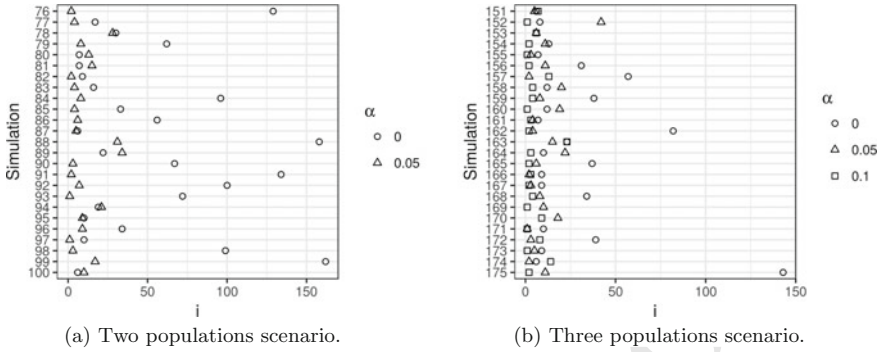
(a) Two-population scenario.
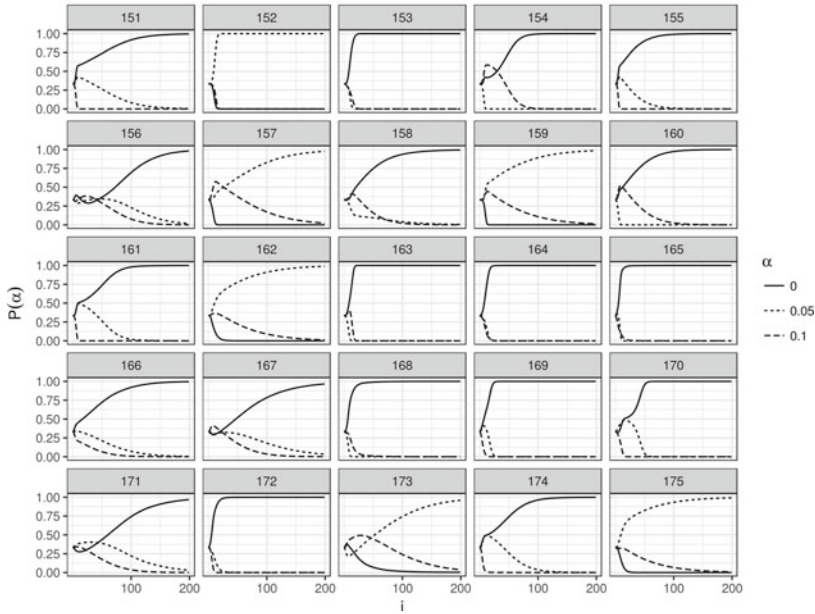
(b) Three-population scenario.

Fig. 4 Evolution of population proportions through time for each simulation trial of the loose interaction model. Numbers on top of each plot identify each trial

(a) Two populations scenario.   (b) Three populations scenario.

Fig. 5 First iteration with convex sender strategies for each simulation trial of the loose interaction model

⁶⁸¹ scenario; 156, 165, 168, and 172 in the three-population scenario). It also does not
⁶⁸² seem to be fully necessary, given a few examples where $\alpha = 0$ reached convexity
⁶⁸³ early on and another type ended up dominating (89 in the two-population scenario
⁶⁸⁴ and 152 in the three-population scenario).

⁶⁸⁵   Another consequence of the populations evolving separately in this model is that
⁶⁸⁶ they do not necessarily evolve towards the same equilibrium. In a sim-max game such
⁶⁸⁷ as the one set up here, there are only two stable equilibria. These are the two Voronoi
⁶⁸⁸ languages of the kind shown in Fig. 1: one where the first message is used for the
⁶⁸⁹ first half of the state space, and the other where the second message is used for this
⁶⁹⁰ region. In the multi-population model with loose interaction, each population evolves
⁶⁹¹ independently towards one of these two equilibria. An important factor determining
⁶⁹² which language strategies converge to is the random initial population configuration.
⁶⁹³ The populations are, however, not fully independent, since the process of selection of
⁶⁹⁴ the level of precision relies on the expected utility of one population playing against
⁶⁹⁵ itself and the others. And this is important because strategies in one equilibrium get
⁶⁹⁶ the lowest payoff possible against strategies in the other equilibrium. Now, if two
⁶⁹⁷ populations evolve towards different equilibria, whatever advantage one population
⁶⁹⁸ has playing against itself could trigger a runaway effect by causing an increase in
⁶⁹⁹ its proportion, which in turn will increase the population's relative expected utility,
⁷⁰⁰ potentially increasing their proportion further in the next round, and so forth. In this
⁷⁰¹ case, one would expect that faster convergence towards convexity would be especially
⁷⁰² important for a population's success.

⁷⁰³   In the two-population scenario, of the 8 trials where $\alpha = 0$ was reduced to 0%, all
⁷⁰⁴ but two (88, 98) ended with the two populations each close to different equilibria. In
⁷⁰⁵ the three-population scenario, the population with $\alpha = 0$ evolved towards the same
⁷⁰⁶ equilibrium as the other two in 1 of the trials where it was eliminated. In the other 3
⁷⁰⁷ trials, in 2 it evolved to a different equilibrium than the other two, and in 1 it evolved
⁷⁰⁸ towards the same equilibrium as $\alpha = 0.05$ (but different to the one $\alpha = 0.1$ evolved
⁷⁰⁹ towards). Again, we do not find a clear case for this being a decisive factor in the

(a) Sender expected utility.   (b) Receiver expected utility.
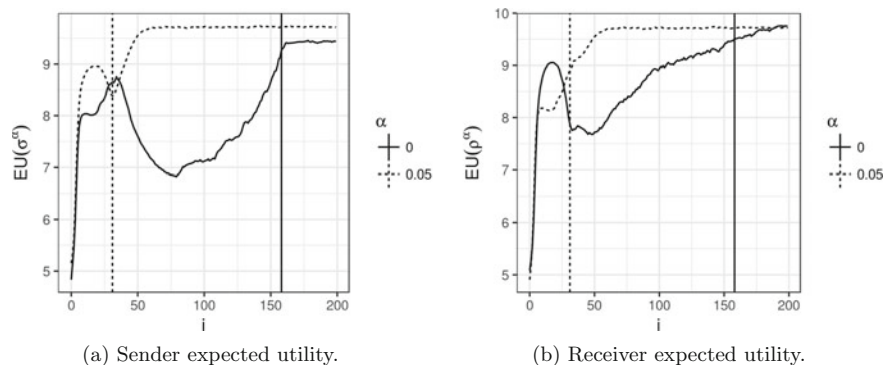
Fig. 6 Expected utilities for trial 88 of the loose interaction model. Vertical lines demarcate, for each type, the iteration where convexity was achieved

success of populations with some imprecision. The same goes for the impact of an initial advantage in expected utility creating the runaway effect we just mentioned. Despite none of these three factors (including reaching convexity sooner) sentencing the demise of $\alpha = 0$ with certainty, they do seem to conspire together to bring it about. In most trials where the type was eliminated, in both the two- and three-population scenarios, $\alpha = 0$ ended up evolving towards a different equilibrium than the other types, and either had an initial disadvantage in expected utility, or reached convexity much later. These factors can thus be seen as indicators at best, and the story of how a population with imprecision ends up dominating one without it seems to be more complicated than expected.

This is not surprising, since we are facing a complex dynamical system with various interacting components (sender and receiver strategies and multiple populations). Just as an example of this, in Fig. 6 we plot the expected utilities of sender and receiver strategies for both populations of trial 88. The curves up to where $\alpha = 0.05$ achieved convexity seem to suggest that the sender strategy of that type was evolving towards the same equilibrium as the receiver strategy of the other. The moment where the population reaches convexity marks the point where the sender strategy of $\alpha = 0.05$ aligns with its type's receiver strategy and this coincides with the moment when the population seems to gain real traction over the other type (see again the plot for trial 88 in Fig. 5). In this particular case, reaching convexity seems to have made an important difference.

6 Conclusions

Vagueness presents a challenge to both procedural and instrumental pictures of rationality alike. In the context of game-theoretical models of language, this takes the form of a question about evolution: how can vagueness persist if it characterizes demonstrably less efficient communication? Most existing proposals in the literature

attempt to explain this by considering agents with some degree of bounded rational-
ity. Despite the many ways in which our rationality is inevitably limited, the argument
for the pervasiveness of vagueness in natural language would be much stronger if
one could also find associated advantages. In this paper, we argued that finding those
might require us to go beyond a local notion of rationality, as two of the proposals
reviewed here (O'Connor 2014; Franke and Correia 2017) suggest. We advocate
moving towards an ecological approach, studying vagueness in more heterogeneous
ecosystems. Language is part of a very complex system (Beckner et al. 2009) that
involves many components interacting in often unpredictable ways. An ecology of
vagueness would involve studying models where different populations can evolve
and interact with each other, where different language games can be played between
individuals, where the environment is uncertain and changing, or anything else that
more closely approximates the real context of language evolution.

In light of this picture, we have proposed two variants of a concrete multi-
population signaling model to test the hypothesis (Franke and Correia 2017) that
certain features of imprecise imitation, like promoting faster convergence and regu-
larity, could prove beneficial in contrast with full precision. Analysing these models
did not provide us with a clear-cut answer, and revealed that the story is much more
nuanced than initially expected. In a variant where populations with and without
imprecision interact tightly, although precision always has the upper hand, vague-
ness seems to take a long time to be weeded out. When we let populations interact
more loosely, we see a more complex pattern of outcomes. These include scenarios
where imprecise imitation dominates over full precision, showing that strategies with
vagueness can actually, under certain circumstances, be more successful. Bringing
several populations together in a more complex ecosystem thus allowed us to not
only spell out and test the original intuition, but also learn about unforeseen effects.
These models thus serve as an example of how moving to a more global perspec-
tive on rationality can allow us to achieve a more detailed awareness of the complex
interactions that might be involved in sustaining vagueness in natural language. They
can be seen, we believe, as a first step towards an ecology of vagueness.   AQ1

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