Cognitive Light Cones in Bacterial Chemotaxis: Capacity Bounds for Gradient-Following Agents

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Abstract

We instantiate a general framework for resource-bounded intelligence in a canonical biological system: bacterial chemotaxis. A single chemotactic bacterium is modeled as a run-and-tumble agent in a one-dimensional chemical gradient, with bounded speed, noisy sensing, and finite sampling time. We define a chemotactic cognitive light cone via an *effective sensing radius* r_{chem} derived from temporal concentration comparisons under noise and resource constraints. By reusing the one-dimensional detect-and-reach capacity theorem from the corridor model, we obtain a simple upper bound on the *chemotactic capacity*—the maximal initial distance from which nutrient can be reliably found. The result exhibits the same three qualitative regimes (detection-limited, coupled, reach-limited) as in the corridor case, but with a biologically interpretable radius r_{chem} that depends on gradient strength, sensory noise, and sampling resources. We briefly discuss how active inference implementations fit inside the same capacity bound.

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1 Introduction

The core theory of resource-bounded intelligence models agents as policies $\pi \in \Pi(R)$ acting in environment classes $\mathcal{E}$, pursuing goal families $\mathcal{G}$ under resource vectors R, and evaluates them via a resource-bounded intelligence functional $I(\pi; \mathcal{E}, \mathcal{G}, R, w, \mu)$. In a first spatial instantiation, a *detect-and-reach* problem for agents with speed v, sensing radius r, actuation radius a, and horizon T yields tight capacity bounds $C(v, T, r, a)$ for the size of the capturable set of target locations. The resulting theory exhibits three regimes — detection limited, coupled, and reach limited — and admits a geometric interpretation in terms of cognitive light cones.

In this paper we provide a second spatial instantiation in a biological setting: bacterial chemotaxis. Instead of perfect detection in a fixed radius r, a chemotactic bacterium must infer the up-gradient direction from noisy temporal comparisons of chemical concentration. This breaks the idealized detection model but fits naturally into the same framework:

- The environment is a chemical concentration field with a nutrient source.
- Goals are "reach the source region by time T".
- Resources specify speed, horizon, sampling rate, sensory noise, and energy.
- Policies are run-and-tumble strategies that bias motion using temporal comparisons.

Our main move is deliberately minimal: we define an *effective chemotactic sensing radius* $r_{\text{chem}}(R, q)$ that encodes, for a given resource vector R and confidence level q, the largest distance from which the bacterium can infer the correct gradient direction with probability at least q. Plugging r_{chem} into the existing capacity theorem yields an upper bound on chemotactic capacity.

The resulting bounds are simple, interpretable, and show that the corridor capacity law is robust to replacing perfect sensing with noisy chemotaxis.

2 Chemotaxis as a Detect-and-Reach Problem

We work in one spatial dimension for clarity. Let the spatial domain be $X = \mathbb{R}$ or a finite interval $[0, L]$.

2.1 Environment class and goals

Definition 2.1 (Chemotactic environment class)**.** An environment $e \in \mathcal{E}^{\text{chem}}$ consists of:

- a chemoattractant concentration field
$$c_e : X \to \mathbb{R}_{\geq 0},$$
- a nutrient source region
$$B(x^\star, a) := \{x \in X : |x - x^\star| \leq a\},$$
for some center $x^\star \in X$ and actuation radius $a \geq 0$.

We often specialize to a time-homogeneous exponential gradient

$$c_e(x) = c_{\max} e^{-k|x-x^\star|}, \qquad c_{\max} > 0,\ k > 0, \tag{2.1}$$

whose local gradient magnitude at distance $d = |x - x^\star|$ is

$$G(d) := \left| \frac{\mathrm{d}}{\mathrm{d}x} c_e(x) \right|_{|x-x^\star|=d} = k c_{\max} e^{-kd}. \tag{2.2}$$

Definition 2.2 (Chemotactic goals)**.** Fix $x^\star$, a, and a time horizon $T > 0$. Given a trajectory $(x_t)_{t\in[0,T]}$, define the source-reaching goal

$$g_{x^\star,a,T}(\mathrm{Traj}) := \begin{cases} 1 & \text{if } \exists t \le T \text{ with } |x_t - x^\star| \le a, \\ 0 & \text{otherwise.} \end{cases} \tag{2.3}$$

The chemotactic goal family is

$$\mathcal{G}^{\mathrm{chem}} := \{g_{x^\star,a,T} : x^\star \in X,\ a \ge 0,\ T > 0\}. \tag{2.4}$$

2.2 Run-and-tumble agent model

A chemotactic bacterium is modeled as a run-and-tumble agent with bounded speed, noisy observations, and finite memory.

Definition 2.3 (Chemotaxis dynamics and observations)**.** Let $x_t \in X$ be the position of the agent at time t, and $d_t \in \{+1, -1\}$ its running direction. In the running state,

$$\dot{x}_t = v d_t, \qquad |d_t| = 1, \tag{2.5}$$

with maximum speed $v > 0$. Runs have characteristic duration τ (e.g. fixed τ or i.i.d. with mean τ), after which the agent tumbles and chooses a new direction.

At the end of each run $n = 1, 2, \ldots$ (at times t_n), the agent samples the local concentration with additive Gaussian noise:

$$o_n = c_e(x_{t_n}) + \eta_n, \qquad \eta_n \sim \mathcal{N}(0, \sigma^2) \tag{2.6}$$

independent across runs.

Definition 2.4 (Chemotaxis policy class)**.** A resource vector R^{chem} (defined below) fixes speed v, horizon T, sensory noise σ, run duration scale τ, energy budget E, and memory bits m. A *chemotaxis policy* is a measurable mapping that, at each tumble, uses the history $h_n = (x_{t_i}, o_i)_{i\le n}$ and an internal memory state of size $\le m$ bits to choose the next direction and run duration, subject to the constraints in R^{chem}. The set of such policies is denoted $\Pi^{\mathrm{chem}}(R) \subseteq \Pi(R)$.

A canonical subclass uses temporal comparisons $\Delta o_n = o_n - o_{n-1}$ to decide whether to maintain or reverse direction.

2.3 Resources and chemotactic capacity

Definition 2.5 (Resource vector)**.** For chemotaxis we take

$$R^{\mathrm{chem}} = (v, T, \tau, \sigma, E, m, a), \tag{2.7}$$

where v is speed, T the horizon, τ the run timescale, σ the noise level, E an energy budget (e.g. max total running time), m memory bits, and a the actuation radius.

We consider initial positions at distance d from the source, i.e. $x_0 = x^\star - d$.

Definition 2.6 (Chemotactic capacity)**.** For a given $d \ge 0$ and resources R, let e_d denote an environment with source center $x^\star$ and initial position $x_0 = x^\star - d$. The best achievable success probability at distance d is

$$p^*(d; R) := \sup_{\pi\in\Pi^{\mathrm{chem}}(R)} \mathbb{P}^{e}_{\pi}\big[g_{x^\star,a,T}(\mathrm{Traj}) = 1\big]. \tag{2.8}$$

Fix a target threshold $p_{\min} \in (0, 1)$. The *chemotactic capacity* at resources R and threshold $p_{\min}$ is

$$d_{\max}(R, p_{\min}) := \sup\{d \ge 0 : p^*(d; R) \ge p_{\min}\}. \tag{2.9}$$

Intuitively, $d_{\max}(R, p_{\min})$ is the largest initial distance from which a bacterium with resources R can reliably find the source before time T.

3 Effective Chemotactic Sensing Radius

In the corridor model, detection was perfect inside a fixed radius r. Chemotaxis replaces this with *noisy inference* of the gradient direction from temporal comparisons.

3.1 Single-sample gradient sign inference

Consider a run of duration τ taken from a position at distance $d = |x - x^\star|$ from the source. Linearizing c_e along the path, we have

$$c_e(x_{t_n}) - c_e(x_{t_{n-1}}) \approx \pm G(d)\, v\tau, \tag{3.1}$$

where the sign is positive for an up-gradient run and negative for a down-gradient run. Thus

$$\Delta o_n := o_n - o_{n-1} \sim \mathcal{N}\big(\pm\, G(d) v\tau,\ 2\sigma^2\big). \tag{3.2}$$

A simple decision rule sets "up" if $\Delta o_n > 0$, "down" otherwise. Conditioned on an up-gradient test run, the probability of a correct decision is

$$p_{\mathrm{dir}}(d; R) = \Phi\left(\frac{G(d) v\tau}{\sqrt{2}\,\sigma}\right), \tag{3.3}$$

where Φ is the standard normal CDF. Define the single-sample SNR

$$\mathrm{SNR}_1(d; R) := \frac{G(d) v\tau}{\sqrt{2}\,\sigma}. \tag{3.4}$$

Far from the source, $G(d)$ is small and $p_{\mathrm{dir}}(d; R) \to 1/2$; close to the source, $p_{\mathrm{dir}}(d; R)$ approaches 1.

3.2 Multi-sample integration and directional confidence

Suppose the agent collects N such differences Δo_i near the same distance scale d and averages them:

$$\bar{\Delta o} := \frac{1}{N} \sum_{i=1}^{N} \Delta o_i. \tag{3.5}$$

Then

$$\bar{\Delta o} \sim \mathcal{N}\big(\pm\, G(d) v\tau,\ 2\sigma^2/N\big), \tag{3.6}$$

and the probability that the sign of $\bar{\Delta o}$ is correct is

$$p^{(N)}_{\mathrm{dir}}(d; R) = \Phi\Big(\sqrt{N}\,\mathrm{SNR}_1(d; R)\Big). \tag{3.7}$$

Resource constraints limit N. If each measurement consumes one run of duration τ, then

$$N\tau \le T, \qquad N \le N_E(R) \tag{3.8}$$

for some energy-derived bound $N_E(R)$. Define

$$N_{\max}(R) := \min\left(\lfloor T/\tau \rfloor,\ N_E(R)\right). \tag{3.9}$$

The best achievable directional confidence at distance d is then

$$p_{\mathrm{dir,max}}(d; R) := \Phi\Big(\sqrt{N_{\max}(R)}\,\mathrm{SNR}_1(d; R)\Big). \tag{3.10}$$

3.3 Definition and explicit form of r_{chem}

Definition 3.1 (Effective chemotactic sensing radius)**.** Fix a confidence threshold $q \in (0.5, 1)$. For resources R, the *effective chemotactic sensing radius* is

$$r_{\text{chem}}(R, q) := \sup \big\{ d \geq 0 : p_{\text{dir,max}}(d; R) \geq q \big\}. \tag{3.11}$$

That is, $r_{\text{chem}}(R, q)$ is the largest distance from which a bacterium with resources R can infer the correct up-gradient direction with probability at least q using at most $N_{\max}(R)$ samples.

In the exponential gradient $c_e(x) = c_{\max} e^{-k|x-x^\star|}$, we have

$$\text{SNR}_1(d; R) = \frac{k c_{\max} e^{-kd} v\tau}{\sqrt{2}\,\sigma}. \tag{3.12}$$

Thus

$$p_{\text{dir,max}}(d; R) = \Phi\left(\sqrt{N_{\max}(R)} \cdot \frac{k c_{\max} e^{-kd} v\tau}{\sqrt{2}\,\sigma} \right). \tag{3.13}$$

Enforcing $p_{\text{dir,max}}(d; R) \geq q$ is equivalent to

$$e^{-kd} \geq \frac{\sqrt{2}\,\sigma}{k c_{\max} v\tau \sqrt{N_{\max}(R)}}\, \Phi^{-1}(q), \tag{3.14}$$

so we obtain:

Proposition 3.2 (Explicit chemotactic sensing radius)**.** *In an exponential gradient $c_e(x) = c_{\max} e^{-k|x-x^\star|}$, the effective chemotactic sensing radius is*

$$r_{\text{chem}}(R, q) = \frac{1}{k} \ln\left(\frac{k c_{\max} v\tau \sqrt{N_{\max}(R)}}{\sqrt{2}\,\sigma\, \Phi^{-1}(q)} \right)_+, \tag{3.15}$$

where $(\cdot)_+ = \max(\cdot, 0)$.

The dependence is intuitive: r_{chem} increases with speed v, run duration τ, gradient amplitude $c_{\max}$, and sampling budget $N_{\max}(R)$, and decreases with sensory noise σ and required confidence q.

4 Chemotactic Capacity Bound

We now reuse the one-dimensional detect-and-reach capacity theorem by substituting r_{chem} for the ideal detection radius r.

4.1 Corridor capacity (recall)

Consider the 1D detect-and-reach POMDP with discrete domain $X = \{1, \ldots, N\}$, uniform prior over target locations, speed v, horizon T, sensing radius r, and actuation radius a. Let $C(\pi)$ denote the capturable set of targets under policy π and

$$C(v, T, r, a) := \max_{\pi} |C(\pi)| \tag{4.1}$$

the capacity. The main theorem in the corridor paper states:

Theorem 4.1 (1D detect-and-reach capacity)**.** *For any policy* π*,*

$$|C(\pi)| \le C(v, T, r, a), \tag{4.2}$$

where

$$C(v,T,r,a) = \begin{cases} \min(N, vT + 2r + 1), & a \ge r \quad \textit{(detection-limited)}, \\ \min(N, vT + r + a + 1), & a < r \le vT + a \quad \textit{(coupled)}, \\ \min(N, 2(vT + a) + 1), & r > vT + a \quad \textit{(reach-limited)}. \end{cases} \tag{4.3}$$

These bounds are tight and are achieved by sweep or detect–then–pursue policies in the respective regimes.

4.2 Chemotactic capacity via effective radius

In the chemotaxis setting, physical reach is still governed by v, T, and a, but detection is now limited by r_{chem} and directional errors.

Theorem 4.2 (Chemotactic capacity bound)**.** *Fix resources* R^{chem}*, confidence levels* $q \in (0.5, 1)$ *and* $p_{\min} \in (0, 1)$*, and consider the chemotaxis model in an exponential gradient as above. Let* $r_{\text{chem}}(R, q)$ *be given by Proposition 3.2. Then there exists a constant* $0 < c(q, p_{\min}) \le 1$ *such that the chemotactic capacity satisfies*

$$d_{\max}(R, p_{\min}) \;\le\; c(q, p_{\min})\, C\big(v, T, r_{\text{chem}}(R, q), a\big), \tag{4.4}$$

where $C(\cdot)$ *is the corridor capacity function from Theorem 4.1.*

Proof sketch. From the construction of $r_{\text{chem}}(R, q)$, any policy $\pi \in \Pi^{\text{chem}}(R)$ can, from any initial position within distance $r_{\text{chem}}(R, q)$ of the source, devote at most $N_{\max}(R)$ samples to estimating the gradient direction and thereby choose the correct direction with probability at least q.

Abstractly, this defines an *effective detection event*: a stopping time at which the agent knows, with probability at least q, whether to move left or right to approach the source, but only if the source lies within distance $r_{\text{chem}}(R, q)$. Conditioned on the event being correct, the agent's subsequent motion reduces to the corridor setting with detection radius $r = r_{\text{chem}}(R, q)$ and speed v, so the set of such starting positions cannot exceed $C(v, T, r_{\text{chem}}(R, q), a)$.

The overall success probability from a given distance d is the probability that (i) sufficient sampling occurs before the horizon to trigger a detection event, (ii) the directional estimate is correct, and (iii) the remaining motion reaches the source region in time. Using standard concentration bounds, one can show that to achieve overall success probability at least $p_{\min}$, d must lie in a subset of the corridor capturable set whose measure is at most $c(q, p_{\min})C(v, T, r_{\text{chem}}(R, q), a)$ for some $c(q, p_{\min}) \le 1$. Taking the supremum over such distances yields the stated bound on $d_{\max}(R, p_{\min})$. □

The important point is that the *shape* of the capacity law is unchanged: the three regimes of Theorem 4.1 reappear, with r replaced by r_{chem}.

4.3 Regimes for chemotaxis

Because Theorem 4.2 factors through $C(v, T, r, a)$, the same regime structure emerges:

- **Noise-limited chemotaxis.** When $r_{\text{chem}}(R, q) \ll vT + a$, capacity is dominated by sensing. Reducing σ or increasing gradient strength greatly increases $d_{\max}$; increasing v or T has limited effect unless it also increases $N_{\max}(R)$ (and thus r_{chem}).

- **Coupled regime.** When $r_{\text{chem}}(R, q) \sim vT$, both information and locomotion matter. Increasing v or T helps twice: the agent can physically reach farther and can afford more samples.
- **Reach-limited chemotaxis.** When sensors are excellent so that $r_{\text{chem}}(R, q) \gg vT + a$, capacity is dominated by locomotion: $d_{\max}$ scales essentially with vT; further improvements in sensing bring diminishing returns.

5 Discussion and Outlook

We have shown how a simple chemotaxis model can be embedded into an existing geometric theory of resource-bounded intelligence. The key idea is to replace an ideal sensing radius r with an *effective* chemotactic radius $r_{\text{chem}}(R, q)$ derived from a noisy sensorimotor model. Once this mapping is in place, the corridor capacity theorem applies almost verbatim, yielding chemotactic capacity bounds and regimes with clear biological interpretation.

This construction is deliberately modest:

- It retains the core framework and capacity function unchanged.
- It requires only a basic run-and-tumble model with Gaussian noise.
- It yields explicit scaling of r_{chem} in terms of gradient strength, noise, speed, and sampling budget.

Relation to active inference. Chemotaxis has been modeled as active inference, where the bacterium minimizes expected free energy under a generative model that encodes preferences for high nutrient concentration states. Such controllers live inside the same policy class $\Pi^{\text{chem}}(R)$: they choose run-and-tumble parameters based on concentration histories. The capacity bound in Theorem 4.2 is therefore *substrate-agnostic*: any implementation—active inference, reinforcement learning, or heuristic control—that operates under resources R and attains the same directional accuracy and locomotor performance inherits the same upper bound on $d_{\max}$.

Next steps. Natural extensions include:

- Moving from 1D to 2D and 3D chemotaxis, including angular noise.
- Studying multi-agent chemotactic collectives and their team capacity.
- Connecting chemotactic capacity to morphogenesis and pattern formation in multicellular systems.
- Adding simple empirical checks by plugging in known parameters for specific bacterial species and comparing the implied $d_{\max}$ to observed chemotactic ranges.

Overall, chemotaxis provides a concrete biological test case where cognitive light cones and capacity bounds can be related both to theory and to data, helping to bridge the gap between abstract intelligence measures and real living systems.

Acknowledgments

(Placeholder.)