

Vagueness, Noise, and Signaling

Abstract

Signaling games are popular models for studying the evolution of meaning, but typical approaches do not incorporate vagueness as a feature of successful signaling. Complementing recent like-minded models, we introduce the replicator diffusion dynamic as a special case of the replicator mutator dynamic that combines evolutionary pressure on optimal signal use with stochastic noise in the processing of similar stimuli. Applying this new dynamic to a generalization of Lewis' signaling games, we show that stochastic imprecision leads to vague, yet communicative efficient signal use, and, moreover, unifies evolutionary outcomes and helps avoid suboptimal categorization.

Many concepts and expressions are vague. A vague category knows clear cases that fall under it, clear cases that do not, and also so-called borderline cases. Borderline cases do not clearly apply, nor do they clearly not apply, and there may be differences between borderline cases in terms of how well they represent the category in question. Vagueness does not seem to dramatically affect the success of everyday communication, but it is troublesome for some of the most prominent theories of language and meaning. This is especially so for the logico-semantic tradition of Frege, Russell and the young Wittgenstein which is challenged by the paradoxes vagueness gives rise to.

But there are other intriguing aspects about vagueness. The puzzle that we are concerned with here is how vagueness could arise and be maintained in the first place.

This is a serious worry to functionalist accounts that maintain that our concepts and language use evolved in order to be efficient. Since the existence of unclear borderline cases seems to entail inefficiency of categorization or communication, the challenge, succinctly put forward by Lipman (2009), is to explain how vagueness can exist despite its obvious and demonstrable functional deficiency under evolutionary pressure to be optimally expressive. We address *Lipman’s problem* as a technical challenge, namely to find a conceptually sound and mathematically coherent formal model of the evolution of categorization and language use that yields by-and-large efficient categories *and* vagueness.

A number of authors have recently tried to explain why vagueness evolved as something that is itself useful (e.g. de Jaegher, 2003; Deemter, 2009; Blume and Board, 2014). Others have argued that vagueness is a natural byproduct of limitations in information processing (e.g. Franke, Jäger, and van Rooij, 2011; O’Connor, 2014). This paper contributes to the latter line of thought. Concretely, we introduce the replicator diffusion dynamics —a novel variant of the replicator mutator dynamic— that integrates stochastic noise on the differential confusability of similar stimuli. The dynamic proposed here generalizes and complements previous like-minded accounts. We show that stochastic noise can not only lead to vague, yet efficient signal use, but can also unify evolutionary outcomes and help avoid suboptimal categorization.

The next section introduces the background against which the work presented here can be appreciated. Section 2 introduces the replicator diffusion dynamic and elaborates on its relation with the replicator mutator dynamic. Section 3 explores the replicator diffusion dynamic. Section 4 reflects and compares our approach to others.

1 Background

1.1 Sim-max games & conceptual spaces

Signaling games, as introduced by Lewis (1969), have a sender and a receiver. The sender knows the true state of the world, but the receiver does not. The sender can select a signal, or message, to reveal to the receiver, who then chooses an act. In Lewis' games, if the receiver chooses the act that corresponds to the actual state, the play is a success, otherwise a failure. Certain regular combinations of sender signaling and receiver reaction make messages meaningful, in the sense that their use is correlated systematically to certain states or acts. To investigate the conditions under which such meaning-generating behavior can evolve is a highly interesting topic that we are only beginning to fully understand (e.g. Wärneryd, 1993; Blume, Kim, and Sobel, 1993; Huttegger, 2007; Pawlowitsch, 2008; Barrett, 2009; Huttegger et al., 2010; Skyrms, 2010).

Similarity-maximizing games, or *sim-max games* for short, are generalizations of Lewis' games where different states are allowed to be more or less similar to one another. While Lewis' games treated communicative success as a matter of black and white, sim-max games allow for a gradient notion of communicative success: the more similar the receiver's interpretation is to the actual state, the more successful the play is taken to be. Signaling games with utility-relevant similarities in the state space are fairly standard in economics (e.g. Spence, 1973; Crawford and Sobel, 1982), but have received particular attention in a more philosophical and linguistic context for reasons that will become clear presently (Jäger, 2007; Jäger and van Rooij, 2007; Jäger, Metzger, and Riedel, 2011).

A sim-max game, in the relevant sense here, consists of a set of states T , a set of messages M with much fewer messages than states, a prior probability distribution $\Pr \in \Delta(T)$ that gives the occurrence probabilities of states, a similarity metric on states

$\text{Sim} : T \times T \rightarrow \mathbb{R}$, and a utility function $U : T \times T \rightarrow \mathbb{R}$. We identify the receiver's acts with the states of the world, so that the game is one of guessing the actual state, so to speak. We also assume that sender's and receiver's interests are alike, so we only have one utility function. We do not consider message costs, so utilities only depend on the actual state and the receiver's response. The similarity function should satisfy the usual requirements for a metric and the utility function should be a monotonically decreasing function of similarity.

Jäger, Metzger, and Riedel (2011) showed that the evolutionarily stable states of sim-max games are remarkably systematic. Their results were obtained for games with infinitely many states in n -dimensional Euclidean space $T \subseteq \mathbb{R}^n$ and a quadratic loss function for utilities $U(t_1, t_2) = -(t_1 - t_2)^2$. For these games, the evolutionarily stable states are demonstrably so-called Voronoi languages. Roughly put, a Voronoi language is a pair of sender and receiver strategies, such that the sender strategy partitions the state space into convex categories, while the receiver's interpretations are the central spots in each category. For example, if T is the unit interval and all states are equiprobable, a Voronoi language with two messages would have the sender use one message exclusively for all points in the lower half of the unit interval and another for all points in the upper half; the receiver's interpretations of messages are the central points, .25 and .75, in the respective intervals.

This result is interesting, because it demonstrates that signaling can impose orderly categories on a metric space, without that being the ulterior purpose of it all. Sender and receiver can be distinct entities, whose purpose is to communicate effectively about the actual state. In that case, evolving Voronoi languages would explain why *linguistic categories* are well-behaved and orderly in the way they appear to be. More abstractly, sender and receiver can also be thought of as distinct modules in a single system, where the first module must discretize the information it is fed by selecting a small sample of, suggestively, category labels. These are passed to a second module that tries to decode

the original information. In this case, evolving Voronoi languages would explain why *conceptual categories* are well-behaved and orderly in the way that they appear to be.

Seen in this light, sim-max games may provide a foundation to approaches in cognitive semantics that rely on the notion of conceptual spaces. Gärdenfors (2000, pp. 70–77), for example, has prominently argued that natural categories are convex regions in conceptual space. If the conceptual space has a suitable metric, convex categories can be derived from a set of prototypes. The category corresponding to prototype p is the set of points that are more similar to p than to any other. In this way, Gärdenfors (2000) argues, an efficient categorization system can be obtained: storing the prototypes lets us recover the categories without having to store each category’s extension. However, what is left unexplained, is where the prototypes come from, and why we would not see just any distribution of prototypes as an equally efficient classification system. This is where sim-max games can contribute a principled approach to deriving, in an independent way, not only convex categories but also prototypical exemplars belonging to them.

1.2 Vague signaling in sim-max games

This outline of an approach to categorization using sim-max games leaves many problems unaddressed. One of them is that natural categories for continuously variable stimuli usually do not have unique, point-valued prototypes and clear category boundaries. We would like to account for the possibility of such vagueness, in particular: (i) clear positive examples of a vague category should show a gradient transition to clear negative examples (so-called *higher-order vagueness*); (ii) prototypes should likewise be gradient regions, peaking at the center of the vague category they represent (e.g. Sainsbury, 1991; Keefe and Smith, 1997).

Douven et al. (2011) show that Gärdenfors’s conceptual spaces approach can be extended to account for the existence of borderline cases. From the assumption that

prototypes are extended, yet convex regions in conceptual space, a construction algorithm is available that yields “collated Voronoi diagrams” with thick boundaries representing borderline regions. Decock and Douven (2012) show further how it is possible to arrive at higher-order vagueness, by weighing in the distance of different borderline cases to various prototypical regions. This accounts for the first of the two desiderata mentioned above, but still assumes that crisp prototype regions must be given.

An alternative approach is taken by Franke, Jäger, and van Rooij (2011) and O’Connor (2014) who show how the above desiderata can be met by evolving strategies in sim-max games. We should distinguish micro- and macro-level approaches. Micro-level approaches look at adaptive behavior of individual agents. Usually, changes in the behavioral dispositions of agents occur after every single interaction. In contrast, macro-level approaches outline more abstract, aggregate dynamics, happening in a population of agents, or otherwise abstracting from seemingly irrelevant detail. Usually, a macro-level dynamic captures changes of frequencies of behavioral types in the population over time.

As for micro-level approaches, Franke, Jäger, and van Rooij (2011) show how limited memory of past interactions can lead to vague signal use, when averaging over a single agent’s behavior over time or over the momentary behavior of a population of several language users. O’Connor (2014) introduces a variant of reinforcement learning that entails a low-level form of stimulus generalization. Agents update their behavior after each round of play in such a way that states similar to the one that actually occurred are subject to behavioral adjustment as well.

Franke, Jäger, and van Rooij (2011) also consider a macro-level approach, using the notion of a *quantal response*. A quantal response function is a probabilistic choice rule that formalizes the idea that agents make small mistakes when calculating the expected utility of choice options. In aggregation, these probabilistic mistakes lead to systematic “trembles” that produce vague signal use.

The approach we take here is superficially similar to the latter, but there are crucial differences. For one, we adopt a dynamic perspective by looking at limited-time outcomes of a dynamic process. For another, we demonstrate in Section 4 that quantal responses can give rise to counterintuitive predictions. These counterintuitive examples suggest that vagueness in sim-max games is not convincingly explained by appeal to mistakes in calculating expected utility, but rather, as we assume here, as the result of confusing similar states. A more in-depth discussion of alternative approaches is deferred until Section 4. Let us first look into our own proposal in more detail.

2 Replicator diffusion dynamic

Against this background, we introduce a new macro-level approach, where the source of vagueness is the agents' natural inability to sharply distinguish similar states. We call our dynamic *replicator diffusion dynamic* (RDD), because it is an extension of the replicator dynamic (RD) that adds diffusion of behavior, and a special case of the replicator mutator dynamic (RMD).

We begin by recapitulating the formulation of the RD in Section 2.1, and introduce the RDD in Section 2.2. Section 2.3 introduces the RMD, and Section 2.4 shows in what sense the RDD is a special case of the RMD.

2.1 Replicator dynamic in behavioral strategies

Fix a signaling game with finite states T , messages M and acts A . Let $\Pr(\cdot) \in \Delta T$ be the prior distribution over states and $U_{S,R} : T \times M \times A \rightarrow \mathbb{R}$ the senders's and receiver's utility functions. The sender's behavioral strategies are functions $\sigma \in \Delta(M)^T$; the receiver's are functions $\rho \in \Delta(A)^M$. Define the expected utility of choices in each

choice point as usual:

$$\begin{aligned} \text{EU}(m, t, \rho) &= \sum_{a \in A} \rho(a | m) \cdot U_S(t, m, a) \\ \text{EU}(a, m, \sigma) &= \sum_{t \in T} \Pr(t) \cdot \sigma(m | t) \cdot U_R(t, m, a). \end{aligned}$$

The *fitness* of a behavioral strategy at a choice point is the frequency-weighted average of expected utilities of each choice, given the opponent's strategy:

$$\begin{aligned} \Phi(t, \sigma, \rho) &= \sum_m \sigma(m | t) \cdot \text{EU}(m, t, \rho) \\ \Phi(m, \sigma, \rho) &= \sum_a \rho(a | m) \cdot \text{EU}(a, m, \sigma). \end{aligned}$$

The discrete-time replicator dynamic maps current strategies σ and ρ to future strategies $\text{RD}(\sigma)$ and $\text{RD}(\rho)$ in such a way that changes in frequency are proportional to expected utilities. For behavioral strategies, the changes take place locally at each of the agents' choice points:

$$\begin{aligned} \text{RD}(\sigma)(m | t) &= \frac{\sigma(m | t) \cdot \text{EU}(m, t, \rho)}{\Phi(t, \sigma, \rho)} \\ \text{RD}(\rho)(a | m) &= \frac{\rho(a | m) \cdot \text{EU}(a, m, \sigma)}{\Phi(m, \sigma, \rho)}. \end{aligned}$$

2.2 Replicator diffusion dynamic in behavioral strategies

The replicator diffusion dynamic is a noise-perturbed variant of the replicator dynamic (RD) in behavioral strategies. Fix a sim-max game with $T = A$ and a confusion matrix $C : T \times T$. C is a row-stochastic matrix whose elements C_{ij} give the probability that t_i is realized as t_j . The confusability of states affects senders and receivers alike, but in slightly different ways (see Figure 1). For the sender, C_{ij} is the probability that the actual state t_i is perceived as t_j . For the receiver, C_{ij} is the probability that t_j is the interpretation that is actually formed when t_i is the intended interpretation.

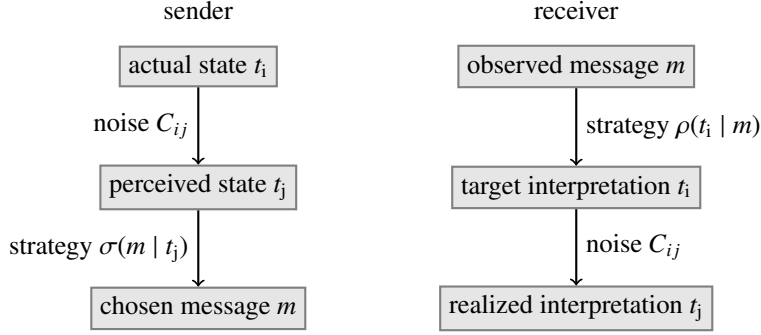


Figure 1: Effect of confusion of states on sender and receiver choices.

The aggregate effect of confusion of states on behavioral strategies can be captured in a function that maps behavioral strategies σ and ρ to their diffusions $D(\sigma)$ and $D(\rho)$. The idea is that σ and ρ are what, on average, the idealized, noise-free behavior would be, while $D(\sigma)$ and $D(\rho)$ are behavioral strategies that describe the agents' actual noise-perturbed probabilistic behavior. Since the effect of confusion of states is that behavior at one choice point percolates to behavior at similar choice points, we speak of *diffusion of behavior under confusion of states*. If we conceive of behavioral strategies σ and ρ as row-stochastic matrices, the diffusion effect of state confusability is easily captured by matrix multiplication as a diffusion function D_C that, based on confusion matrix C , maps behavioral strategies onto their diffused realizations:

$$D_C(\sigma) = C\sigma \qquad D_C(\rho) = \rho C. \qquad (1)$$

The discrete-time replicator diffusion dynamic takes the replicator dynamic as basic, but factors in the confusion of states at each update step in a sequential update:

$$RDD(\sigma) = D_C(RD(\sigma)) \qquad RDD(\rho) = D_C(RD(\rho)).$$

This is equivalent to the following, perhaps more transparent formulation:

$$\begin{aligned}\text{RDD}(\sigma)(m \mid t_i) &= \sum_j C_{ij} \cdot \frac{\sigma(m \mid t_j) \cdot \text{EU}(m, t_j, \rho)}{\Phi(t_j, \sigma, \rho)} \\ \text{RDD}(\rho)(t_i \mid m) &= \sum_j C_{ji} \cdot \frac{\rho(t_j \mid m) \cdot \text{EU}(t_j, m, \sigma)}{\Phi(m, \sigma, \rho)}.\end{aligned}$$

The idea behind the sequential definition of the `RDD` is that, at each time step, strategies are gradually optimized along the current fitness landscape, as described by the `RD`, but the realization of optimized strategies is noisy, due to similarity of states. It is obvious that other (discrete-time) evolutionary dynamics can be subjected to state-confusability in an analogous fashion (see also Section 4.1). In the case of the `RD`, however, perturbation by state-confusion has a prominent close relative in the replicator mutator dynamic.

2.3 The replicator mutator dynamic

The `RMD` has been proposed first in the context of signaling game models for the evolution of grammar (e.g. Komarova, Niyogi, and Nowak, 2001; Nowak, Komarova, and Niyogi, 2001). It adds stochastic mutation to the replicator dynamic in order to capture, in the context of grammar evolution, the differential learning success of different dispositions in first language acquisition. The relation of the `RMD` to other prominent evolutionary dynamics is well understood (Page and Nowak, 2002). The `RMD` has seen further fruitful applications in the context of signaling games (e.g. Huttegger et al., 2010). It is therefore desirable to relate the `RDD` that we propose here to the `RMD`.

A direct comparison between `RDD` and the `RMD` is not possible, because the usual formulation of the latter is in its continuous-time form, and based on mixed strategies, not behavioral strategies. We therefore give a discrete-time formulation of the `RMD` in behavioral strategies, which is independently useful. The relation between `RDD` and `RMD` will then be plain to see. We start with a delineation of mixed and behavioral strategies.

Mixed strategies. Pure sender (receiver) strategies are functions $s \in M^T$ ($r \in A^M$). Mixed sender (receiver) strategies are functions $\mathbf{s} \in \Delta(M^T)$ ($\mathbf{r} \in (A^M)$). The latter give the relative population frequencies of the former. We write \mathbf{s}_i for the frequency $\mathbf{s}(s_i)$ of pure strategy s_i . Likewise for the receiver.

Every mixed strategy \mathbf{s} converts to a unique behavioral strategy defined by:

$$\sigma(m | t) = \sum_{s(t)=m} \mathbf{s}(s). \quad (2)$$

Let G be this mapping from mixed to behavioral strategies. Note that G is *not* an injection, as many mixed strategies map onto the same behavioral strategy. In this sense, mixed strategies hold more information than behavioral strategies about the distribution of pure strategies in a population.

In the context of evolutionary dynamics, we can think of behavioral strategies as treating each choice point as an independent update site (e.g. Cressman, 2003; Sandholm, 2013). Dynamics for mixed strategies conceive of agents as adjusting their whole pure strategy globally. In this way, dependencies between different choices at different choice points can matter to the evolutionary path. In contrast, dynamics for behavioral strategies conceive of agents as adjusting their strategies locally at each choice point, regardless of what they are disposed to do at other choice points. The latter perspective is then more coarse-grained, and not necessarily equivalent to the former. But an advantage of dynamics in behavioral strategies is that they reduce complexity, which is especially helpful for computational simulations of larger games, like the ones we are interested in here.

Replicator dynamic in mixed strategies. To understand the RMD in mixed strategies, we first introduce the RD in mixed strategies. Let $F_i^{\mathbf{r}}$ be s_i 's fitness given \mathbf{r} and $F_i^{\mathbf{s}}$ be r_i 's fitness given \mathbf{s} . Then $\Phi(\mathbf{s}, \mathbf{r}) = \sum_k \mathbf{s}_k \cdot F_k^{\mathbf{r}}$ is the average fitness in the sender population and $\Phi(\mathbf{r}, \mathbf{s}) = \sum_k \mathbf{r}_k \cdot F_k^{\mathbf{s}}$ the average fitness in the receiver population.

The continuous-time replicator dynamic defines the change of frequency of mixed strategies. Its so-called two-population version, that takes sender and receiver roles as separate update sites:

$$\dot{\mathbf{s}}_i = \mathbf{s}_i \cdot (F_i^{\mathbf{r}} - \Phi(\mathbf{s}, \mathbf{r})) \quad \mathbf{r}_i = \mathbf{r}_i \cdot (F_i^{\mathbf{s}} - \Phi(\mathbf{r}, \mathbf{s})) .$$

Similar to the version for behavioral strategies, a discrete-time formulation of the RD for mixed strategies can be conceptualized as an update function that maps mixed strategies \mathbf{s} and \mathbf{r} onto mixed strategies $\text{RD}(\mathbf{s})$ and $\text{RD}(\mathbf{r})$ respectively, such that:

$$\text{RD}(\mathbf{s})_i = \frac{\mathbf{s}_i \cdot F_i^{\mathbf{r}}}{\Phi(\mathbf{s}, \mathbf{r})} \quad \text{RD}(\mathbf{r})_i = \frac{\mathbf{r}_i \cdot F_i^{\mathbf{s}}}{\Phi(\mathbf{r}, \mathbf{s})} .$$

Replicator mutator dynamic. The replicator mutator dynamic extends the RD by adding probabilistic mutation. Let Q be a row-stochastic mutation matrix where Q_{ji} gives the probability that pure sender strategy s_j mutates into s_i . Similarly, let R be a row-stochastic mutation matrix where R_{ji} gives the probability that pure receiver strategy r_j mutates into r_i .

The RMD is usually given only in its continuous-time form. A two-population (non-payoff adjusted) formulation is then:

$$\dot{\mathbf{s}}_i = \sum_j Q_{ji} \cdot \mathbf{s}_j \cdot F_j^{\mathbf{r}} - \mathbf{s}_i \cdot \Phi(\mathbf{s}, \mathbf{r}) \quad \dot{\mathbf{r}}_i = \sum_j R_{ji} \cdot \mathbf{r}_j \cdot F_j^{\mathbf{s}} - \mathbf{r}_i \cdot \Phi(\mathbf{r}, \mathbf{s}) .$$

A discrete-time, two-population formulation of the RMD can be defined as follows (c.f. Page and Nowak, 2002, p. 97):

$$\text{RMD}(\mathbf{s})_i = \sum_j Q_{ji} \frac{\mathbf{s}_j \cdot F_j^{\mathbf{r}}}{\Phi(\mathbf{s}, \mathbf{r})} \quad \text{RDD}(\mathbf{r})_i = \sum_j R_{ji} \frac{\mathbf{r}_j \cdot F_j^{\mathbf{s}}}{\Phi(\mathbf{r}, \mathbf{s})} .$$

The discrete-time RMD has the same sequential nature as the RDD: first we com-

pute the fitness-driven change according to the standard replicator dynamic; then we compute the perturbation from mutation. This becomes clear if we define independent mutation functions $M_{Q,R}$ that map mixed strategies onto the outcome of (one round of) mutation:

$$M_Q(\mathbf{s})_i = \sum_j \mathbf{s}_j \cdot Q_{ji} \quad M_R(\mathbf{r})_i = \sum_j \mathbf{r}_j \cdot R_{ji}. \quad (3)$$

The discrete-time RMD given above can then be rewritten as:

$$\text{RDD}(\mathbf{s}) = M_Q(\text{RD}(\mathbf{s})) \quad \text{RDD}(\mathbf{r}) = M_R(\text{RD}(\mathbf{r})).$$

2.4 Diffusion as mutation

The sequential reformulation of the RMD in terms of a sequence of updates already closely resembles our initial formulation of the RDD. However, there is a major difference between these. While the RDD is a dynamic for behavioral strategies, the RMD, as taken from the literature, applies to mixed strategies. Similarly, the confusability of states is an operation that is straightforwardly definable for behavioral strategies (see Equations (1)), while mutation in the RMD is defined as mutation from one pure strategy to another. Nonetheless, it is possible to map each confusion matrix to a mutation matrix in a natural way so that the correspondence between mixed and behavioral strategies are conserved. This is the main result presented in this section, and it is our justification for regarding the RDD as the behavioral-strategy analogue of the replicator-mutator dynamic when the only source of mutation is confusability of states.

More concretely, since there is a non-injective mapping G from behavioral to mixed strategies, we would like to show that there is a systematic translation from a confusion matrix C to a mutation matrices Q^C such that for each mixed sender strategy \mathbf{s} , its unique corresponding behavioral strategy $\sigma = G(\mathbf{s})$ has a confusion-perturbation $C(\sigma)$ that corresponds, via G , to the mutation-perturbation $Q^C(\mathbf{s})$ (see Figure 2). In other

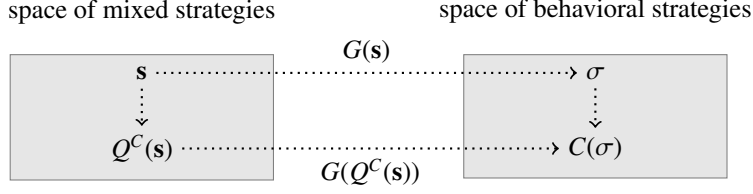


Figure 2: Correspondence between state-confusion and mutation.

words, we hypothesize that a confusion matrix C should give rise to a unique mutation matrix Q^C so that whenever $G(s) = \sigma$ we also have $G(Q^C(s)) = C(\sigma)$. Similarly, for the receiver.

Confusion-based mutations. There are natural conversions of C into Q^C and R^C . The case for the receiver is easier, so we start with that.

The probability that r_i is realized as r_j under diffusion with C is the product of the probabilities, for each m , that the state $r_i(m)$ is perceived as state $r_j(m)$. Abusing notation by referring to the indices of states $r_i(m)$ and $r_j(m)$ with $r_i(m)$ and $r_j(m)$ directly, we define:

$$R_{ij}^C = \prod_m C_{r_i(m)r_j(m)}. \quad (4)$$

Now look at the sender. The probability that s_j is realized as s_i under diffusion with C is the product of the probabilities, over all states t_k , that the message $s_i(t_k)$, that s_i would produce at state t_k in the absence of noise, is produced by a noisy realization of s_j , which is the probability $\sum_{t_l \in s_j^{-1}(s_i(t_k))} C_{kl}$ that the state t_k is realized as a state t_l which s_j would map unto $s_i(t_k)$. So, define:

$$Q_{ji}^C = \prod_{t_k} \sum_{t_l \in s_j^{-1}(s_i(t_k))} C_{kl}. \quad (5)$$

Based on Equations (4) and (5), we can formulate the desired correspondence re-

sult. A proof is given in Appendix A.

Theorem 1. (i) If $G(\mathbf{s}) = \sigma$, then $G(M_{Q^c}(\mathbf{s})) = D_C(\sigma)$. And, (ii) if $G(\mathbf{r}) = \rho$, then $G(M_{R^c}(\mathbf{r})) = D_C(\rho)$.

3 Exploring the RDD

The previous section showed that the RDD can be conceptualized as a two-step procedure: we first calculate where the replicator dynamic would take us, then apply diffusion. To understand the RDD better, in Section 3.1 we look closer at the novel diffusion part, before exploring in Section 3.2 how diffusion and fitness-based selection interact.

3.1 (Iterated) Diffusion

Confusion of states should intuitively be a function of their perceptual similarity. To make this concrete, let us assume that the state space of the sim-max game consists of $n \geq 2$ states that are equally spaced across the unit interval, including 0 and 1. The distance $|t_i - t_j|$ is the objective, physical similarity between two states t_i and t_j . Distance in physical space feeds into a perceptual similarity function, as described by Nosofsky (1986):

$$\text{Sim}(t_i, t_j; \alpha) = \begin{cases} 1 & \text{if } \alpha = 0 \text{ and } t_i = t_j \\ 0 & \text{if } \alpha = 0 \text{ and } t_i \neq t_j \\ \exp\left(-\frac{|t_i - t_j|^\alpha}{\alpha^2}\right) & \text{otherwise,} \end{cases}$$

where $\alpha \geq 0$ is an imprecision or indiscriminability parameter. When $\alpha = 0$ agents perfectly discriminate between states; when $\alpha \rightarrow \infty$ agents cannot discriminate states at all. The top part of Figure 3 gives an impression of Nosofsky-similarity for different parameter values. Other formalizations of perceptual similarity are possible, including

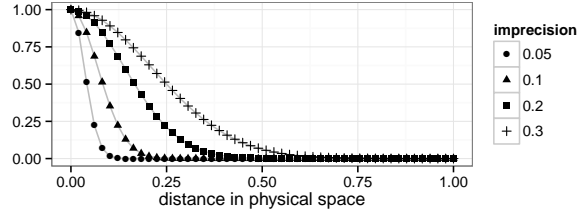


Figure 3: Examples of Nosofsky-similarity (non-normalized) and derived probability of state confusion (normalized) for $|T| = 50$ and different values of imprecision α .

ones that allow for different discriminability in different areas of the state space, but we stick with Nosofsky’s similarity function for the time being, because it is mathematically simple, yet an established notion in mathematical psychology.

We further assume that the probability of confusing any two states t_i and t_j is proportional to their perceived similarity and therefore obtained by normalization:

$$C_{ij} = \frac{\text{Sim}(t_i, t_j; \alpha)}{\sum_j \text{Sim}(t_i, t_j; \alpha)}.$$

Confusion of states is then also a function of imprecision parameter α (see Figure 3, bottom part). For $\alpha = 0$ the confusion matrix has $C_{ii} = 1$; for $\alpha > 0$, C is positive, i.e., $C_{ij} > 0$ for all i and j ; for $\alpha \rightarrow \infty$ we find $C_{ij} = 1/|T|$ for all i and j .

If states are confused on average with a probability proportional to their similarity, repeated applications of diffusion of behavior as defined in Equation (1) causes behavioral dispositions gradually to diffuse along a gradient of similarity of states as well. Consequently, iterated diffusion leads to a smoothing out and an eventual equalization of behavioral strategies, for both the sender and the receiver. After n steps of iterated diffusion an initial sender strategy σ will be $C^n \sigma$, and an initial receiver strategy ρ will be ρC^n . If we assume that $\alpha > 0$, C is a positive row-stochastic matrix (each state could in principle be confused as any other state, albeit with possibly a very low probability). It then follows from the Perron-Frobenius theorem that the limit $C^\infty = \lim_{n \rightarrow \infty} C^n$

exists (it is the Perron-projection) and all of its rows are identical. That is why, in the limit of iterated diffusion, $C^\infty S$ has identical rows (messages are sent with the same probability in each state), and RC^∞ likewise has identical rows (every message is interpreted equally). All of this is in line with the intuition that diffusion of behavior under confusion of states iteratively equalizes behavior for similar states; if all states can be confused for one another in principle, behavior smoothes out entirely in the limit.

3.2 Exploring the replicator diffusion dynamics

Now that we understand better what diffusion does, let us look at how diffusion interacts with optimization of behavior, as described by the replicator dynamics. The main question we should address is whether diffusion and fitness-based replication reasonably interact, and if so, whether the inclusion of diffusion has any noteworthy effects on the evolving meaning of signals. Clearly, if $\alpha = 0$, the \mathbf{RDD} reduces to the \mathbf{RD} . If $\alpha \rightarrow \infty$, the diffusion component takes over and the \mathbf{RDD} reduces to the trivial iterated diffusion process that we looked at in the previous section. We will show presently that for reasonable in-between levels of imprecision α , the \mathbf{RDD} leads to communicatively successful, yet vague signal meaning. Suitable levels of imprecision can have further accelerating and unifying effects on meaning evolution.

3.2.1 Experimental set-up

To explore the \mathbf{RDD} , we turn to numerical simulations. Let states be evenly spaced elements of the unit interval that always include 0 and 1, and let priors be flat: $\Pr(t) = \Pr(t')$ for all t, t' . As for utility functions, we take another Nosofsky-style function:

$$U(t_i, t_j; \beta) = \begin{cases} 1 & \text{if } \alpha = 0 \text{ and } t_i = t_j \\ 0 & \text{if } \alpha = 0 \text{ and } t_i \neq t_j \\ \exp\left(-\frac{|t_i - t_j|^\beta}{\beta^2}\right) & \text{otherwise.} \end{cases}$$

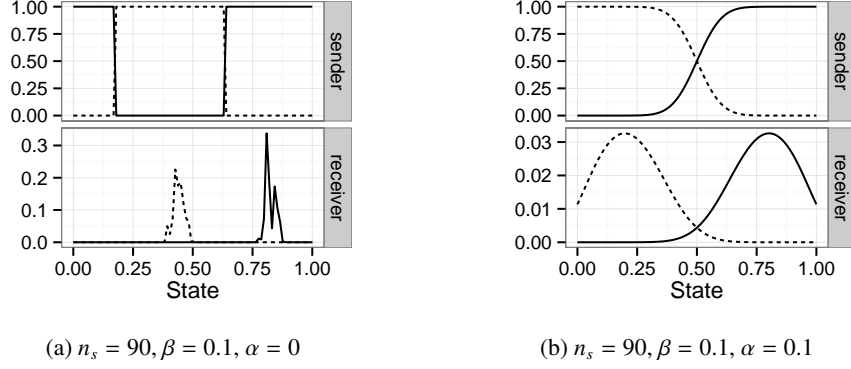


Figure 4: Example strategies under RDD at stopping time of our simulations.

The tolerance parameter $\beta \geq 0$ models the amount of tolerable pragmatic slack or communicative imprecision. This choice of utility function is governed partly by convenience in parallel to the similarity function, but also because it has, as we believe, the right general properties for a communicative payoff function. Unlike utilities that are, say, linearly or quadratically decreasing in physical distance (c.f. Jäger, Metzger, and Riedel, 2011; Franke, Jäger, and van Rooij, 2011), utilities that are exponentially decreasing in negative quadratic distance can model situations where a small amount of imprecision in communication is tolerable, whereas similarly small differences in intolerably far away interpretations matter very little, with a smooth transition between these regimes (c.f. O’Connor, 2014).

We ran 50 trials of the RDD, starting with randomly sampled sender and receiver strategies, for each triplet of independent parameter values: $n_s = |T| \in \{6, 10, 50, 90\}$, $\alpha \in \{0, 0.05, 0.1, 0.2, 0.3\}$, $\beta \in \{0.05, 0.1, 0.2, 0.3\}$. Each trial ran for a maximum of 200 update steps of the RDD. This may not be enough to guarantee convergence to the eventual attracting state, but we were interested especially in whether the dynamics could lead to reasonable outcomes in reasonable time. A trial was considered converged and stopped before the maximum of 200 rounds if the total amount of change between

strategies before and after the current RDD step was smaller than 0.001, i.e., if:

$$\sum_{t \in T} \sum_{m \in M} |\text{RDD}(\sigma)(t, m) - \sigma(t, m)| < 0.001, \text{ and}$$

$$\sum_{m \in M} \sum_{t \in T} |\text{RDD}(\rho)(m, t) - \rho(m, t)| < 0.001.$$

Representative examples for resulting strategy pairs at stopping time are given in Figure 4. Figure 4a shows a strategy pair at stopping time with 90 states, tolerance $\beta = 0.1$ and imprecision $\alpha = 0$. Zero imprecision means that the trial was effectively an application of the plain RD. Noteworthy, the given sender strategy approximates a pure sender strategy that crisply partitions the state space into non-convex sets. The irregular shape of the receiver strategy shows clearly that the pictured strategy pair has not yet reached a dynamically stable state under the RD. Indeed, the trial did not converge in our technical sense, but was stopped after the maximum of 200 rounds. In contrast, the outcome of a trial with identical parameters, except with imprecision $\alpha = 0.1$, which is shown in Figure 4b, had converged (in our technical sense) after 99 rounds of iterated RDD. The sender strategy shows a smooth blending from one “category” to the other, and the receiver’s interpretations are rather extended curves, peaking at a central point in the relevant “categories.”

These examples already show two interesting things. Firstly, inclusion of imprecision can lead to seemingly well-behaved, yet vague strategies in the sense that we are after. The sender strategy in Figure 4b identifies clear positive and clear negative cases for each signal, with a smooth transition in-between. The receiver’s interpretations of signals can be seen as smoothed-out prototype regions. Secondly, (sender) strategies can approach non-convex pure strategies under the replicator dynamic. We see this in our limited-time simulations, but this also holds, for some types of utility functions, for the limiting case. This was first observed by Elliott Wagner in unpublished work (c.f. O’Connor, 2013). A full analysis of the dynamics of sim-max games is beyond

the scope of this paper, but we will see shortly that diffusion from confusability of states clearly prevents evolutionary paths that meander for a long time in the vicinity of non-convex strategies.

3.2.2 Dependent measures

To further explore our simulation results, we defined a small number of metrics that aim to capture numerically how vague, communicatively efficient and generally well-structured the resulting strategy pairs were. *Entropy* captures the amount of systematicity or regularity in signal use. *Voronoiness* and *convexity* are, respectively, gradient and categorical measures for the coherence of signal-induced categories. *Expected utility* measures the communicative efficiency of evolved strategy pairs.

Entropy. This classic information-theoretic notion captures the amount of uncertainty in a probability distribution. Roughly put, entropy of a signaling strategy captures inverse distance from a pure strategy. The usual definition of entropy applies directly to mixed strategies:

$$E(\mathbf{s}) = \sum_{s \in M^T} \mathbf{s}(s) \cdot \log(\mathbf{s}(s)) \quad E(\mathbf{r}) = \sum_{r \in T^M} \mathbf{r}(r) \cdot \log(\mathbf{r}(r)).$$

These measures are computationally expensive to calculate, since the size of the domain over which the sum is computed grows exponentially with the number of choice points. But provably equivalent metrics for behavioral strategies are ready to hand:

$$E(\sigma) = - \sum_{t \in T} \sum_{m \in M} \sigma(m \mid t) \cdot \log(\sigma(m \mid t))$$

$$E(\rho) = - \sum_{m \in M} \sum_{t \in T} \rho(t \mid m) \cdot \log(\rho(t \mid m)).$$

Values obtained by these definitions are lower bounded by 0 and upper bounded by, respectively, $\log(|M^T|) = |T| \cdot \log(|M|)$ and $\log(|T^M|) = |M| \cdot \log(|T|)$. We work with normalized values.

The sender strategies in Figures 4a and 4b have entropy $1.19e^{-5}$ and 0.21, respectively. The receiver strategies have respective entropies 0.43 and 0.84. In general, we expect that vague languages will have higher entropy than crisp ones and that increasing imprecision will increase entropy, all else equal.

Voronoiness. This metric aims to quantify how close a strategy is to being a part of a Voronoi tessellation of the state space. Bearing in mind that this metric is specifically geared towards the sim-max games in our set-up, we define Voronoiness as follows. Let $\pi(\rho, m) = \arg \max_{t \in T} \rho(m, t)$ be the prototype of a message $m \in M$, then

$$V(\sigma, \rho) = \sum_{t \in T} \sum_{m_1 \in M} \sum_{m_2 \in M} F(\sigma, \rho, t, m_1, m_2) \quad \text{with}$$

$$F(\sigma, \rho, t, m_1, m_2) = \begin{cases} 1 & \text{if } \text{Sim}(t, \pi(\rho, m_1)) > \text{Sim}(t, \pi(\rho, m_2)) \Leftrightarrow \\ & \sigma(t, m_1) > \sigma(t, m_2) \\ 0 & \text{otherwise} \end{cases}$$

is the Voronoiness of the sender strategy σ given the receiver strategy ρ and

$$V(\rho) = \sum_{m \in M} \sum_{t_1 \in T} \sum_{t_2 \in T} F(\rho, m, t_1, t_2) \quad \text{with}$$

$$F(\rho, m, t_1, t_2) = \begin{cases} 1 & \text{if } \text{Sim}(t_1, \pi(\rho, m)) > \text{Sim}(t_2, \pi(\rho, m)) \\ & \Leftrightarrow \rho(m, t_1) > \rho(m, t_2) \\ 0 & \text{otherwise} \end{cases}$$

is the Voronoiness of the receiver strategy ρ . The metrics are lower bound by 0 and upper bound by, respectively, $|T| \times |M|^2$ and $|M| \times |T|^2$, thus we will normalize them by

dividing by these values.

Despite being vague, a language can have maximal Voronoiness according to this gradient measure. For instance, the sender strategies in Figures 4a and 4b have Voronoiness 0.91 and 1, respectively. The receiver strategies have Voronoiness 0.78 and 1, respectively. We do not *prima facie* expect a conceptual connection between vagueness and Voronoiness, and so do not predict a clear effect of imprecision on Voronoiness either.

Convexity. At least for sender strategies, which converge faster, it also makes sense to define a categorical measure of convexity, that compensates for potential vagueness. To determine whether a sender strategy σ is convex despite possibly being vague, we look at the derived pure strategy s for which $s(t) = \arg \max_{m' \in M} \sigma(t, m')$. If that s is convex, we also count σ as convex. The sender strategy in Figure 4a is not convex, while the one in Figure 4b is. Again, we do not expect a conceptual relation between vagueness and imprecision, on the one hand, and convexity on the other.

Expected utility. We also record the expected utility of a strategy pair:

$$\text{EU}(\sigma, \rho; \beta) = \sum_{t \in T} \sum_{m \in M} \sum_{t' \in T} \text{Pr}(t) \cdot \sigma(t, m) \cdot \rho(m, t') \cdot U(t, t'; \beta).$$

To make direct comparisons across different parameter settings, we normalize expected utility by the maximal amount of expected utility obtainable in the relevant game.

The strategy pair in Figure 4a has a normalized expected utility of 0.992, the pair in Figure 4b has 0.901. Generally, vagueness and imprecision can be expected to decrease expected utility (c.f. Lipman, 2009). The crucial question is whether communicative success drops unacceptably fast with moderate levels of vagueness and imprecision.

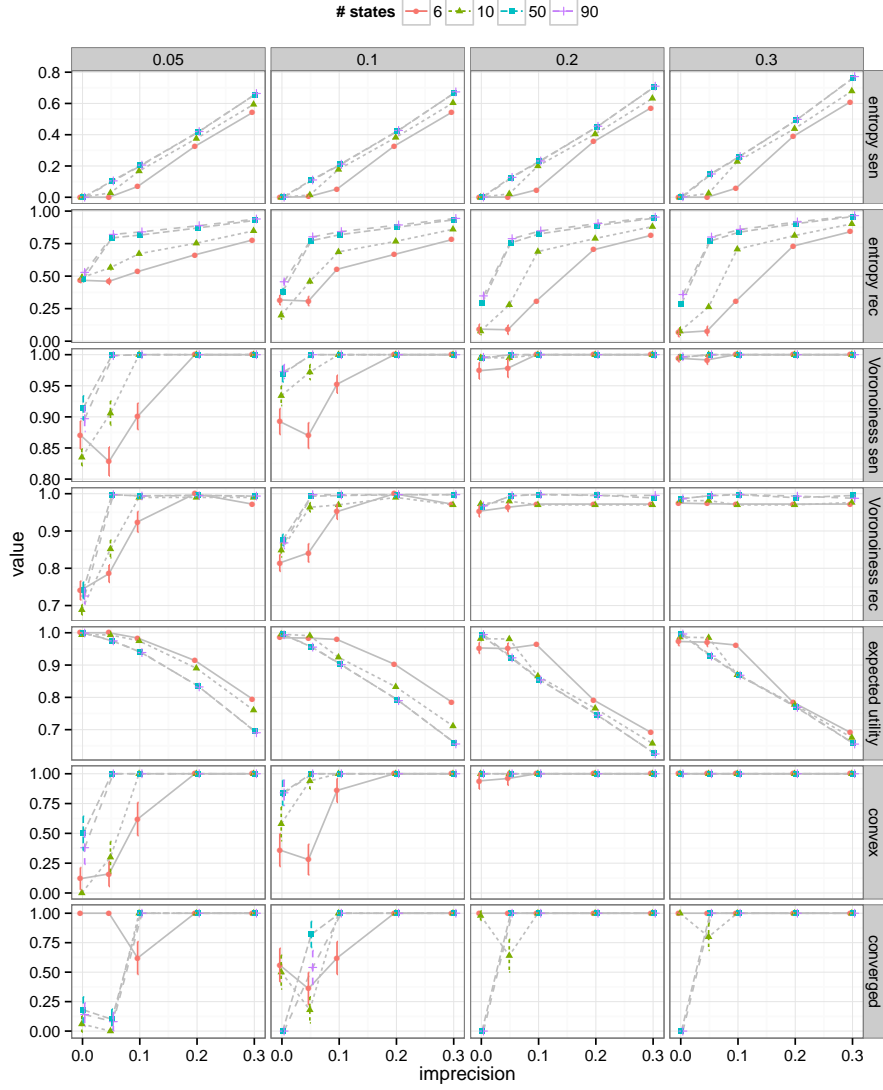


Figure 5: Means of gradient and proportions of categorical metrics for each triple of independent variables, with estimated 95% confidence intervals.

3.2.3 Results

Figure 5 shows the means of the recorded metrics, together with their estimated 95% confidence intervals, for each triple of independent variables. Most trends are quite clear, and not unexpected. Entropy of sender and receiver strategies are increasing with imprecision. Entropy of sender strategies seems independent of tolerance levels and the number of states, while there seems to be an effect of both parameters on the entropy of the receiver strategy. What was not expected *a priori* was that Voronoiness of both sender and receiver strategies appear to be non-decreasing under increasing impairment, with the only exception being games with 6 states and the transition from $\alpha = 0$ to $\alpha = 0.05$. Expected utility of evolved languages mostly decreases in increasing imprecision, with the sole exception of a rise for $\beta = 0.2$ and $n_s = 6$ at $\alpha = 0.1$. Still, normalized expected utility of evolved strategy pairs under mild values of imprecision are not necessarily much worse than their respective perfect precision RD-analogues. This is especially so for lower numbers of states (see discussion below).

Taken together, these findings suggest the following effect of confusability of states and diffusion of behavior. Mild forms of perceptual imprecision lead to slightly less communicative efficiency, more vagueness and more regular, well-behaved languages. This impression is also supported by the proportion of converged and convex trials, pictured in Figure 5. Non-converged and non-convex sender strategies showed for several values of parameters in the absence of impairment. Increasing impairment always assured convergence and convexity: diffusion speeds up convergence to a convex and regular language.

Diffusion also has another interesting regularizing effect on the evolution of signaling. If we look at the confidence intervals in Figure 5, we see that there is very little variation in the recorded metrics for evolved strategies, at least for higher values of impairment. On closer inspection, it turns out that variability in low-impairment conditions is not only due to non-convergence or non-convexity. Figure 6 gives two more

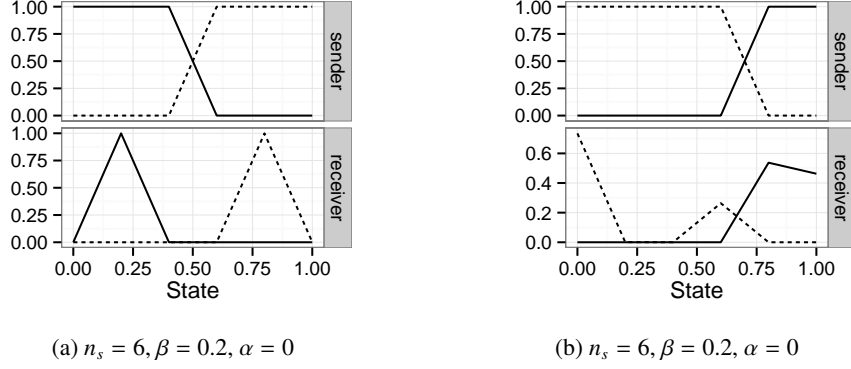


Figure 6: More example strategies under RDD at stopping time of our simulations.

examples of strategy pairs at stopping time. Both are obtained for the same triple of parameters, both converged and are convex. However, they are not equally efficient. In fact, the pair in Figure 6a has a normalized expected utility of 0.99 while the one in Figure 6b only has 0.89.

Interestingly, this type of variability in evolutionary outcomes is also weeded out by mild increases in impairment. To see this, we calculated the average distance between evolved sender strategies within each group of trials that had identical independent parameter values. We determined the distance between sender strategies σ and σ' as the average Hellinger distance between probability distributions $\sigma(t)$ and $\sigma'(t)$ over messages at each choice point t :

$$\text{HD}(\sigma, \sigma') = \frac{1}{|T| \cdot \sqrt{2}} \cdot \sum_{t \in T} \sqrt{\sum_{m \in M} (\sqrt{\sigma(t, m)} - \sqrt{\sigma'(t, m)})^2}.$$

To compensate for the arbitrariness of message use, we set the distance between strategies σ and σ' to be the maximum of $\text{HD}(\sigma, \sigma')$ and $\text{HD}(\sigma^*, \sigma')$ where σ^* is σ with reversed message indices. The *within group distances*, i.e., the average distances between all sender strategies in each parameter group, are plotted in Figure 7. This clearly

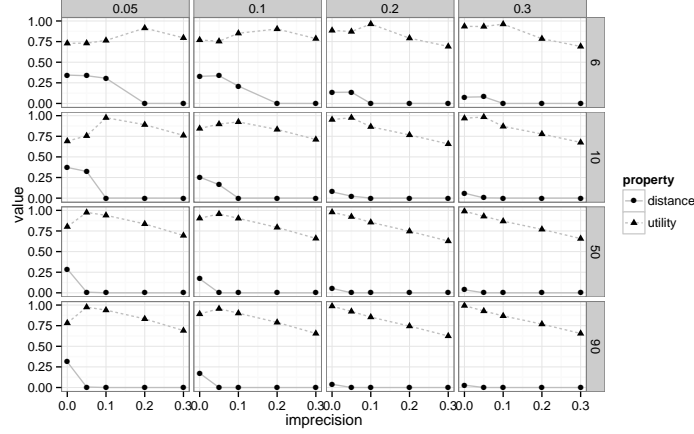


Figure 7: For each group, i.e., triplet of independent variables, the plots show (i) within group distances and (ii) within group expected utilities, as defined in the main text.

shows that with increasing imprecision, the sender strategies evolving under `RDD` were exactly alike (modulo swapping of messages). In other words, perceptual imprecision unifies evolutionary outcomes, and in fact guarantees sender strategies that are not only convex, but also maximally efficient in that they induce a vague category split exactly in the middle of the unit interval.

This unifying property of perceptual imprecision could be considered an evolutionary beneficial side-effect. Higher imprecision can lead to higher *within group expected utility*, i.e., the average expected utility that each evolved language (i.e., pair of sender and receiver strategy at stopping time) scored when playing against an arbitrary other language of the same parameter group (see Figure 7). What this means is that, while imprecision might decrease the communicative efficiency of individual languages, it increases the conceptual coherence and communicative success between independently evolving strategies. It is almost as if mere confusability of states imposes a regularity constraint on evolving categories.

4 Discussion

4.1 Model interpretation

We mentioned in passing in Section 2.2 that adding diffusion to other discrete-time evolutionary dynamics that operate on behavioral strategies is entirely straightforward. We could, for instance, easily diffuse the outcome of a best-response dynamic at each time step. That would make sense if we thought of agents as prone to confusing similar states, but otherwise rationally optimizing behavior. The reason that we chose the RD to combine with diffusion is twofold. A minor reason is that it makes for a conceptually interesting link with the RMD. A more important reason is that the RD is especially versatile and non-committal about what the exact process of adaption is that is being modelled. Generally, the RD could be a biological or a cultural dynamic, and so could the RDD.

Under a biological interpretation of the RD, signaling strategies are innate behavioral dispositions of organisms, steps in the evolutionary process are successive generations, and selection captures the reproductive advantage of fitter individuals. This interpretation, strictly speaking, requires formalization in terms of mixed strategies via the RMD, and possibly also a symmetrizing of the game, so that every agent is assumed to have a unique sender and receiver role at the same time, the pair of which is bequeathed onto the next generation.

Diffusion, in this context, could be either of two things. It could be differential mutation probabilities in line with other interpretations of mutation in the RMD (e.g. Nowak, Komarova, and Niyogi, 2001; Komarova, Niyogi, and Nowak, 2001): pure strategies are not faithfully reproduced, e.g. by imperfections of learning or underdetermination of input, and mistakes in inheriting pure strategies are more likely if they result from the confusion of similar states. Another possible biological interpretation of the RDD is that inheritance is faithful, but strategies are noisily realized. Strategies

are not necessarily selected for what they are, but rather for how they are realized, once noise is factored in.

As a cultural dynamic, the RD can also be interpreted as a high-level description of the likely development of other behavioral adaptation processes, like differential imitation (see Sandholm, 2013, for various derivations of the RD). Under this interpretation, what is subject to evolutionary optimization is behavior, not genetic make-up. Fitness captures the success of behavioral patterns. Differential reproduction represents the tendency of more successful ways of behavior to be more likely realized, be it through imitation or some solitary learning or optimization mechanism.

Under this non-biological interpretation, we have again two options of picturing what diffusion is. One possibility is that diffusion is noise in the adoption of strategies, e.g. by conditional imitation, of strategies by other signalers. Another possibility is, again, that diffusion is noise on the realization of strategies: while behavior is optimized to be efficient (be it due to learning, introspection or imitation), realization of strategies is bound to be noisy due to confusability of similar states.

We believe that all of the four mentioned interpretations are, on first approximations, feasible conceptualizations of the RDD, and that it is a good thing to know of a working account of vague signaling that sketches where fitness-based selection under state-confusability will take us, abstracting away from the details of actually playing the game, inheriting, imitating or otherwise optimizing behavior in whatever particular way. It is a good thing to know this on the macro-level, especially since there are also micro-level accounts that nicely complement the picture. We turn to one such next.

4.2 Relation with previous accounts

Generalized reinforcement. Under *Herrnstein reinforcement learning*, sender and receiver adjust their dispositions to act after each round of play, in such a way that the actual (non-negative) payoff gained in the previous interaction is added to the non-

normalized propensities for repeating the same behavior. For sim-max games, this means that, when the sender chose m in state t , and this resulted in some non-negative payoff, the probability that the sender chooses m again in t is increased, but nothing else changes. In particular, the sender’s behavior in other choice points does not change. *Generalized reinforcement learning* is different. When the use of m in t gave positive payoff, then not only will its future use probability be promoted at t but also at other states, proportional to how similar these are to t . Similar amendments take care of the way that the receiver updates his choice dispositions.

O’Connor (2014) shows that this extension not only leads to vague signaling of the appropriate kind, but also speeds up learning in such a way that, especially for sim-max games with higher numbers of states, higher levels of communicative success are reached in shorter learning periods. Technically, this result is partly due to the fact that signalers make bigger changes to their behavioral strategies after each round of play under generalized reinforcement than under the Herrnstein variety. But that only explains the speed of adaptation, not necessarily that generalization also leads to regularity and communicative efficiency, but it does.

Diffusion of strategies in the RDD can also be conceived of as a form of generalization, and works in large part quite analogous to stimulus generalization in O’Connor’s approach. But there are still differences. O’Connor showed that generalized reinforcement learning can speed up the development of efficient signaling, especially for higher numbers of states. Complementing this, we showed that diffusion regularizes evolutionary outcomes and prevents meandering around sub-optimal and non-convex strategies, also for low numbers of states.

Conceptually, the RDD is a more abstract framework than generalized reinforcement learning. The latter is foremost motivated as a learning dynamic that has two players adapt their individual strategies after each concrete round of play. In contrast, the RDD describes a more abstract, average dynamical change in behavioral dispositions.

Although the behavior of (some forms of) reinforcement learning mirror those of the RD (at some stage in time) (Börger and Sarin, 1997; Hopkins and Posch, 2005; Beggs, 2005), this does not mean that *generalized* reinforcement learning is also necessarily a plausible high-level description of, say, generalized learning in a population of agents. Seen in this light, generalized reinforcement learning and the RDD nicely complement each other, as similarly-minded accounts operating at different levels of abstraction.

Quantal response equilibria. Franke, Jäger, and van Rooij (2011) suggested a number of ways in which information-processing limitations could lead to vague strategies. The model that is most clearly related to the present approach uses the notion of a *quantal response*, also known as a *soft-max function* (e.g. Luce, 1959; McFadden, 1976; Goeree, Holt, and Palfrey, 2008). A quantal response function is a parameterized generalization of the classic best response function. For example, if $U : A \rightarrow \mathbb{R}$ is the measure of expected utility over choices A of an agent, then a best response function would have the agent choose a only if $U(a) = \max_{a' \in A} U(a')$. A quantal response function rather assumes that agents would choose a with a probability proportional to $\exp(\lambda \cdot U(a))$, where λ is a rationality parameter. If $\lambda \rightarrow \infty$ we retrieve the behavior of the best response function, but if it is positive but finite, any choice a will receive a positive probability, but acts with higher expected utility will be more likely. The underlying motivation is the assumption that there is noise in the computation of expected utilities and/or in maximization of expected utilities. Consequently, choices with almost equal expected utilities will be chosen with almost equal probability (for moderate values of λ).

Franke, Jäger, and van Rooij (2011) show that quantal response equilibria of sim-max games, i.e., pairs of sender and receiver strategies such that the sender strategy is the quantal response to the expected utilities under the receiver strategy and vice versa, can show the desired marks of vague signaling. Figure 8a gives an example of a quantal response equilibrium for a sim-max game, as used in our set-up but with

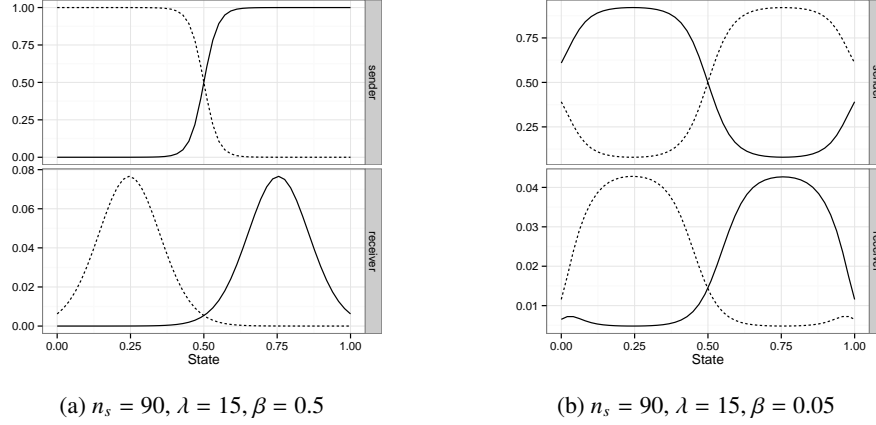


Figure 8: Examples of vague quantal response equilibria.

higher tolerance $\beta = 0.5$. Sender and receiver strategies look very much like what evolves under RDD with modest values of perceptual imprecision.

But not all quantal response equilibria are equally plausible, and the ones that are not suggest that it is less natural to think of vagueness as arising from errors in the computation or maximization of expected utility than that it arises from confusion of similar states. To see what the problem is, we can look at cases like given in Figure 8b, which is the quantal response equilibrium for a game with lower tolerance $\beta = 0.05$. Unlike what evolves under RDD in this case, sender strategies have vague boundaries also towards the end of the unit intervals. Technically, this is because quantal responses equalize message use far away from the “prototypical” interpretation, not just in-between categories, so to speak. This, in turn, is because quantal responses introduce noise into the decision making at the level of computing or maximizing expected utility of choices.

That this is conceptually odd shows even more clearly in a case where the state space is intuitively unbounded, as for instance for the property “tall”. If the usual interpretation of a “tall man” peaks at around, say, 195cm then when meeting a giant

of n meters, senders would, according to the quantal response approach, be ever more inclined to describe the giant indifferently as either “tall” or “short” the larger n gets. This is because, as the distance from the prototype increases for larger n , the difference between the expected utilities of saying “short” or “tall” will converge to zero.

Admittedly, this argument hinges on the choice of utility function. Still, to the extent that the chosen lower bounded utility functions are reasonable—and we think they are very reasonable—the case suggests that quantal responses are not a good model, intuitively speaking, for why linguistic categories are vague. Vagueness is more plausibly an effect of perceptual confusion of similar states than of computational errors in maximizing expected utility.

5 Conclusions

Vagueness is a pervasive but seemingly non-disruptive feature of natural communication and classification systems. From an evolutionary point of view, the challenge arises to explain how vagueness can persist under selective pressure for precision. This is foremost a technical challenge, probing for the possibility of integrating into a consistent model forces that lead to vagueness and forces that lead to efficient information transfer. This paper proposed one such model in the context of sim-max games and explored some of its consequences.

We introduced the replicator diffusion dynamic as a special case of the replicator mutator dynamic. The RDD incorporates a special kind of stochastic mutation due to differential confusability of similar states. Based on data from numerical simulations, we demonstrated that the inclusion of mild levels of stochastic confusability of states does not undermine the possibility of evolving communicatively successful signaling strategies at all. On the contrary, strategies that evolved under mild diffusion induce a highly regular and systematic category structure that shows the signs of vagueness as desired. There might even be a higher-order benefit to the presence of imprecision,

in that it can accelerate convergence to optimal categorization, by preventing evolutionary paths to stay near inefficient non-convex strategies for a long time. Diffusion thus also unifies and regularizes evolutionary outcomes in such a way that sub-optimal categorization is avoided.

A Proof of Theorem

Part (i). Fix \mathbf{s} and $\sigma = G(\mathbf{s})$. Look first at the rhs of the consequent:

$$\begin{aligned} D_C(\sigma)(m_y | t_x) &= \sum_{t_l} C_{xl} \cdot \sigma(m_y | t_l) && \text{(by Equation (1))} \\ &= \sum_{t_l} C_{xl} \cdot \sum_{s_i(t_l)=m_y} \mathbf{s}_i && \text{(by Equation (2))} \\ &= \sum_{t_l} \sum_{s_i(t_l)=m_y} \mathbf{s}_i \cdot C_{xl} \cdot \end{aligned}$$

Next consider the lhs of the consequent:

$$\begin{aligned} G(M_{Q^C}(\mathbf{s}))(m_y | t_x) &= \sum_{s_i(t_x)=m_y} M_{Q^C}(\mathbf{s}_i) && \text{(by Equation (2))} \\ &= \sum_{s_i(t_x)=m_y} \sum_{s_j} \mathbf{s}_j \cdot Q_{ji}^C && \text{(by Equation (3))} \\ &= \sum_{s_i(t_x)=m_y} \sum_{s_j} \mathbf{s}_j \cdot \prod_{t_l} \sum_{t_m \in S_j^{-1}(s_i(t_l))} C_{lm} && \text{(by Equation (5))} \\ &= \sum_{s_j} \mathbf{s}_j \cdot \sum_{s_i(t_x)=m_y} \prod_{t_l} \sum_{t_m \in S_j^{-1}(s_i(t_l))} C_{lm} \end{aligned}$$

To simplify this further we look at a fixed s_j and consider the term:

$$\sum_{s_i(t_x)=m_y} \prod_{t_l} \sum_{t_m \in S_j^{-1}(s_i(t_l))} C_{lm} \cdot \tag{6}$$

Let Y be the row-stochastic matrix with $Y_{kl} = \sum_{t_m \in s_j^{-1}(m_l)} C_{km}$. Every pure sender strategy maps each state t_k onto exactly one Y_{kl} . If we quantify over all pure strategies, we essentially look at each such mapping. Term (6) above sums over all pure strategies that map t_k onto m_y . The above term then sums over all products whose factors are tuples in $\times_{k \geq 2} \{y \mid \exists l : y = Y_{kl}\}$. So term (6) expands to (where $e = |T|$ and $d = |M|$):

$$\begin{aligned} & (Y_{11} \cdot Y_{21} \cdot Y_{31} \cdot \dots \cdot Y_{e1}) + (Y_{11} \cdot Y_{21} \cdot Y_{31} \cdot \dots \cdot Y_{e2}) + \dots \\ & + (Y_{11} \cdot Y_{2d} \cdot Y_{3d} \cdot \dots \cdot Y_{ed}) \end{aligned}$$

But since Y is a row-stochastic matrix, this simplifies to Y_{xy} . Continuing the derivation with this:

$$\begin{aligned} G(M_{QC}(\mathbf{s}))(m_y \mid t_x) &= \sum_{s_j} \mathbf{s}_j \cdot \sum_{t_l \in s_j^{-1}(m_y)} C_{xl} \\ &= \sum_{s_j} \sum_{t_l \in s_j^{-1}(m_y)} \mathbf{s}_j \cdot C_{xl} \\ &= \sum_{t_l} \sum_{s_i(t_l)=m_y} \mathbf{s}_i \cdot C_{xl}. \end{aligned}$$

□

Part (ii). Fix \mathbf{r} and $\rho = G(\mathbf{r})$. The rhs of the consequent expands to:

$$\begin{aligned} D_C(\rho)(t_x \mid m_y) &= \sum_{t_j} \sum_{i \in \{k \mid r_k(m_y)=t_x\}} \mathbf{r}_i \cdot C_{jx} && \text{(by Equation (1))} \\ &= \sum_{r_i} \sum_{j \in \{k \mid r_i(m_y)=t_j\}} \mathbf{r}_i \cdot C_{jx} \\ &= \sum_{r_i} \mathbf{r}_i \cdot C_{r_i(m_y)x}. \end{aligned}$$

The rhs expands to (by Equations (2), (3) and (4)):

$$G(\mathbf{M}_{RC}(\mathbf{r}))(t_x | m_y) = \sum_{r_i} \mathbf{r}_i \cdot \sum_{j \in \{j | r_k(m_y)=t_x\}} \prod_m C_{r_i(m)r_j(m)}.$$

Without loss of generality, assume that $x = y = 1$, and fix $|M| = d$ and let e be the number of pure receiver strategies. Then the last term can be rewritten as:

$$\begin{aligned} &= \sum_i \mathbf{r}_i \cdot (C_{r_i(m_1)1} \cdot C_{r_i(m_2)r_1(m_2)} \cdot \dots \cdot C_{r_i(m_d)r_1(m_d)} + \dots \\ &\quad + C_{r_i(m_1)1} \cdot C_{r_i(m_2)r_2(m_2)} \cdot \dots \cdot C_{r_i(m_d)r_2(m_d)} + \dots \\ &\quad + C_{r_i(m_1)1} \cdot C_{r_i(m_2)r_e(m_2)} \cdot \dots \cdot C_{r_i(m_d)r_e(m_d)})). \end{aligned}$$

For every messages m_l , $C_{r_i(m_l)}$ is a stochastic vector. For $l > 1$, all elements of these vectors appear equally often. But that means that these cancel out. What remains is:

$$= \sum_{r_i} \mathbf{r}_i \cdot C_{r_i(m_y)x}.$$

□

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