

# Measurements of drag and lift on tennis balls in flight

Rod Cross · Crawford Lindsey

Published online: 7 November 2013  
© International Sports Engineering Association 2013

**Abstract** Measurements are presented of drag and lift on new tennis balls in flight. Two video cameras were used to measure the velocity and height of the balls at two positions separated horizontally by 6.4 m. The balls were fired from a ball launcher at speeds between 15 and 30 m/s and with topspin or backspin at rates up to 2,500 rpm. Significant shot-to-shot variations were found in both the drag and lift coefficients. The average drag coefficient was  $0.507 \pm 0.024$ , independent of ball speed or spin, and lower than the value usually observed in wind tunnel experiments. The lift coefficient increased with ball spin, on average, but significant lift was observed even at very low spin. The latter effect can be attributed to a side force arising from asymmetries in the ball surface, analogous to the side force responsible for the erratic path of a knuckleball in baseball.

## 1 Introduction

The trajectory of a tennis ball can be estimated from its initial launch parameters, provided that the aerodynamic drag and lift coefficients are known. The coefficients have been measured in wind tunnel experiments [1–11], indicating that tennis balls behave as rough rather than smooth spheres. For a relatively smooth or slightly rough sphere such as a soccer or golf ball, the drag coefficient drops suddenly at high ball speeds, an effect known as the drag

crisis. Increased surface roughness usually acts to lower the Reynold's number at which drag crisis is observed. No such drop has been observed for tennis balls. Instead, the drag coefficient measured in wind tunnels remains relatively constant at about 0.6 at all ball speeds of interest, although the coefficient can increase or decrease slightly depending on the condition of the ball cloth. For that reason, top players like to hand pick balls when serving, discarding balls that are perceived to be too fluffy.

An interesting question is whether the drag and lift coefficients for a tennis ball traveling through still air are the same as those for a stationary tennis ball supported on a rod in a wind tunnel. There has been no evidence to suggest that the coefficients might be different, nor has there been any experimental evidence that they are the same. Wind tunnel measurements are conducted under conditions where the air flow is allowed time to stabilise, whereas tennis balls are launched under game conditions by accelerating the ball rapidly into stationary or low speed air. It is not obvious that identical air flow conditions should prevail in both cases, nor is it obvious that drag and lift coefficients should remain constant in time under the turbulent flow conditions that prevail around a high-speed, fluffy tennis ball. If significant fluctuations were to occur over a time scale of order 0.1 s, then drag and lift forces could vary significantly from one tennis shot to the next, even if the launch speed and spin remain the same. The present work describes the results of an experiment designed to measure the drag and lift coefficients when a tennis ball is launched into stationary air.

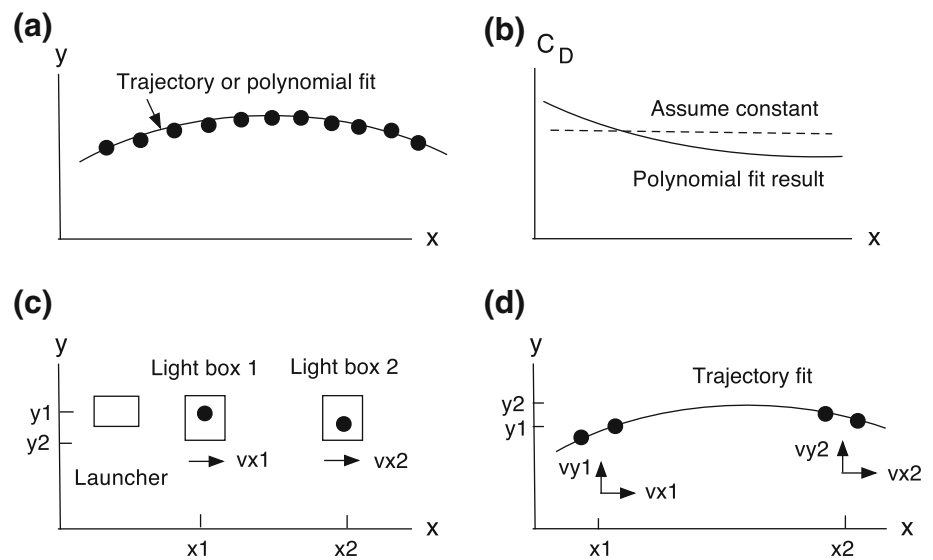
Experiments on the aerodynamics of other sports balls in flight have been described previously [12–18]. Three different experimental approaches have been adopted, as summarised in Fig. 1. One approach, shown in Fig. 1a, is to film the trajectory or several segments of the trajectory and to match the observed trajectory to numerical predictions

---

R. Cross (✉)  
Physics Department, University of Sydney, Sydney, NSW,  
Australia  
e-mail: cross@physics.usyd.edu.au

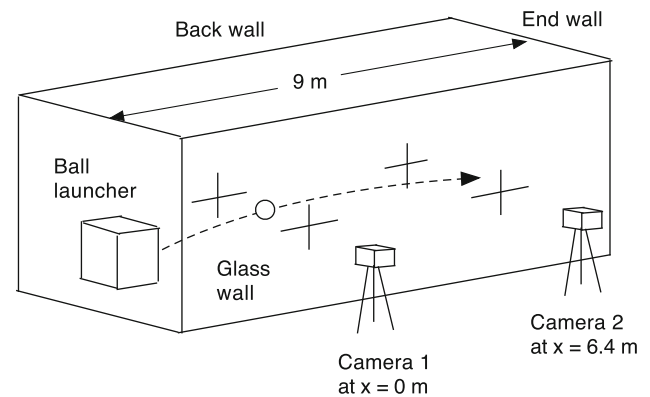
C. Lindsey  
Tennis Warehouse, San Luis Obispo, CA, USA

**Fig. 1** A summary of various methods that can be used to extract drag and lift coefficients from trajectory data. If a polynomial is used to fit  $(x, y)$  trajectory data then the variation of the drag and lift coefficients can be determined from a single shot, at least in principle



[12–14]. Different trajectories can be calculated based on the assumption that the drag and lift coefficients remain constant over the flight path. In that manner, best fit values of the coefficients can be obtained. An alternative approach [15] is to find a best fit polynomial curve to say measured  $x(t)$  trajectory data, in which case  $dx/dt$  yields  $v_x$  vs.  $t$  and  $dv_x/dt$  yields the acceleration,  $a_x$  vs.  $t$ . Multiplying  $a_x$  by the ball mass yields the aerodynamic force in the  $x$  direction. The force in the  $y$  direction is obtained in a similar manner, in which case a direct measurement is obtained of the drag and lift forces on the ball. The latter approach is more challenging, but in principle allows for a direct determination of the change in the coefficients with time or with ball speed, as indicated in Fig. 1b. The main problem is that small errors in the measured  $x$  and  $y$  coordinates can lead to unacceptably large errors in the inferred values of  $a_x$  and  $a_y$ . A symptom of the latter problem, observed by the authors in several preliminary experiments, is that the final results can depend significantly on the order of the polynomials used to fit the  $x(t)$  and  $y(t)$  data.

The third approach is the one adopted in the present paper. The ball is launched in an approximately horizontal direction and accurate measurements are obtained of the change in ball speed and vertical height of the ball at two or more different locations spaced several metres apart horizontally. The change in horizontal speed depends primarily on the drag coefficient and the change in vertical height depends primarily on the lift and gravitational forces. This is the preferred approach used by the US Golf Association to test the aerodynamic properties of golf balls, since the measurements can be conducted indoors under controlled conditions and can be adapted to test thousands of balls annually [16]. The same approach was used by Kensrud and Smith [17, 18] to measure drag and lift coefficients for baseballs, softballs and cricket balls. As is the case with



**Fig. 2** Experimental arrangement used to measure ball velocity, spin and height at two points separated horizontally by 6.4 m. A third camera, Camera 3, was used to measure the impact point on the end wall

golf ball measurements, the latter experiments were conducted using light gates to measure the horizontal speed and vertical height of the balls at two locations, as indicated in Fig. 1c. The present experiment is similar in principle, but was conducted using two video cameras to measure the horizontal ( $x$ ) and vertical ( $y$ ) speeds of a tennis ball and the vertical height of the ball at two locations separated horizontally by 6.4 m, as indicated in Fig. 1d. A third video camera was used to monitor small changes in the  $z$  coordinate between the two locations, the latter effect being caused by small variations in the horizontal launch angle.

## 2 Experimental procedure

The arrangement used in the present experiment is shown in Fig. 2. To avoid the effects of wind, the experiment was

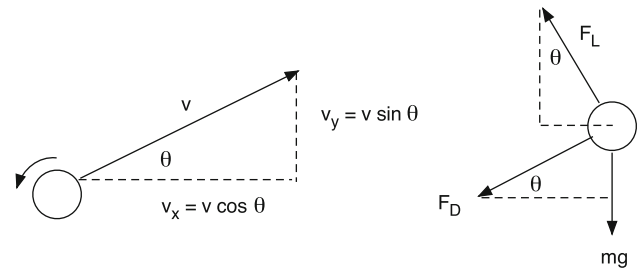
conducted indoors in a relatively long, 2.5 m wide corridor bounded by three brick walls and one glass wall containing several sliding glass doors. New balls were launched at one end of the corridor, in a direction parallel to the side walls, so that they impacted on the end wall. Each new ball was fired from the ball launcher about 20 times before replacing it with another new ball of the same type (Penn ATP Professional, Extra-duty felt).

The ball launcher was a Tennis Tutor brand, battery-operated device containing two counter-rotating wheels. Each wheel was separately controlled so that balls could be launched with no spin or with topspin or backspin, at speeds between 10 and 30 m/s, and with spins up to about 2,500 rpm. The ball was projected slightly above the horizontal so that it would fall to a height approximately equal to the launch height. In that way, both cameras could be zoomed in, without losing sight of the ball, to obtain more accurate measures of the speed and height of the ball.

Four large crosses were marked on the side walls, indicated by the large + symbols in Fig. 2, all at the same height, two at  $x = 0$  and two at  $x = 6.4$  m. The crosses on the glass wall were used to align the two cameras accurately, by adjusting the height and tilt angles of the cameras so that opposite pairs of crosses overlapped in the centre of the field of view. Both cameras were Casio EX-F1 video cameras set to record at 300 frames/s with a shutter speed of  $1/5,000$  s and resolution  $640 \times 480$  pixels. The cameras were calibrated by filming a large rectangular grid of known dimensions located in the same plane as the ball. The grid was removed prior to firing the balls. The balls were illuminated by two 500 W halogen lamps at each location. Equator lines drawn on each ball were used to measure the rotation angle and hence determine the angular velocity of the ball.

The most difficult and the most important part of the experiment was to measure the ball speed at each location to an accuracy of about 1 % or better. The decrease in ball speed over the 6.4 m measurement path was typically about 11 % for a drag coefficient of 0.5, or about 13 % for a drag coefficient of 0.6. A change of that magnitude is easy to measure, but an accurate determination of the drag and lift coefficients is more difficult. For that reason, particular care was taken when aligning and calibrating the two cameras and when processing the experimental data. Motion analysis software was used to manually digitise the coordinates of the centre of the ball, using several different techniques. The accuracy was limited primarily by pixel resolution and by the sharpness of the video image of the ball. All images were digitised by both authors independently and discrepancies exceeding 1 % resulted in the discarding of some data.

The largest potential source of error turned out to be the fact that balls tended to deviate slightly from the path



**Fig. 3** The velocity components and forces acting on a ball rising upward at angle  $\theta$  to the horizontal with backspin.  $F_D$  is the drag force and  $F_L$  is the lift force

exactly parallel to the back wall. If the ball at  $x = 6.4$  m passed up to 5 cm in front of or behind the parallel plane, then the distance calibration was found to be in error by up to 1.7 %. The problem was solved by recording the out-of-plane deviation for each ball. For that purpose, a third camera was used to film the impact point on the end wall. A distance calibration scale was measured in ten different vertical planes and the results were fitted by a quadratic, so that the relevant distance calibration factor on the video film could be determined for each shot from the measured impact point on the end wall.

### 3 Trajectory analysis

The trajectory of a ball in the  $x, y$  plane can be calculated in terms of the drag and lift coefficients using the diagrams shown in Fig. 3. We assume that the ball rises at angle  $\theta$  to the horizontal at speed  $v$  and has velocity components  $v_x = v \cos \theta$  and  $v_y = v \sin \theta$ . The drag force  $F_D = 0.5 C_D \rho A v^2$  acts backward and the lift force  $F_L = 0.5 C_L \rho A v^2$  acts at right angles to the path of the ball, where  $C_D$  is the drag coefficient,  $C_L$  is the lift coefficient,  $\rho$  is the air density and  $A = \pi R^2$  is the cross-sectional area of the ball,  $R$  being the ball radius. The equations describing the trajectory of a ball of mass  $m$  are then

$$m dv_x/dt = -F_D \cos \theta - F_L \sin \theta \quad (1)$$

and

$$m dv_y/dt = F_L \cos \theta - F_D \sin \theta - mg \quad (2)$$

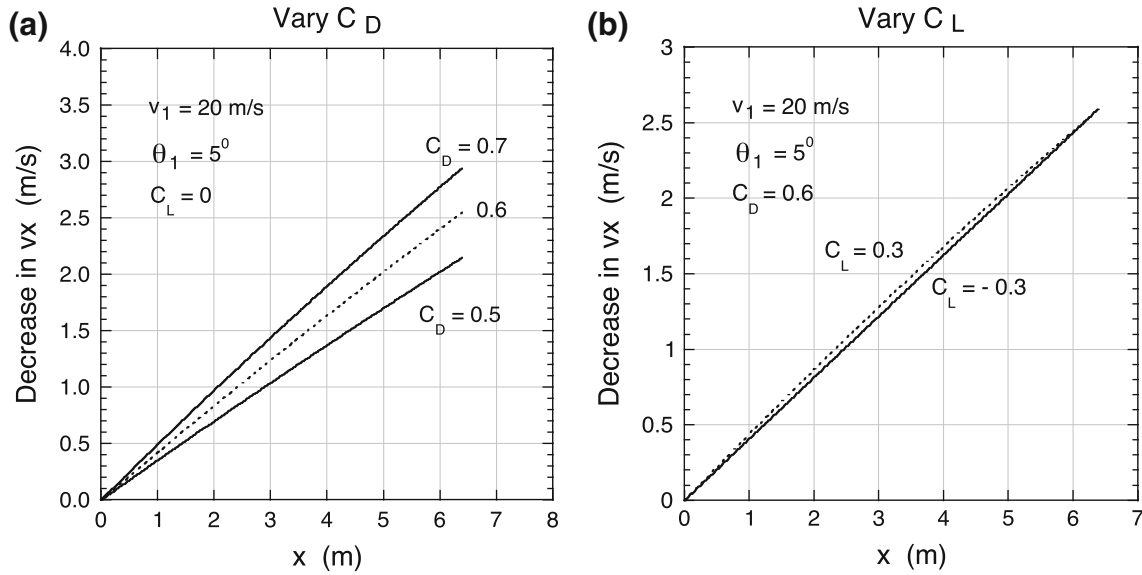
which can be expressed in the form

$$dv_x/dt = -kv(C_D v_x + C_L v_y) \quad (3)$$

and

$$dv_y/dt = kv(C_L v_x - C_D v_y) - g \quad (4)$$

where  $k = 0.5 \rho \pi R^2/m$ . If the ball has topspin then  $C_L$  can be taken as a negative quantity. Equations (3) and (4) need to be solved numerically in general, although analytical solutions can be obtained if  $v_x = 0$  or if  $v_y = 0$ . Numerical



**Fig. 4** The decrease of  $v_x$  with  $x$  for a typical shot with  $v_1 = 20$  m/s and  $\theta_1 = 5^\circ$  showing **a** the sensitivity to  $C_D$  and **b** the insensitivity to  $C_L$

solutions were obtained to find values of  $C_D$  and  $C_L$  that best fit the experimental data obtained for each shot.

Approximate analytical solutions of Eqs. (3) and (4) can be obtained if it is assumed that  $v_y \ll v_x$ , as was the usual case in this experiment. In that case, the drag force is approximately horizontal and the lift force is approximately vertical. The horizontal and vertical accelerations are then approximately  $a_x = -F_D/m$  and  $a_y = F_L/m - g$ , respectively. Using subscripts 1 and 2 to denote parameters at  $x = 0$  and  $x = 6.4$  m, respectively, we find that  $v_{x2}$  and  $y_2$  are given approximately by

$$v_{x2}^2 = v_{x1}^2 - 2F_D X/m \quad (5)$$

and

$$y_2 = y_1 + v_{y1}t + 0.5(F_L/m - g)t^2 \quad (6)$$

where  $X = 6.4$  m and  $t = 2X/(v_{x1} + v_{x2})$  is the approximate transit time of the ball from  $x = 0$  to  $x = 6.4$  m. Equations (5) and (6) provided useful guides when solving the equations numerically and show that the decrease in  $v_x$  depends primarily on the drag coefficient, while the change in ball height depends primarily on the vertical launch speed and the lift coefficient. The procedure adopted to fit the experimental data was therefore simplified as a first approximation by adjusting  $C_D$  to fit the change in  $v_x$  and by adjusting  $C_L$  to fit the change in ball height. Only very minor adjustments to  $C_D$  and  $C_L$  were subsequently needed to fit both the change in  $v_x$  and the change in  $y$ .

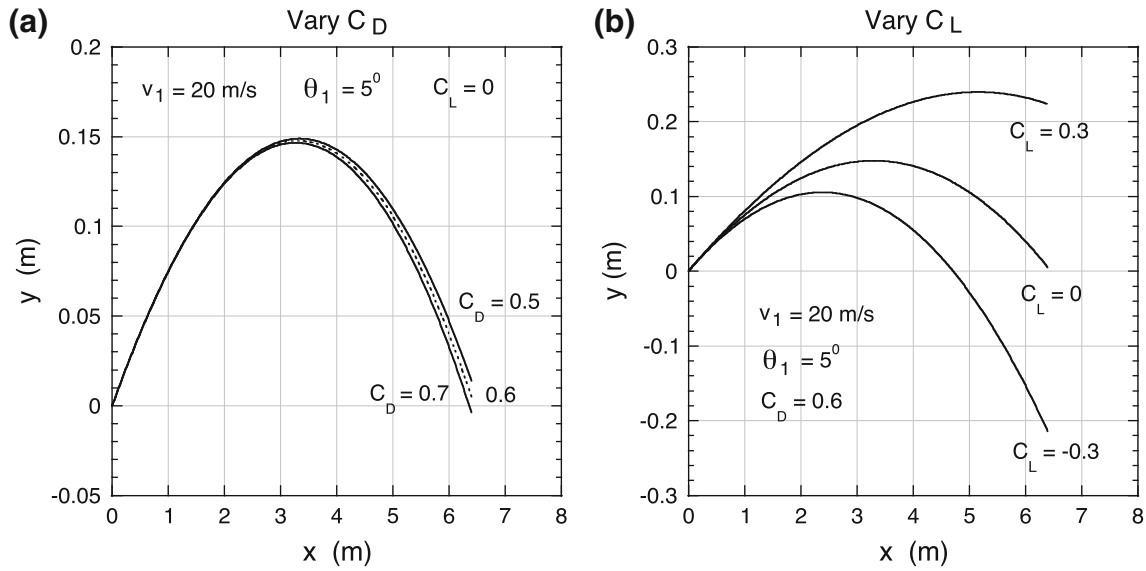
Numerical solutions of Eqs. (3) and (4) are shown in Figs. 4, 5, 6 to illustrate the sensitivities involved in fitting the experimental data. Figure 4 shows the change in the  $v_x$  velocity component with  $x$  for a typical launch speed

$v_1 = 20$  m/s and launch angle  $\theta_1 = 5^\circ$ . The decrease in  $v_x$  is typically about 10 % over the 6.4 m path length, being sensitive to  $C_D$  and relatively insensitive to  $C_L$ . The results in Fig. 4 indicate that a 2 % error in the measurement of  $v_x$  at  $x = 0$  or at  $x = 6.4$  m would result in a 20 % error in the measurement of  $C_D$ . A +2 % error at both  $x = 0$  and  $x = 6.4$  m would result in no significant error, but a -2 % error at  $x = 0$  combined with a +2 % error at  $x = 6.4$  m would result in a 40 % error in  $C_D$ .

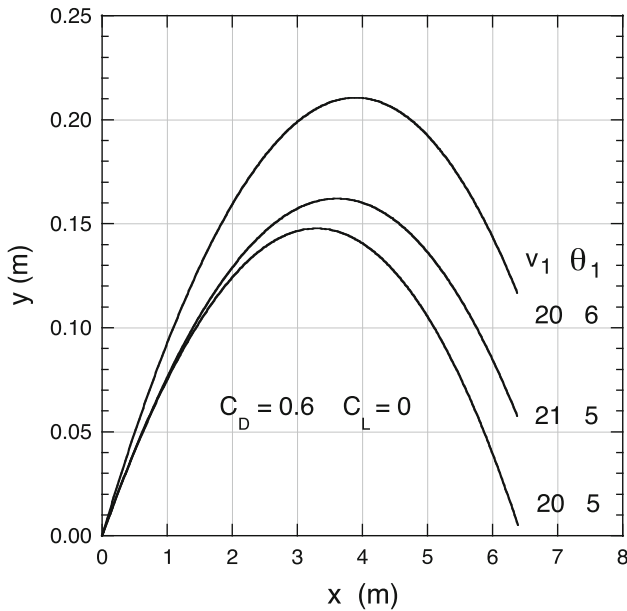
Figure 5 shows the change in height of the ball with  $x$  for the same launch conditions as Fig. 4. The height is sensitive to  $C_L$  and relatively insensitive to  $C_D$ . However, the height is also sensitive to the launch speed and launch angle, as indicated in Fig. 6. Consequently, the measurement accuracy of  $C_L$  is limited by the accuracy with which  $v_1$  and  $\theta_1$  can be determined. It is clear from Figs. 5 and 6 that  $v_1$  needs to be determined to within about  $\pm 0.5$  m/s and  $\theta_1$  needs to be determined to within about  $\pm 0.3^\circ$  if  $C_L$  is to be determined within  $\pm 0.05$ .

#### 4 Error analysis

Each shot recorded on video film was analysed to locate the centre of the ball to within  $\pm 0.3$  mm, by expanding the on-screen image of the ball to about 60 mm diameter. The main limitation was the slightly blurred image of the edge of the ball. The ball speed was calculated from two ball centres spaced about 400 mm horizontally, typically about 40 mm vertically, and separated in time by between 6 and 9 video frames. The horizontal speed could therefore be estimated to within  $\pm 0.15$  % and the vertical speed to



**Fig. 5**  $y$  vs.  $x$  for a typical shot with  $v_1 = 20$  m/s and  $\theta_1 = 5^\circ$  showing **a** the insensitivity to  $C_D$  and **b** the sensitivity to  $C_L$



**Fig. 6**  $y$  vs.  $x$  for a typical shot with  $v_1 = 20$  m/s or  $v_1 = 21$  m/s and  $\theta_1 = 5^\circ$  or  $\theta_1 = 6^\circ$  showing the sensitivity of the ball height to  $v_1$  and  $\theta_1$

within about  $\pm 1.5\%$ , ignoring calibration errors. The vertical launch angle and the angle of descent at the camera 2 location was determined to within about  $\pm 0.1^\circ$ .

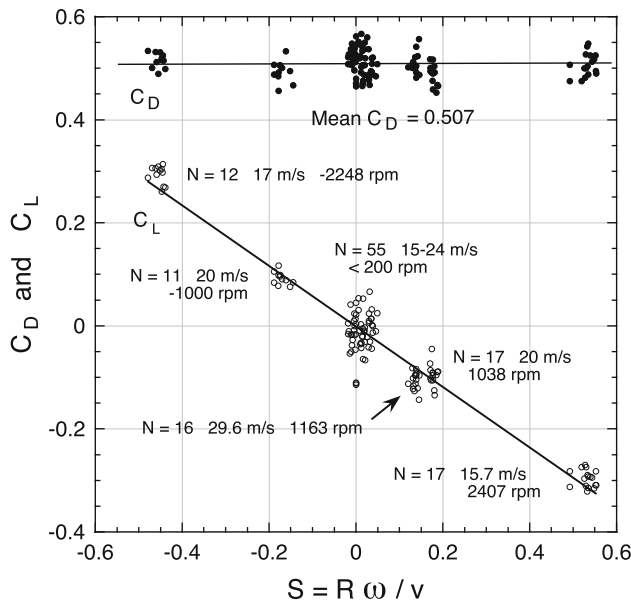
The horizontal and vertical distance on the video image was calibrated to within  $\pm 0.15\%$  by filming a 600 mm-long grid located in the same plane as the ball. The same procedure was used to calibrate the distance scale in other parallel planes in front of camera 2, to account for the fact that the ball sometimes veered slightly out of the path exactly parallel to the back wall.

If the maximum errors in  $v_{x1}$  and  $v_{x2}$  are each taken as  $0.3\%$ , then the maximum error in  $C_D$  is about  $5\%$  from Eq. (5), or about  $\pm 0.025$  for any given shot, in the most pessimistic case. Independent estimates of  $C_D$  obtained by both authors agreed in almost all cases within  $\pm 0.01$  when using the same calibration factors.

The remaining uncertainties concern the ball mass and diameter. The diameter of a tennis ball is not well defined, since it is not a perfect sphere and since the outer surface is not smooth. The diameters of all 12 new balls used in this study were measured with a digital caliper, across three perpendicular axes, allowing the caliper to grip the ball lightly. Individual diameters ranged from 64.94 to 66.72 mm. The average diameter of each ball ranged from 65.36 to 66.27 mm. The average diameter of all 12 balls was 65.68 mm. Ball masses ranged from 56.68 to 60.25 g, the average mass of all 12 balls being 58.03 g.

Of greater concern is the quantity  $R^2/m$  that multiplies both  $C_D$  and  $C_L$  in Eqs. (3) and (4). The value of  $R^2/m$  averaged over all 12 balls was  $0.0186 \text{ m}^2/\text{kg}$ . Most balls had a value of  $R^2/m$  within  $1\%$  of the average, although one ball was  $3\%$  lower and three were  $2\%$  higher. Individual balls were not separately identified when filming them in batches of six at a time. Drag and lift coefficients were calculated for all balls assuming the same average value of  $R^2/m$  for each ball, leading to an error of up to  $1\%$  for most balls,  $2\%$  for three of the balls and  $3\%$  for another ball. In other words, the error resulting from variations in ball mass and diameter was typically about  $\pm 0.005$  for  $C_D$  and up to  $\pm 0.003$  for  $C_L$ .

The ball spin does not enter directly into calculations of the drag and lift coefficients. The spin at each location was measured to an accuracy of better than  $1\%$ , and decreased



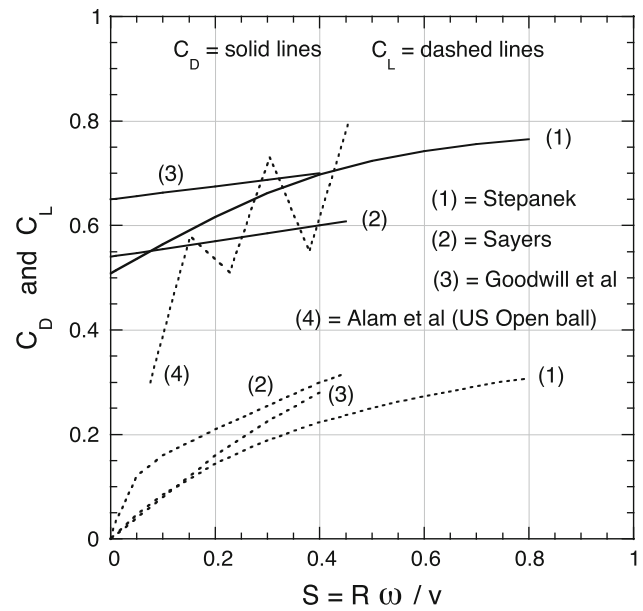
**Fig. 7** Measured values of  $C_D$  and  $C_L$  vs. the spin parameter  $S = R\omega/v$ , for each of the 128 shots analysed. Labels show the average speed and spin for each group of shots and the number of shots in each group. The straight lines represent **a** the mean value of  $C_D$  for all 128 shots and **b** a linear fit to the  $C_L$  data

by about 2 % over the 6.4 m path length. Accurate measurements of the spin axis alignment were not attempted. Visual inspection indicated that the axis was within  $2^\circ$  of the horizontal in all cases.

## 5 Experimental results

The main results of this experiment are shown in Fig. 7. The results for  $C_D$  and  $C_L$  are plotted as a function of the spin parameter  $S = R\omega/v$  (measured at  $x = 0$ ), where  $\omega$  is the ball spin and  $v$  is the ball speed. Negative values of  $\omega$  shown in Fig. 7 correspond to backspin, and positive values correspond to topspin. Eight separate combinations of ball spin and speed were selected by varying the speeds of the two wheels in the ball launcher. The average spin and speed recorded for each group are shown in Fig. 7, together with the number of shots recorded for each group. A total of 128 shots were analysed. Three of the groups, all with spins varying from  $-120$  rpm to  $+200$  rpm but with different average speeds, overlap in Fig. 7 and are labelled as a large group of 55 shots with speeds ranging from 15 to 24 m/s. These three groups were obtained at three different wheel speed settings, with both wheels set to the same nominal speed. Small differences in the rotation speed of each wheel caused the ball to be launched with a small amount of topspin or backspin.

The values of  $C_D$  for all 128 shots ranged from 0.453 to 0.567, with an average value of 0.507 and a standard



**Fig. 8** Results for tennis balls obtained previously from wind tunnel experiments [1, 4, 6, 7] showing the drag coefficient,  $C_D$ , and the lift coefficient,  $C_L$  vs the spin parameter  $S = R\omega/v$  at wind speeds in the range  $20 < v < 30$  m/s

deviation of 0.024. There was no significant variation of  $C_D$  with ball speed, or with spin or with  $S$ . Of greater significance was the variation of  $C_D$  within each group of shots, showing that large variations in  $C_D$  can occur from one shot to the next even when the ball speed and spin are held relatively constant. Part of that variation can be attributed to experimental errors, but the variation from 0.453 to 0.567 is well beyond the  $\pm 0.025$  estimated maximum experimental error for any given shot.

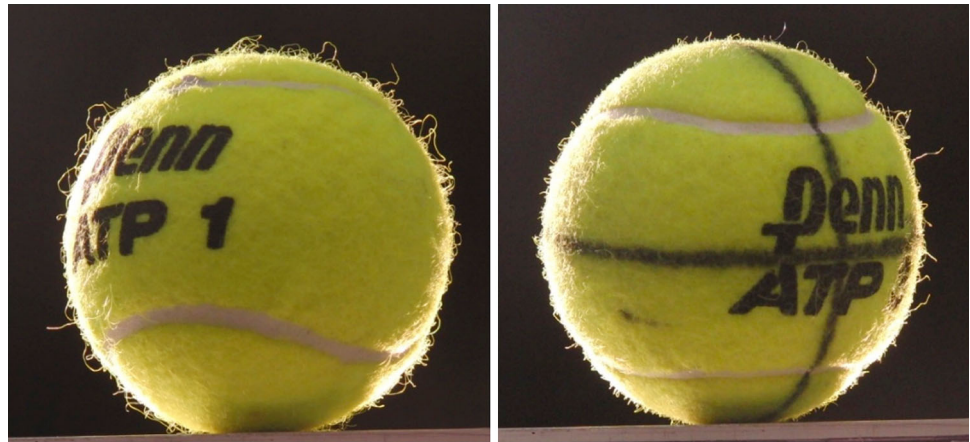
The lift coefficient,  $C_L$ , was found to vary linearly with  $S$  but there were significant variations within each group of shots, as indicated in Fig. 7. The variation was largest within the three combined low spin groups. In theory, the lift coefficient should be zero when the ball spin is zero if one assumes that the lift force arises from spin alone. In fact, some of the lift coefficients obtained at very low spin values were as large as those obtained when the ball was spinning at about 1,000 rpm, indicating that some other factor contributed to the total lift force on the ball.

## 6 Discussion

The results in Fig. 7 can be compared with data obtained previously in wind tunnel experiments. A selection of results is shown in Fig. 8, all obtained at wind speeds comparable to the ball speeds measured in the present experiment. The first such data were obtained in 1986 by Stepanek [1] by dropping a spinning tennis ball vertically



**Fig. 9** Comparison of a new ball (at *left*) with a ball fired 20 times from the ball launcher



through a horizontal stream of air. He found that  $C_D = 0.508$  at zero spin and that  $C_D$  increases significantly with spin, to about 0.75 at  $S = 0.6$ . He also found that  $C_L$  increases with spin, from zero at  $S = 0$  to about 0.28 at  $S = 0.6$ . In 2000, Chadwick and Haake [2] found that  $C_D$  is about 0.53 for a non-spinning, standard new ball, regardless of ball speed. Mehta and Pallis [3] found that  $C_D$  varies from about 0.6 to 0.7 for new balls, but can be as low as 0.5 for a used ball.

In an attempt to improve the accuracy of wind tunnel data, Sayers [4] used a 235 mm-diameter sphere covered with tennis ball cloth, obtaining the results shown in Fig. 8. He found that  $C_D$  increased from about 0.54 to about 0.6 as  $S$  increased from zero to 0.4 and that  $C_L$  increased to 0.3 at  $S = 0.4$ . Others have obtained even larger values of  $C_D$  and  $C_L$ , such as those reported by Alam et al. [5, 6]. The very large values of  $C_L$  shown in Fig. 8 appear to be subject to larger than usual experimental errors when compared with other wind tunnel data.

Only one previous ball trajectory measurement of  $C_D$  for a tennis ball has been reported, as far as the authors are aware. Zayas [12] found that  $C_D = 0.51 \pm 0.08$  at  $v = 26.8$  m/s by measuring the launch speed and the landing speed of a new ball fired horizontally so that it impacted the floor about 8 m from a ball launcher. His result is consistent with ours.

Our results for  $C_L$  are broadly consistent with the wind tunnel data shown in Fig. 8, apart from the large shot-to-shot variations found at small values of  $S$ . Our results for  $C_D$  are lower than most wind tunnel results, show strong shot-to-shot variations and do not indicate any increase with  $S$ . A direct comparison with the wind tunnel data might suggest that we tested used rather than new balls, or a smooth sphere, given the low value  $C_D$  results reported in Refs. [3, 7]. After about 20 shots, the surface appearance of each of the new balls in our experiment did change noticeably, as indicated by the back-lit

photographs shown in Fig. 9. However,  $C_D$  remained low in our experiment even when a new ball was fired the very first time.

The new ball in Fig. 9 shows many fibres extending out from the ball surface by up to about 5 mm. The matted fibres of a new ball are easily pulled out of the surface by hand. When impacted a few times at high speed on a solid surface, additional fibres untangle and extend even further out from the surface, giving a new ball a fuzzy appearance. However, further impacts act to break the fibres near the ball surface, giving the ball a “crew-cut” appearance, as shown by the ball on the right in Fig. 9. A few long fibres are still visible on the right side of the latter ball, implying that some parts of the ball experienced fewer impacts than others. The ball cloth remained intact and relatively undamaged after only 20 shots, but the bulk of the loose fibres extending beyond the surface were effectively shorn off by the abrasive effects of the ball launcher wheels and subsequent impacts on the end brick wall.

Back lighting of balls in flight indicated that new balls did not wear uniformly over their surface during the first 20 shots. Instead, tufts of raised cloth were observed in isolated patches over the ball surface, extending up to about 5 mm beyond the ball surface. In addition, a few isolated fibres were observed, extending up to about 20 mm beyond the surface. The air drag or lift on an isolated fibre is presumably negligible, given its very small surface area. However, a relatively thick tuft of ball cloth could introduce a significant asymmetry in the air flow around the ball, comparable to the effect of the seam on a baseball or a cricket ball, even if the tuft is only 0.5 or 1 mm thick. We suspect that those tufts were responsible for both the relatively large variations observed in the drag coefficient and the relatively large values of the lift coefficient observed even when the ball was spinning slowly. Large shot-to-shot variations in the drag and lift coefficients were also observed by Kensrud and Smith [17, 18] in their trajectory

data for baseballs, particularly when using balls with a raised rather than a flat seam.

In baseball, pitchers usually spin the ball rapidly to maximise the effect of the lift force on the ball. However, some pitchers deliberately spin the ball slowly to maximise the effect of the side force. A side force arises from an asymmetry in the air flow generated by an asymmetry in the location of the ball seam. If the ball spins rapidly then the side force alternates in direction rapidly and has no significant average effect on the flight of the ball. However, if the ball spins slowly, then the side force acts in any given direction for a longer period of time, allowing the ball to change direction once or twice on its way to the batter as the ball slowly rotates. The result is a difficult to hit knuckleball. The side force acts in a sideways direction, but it can be up or down, left or right, depending on the orientation of the seam. The magnitude of the side force on a baseball is comparable to the magnitude of the spin-induced lift force. In the same way, one or more tufts of ball cloth on a tennis ball could generate a significant side force, in any sideways direction, and the effect would be more noticeable if the ball spins slowly.

## 7 Conclusions

Measurements of the trajectory of new tennis balls in flight were described over a speed range from 15 to 30 m/s at spins ranging from  $-2,250$  rpm to  $+2,500$  rpm. The results show that the drag coefficient varies on a shot-to-shot basis from about 0.45 to about 0.57, independent of the ball speed and spin. The statistical mean and standard deviation of 128 shots was 0.507 and 0.024, respectively. The lift coefficient was found to be proportional to the spin parameter  $S = R\omega/v$ , but it also varied on a shot-to-shot basis by about  $\pm 0.05$  even when  $S$  was approximately zero. The latter result appears to be due to the uneven surface of a new tennis ball that becomes fluffed up as a result of repeated impacts, introducing a side force that is especially evident when the ball spins slowly.

We are unable to explain why our measured drag coefficient for a new tennis ball is significantly lower than values normally observed with wind tunnels. The inference is that the sting attached to a tennis ball in a wind tunnel has a stronger effect on the air flow than previously suspected. Only one other measurement of the drag coefficient obtained from trajectory data has been reported [12], and

the result was essentially the same as ours. Further investigations are clearly warranted.

## References

1. Stepanek A (1988) The aerodynamics of tennis balls—the topspin lob. *Am J Phys* 56(2):138–142
2. Chadwick S, Haake S (2000) The drag coefficient of tennis balls. In: Subic A, Haake S (eds) *Engineering of sport, research, development and innovation: Proceedings of the 3rd international conference on the engineering of sport*; Sydney, Australia. Blackwell Science, Oxford, pp 169–176
3. Mehta R, Pallis J (2001) The aerodynamics of a tennis ball. *Sports Eng* 4(4):1–13
4. Sayers A (2003) Aerodynamics of a tennis ball—stationary and spinning. In: Miller S (ed) *Tennis Science and Technology 2*, International Tennis Federation, pp 123–132
5. Alam F, Subic A, Watkins S (2003) An experimental study on the aerodynamic drag of a series of tennis balls. In: *Proceedings of the sports dynamics-discovery and application: the International Congress on Sports Dynamics*, Melbourne, 1–3 Sept 2003, p 22
6. Alam F, Watkins S, Subic A (2004) Effects of spin on aerodynamic properties of tennis balls. In: *Proceedings of the ISEA 5th International Conference on Sports Engineering*, Davis, 13–16 Sept 2004
7. Goodwill S, Chin S, Haake S (2004) Aerodynamics of spinning and non-spinning tennis balls. *J Wind Eng Ind Aerodyn* 92:935–958
8. Goodwill S, Haake S (2004) Aerodynamics of tennis balls—effect of wear. In: Hubbard M, Mehta RD, Pallis JM (eds) *The engineering of sport 5: Proceedings of the 5th International Sports Engineering Association Conference*. Springer: Hoboken, pp 35–41
9. Djamovski V, Pateras J, Chowdhury H, Alam F, Steiner T (2012) Effects of seam and surface texture on tennis balls aerodynamics. *Procedia Eng* 34:140–145
10. Cooke AJ (2000) An overview of tennis ball aerodynamics. *Sports Eng* 3(2):123–129
11. Mehta R, Alam F, Subic A (2008) Review of tennis ball aerodynamics. *Sports Technol* 1:7–16
12. Zayas J (1986) Experimental determination of the coefficient of drag of a tennis ball. *Am J Phys* 54(7):622–625
13. Nathan A (2008) The effect of spin on the flight of a baseball. *Am J Phys* 76(2):119–124
14. Goff J, Carre M (2009) Trajectory analysis of a soccer ball. *Am J Phys* 77(11):1020–1027
15. Cross R (2012) Aerodynamics in the classroom and at the ball park. *Am J Phys* 80(4):289–297
16. Lieberman B, Smits A, Quintavalla S, Thomas F, Winfield D (2001) Method for determining coefficients of lift and drag of a golf ball. US Patent Number 6,186,002.
17. Kensrud J, Smith L (2010) In situ drag measurements of sports balls. *Procedia Eng* 2:2437–2442
18. Kensrud J, Smith L (2011) In situ lift measurement of sports balls. *Procedia Eng* 13:278–283