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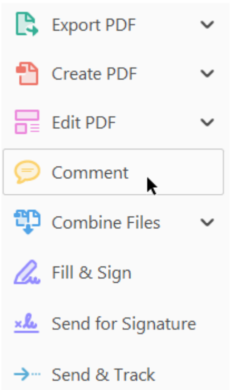

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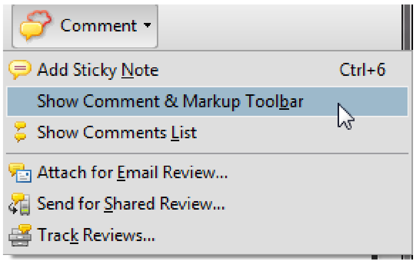

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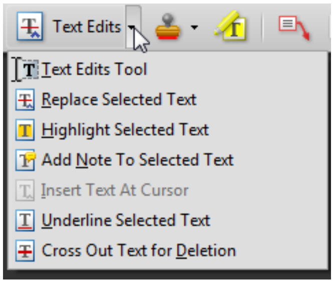

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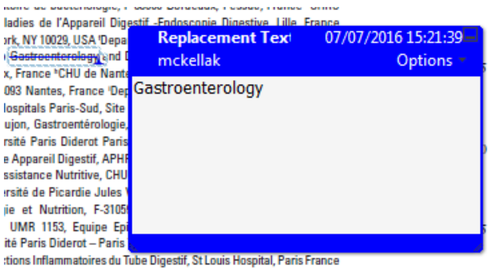

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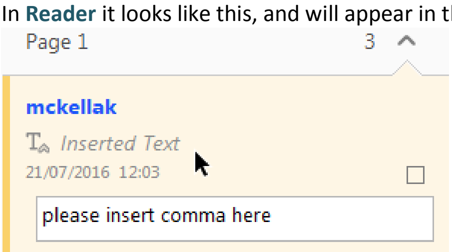

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Vagueness and Imprecise Imitation in Signalling Games

Michael Franke and José Pedro Correia

ABSTRACT

Signalling games are popular models for studying the evolution of meaning, but typical approaches do not incorporate vagueness as a feature of successful signalling. Complementing recent like-minded models, we describe an aggregate population-level dynamic that describes a process of imitation of successful behaviour under imprecise perception and realization of similar stimuli. Applying this new dynamic to a generalization of Lewis's signalling games, we show that stochastic imprecision leads to vague, yet by-and-large efficient signal use, and, moreover, that it unifies evolutionary outcomes and helps avoid sub-optimal categorization. The upshot of this is that we see 'as-if'-generalization at an aggregate level, without agents actually generalizing.

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1 Introduction

Many concepts and expressions are vague. A vague category knows clear cases that fall under it, clear cases that do not, and also so-called borderline cases. Borderline cases do not clearly belong to the category, nor do they clearly not belong, and there may be differences between borderline cases in terms of how well they represent the category in question. Vagueness does not seem to dramatically affect the success of everyday communication, but it is troublesome for some of the most prominent theories of language and meaning. This is especially so for the logico-positivist tradition of Frege, Russell, and early Wittgenstein, which is challenged by the paradoxes vagueness gives rise to.

There are other intriguing aspects about vagueness. One perplexing issue is how vagueness could ~~arise and be~~ maintained in the first place. This is an apparent puzzle for functionalist accounts that maintain that concepts and linguistic meanings evolved towards efficiency. Lipman ([unpublished]) argues that in common interest signalling situations, the existence of unclear borderline cases entails inefficiency of categorization or communication, or at least no advantage. The challenge is then to explain how vagueness can persist both: (i) under evolutionary pressure to be optimally discriminative, and (ii) without undermining the possibility of evolving, learning, and communicating with a meaningful language. A number of authors have consequently tried to explain why vagueness evolved as something that is itself useful (de Jaegher [2003]; van Deemter [2009]; Blume and Board [2014]). Others have argued that vagueness is a natural by-product of limitations in information processing (Franke *et al.* [2011]) or of generalization in low-level learning strategies (O'Connor [2014a]). This article contributes to the latter line of thought.

We believe that vagueness in language and thought may have many ~~reasons,~~ not just one. We focus here on one *a priori* plausible reason for why vagueness is natural and pervasive. The idea is that vagueness is, at least in part, due to imprecision in the perception of similar stimuli and imprecision in the realization of similar responses. On this view, vagueness in language may be seen as a necessary sub-optimality due to limitations in another domain of cognition first and foremost, namely, perception. It could then be speculated that the whole system, perception and language together, may be an almost optimal adaptation in a larger frame of reference, for example, if we take into account the metabolic costs for increased perceptual accuracy. We will not engage in such speculation here. Instead, we will explore the consequences of perceptual limitations on processes of meaning evolution in a suitable formal framework. In other words, in order to address Lipman's challenge seriously, and not just hand-wave it away by appeal to the naturalness of vagueness, one needs to spell out how exactly confusability of stimuli could come into play in a process of meaning evolution and how it could lead to vague but by-and-large informative signal meaning. The formal model that this article introduces does exactly that. But it also does more. We find that confusability of stimuli can regularize and systematize evolving meaning. This suggests a possibly advantageous side effect that a natural cause of vagueness may have, and that might compensate for some of the disadvantages for communicative efficiency (see O'Connor [2014a]).

The next section introduces the background against which the work presented here can be appreciated. Section 3 introduces a generalization of the replicator dynamic that is derived from the idea that agents imitate other agents' behaviour while possibly confusing similar states.[1] This imprecise imitation dynamic is explored in Section 4. Section 5 reflects and compares our approach to that of others. The appendix provides formal detail.

2 Background

2.1 Sim-max games and conceptual spaces

Signalling games, as introduced by Lewis ([1969]), have a sender and a receiver. The sender knows the true state of the world, but the receiver does not. The sender can select a signal, or message, to reveal to the receiver, who then chooses an act. In Lewis's games, if the receiver chooses the act that corresponds to the actual state, the play is a success; otherwise, it is a failure. Certain regular combinations of sender signalling and receiver reaction make messages meaningful, in the sense that their use is correlated systematically to certain states or acts. The conditions under which such meaning-generating behaviour can evolve is a topic that we are only beginning to fully understand (Blume *et al.* [1993]; Wärneryd [1993]; Huttegger [2007a]; Pawlowitsch [2008]; Barrett [2009]; Huttegger *et al.* [2010]; Skyrms [2010]).

Similarity-maximizing (henceforth, sim-max) games are a variation of Lewis's games, where the receiver's actions are equated with the state space (one can think of the actions as choosing states) and different states are allowed to be more or less similar to one another. While Lewis's games treated communicative success as a black and white matter, sim-max games allow for shades of grey: the more similar the receiver's interpretation is to the actual state, the better. Signalling games with utility-relevant similarities in the state

[1] Much previous work has investigated the interplay of adaptive dynamics, including imitation-based update protocols, on the one hand, and noise or mutation, on the other hand (Foster and Young [1990]; Fudenberg and Harris [1992]; Kandori *et al.* [1993]; Young [1993]; Fudenberg and Imhof [2006]). While this line of research often looks at finite populations and the extreme long-term behaviour of the system under generic randomness, the focus here is on a quite particular source of stochastic noise and its quite particular role in the evolution of meanings through signalling.

space are fairly standard in economics (Spence [1973]; Crawford and Sobel [1982]), but have received particular attention in a more philosophical context for reasons that will become clear presently.

Formally, a sim-max game consists of a set of states, T, a set of messages, M (typically with much fewer messages than states), a probability distribution, $Pr \in \Delta(T)$, such that $Pr(t)$ gives the probability that state t occurs, a similarity metric on states, $\mathrm{Sim} : T \times T \rightarrow \mathbb{R}$, such that $\mathrm{Sim}(t_1, t_2)$ is the (physical) similarity between t_1 and t_2, and a utility function, $\mathrm{U} : T \times T \rightarrow \mathbb{R}$, such that $\mathrm{U}(t_1, t_2)$ is the payoff for sender and receiver for a play with actual state t_1 and receiver interpretation t_2.[2] We identify the receiver's acts with the states of the world, so that the game is one of guessing the actual state, so to speak. For the modelling purposes of this article, we make the simplifying assumption that T is a set of points in Euclidean space, whose closeness to each other tracks physical similarity. Perceived similarity, where it is necessary, would be a monotonic function of physical similarity. Likewise, the utility function should be a monotonically increasing function of physical similarity.[3] Non-probabilistic sender behaviour can be represented by pure strategy $s \in M^T$, which deterministically defines which message would be used for each state. Similarly, a pure receiver strategy is function $r \in T^M$.

Jäger *et al.* ([2011]) showed that the evolutionarily stable states of sim-max games with infinitely many states in n-dimensional Euclidean space, $T \subseteq \mathbb{R}^n$, and quadratic loss function for utilities $\mathrm{U}(t_1, t_2) = -(t_1 - t_2)^2$ are remarkably systematic: the evolutionarily stable states are demonstrably so-called Voronoi languages. Roughly put, a Voronoi language is a pair of sender and receiver strategies, such that the sender strategy partitions the state space into convex categories, while the receiver's interpretations are the central spots in each category. A subset X of $\mathbb{R}^n$ is convex if, informally put, all points in X are connected via a straight line that lies entirely in X; X has no gaps or dents. For example, if T is the unit interval and all states are equiprobable, a Voronoi language with two messages could have the sender use one message exclusively for all points in the lower half of the unit interval and another for all points in the upper half; the receiver's interpretations of messages are the central points, 0.25 and 0.75, in the respective intervals (see Figure 1 for an illustration).

[2] A metric is a function of distance between any two points in a given space. It should satisfy certain axioms that ensure behaviour that one would intuitively expect from the words 'similarity' and 'distance' alone, but these details do not matter for the purposes of this article. The assumption that state similarity forms a metric is conceptually loaded, but we follow the literature here and conceive of it as a first and pragmatic simplification, possibly to be dispensed with later.

[3] Section 4.1 motivates particular choices of similarity and utility functions that we will explore in more detail.

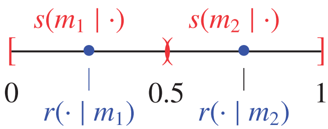


Figure 1. Example of a Voronoi language on $T = [0, 1]$. Pure sender strategy s uses one signal for the lower half, and another for the upper half of the unit interval. Pure receiver strategy r selects the central elements in the respective intervals. See (Jäger *et al.* [2011]) for further details.

It is intuitive to think that linguistic and conceptual categories are orderly in such a manner. For example, if you consider two people tall, one with a height of 2m and another with a height of 2.2m, it would be difficult to defend not considering a person with a height of 2.1m tall as well. Another example where this intuition is additionally supported by empirical data is colour categorization. The World Color Survey project (Cook *et al.* [2005]; Kay *et al.* [2009]) collected colour naming data for 110 unwritten languages of forty-five language families. For the great majority of these languages, a pattern can be observed: basic colour terms are by and large convex (Regier *et al.* [2007]; Jäger [2010]). It is premature to argue that these observations can be extended to all cases of categorization. However, for the cases where it does apply, the result of (Jäger *et al.* [2011]) is interesting because it demonstrates that signalling can impose this kind of orderly category on a metric space without that being the ulterior purpose of it all.[4] Finally, these considerations are in line with a prominent school of thought in comparative linguistics that also assumes that more abstract conceptual domains (for example, spatial-topological relations, temporal reference, or the meanings expressible by indefinite pronouns) are preferably carved up by the languages of the world in such a way that meanings are connected regions on a 'semantic map' (Croft [2003]; Haspelmath [2003]; Levinson *et al.* [2003]).

There are at least two ways of interpreting the signalling set-up. Sender and receiver can be distinct entities, whose purpose is to communicate effectively about the actual state. In that case, evolving Voronoi languages would explain why linguistic categories are well-behaved and orderly in the way they appear to be. More abstractly, sender and receiver can also be thought of as distinct modules in a single system, where the first module must discretize the information it is fed by selecting a small sample of, suggestively, category labels. These are passed to a second module that tries to decode the original information. In this case, evolving Voronoi languages would explain why conceptual categories are well-behaved and orderly in the way that they appear to be. Seen in this light, sim-max games may provide a foundation to those

[4] For the concrete case of colour categorization, see also (Jäger and van Rooij [2007]; Correia *et al.* [unpublished]). insert hyperlink

approaches in cognitive semantics that rely on the notion of conceptual spaces. Gärdenfors ([2000], pp. 70–7), for example, has prominently argued that natural categories are convex regions in conceptual space. If the conceptual space has a suitable metric, convex categories can be derived from a set of prototypes. The category corresponding to prototype p is the set of points that are more similar to p than to any other. In this way, Gärdenfors argues, an efficient categorization system can be obtained: storing the prototypes lets us recover the categories without having to store each category's extension. However, what is left unexplained is where the prototypes come from, and why we would not see any distribution of prototypes as an equally efficient classification system. This is where sim-max games can contribute a principled approach to deriving, in an independent way, not only convex categories but also prototypical exemplars belonging to them. These ideas and more are developed further by Jäger ([2007]), Jäger and van Rooij ([2007]), Jäger *et al.* ([2011]), and O'Connor ([2014b]), among others.

2.2 Vagueness in sim-max games and conceptual spaces

This outline of an approach to categorization using sim-max games leaves some problems unaddressed. One of them is that, usually, natural categories for continuously variable stimuli have neither clear boundaries nor unique, point-valued prototypes. We would like to account for the possibility of such vagueness. In particular: (i) clear positive examples of a vague category should show a gradient transition to clear negative examples; and (ii) prototypes should likewise be gradient regions, peaking at the centre of the vague category they represent.

Douven *et al.* ([2011]) show that Gärdenfors's conceptual spaces approach can be extended to account for the existence of borderline cases. From the assumption that prototypes are extended yet convex regions in conceptual space, a construction algorithm is available that yields collated Voronoi diagrams with thick boundaries representing borderline regions. Decock and Douven ([2012]) show further how it is possible to arrive at a gradient transition between categories, by weighing in the distance of different borderline cases to various prototypical regions. This accounts for the first of the two desiderata mentioned above, but still assumes that crisp prototype regions must be given.

Alternative approaches are taken by, for example, Franke *et al.* ([2011]) and O'Connor ([2014a]), who show, in different ways, how the above desiderata can be met by evolving strategies in sim-max games. To illustrate what vague signalling would look like, let us briefly consider a sim-max game with six equiprobable states and two messages, and what its equilibria would be like (for a more thorough discussion of equilibria of sim-max games, see Jäger [2007];

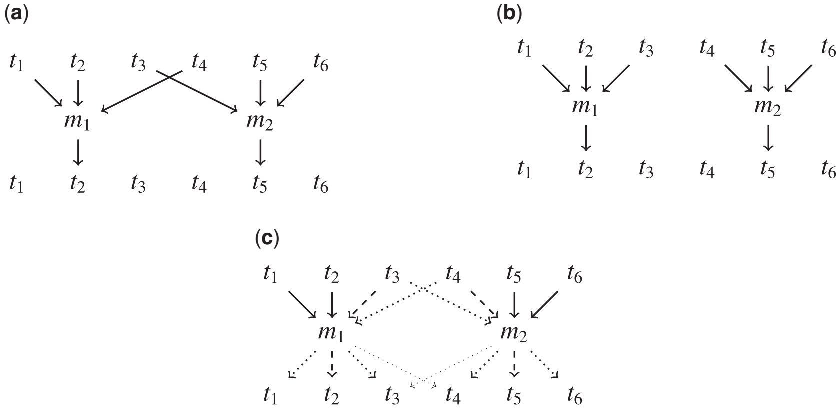


Figure 2. Examples of strategy pairs in sim-max games. Sender strategies map states onto messages (top two rows); receiver strategies map messages onto states (bottom two rows). The thickness of arrows indicates the probability of a choice in a probabilistic strategy. (a) Inefficient, non-convex, non-equilibrium; (b) maximally efficient equilibrium, partial pooling; (c) almost efficient, probabilistic, intuitively vague.

O'Connor [2014a]). We assume that utilities are linearly decreasing with decreasing similarity. Figure 2 shows three pairs of sender and receiver strategies. States are arranged according to their similarity: the closer they are to each other, the more similar they are. The pair in Figure 2a is not an equilibrium, because the sender's non-convex use of signals is sub-optimal given the receiver's behaviour. Namely, if m_1 is interpreted as t_2 and m_2 as ~~t_4~~, then the sender would get a higher payoff from sending m_1 in t_3 than from sending m_2, because (by assumption) t_3 is more similar to t_2 than t_5 is. In contrast, Figure 2b shows a maximally efficient equilibrium. This is a partial pooling equilibrium in the sense that the speaker uses the same message for several states. Partial pooling equilibria can be less ~~inefficient~~ than other non-pooling equilibria, if there are enough messages. The strategy pair of Figure 2b is maximally efficient for the two-message case. So, while partial pooling may entail inefficiency in some sense, and while partial pooling can hamper the evolution of maximally efficient signal use (Huttegger [2007b]; Pawlowitsch [2008]; Huttegger *et al.* [2010]), this is orthogonal to our concerns about vagueness. Regular, natural, but vague signal use would look like the pair in Figure 2c. This is not an equilibrium, but it gets close, so to speak. It shows smooth transitions across similar states at the boundaries of categories and across acts around the most prototypical instances (as indicated by decreasing thickness of arrows in Figure 2c).

2.3 Vagueness, functional pressure, and transmission biases

The approach we take here is similar in spirit to that of Franke *et al.* ([2011]) and O'Connor ([2014a]), but different in relevant detail. A more in-depth comparison is deferred until Section 5. Let us first motivate our approach here, and spell it out in more detail in the following section.

Our conceptual starting point is the widely shared conviction that language is shaped by at least two forces, which may, on occasion, pull in opposite direction. On the one hand, there is functional pressure towards efficient communication. On the other hand, there is systematic error, noise, or imprecision in the transmission of linguistic behaviour, knowledge, or traits. As an example of the latter, consider a child learning syntactic rules from a parent generation. The child must infer these unobservable rules from observable speech. Inductive biases may influence which syntactic rules are likely to be inferred from (finite) parental input. Over the course of many generations, the effects of such biases can, in a manner of speaking, 'accumulate' and lead to surprising results, such as the evolution of compositional form-meaning mappings or the use of regular recursive syntactic structure. This can happen when the effects of transmission biases are isolated, as in iterated learning models (Kirby and Hurford [2002]; Smith *et al.* [2003]; Griffiths and Kalish [2007]; Kirby *et al.* [2014]), or when they interact with functional pressure towards efficient communication, for example, as formalized in the replicator mutator dynamic (Nowak *et al.* [2000], [2001]).

The emphasis of previous models that studied the effects of transmission infidelity on the evolution of language has been on inductive biases and the systematicity, compression, and regularization that they can introduce. Here, we would like to show that transmission noise of a different kind can lead to regularization as well and can also give rise to vague meaning. We find that shared perceptual biases that perturb the transmission of successful signalling behaviour can regularize, facilitate, and accelerate the evolution of meaning conventions, albeit at the cost of vagueness. Concretely, we formalize the expected change in the behaviour of a population of agents that try to imitate other agents' signalling behaviour. We assume, however, that both observation of ~~others' behaviour~~ and realization of behaviour are systematically perturbed by noise. The resulting population-level dynamic generalizes the replicator dynamic (Taylor and Jonker [1978]) in its interpretation as a cultural evolutionary dynamic based on imitation (Helbing [1996]; Schlag [1998]). The inclusion of confusability of stimuli does not undermine the possibility of evolving communicative signalling behaviour. Instead, it leads to the evolution of vague meanings. Moreover, it accelerates the emergence of communicative signalling because it unifies and regularizes evolutionary outcomes, making it appear as if agents were applying inductive biases or generalizing over partial observations, when this is actually the sole ~~effect~~ of confusion of similar stimuli.

3 Imprecise Imitation

Signalling agents can adapt their dispositions to act, given some feedback about their past success, in multiple ways. Usually, we would assume that changes in behaviour should, at least on average, lean towards increasing chances of communicative success. Such behavioural adaptations can be described at different levels of abstraction. At the level of individual agents, we can picture a more or less idealized process of how each agent adapts dispositions for future actions based on various pieces of information available to the agent. More abstractly, at the level of a population of agents, we can describe how average behavioural dispositions will evolve. The population-level perspective abstracts over small stochastic fluctuations and zooms in on the general tendency or direction of evolution that ensues from behaviour at the agent level.

To better understand the interaction of confusability of stimuli and selective pressure towards successful communication, we look at a population-level dynamic that describes the most likely evolutionary path of a population of signalling agents who imitate other agents' behaviour, but are liable to confuse states for one another. In the special limiting case where state confusability vanishes, the process is just the well-known replicator dynamic (Taylor and Jonker [1978]), which should therefore be briefly reviewed first.

3.1 Replicator dynamic in behavioural strategies

Fix a sim-max game with finite states T and messages M. As usual, we assume that the receiver chooses states in T in response to messages. Let $Pr(\cdot) \in \Delta(T)$ be the prior distribution over states and $\mathrm{U} : T \times T \to \mathbb{R}$ the utility function shared by senders and receivers in the population. A behavioural strategy is a function that maps an agent's choice points to a probability distribution over available choices.[5] The sender's behavioural strategies are

[5] Our focus is on behavioural strategies not mixed strategies, that is, probability distributions over functions from each choice point to an act, such as $s \in \Delta(M^T)$. Dynamics on behavioural strategies assume that agents can adapt their behaviour locally, that is, independently at each choice point. Our focus on behavioural strategies greatly reduces the complexity of the dynamic and simplifies numerical simulations. But it also seems the more plausible choice for imitation-based update protocols of the kind we consider here: agents only observe how, on some occasion, some other agent behaved in one particular situation, not how that agent would behave in all relevant choice situations; they imitate the use of a single word, so to speak, not a whole lexicon (for more on the difference between dynamics on mixed or behavioural strategies, see Cressman [2003]). It is in this respect that the cultural evolutionary dynamic introduced here differs most visibly from the replicator mutator dynamic (Nowak *et al.* [2000], [2001]), which operates on mixed strategies and is motivated by assumptions of (asexual) biological inheritance with transmission infidelity.

$$\xrightarrow{\text{N}} t_1 \in T \xrightarrow{\text{S}} m \in M \xrightarrow{\text{R}} t_2 \in T$$
$$P(t_1) \qquad \sigma(m|t_1) \qquad \rho(t_2|m)$$

Figure 3. A round of play in a sim-max game with behavioural strategies: Nature (N) chooses state t_1 with probability $Pr(t_1)$; a random sender (S) selects message m with probability $\sigma(m|t_1)$; a random receiver (R) selects state t_2 with probability $\rho(t_2|m)$. Payoff for both sender and receiver is given by $U(t_1, t_2)$.

functions $\sigma \in \Delta(M)^T$, thus mapping each state $t \in T$ to a probability of each message $m \in M$ being sent in t; the receivers are functions $\rho \in \Delta(T)^M$, thus mapping each message $m \in M$ to a probability of each interpretation $t \in T$ being chosen in response to m. Although behavioural strategies are probabilistic, evolutionary modelling usually imagines that every individual agent has a non-probabilistic strategy. Behavioural strategies then capture average population behaviour. Assuming a virtually infinite population, the number $\sigma(m|t)$, for instance, is then the probability that a randomly sampled sender would send message m if the actual state was t. Similarly, $\rho(t|m)$ is then the probability with which a randomly sampled receiver interprets m as t. A play of a single evolutionary game with behavioural strategies is illustrated in Figure 3.

The expected utility of choices at each choice point is[6]:

$$\mathrm{EU}(m, t, \rho) = \sum_{t' \in T} \rho(t'|m)\, U(t, t'),$$

$$\mathrm{EU}(t', m, \sigma) = \sum_{t \in T} P(t|m)\, U(t, t'), \quad \text{where } P(t|m) \propto Pr(t)\, \sigma(m|t)\,.$$

Expected utilities are sums, over all possible concrete outcomes, of the utilities of these outcomes, weighted by how likely they are to occur. For instance, the expected utility for the sender of choosing message m, given state t and receiver strategy ρ, is the sum of the probability that some random receiver will choose t' given m times the actual utility of pair t and t', for each interpretation t'.

The discrete-time replicator dynamic tracks changes in frequency of choices in the population as proportional to their expected utilities (Hofbauer and Sigmund [1998])[7]:

$$\sigma'(m|t) \propto \sigma(m|t)\, \mathrm{EU}(m, t, \rho)\,, \; \rho'(t|m) \propto \rho(t|m)\, \mathrm{EU}(t, m, \sigma)\,. \tag{1}$$

6 The notation $\propto$, for 'proportional to', that is used here and hereafter means that the right-hand side might still need to be normalized so as to have probabilities sum to one. In general, writing $P(x) \propto f(x)$ for any function $f : X \to \mathbb{R}$ is shorthand for $P(x) = \frac{f(x)}{\sum_{x' \in X} f(x')}$.

7 The formulation given in Equation (1) is adequate only for cases like the one we will be looking at in Section 4, where utilities are always non-negative and expected utilities are always positive.

For a given choice point, say state t, the probability of seeing m played by an average agent in the population after the update is proportional to the probability of seeing it before the update, which is $\sigma(m|t)$, times the expected utility of m at state t. Intuitively put, frequencies of choices change by a gradient of current frequency and a measure of how good they are.

The replicator dynamic is an abstract population-level dynamic that describes the mean expected change of behavioural dispositions in a population of signallers. There are several ways of deriving the replicator dynamic from agent-level processes of behavioural adaptation. We focus here on one of the simplest: imitation of success (Sandholm [2010]). The intuitive idea is the following: every now and then, a random agent gets a chance to alter his behaviour for one of his choice points (say this is a sender who gets to 'reconsider' his choice of message for state t); the revising agent then observes what some random agent does at t, say m, and will henceforth play m in t with a probability given by the expected utility of m for state t (see the appendix for a derivation of the standard replicator dynamic from this update scheme).

3.2 Noise-perturbed conditional imitation

Imitation of success, as described above, presupposes that agents make no mistakes when observing states, or choosing interpretations. This may not always be an appropriate assumption, especially when some states can be perceptually similar and therefore likely to be confused for one another. Human performance in these situations has been studied by a number of authors in experimental psychology. In a stimulus identification experiment (Luce [1963]), subjects are presented in each trial with a stimulus to be identified out of a fixed set. The more similar the stimuli are to each other, the larger the number of errors subjects make. For example, in Robert Nosofsky's ([1986]) experiments, stimuli consisted of sixteen semi-circles with radial line varying in length (0.478, 0.500, 0.522, or 0.544 cm) and angle (50°, 53°, 56°, or 59°). In over 9000 trials, the two subjects identified the stimulus correctly only approximately 44% and 35% of the time. Similar experiments have been conducted with other identification tasks, for example, for frequency and intensity of tones, taste, hue of colours, and magnitude of lines and areas (Donkin *et al.* [2015]). The empirical data confirm not only the pervasiveness of variation in the subjects' ability to correctly identify stimuli, but also the relation between similarity and likelihood of stimulus confusion, and further suggests that the phenomenon might extend to all types of perception.

In the context of imitation of success in a game where states have a degree of similarity between them, the possibility of agents mistaking one state for

$$\begin{array}{ccccccccc} \xrightarrow{\text{N}} & t_{\mathrm{a}} \in T & \longrightarrow & t_{\mathrm{o}} \in T & \xrightarrow{\text{S}} & m \in M & \xrightarrow{\text{R}} & t_{\mathrm{i}} \in T & \longrightarrow & t_{\mathrm{r}} \in T \\ & P_a(t_{\mathrm{a}}) & & P_o(t_{\mathrm{o}}|t_{\mathrm{a}}) & & \sigma(m|t_o) & & \rho(t_i|m) & & P_r(t_r|t_i) \end{array}$$

Figure 4. A round of play in a sim-max game with probabilistic confusability of states: Nature (N) chooses an actual state with probability $P_a(t_a)$; a randomly sampled sender (S) observes t_o with probability $P_o(t_o|t_a)$ and subsequently selects m with probability $\sigma(m|t_o)$; a randomly sampled receiver (R) intends to realize interpretation t_i with probability $\rho(t_i|m)$ but actually realizes interpretation t_r with probability $P_r(t_r|t_i)$. Payoff for both sender and receiver is given by $\mathrm{U}(t_a, t_r)$.

another is thus something that should be taken into account. Imitation dynamics could be affected by at least two sources of probabilistic noise:

(1) Observation noise: whenever state t_a actually occurs, the probability that an agent observes it as t_o is $P_o(t_o|t_a)$;

(2) Realization noise: whenever an agent intends to realize interpretation t_i, the probability that t_r is realized is $P_r(t_r|t_i)$.

A round of play of a sim-max game with these two sources of confusability of states is pictured in Figure 4. Note that, and this is crucial, senders respond to observations not actual states, receiver strategies determine intentions not realized states, but payoff is calculated based on actual and realized states. Behavioural strategies thus encode what agents actually do from their subjective point of view or, put differently, what they would do in a noise-free world. The noise-free situation depicted in Figure 3 is then the special case where $P_o(t_o|t_a) = 1$ if and only if $t_a = t_o$, and $P_r(t_r|t_i) = 1$ if and only if $t_i = t_r$.

The presence of observation and realization noise also affects imitation of successes. Here, we focus on the main ideas, formal detail is provided in the appendix. Take a sender who gets to revise behaviour for state t. What agents can plausibly revise by imitation is their pure strategy, which maps perceived states onto messages. When an agent gets to revise his strategy for perceived state t, this need not necessarily be the actual state. Moreover, when that agent observes t, another agent may perceive yet another state. Given what that latter agent perceives, his actual (pure) strategy will determine what he plays. In sum, to describe how likely the potential imitator observes message choice m, we are interested in the conditional probability, $P_o(m|t)$, that some other random agent selects m when the first agent perceives t. This $P_o(m|t)$ is derived from the prior probability of states, the current sender population behaviour, σ, and the given observation noise (see the appendix). Eventually, the imitating agent adopts m as his choice for t with a probability given by the expected utility of sending m when perceiving state t. Expected utility should, of course, take the probabilistic confusability of states into account as well.

Similar considerations apply to the receiver side. If an agent gets a chance to change his intended interpretation of m, we need to look at the conditional probability, $P_o(t|m)$, of observing another agent realize interpretation t, given that the first agent (and therefore the second as well) perceived m. This depends on observation and realization noise, as well as on the current receiver population behaviour, ρ.

The appendix shows how imprecise imitation of this sort leads to mean changes in population frequencies of choices that can be covered by the following discrete-time formulation:

$$\sigma'(m|t) \propto P_o(m|t)\,\mathrm{EU}(m,t,\rho)\,, \qquad \rho'(t|m) \propto P_o(t|m)\,\mathrm{EU}(t,m,\sigma)\,. \qquad (2)$$

This looks very much like the discrete-time formulation of the standard replicator dynamic in Equation (1), but there are, of course, the aforementioned differences. First, expected utility here takes stochastic confusability of states into account. Second, where the standard replicator dynamic had probabilities $\sigma(m|t)$ and $\rho(t|m)$, we now have $P_o(m|t)$ and $P_o(t|m)$, respectively. If there is no observation or realization noise, $P_o(m|t)$ reduces to $\sigma(m|t)$ and $P_o(t|m)$ reduces to $\rho(t|m)$. The imprecise imitation dynamic in Equation (2) conservatively extends the classic case in Equation (1).

4 Exploring Imprecise Imitation

How does a tendency to confuse similar states interact with selective pressure towards more efficient signalling strategies under the imprecise conditional imitation dynamic? If confusion probabilities are moderate and regular, in that they track similarity of states, we might also expect a regularizing effect on evolving signalling strategies. Indeed, we hypothesize that confusion of states can give rise to population-level aggregate behaviour that looks as if information about what is good for one state percolates to similar states. In other words, imprecise imitation may make signalling behaviour look, on average, as if agents generalize across similar states, even if no agent actually generalizes. To explore whether confusion of states can have this effect, we turn to numerical simulation.

4.1 Setting the stage

To obtain concrete results, we must fix how to represent states, similarity of states, the conditional confusion probabilities (P_o and P_r), and the utilities of our sim-max games. Confusion probabilities between states should be a function of perceptual similarity: the more similar two states are, the more likely they could be mistaken for each other.

Let the state space consist of $n_s \geq 2$ states that are equally spaced across the unit interval, including 0 and 1. All states occur, for simplicity, with the same probability (that is, P_a is uniform). Distance $|t_\mathrm{i} - t_\mathrm{j}|$ is the objective, physical

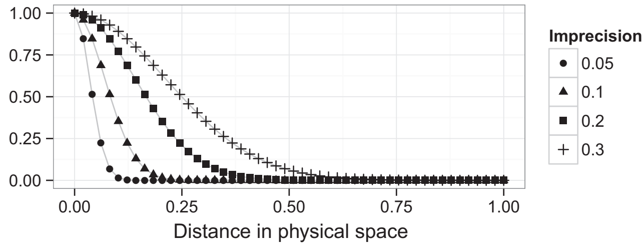


Figure 5. Examples of Nosofsky similarity for different values of imprecision.

similarity between two states, t_{i} and t_{j}. Distance in physical space feeds into a perceptual similarity function, as described by Nosofsky ([1986]):

$$\mathrm{Sim}(t_{\mathrm{i}}, t_{\mathrm{j}}\,;\, \alpha) = \begin{cases} 1 \text{ if } \alpha = 0 \text{ and } t_{\mathrm{i}} = t_{\mathrm{j}} \\ 0 \text{ if } \alpha = 0 \text{ and } t_{\mathrm{i}} \neq t_{\mathrm{j}} \\ \exp\left(-\dfrac{|t_{\mathrm{i}} - t_{\mathrm{j}}|^2}{\alpha^2}\right) \text{otherwise,} \end{cases}$$

where $\alpha \geq 0$ is an imprecision parameter. When $\alpha = 0$, agents perfectly discriminate between states; when $\alpha \to \infty$, agents cannot discriminate states at all. Figure 5 gives an impression of Nosofsky similarity for different parameter values. Other formalizations of perceptual similarity are possible, including ones that allow for different discriminability in different areas of the state space, but we stick with Nosofsky's similarity function for the time being, because it is mathematically simple and an established notion in mathematical psychology.

We further assume that the probability of confusing any two states, t_i and t_j, is proportional to their perceived similarity and, to keep matters simple, that observation noise P_o is simply the same as realizational confusability P_r, and that both are governed by the same imprecision parameter, α:

$$P_o(t_o|t_a) \propto \mathrm{Sim}(t_o, t_a\,;\, \alpha)\,, \quad P_r(t_r|t_i) \propto \mathrm{Sim}(t_r, t_i\,;\, \alpha)\,.$$

For $\alpha = 0$, we obtain trivial confusion probabilities: everything is reduced to perfect imitation and the replicator dynamic. For $\alpha > 0$, any state can be confused for any other state with some positive probability. For the uninteresting case of $\alpha \to \infty$, confusion is maximal and every state can be perceived or realized as any other state with probability $1/|T|$.

As for utility, we define it in terms of similarity and introduce another free parameter, $\beta \geq 0$. The intention is for this parameter to model the amount of tolerable pragmatic slack, which should be allowed to vary separately from perceptual imprecision:

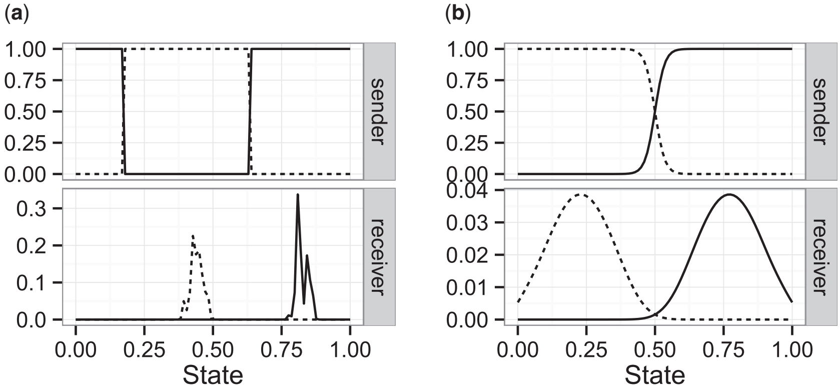
$$U(t_i, t_j\,;\,\beta) = \mathrm{Sim}(t_i, t_j\,;\,\beta)\,.$$

With $\beta = 0$, we return to the case of Lewis's games, where only a perfect match of actual state t_i with receiver interpretation t_j leads to a positive payoff. The higher β, the more acceptable a wider environment of interpretations, t_j around t_i, is (Figure 5). This choice of utility function is governed partly by convenience, but also because we believe it has the right general properties for a communicative payoff function. Unlike utilities that, say, linearly or quadratically decrease in physical distance (compare Franke *et al.* [2011]; Jäger *et al.* [2011]), utilities that exponentially decrease in negative quadratic distance can model situations where a small amount of imprecision in communication is tolerable, whereas similarly small differences in intolerably far away interpretations matter very little, with a smooth transition between these regimes (compare O'Connor [2014a]).

In order to simplify the analysis, we focus on games with two messages, that is, we fix $|M| = 2$. In sum, to structure thinking about the behaviour of our imprecise imitation dynamic, the system is governed by three parameters: the number of states in the state space, $n_s = |T|$; imprecision, α; and tolerance, β.

4.2 Simulation set-up

We ran fifty trials of the discrete-time dynamic in Equation (2), starting with randomly sampled sender and receiver strategies, for each triplet of independent parameter values: $n_s \in \{6, 10, 50, 90\}$, $\alpha \in \{0, 0.05, 0.1, 0.2, 0.3\}$, $\beta \in \{0.05, 0.1, 0.2, 0.3\}$. Each trial ran for a maximum of 200 update steps. A trial was considered converged, and thus stopped before the maximum of 200 rounds, if the total amount of change between strategies before and after an update step was smaller than a suitably chosen threshold. It is not guaranteed that strategies at halting time had converged to the eventual attracting state, whether they ran for 200 rounds or not. Our notion of convergence is therefore only a categorical measure for reaching a certain (well-considered, but eventually arbitrary) degree of stability. In other words, our notion of 'convergence' is a measure of relative speed: is it true that the system reached a state in which evolutionary adaptations had slowed down almost to a halt before 200 update steps? This is motivated by practical concerns regarding length of simulation time, but is also theoretically justifiable, because we hypothesize that confusability of states leads to regularization of evolving strategies, which would show exactly in an increased speed of evolutionary trajectories towards well-behaved and regular signalling behaviour.



$n_s = 90, \beta = 0.1, \alpha = 0$ $n_s = 90, \beta = 0.1, \alpha = 0.05$

Figure 6. Example strategies at stopping time. Each line corresponds to a message and plots, for each state, the probability that the message is used. (a) $n_s = 90,\ \beta = 0.1,\ \alpha = 0$; (b) $n_s = 90,\ \beta = 0.1,\ \alpha = 0.05$.

Representative examples for resulting strategy pairs are given in Figure 6. Figure 6a shows a strategy pair at stopping time with ninety states, tolerance $\beta = 0.1$, and imprecision $\alpha = 0$. Zero imprecision means that the trial was effectively an application of the standard replicator dynamic. Noteworthily, the given sender strategy approximates a pure sender strategy that crisply partitions the state space into non-convex sets. The irregular shape of the receiver strategy suggests that the pictured strategy pair has not yet reached a stable state. Indeed, the trial was stopped when reaching the maximum of 200 rounds. In contrast, the outcome of a trial with identical parameters, but with imprecision $\alpha = 0.05$, had converged (in our technical sense) after ninety-nine rounds, as shown in Figure 6b. The sender strategy shows a smooth blending from one 'category' to the other, and the receiver's interpretations are rather extended curves, peaking at a central point in the relevant categories.

These examples illustrate two interesting things. First, inclusion of imprecision can lead to seemingly well-behaved yet vague strategies in the sense that we are after (again, see Section 2.2). The sender strategy in Figure 6b identifies clear positive and clear negative cases for each signal, with a smooth transition in between. The receiver's interpretation~~s~~ of signals can be seen as smoothed-out prototype regions. Second, (sender) strategies can approach non-convex pure strategies under the replicator dynamic and linger there for vast amounts of time, possibly indefinitely. We see this in our limited-time simulations (for example, Figure 6a), but this also holds, for some types of utility function, in the limiting case. This was first observed by Elliott Wagner, as mentioned by

O'Connor ([2014b]). A full analysis of the dynamics of sim-max games is beyond the scope of this article, but we will see shortly that diffusion from confusability of states clearly prevents evolutionary paths that meander for a long time in the vicinity of non-convex strategies.

4.3 Measures of interest

To further explore our simulation results, we calculated metrics that aim to numerically capture how vague, generally well structured, and communicatively efficient the recorded strategy pairs were. Entropy captures the amount of systematicity or regularity in signal use. Convexity captures whether a behavioural strategy would project onto a convex pure strategy. Expected utility measures the communicative efficiency of evolved strategy pairs.

4.3.1 Entropy

This classic information-theoretic notion captures the amount of uncertainty in a probability distribution. Roughly put, entropy of a signalling strategy captures inverse distance from a pure strategy. The usual definition of entropy applies directly to mixed strategies (see Footnote 5), but provably equivalent metrics for behavioural strategies are ready to hand:

$$E(\sigma) = -\sum_{t \in T} \sum_{m \in M} \sigma(m|t) \cdot \log(\sigma(m|t)),$$
$$E(\rho) = -\sum_{m \in M} \sum_{t \in T} \rho(t|m) \cdot \log(\rho(t|m)).$$

Values obtained by these definitions have a lower bound of zero and an upper bound of, respectively, $\log(|M^T|) = |T| \cdot \log(|M|)$ and $\log(|T^M|) = |M| \cdot \log(|T|)$. We work with values rescaled to lie in $[0;1]$ for cross-comparability. The sender strategies in Figure 6a and b have entropy $1.19e^{-5}$ and 0.08, respectively. The receiver strategies have respective entropies 0.43 and 0.81. In general, we expected that vague languages would have higher entropy than crisp ones, and that increasing imprecision would lead to increased entropy, all else being equal.

4.3.2 Convexity

At least for sender strategies, which develop faster than receiver strategies, it also makes sense to define a categorical measure of convexity that ~~compensates for~~ potential vagueness. To determine whether sender strategy σ is convex despite possibly being vague, we look at derived pure strategy s for which $s(t) = \arg\max_{m' \in M} \sigma(t, m')$. If ~~that~~ s is convex, we also count σ as convex. The sender strategy in Figure 6a is not convex, while that in

Figure 6b is. If confusion of states can regularize signalling strategies, as we hypothesized, we should see more convexity with increasing imprecision all else being equal.

4.3.3 Expected utility

We also recorded the expected utility of a strategy pair:

$$\mathrm{EU}(\sigma, \rho\,;\ \beta) = \sum_{t \in T} \sum_{m \in M} \sum_{t' \in T} Pr(t) \cdot \sigma(t, m) \cdot \rho(m, t') \cdot \mathrm{U}(t, t'\,;\ \beta)\,.$$

To make direct comparisons across different parameter settings, we normalize expected utility by the maximal amount of expected utility obtainable in the relevant game. The strategy pair in Figure 6a has a normalized expected utility of 0.99, and the pair in Figure 6b has 0.95. Generally, vagueness and imprecision can be expected to decrease expected utility (compare Lipman [unpublished]). The crucial question is whether communicative success drops unacceptably fast with moderate levels of vagueness and imprecision.

4.4 Results

Figure 7 shows plots summarizing a selected part of our findings. For perspicuity, we only plot results for one level of tolerance, $\beta = 0.1$, and leave out the case of $n_s = 90$. Still, every qualitative trend mentioned in the following applies to the whole set of results.

As expected, increasing imprecision leads to higher entropy and lower expected utility. Importantly, however, imprecision does not necessarily lead to a disastrous decline of communicative success. What is more, in line with our hypothesis that mere imprecision in imitation behaviour can lead to behaviour that looks as if agents generalize across similar stimuli, higher imprecision led to a higher number of outcomes with convex sender strategies. It also led to higher rates of convergence. In fact, sufficient imprecision always ensured convergence and convexity. It appears that perceptual imprecision leads to more vagueness and slightly less communicative efficiency, but more regular, well-behaved languages in shorter time.

Beyond promoting convexity and convergence, diffusion also has another interesting regularizing effect on the evolution of signalling. There is very little variation in the recorded metrics for evolved strategies, at least for higher values of imprecision. On closer inspection, it turns out that variability in low-imprecision conditions is not only due to non-convergence or non-convexity. Figure 8 gives two more examples of strategy pairs at stopping time. Both are obtained for the same triple of parameters, both converged before the maximum number of rounds, and both have convex sender

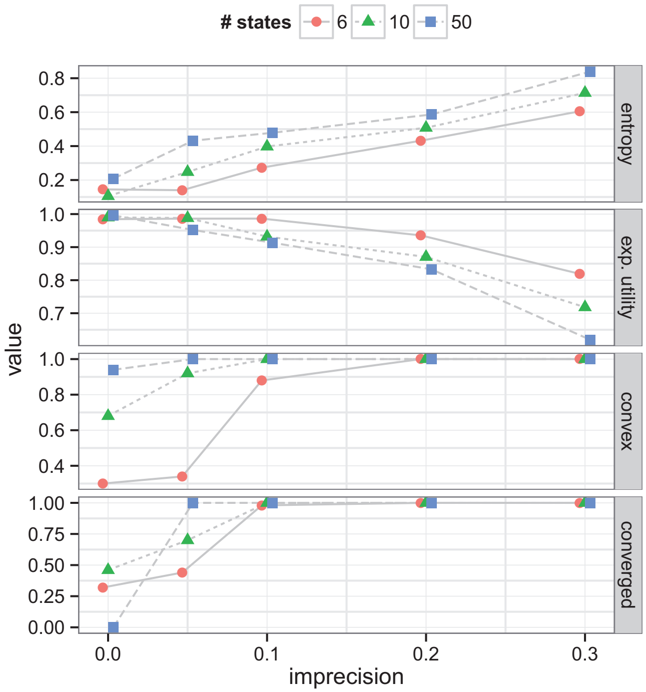
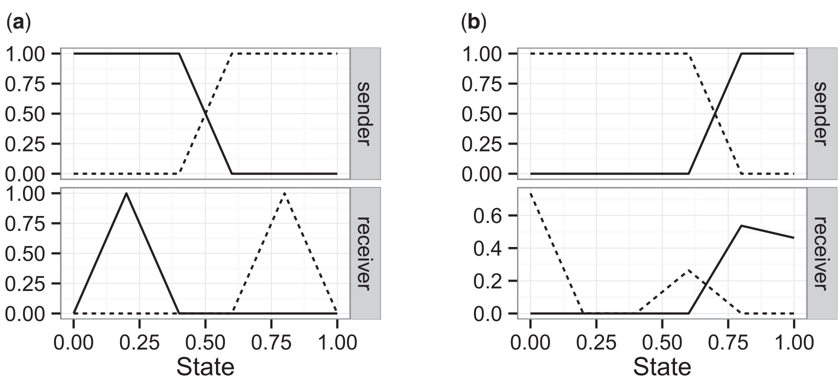
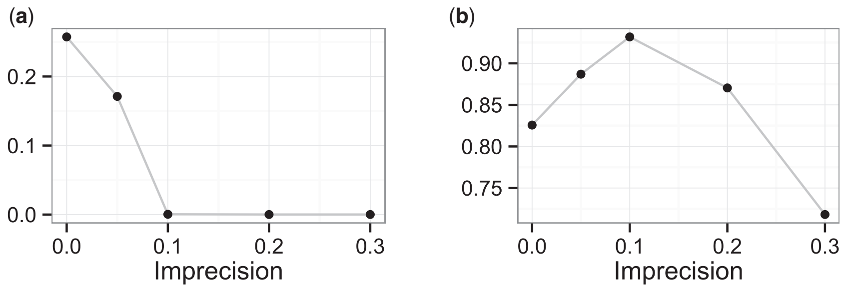


Figure 7. Means of gradient and proportions of categorical measures for $\beta = 0.1$, $n_s \in \{6, 10, 50\}$, and $\alpha \in \{0, 0.05, 0.1, 0.2, 0.3\}$. The plot shows the average of the entropies for the sender and receiver strategy.



$n_s = 6, \beta = 0.2, \alpha = 0$ $n_s = 6, \beta = 0.2, \alpha = 0$

Figure 8. More example strategies at stopping time of our simulations.



Within-group distance

Within-group expected utility

Figure 9. Within-group measures for all runs with $\beta = 0.1$ and $n_s = 10$. (a) Within-group distance; (b) within-group expected utility.

strategies. However, they are not equally efficient. In fact, the pair in Figure 8a has a normalized expected utility of 0.99, while the pair in Figure 8b has a normalized expected utility of only 0.89.

Interestingly, this type of variability in evolutionary outcomes can be weeded out by imprecision. To investigate this, we calculated the average distance between evolved sender strategies within each group of trials that had identical parameter values. We determined the distance between strategies σ and σ' as the average Hellinger distance between distributions $\sigma(t)$ and $\sigma'(t)$ at each choice point t:

$$\mathrm{HD}(\sigma, \sigma') = \frac{1}{|T| \cdot \sqrt{2}} \cdot \sum_{t \in T} \sqrt{\sum_{m \in M} \left(\sqrt{\sigma(t, m)} - \sqrt{\sigma'(t, m)} \right)^2}.$$

To compensate for the arbitrariness of message use, we set the distance between strategies σ and σ' to be the maximum of $\mathrm{HD}(\sigma, \sigma')$ and $\mathrm{HD}(\sigma^*, \sigma')$ where σ^* is σ with reversed message indices. An example of the within-group distance, that is, the average distances between all sender strategies obtained for the same parameter values, is plotted in Figure 9a for $\beta = 0.1$ and $n_s = 10$. Despite some quantitative differences, the general trend is the same for all other parameter settings that we tested: with increasing imprecision, the resulting sender strategies were much more alike (modulo swapping of messages). This means that perceptual imprecision can speed up and unify evolutionary outcomes. It can amplify the emergence of sender strategies that are not only convex, but also regular, in that they induce a vague category split exactly in the middle of the unit interval. This is then reflected in the within-group expected utility, defined as the average expected utility that each evolved language scored when playing against an arbitrary other language obtained for the same parameter values. Figure 9b gives a representative example.

5 Discussion

Our imprecise imitation dynamic leads to by-and-large successful signalling behaviour, even in the presence of noise, that shows the hallmarks of vagueness as desired. It also gives rise to population-level behaviour that looks as if agents are generalizing across similar stimuli. Here, we would like to reflect briefly on some further conceptually relevant points and compare our approach to related work.

5.1 Levels of vagueness

The imprecise imitation dynamic was introduced in Section 3 as tracing changes in the overall distribution of pure strategies in a population: $\sigma(m|t)$ was said to represent the probability that a randomly sampled sender would have a strategy that responds with m to t (likewise for the receiver). This is in line with the standard interpretation of the replicator equation, but we should consider its philosophical implications. Based on this picture, vagueness in signal use would seem to be characterized as a strictly population-level phenomenon since it arises in a signalling system from the inability of individual agents to fully align their (non-vague) strategies as a result of imprecision. This is, we believe, a plausible mechanism that can already explain the existence of vagueness in a language, even if we assume that each agent commands a non-vague idiolect.

We would not, however, want to commit to the idea that vagueness does not exist at the level of individual agents. True, our derivation of the imprecise imitation dynamic assumed that agents carry and revise pure strategies. But that was an assumption of convenience, not of conviction. Moreover, even if individual agents command a non-vague pure strategy, the realization of that pure strategy, according to our model, is bound to be vague: the same agent could send different signals with repeated exposure to the same state because of the non-deterministic nature of observation noise. We have used the term ‘observation noise’ here, but this could equally well be interpreted as an inseparable component of an agent’s ‘signalling faculty’. In this sense, then, the model might be compatible with a picture of agents who have internalized a vague signalling strategy. It would need to be seen, however, how revision of non-deterministic, individual-level behaviour ~~will~~ be spelled out rigorously and whether the resulting population-level dynamic ~~will~~ be equivalent to our present proposal in all relevant respects.

5.2 Evolutionary benefits of imprecision

The inclusion of confusability of similar states has noteworthy effects on the evolving meaning of signals. It transpired from our results that imprecision can have further accelerating and, surprisingly, unifying effects on meaning evolution. The unifying property of perceptual imprecision could be considered an evolutionarily beneficial side effect. A certain degree of imprecision can lead to higher within-group expected utility, defined as the average expected utility that each evolved language scored when playing against an arbitrary other language obtained for the same parameter values. Figure 9b gives a representative example. The observation repeats for other parameter values: while imprecision might decrease the communicative efficiency of individual languages, it increases the conceptual coherence and communicative success between independently evolving strategies. It is as if mere confusability of states imposes a regularity constraint on evolving categories.

The phenomenon could potentially be more than just a side effect. Given the benefit of a certain amount of imprecision that we observe when comparing within-group expected utility, it would be interesting to study whether, under certain conditions, this group-level advantage could trump the individual-level disadvantage of a vague language and thus actually select for a certain amount of imprecision. This could be achieved by letting the imprecision parameter be an evolving part in the dynamics ~~as well~~. For now, this idea is mostly speculative, but we consider it an interesting avenue for future research, especially from the perspective of multilevel selection theory (Wilson and Sober [1994]; O'Gorman *et al.* [2008]).

bad break

5.3 Related work

O'Connor ([2014a]) makes a proposal related to ours, based on a version of reinforcement learning for sim-max games in which successful play leads to reinforcement of choice options for states similar to the ones that actually occurred. This not only leads to vague signalling of the appropriate kind, but also speeds up learning in such a way that, especially for sim-max games with higher numbers of states, higher levels of communicative success are reached in shorter learning periods. Our results complement and extend O'Connor's. The most important differences are that (i) we obtain similar regularizing effects for cases with low numbers of states, and (ii) we do not assume that agents have any kind of generalizing capacity in and of themselves, even if that is only implicit in O'Connor's generalized reinforcement learning. State confusability has an effect on aggregate signalling behaviour that can be described as generalization without generalizers: the dynamics of imprecise imitation look as if conclusions about what works for one state are carried over to similar states. This, however, is merely an epiphenomenon, in the sense that no single agent genuinely generalizes over stimuli or reasons about what a more systematic signalling strategy would be.

Franke *et al.* ([2011]) suggested a number of ways in which information-processing limitations could lead to vague strategies. The model that is most clearly related to the present approach uses the notion of a quantal response, also known as a soft-max response function (Luce [1959]; McFadden [1976]; Goeree *et al.* [2008]). The main difference between this and our present approach lies in where stochastic noise is assumed to reside. In the case of a quantal response dynamic, it resides in the computation of expected utilities; in the case of imprecise imitation, it resides in perception and realization of similar states. There are cases, then, where evolving signalling behaviour, as predicted by these two approaches, is quite different. Intuitively speaking, for a case with two messages, the further we venture away from a prototypical interpretation of either message, the less discriminative a signalling strategy would be when the source of 'trembles' is the computation of expected utilities. To wit, since both 'tall' and 'short' are almost equally bad descriptions for a giant, quantal response dynamics predicts that senders would be almost indifferent. Sender behaviour that evolves under a confusion of states does not have this puzzling property, because a giant would not likely be confused for a dwarf.

6 Conclusion

We set out to meet a technical challenge posed by Lipman's ([unpublished]) problem: is there a conceptually sound and mathematically coherent formal model that shows how vague language can evolve under selective pressure for efficient communication if agents tend to confuse similar stimuli? To address this, we derived a generalization of the replicator dynamic from an agent-level process of imprecise imitation. The resulting population-level dynamic produced signalling behaviour that is at the same time regular and by-and-large communicatively efficient, while also showing the crucial marks of vagueness. In a sense, the model derives vagueness as a by-product of an arguably natural limitation on the discriminatory power of signalling agents. Although the inability to sharply discriminate similar stimuli may lead to vagueness and bring about a (slight) decrease in communicative efficiency, it may also be advantageous; not due to vagueness itself, but due to its cause. Systematicity in the confusability of states (which may be a natural by-product of the perceptual system) supports 'as-if' generalizations at the population-level without having to assume that agents themselves have any generalization power. In this way, the presented model extends research into the effects of transmission biases on processes of meaning evolution. While most previous models have focused on inductive biases of language learners and the regularization that these may effect (Nowak *et al.* [2000], [2001]; Kirby and Hurford [2002]; Smith *et al.* [2003]; Griffiths and Kalish [2007]; Kirby *et al.* [2014]), we have shown here that shared perceptual biases, of which such an effect was not necessarily expected, can also regularize, facilitate, and accelerate the evolution of meaning conventions.

Appendix
A.1 Imprecise Conditional Imitation

The goal of this section is to provide technical details for Section 3. We first show, in Section A.1.1, how to derive the standard replicator dynamic from noise-free imitation of success. Then, in Section A.2, we derive the imprecise imitation dynamic by a parallel chain of arguments.

A.1.1 Deriving the replicator dynamic from imitation of success

The main idea behind imitation of success is that agents imitate the behaviour of other agents at some choice point with a probability that is proportional to the expected utility of the latter agents' choice. Since the sim-max games that we are looking at here have positive utilities upper ~~bound~~ by 1, we can identify the switching probability with the expected utility.

Let's consider sender strategies, as the receiver case is parallel. Call (misleadingly!) the agent who gets a chance to change behaviour 'learner' and the possibly to-be-imitated agent 'teacher'. A random learner is drawn from the population and given a chance to change behaviour at choice point t. The probability that our learner plays m is $\sigma(m|t)$. The learner observes what a randomly sampled teacher does at t. That would be m' with probability $\sigma(m'|t)$. The learner then starts using m' instead of m with probability $\mathrm{EU}(m', t, \rho)$. (Of course, m' and m could be the same; the learner could even be the teacher as well, by random sampling.)

If agents get repeated update chances for their choice points, the expected change of frequency of m-choices at t becomes:

$$\dot{\sigma}(m|t) = P(m' \to m, t) - P(m \to m', t)\,, \tag{3}$$

where $P(m' \to m, t)$ is the 'inflow' probability that agents switch from any m' to m and $P(m \to m', t)$ is the 'outflow' probability that agents switch from m to any m'. Since we are dealing with expectations in a huge population, these can be spelled out as:

$$P(m' \to m, t) = \sum_{m'} \underbrace{\sigma(m'|t)}_{\text{learner plays } m'} \cdot \underbrace{\sigma(m|t)}_{\text{teacher plays } m} \cdot \underbrace{\mathrm{EU}(m, t, \rho)}_{\text{EU teacher choice}},$$

$$P(m \to m', t) = \sum_{m'} \underbrace{\sigma(m|t)}_{\text{learner plays } m} \cdot \underbrace{\sigma(m'|t)}_{\text{teacher plays } m'} \cdot \underbrace{\mathrm{EU}(m', t, \rho)}_{\text{EU teacher choice}}.$$

From this, we can simplify the expression of expected change in Equation (3) to:

$$\begin{aligned}\dot{\sigma}(m|t) &= \sigma(m|t) \cdot \mathrm{EU}(m, t, \rho) - \sigma(m|t) \cdot \textstyle\sum_{m'} \sigma(m'|t) \cdot \mathrm{EU}(m', t, \rho) \\ &= \underbrace{\sigma(m|t)}_{\text{frequency of } m \text{ at } t} \ (\underbrace{\mathrm{EU}(m, t, \rho)}_{\text{EU of } m \text{ at } t} - \underbrace{\sum_{m'} \sigma(m'|t) \cdot \mathrm{EU}(m', t, \rho)}_{\text{average EU at choice point } t})\,.\end{aligned}$$

This latter formulation is the continuous-time version of the replicator dynamic. We obtain a discrete-time formulation from it by assuming that discrete update steps are infinitesimally small, so that

$$\begin{aligned}\dot{\sigma}(m|t) &= \sigma'(m|t) - \sigma(m|t) \\ &= \frac{\sigma(m|t)\,\mathrm{EU}(m, t, \rho)}{\sum_{m'} \sigma(m'|t)\,\mathrm{EU}(m, t, \rho)} - \sigma(m|t) \\ &= \frac{\sigma(m|t)\,\mathrm{EU}(m, t, \rho) - \sigma(m|t) \sum_{m'} \sigma(m'|t)\,\mathrm{EU}(m, t, \rho)}{\sum_{m'} \sigma(m'|t)\,\mathrm{EU}(m, t, \rho)}\,.\end{aligned}$$

By dropping the denominator, which is constant for all m for fixed t, we obtain the above continuous-time formulation.

A.2 Imitation of success with imprecision

The above derivation of the replicator dynamic assumes that agents can discriminate choices and choice points perfectly. Let's dispense with that assumption. With an eye toward sim-max games, we will assume that states, but not messages, may be confused for one another.[8] Confusability of states will affect how agents behave, how they perceive the behaviour of others, and the expected utilities of behavioural dispositions.

To keep matters simple, let us assume that agents carry pure dispositions to act. Noise can affect the realization of these strategies. As a sender, every agent maps states to messages: these are subjectively perceived states, and no longer necessarily the actually occurring states. As a receiver, every agent maps messages to state interpretations: these are intended interpretations that need not always be faithfully realized. This means that behavioural strategies σ and ρ

[8] It is relatively straightforward to also incorporate confusability of messages, but this is irrelevant to our present purposes.

represent the average proportions of actual behavioural dispositions in the population, the realization and observation of which can be distorted by agents' confusion of similar states.

If t_a is the actual state, let $P_o(t_o|t_a)$ be the probability that a given agent observes state t_o. Similarly, if a given receiver intends to select interpretation t_i, let $P_r(t_r|t_i)$ be the probability with which state t_r is realized. A single round of play of a sim-max game is then governed by five pieces of stochastic information, where previously there were only three (see Figures 3 and 4).

Expected utilities of choices at choice points should likewise take into account that actual states need not be observed states, and intended interpretations need not be realized interpretations. First, note that

$$P_{\bar{o}}(t_a|t_o) \propto P_a(t_a)P_o(t_o|t_a)$$

is the probability that t_a is actual if t_o is observed by an agent. The probability that a random sender produces m when the actual state is t_a is:

$$P_\sigma(m|t_a) = \sum_{t_o} P_o(t_o|t_a)\sigma(m|t_o)\,.$$

The probability that the actual state is t_a if a random sender produced m is:

$$P_{\bar{\sigma}}(t_a|m) \propto P_a(t_a)P_\sigma(m|t_a)\,.$$

The probability that t_r is realized by a random receiver in response to message m is:

$$P_\rho(t_r|m) = \sum_{t_i} P_r(t_r\,|t_i)\rho(t_i|m)\,.$$

This lets us capture the expected utilities for observed states (sender) and intended interpretations (receiver) by taking into consideration what the likely actual states and realized interpretations will be:

$$\mathrm{EU}(m, t_o, \rho) = \sum_{t_a} P_{\bar{o}}(t_a|t_o) \sum_{t_r} P_\rho(t_r|m)U(t_a, t_r)\,,$$

$$\mathrm{EU}(t_i, m, \sigma) = \sum_{t_a} P_{\bar{\sigma}}(t_a|m) \sum_{t_r} P_r(t_r|t_i)U(t_a, t_r)\,.$$

If conditional probabilities P_o and P_r are trivial—that is, assign probability 0 to the confusability of non-identical states—the above definitions reduce to the previous definitions of expected utilities. This also legitimates the overload of notation.

Presence of potential imprecision in the form of non-trivial P_o and P_r will also affect the dynamic that ensues from imitation of successes. Since imprecision works slightly differently on senders and receivers (the former confuse choice points, the latter confuse choices), we need to look separately at each case.

As before, suppose that senders receive a chance to change their behaviour independently for a given choice point. In the present case, this would be a chance to change how to respond to subjectively perceived state t_o, which need not be the actual one. We must then consult probability $P_o(m|t_o)$ that, given that the learner observed t_o, he will simultaneously observe a randomly sampled teacher play m. This is (with $P_{\bar{o}}$ and P_σ as defined above):

$$P_o(m|t_o) = \sum_{t_a} P_{\bar{o}}(t_a|t_o)\, P_\sigma(m|t_a)\,.$$

The inflow and outflow probabilities, $P(m' \to m, t)$ and $P(m' \to m, t)$, that a randomly sampled learner switches from any m' to m or from m to any m' in subjectively perceived state t_o are therefore:

$$P(m' \to m, t_o) = \sum_{m'} \underbrace{\sigma(m'|t_o)}_{\text{learner plays } m' \text{ at } t_o} \cdot \underbrace{P_o(m|t_o)}_{\text{observe teacher play } m} \cdot \underbrace{\text{EU}(m, t_o, \rho)}_{\text{EU teacher choice in learner's view}}$$

$$P(m \to m', t_o) = \sum_{m'} \underbrace{\sigma(m|t_o)}_{\text{learner plays } m \text{ at } t_o} \cdot \underbrace{P_o(m'|t_o)}_{\text{observe teacher play } m'} \cdot \underbrace{\text{EU}(m', t_o, \rho)}_{\text{EU teacher choice in learner's view}}$$

The mean change to the proportion of m choices at state t are then:

$$\begin{aligned}\dot{\sigma}(m|t_o) &= P(m' \to m, t_o) - P(m \to m', t_o)\\ &= \textstyle\sum_{m'} \sigma(m'|t_o) P_o(m|t_o) \text{EU}(m, t_o, \rho) - \sum_{m'} \sigma(m|t_o) P_o(m'|t_o) \text{EU}(m', t_o, \rho)\\ &= P_o(m|t_o) \text{EU}(m, t_o, \rho) - \sigma(m|t_o) \textstyle\sum_{m'} P_o(m'|t_o) \text{EU}(m', t_o, \rho).\end{aligned} \tag{4}$$

The case of the receiver is mostly analogous. Presented with an update opportunity for choice point m, a learner will observe a random teacher choose interpretation t_o with probability[9]

$$P_o(t_o|m) = \sum_{t_r} P_o(t_o|t_r) P_\rho(t_r|m).$$

Parallel to the sender case, this gives rise to:

$$\begin{aligned}\dot{\rho}(t|m) &= \textstyle\sum_{t'} \rho(t'|m) P_o(t|m) \text{EU}(t, m, \sigma) - \sum_{t'} \rho(t|m) P_o(t'|m) \text{EU}(t', m, \sigma)\\ &= P_o(t|m) \text{EU}(t, m, \sigma) - \rho(t|m) \textstyle\sum_{t'} P_o(t'|m) \text{EU}(t', m, \sigma).\end{aligned} \tag{5}$$

[9] We assume that the noise, $P_o(t_o|t_a)$, that applies to the sender's observations also applies to the receiver's observations when trying to imitate other agents' strategies. We could easily make the model more complex and introduce another measure of noise for state confusability during receiver attempts to imitate. We refrain from it here, because we see no immediate theoretical gain.

The continuous-time formulations in Equations (4) and (5) have elegant and practical discrete-time solutions in

$$\sigma'(m|t) \propto P_o(m|t)\,\mathrm{EU}(m,t,\rho)\,, \quad \rho'(t|m) \propto P_o(t|m)\,\mathrm{EU}(t,m,\sigma)\,,$$

which is the discrete-time formulation of the imprecise imitation dynamic given in Equation (2). To see how the discrete-time formulation gives rise to the continuous-time formulations above, let's assume that update steps are infinitesimally small, so that, for the sender case:

$$\begin{aligned}\dot{\sigma}(m|t) &= \sigma'(m|t) - \sigma(m|t)\\ &= \frac{P_o(m|t)\,\mathrm{EU}(m,t,\rho) - \sigma(m|t)\sum_{m'} P_o(m'|t)\,\mathrm{EU}(m',t,\rho)}{\sum_{m'} P_o(m'|t)\,\mathrm{EU}(m',t,\rho)}\,.\end{aligned}$$

As before, we drop the denominator, which is constant for all m for fixed t, and obtain the above continuous-time formulation.

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