



Calculation of predictions for non-identical particle correlations in AA collisions at LHC energies from hydrodynamics-inspired models

MASTER OF SCIENCE THESIS

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Obliczenia teoretycznych przewidywań korelacji cząstek nieidentycznych w zderzeniach AA przy energiach LHC pochodzących z modeli hydrodynamicznych

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Abstract

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²³ Introduction

Chapter 1

Theory of heavy ion collisions

1.1 The Standard Model

In the 1970s, a new theory of fundamental particles and their interaction emerged. A new concept, which concerns the electromagnetic, weak and strong nuclear interactions between know particles. This theory is called *The Standard Model*. There are seventeen named particles in the standard model, organized into the chart shown below (Fig. 1.1). Fundamental particles are divided into two families: *fermions* and *bosons*.

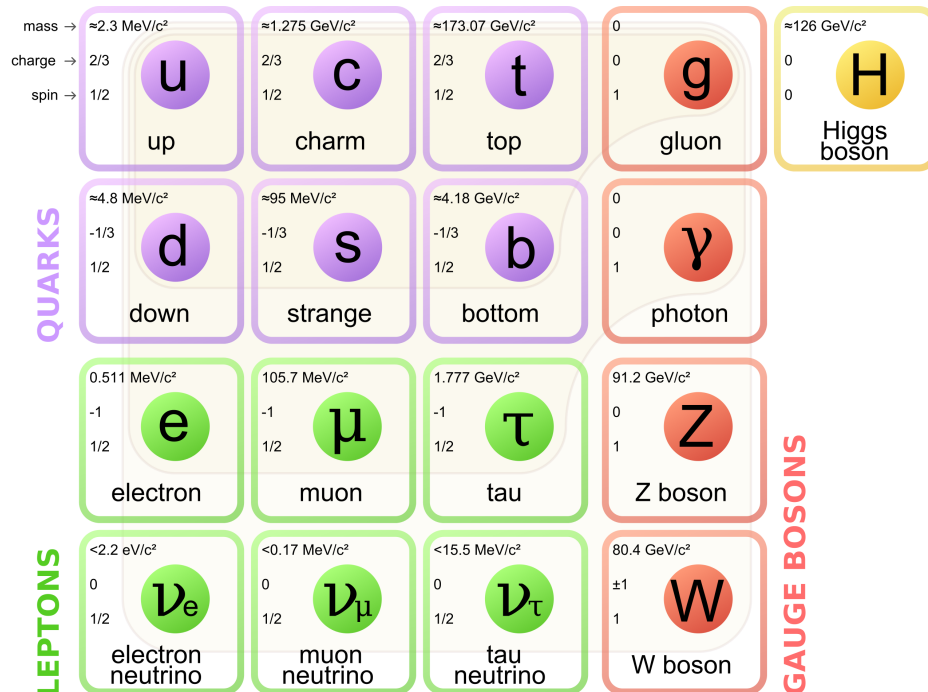


Figure 1.1: The Standard Model of elementary particles [1].

32 Fermions are the building blocks of matter. They are divided into two groups.
 33 Six of them, which must bind together are called *quarks*. Quarks are known to
 34 bind into doublets (*mesons*), triplets (*baryons*) and recently confirmed four-quark
 35 states.¹ Two of baryons, with the longest lifetimes, are forming a nucleus: a pro-
 36 ton and a neutron. A proton is build from two up quarks and one down, and
 37 neutron consists of two down quarks and one up. A proton is found to be a stable
 38 particle (at least it has a lifetime larger than 10^{35} years) and a free neutron has a
 39 mean lifetime about 8.8×10^2 s. Fermions, that can exist independently are called
 40 *leptons*. Neutrinos are a subgroup of leptons, which are only influenced by weak
 41 interaction. Fermions can be divided into three generations (three columns in
 42 the Figure 1.1). Generation I particles can combine into hadrons with the longest
 43 life spans. Generation II and III consists of unstable particles which form also
 44 unstable hadrons.

45 Bosons are force carriers. There are four fundamental forces: weak - respons-
 46 ible for radioactive decay, strong - coupling quarks into hadrons, electromagnetic
 47 - between charged particles and gravity - the weakest, which causes the attraction
 48 between particles with a mass. The Standard Model describes the first three. The
 49 weak force is mediated by W^\pm and Z^0 bosons, electromagnetic force is carried by
 50 photons γ and the carriers of a strong interaction are gluons g . The fifth boson is
 51 a Higgs boson which is responsible for giving other particles mass.

52 1.2 Quantum Chromodynamics

53 1.3 Relativistic heavy ion collisions

¹The LHCb experiment at CERN in Geneva confirmed recently existence of $Z(4430)$ - a particle consisting of four quarks [2].

54 Chapter 2

55 Terminator model

56 THERMINATOR [3] is a Monte Carlo event generator designed to investigate
57 the particle production in the relativistic heavy ion collisions. The functionality
58 of the code includes a generation of the stable particles and unstable resonances
59 at the chosen hypersurface model. It performs the statistical hadronization which
60 is followed by space-time evolution of particles and the decay of resonances. The
61 key element of this method is an inclusion of a complete list of hadronic reson-
62 ances. The second version of THERMINATOR [4] comes with a possibility to in-
63 corporate any shape of freeze-out hypersurface and the expansion velocity field,
64 especially those generated externally with various hydrodynamic codes.

65 2.1 (3+1)-dimensional viscous hydrodynamics

66 Most of the relativistic viscous hydrodynamic calculations are done in
67 (2+1)-dimensions. Such simplification assumes boost-invariance of a matter
68 created in a collision. Experimental data reveals that no boost-invariant region is
69 formed in the collisions [5]. Hence, for the better description of created system a
70 (3+1)-dimensional model is required.

71 In the four dimensional relativistic dynamics one can describe a system
72 using a space-time four-vector $x^\nu = (ct, x, y, z)$, a velocity four-vector
73 $u^\nu = \gamma(c, v_x, v_y, v_z)$ and a energy-momentum tensor $T^{\mu\nu}$. The particular
74 components of $T^{\mu\nu}$ have a following meaning:

- 75 • T^{00} - an energy density,
- 76 • $cT^{0\alpha}$ - an energy flux across a surface x^α ,
- 77 • $T^{\alpha 0}$ - an α -momentum flux across a surface x^α multiplied by c ,
- 78 • $T^{\alpha\beta}$ - components of momentum flux density tensor,

79 where $\gamma = (1 - v^2/c^2)^{-1/2}$ is Lorentz factor and $\alpha, \beta \in \{1, 2, 3\}$. Using u^ν one can
80 express $T^{\mu\nu}$ as follows [6]:

$$T_0^{\mu\nu} = (e + p)u^\mu u^\nu - pg^{\mu\nu} \quad (2.1)$$

81 where e is an energy density, p is a pressure and $g^{\mu\nu}$ is an inverse metric tensor:

$$g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad (2.2)$$

82 The presented version of energy-momentum tensor (2.1) can be used to describe
83 dynamics of a perfect fluid. To take into account influence of viscosity, one has to
84 apply the following corrections coming from shear $\pi^{\mu\nu}$ and bulk Π viscosities [7]:

$$T^{\mu\nu} = T_0^{\mu\nu} + \pi^{\mu\nu} + \Pi(g^{\mu\nu} - u^\mu u^\nu). \quad (2.3)$$

86 The stress tensor $\pi^{\mu\nu}$ and the bulk viscosity Π are solutions of dynamical equa-
87 tions in the second order viscous hydrodynamic framework [6].

88 The comparison of hydrodynamics calculations with the experimental results
89 reveal, that the shear viscosity divided by entropy η/s has to be small and close
90 to the AdS/CFT estimate $\eta/s = 0.08$ [7, 8]. When using $T^{\mu\nu}$ to describe system
91 evolving close to local thermodynamic equilibrium, relativistic hydrodynamic
92 equations in a form of:

$$\partial_\mu T^{\mu\nu} = 0 \quad (2.4)$$

93 can be used to describe the dynamics of the local energy density, pressure and
94 flow velocity.

95 Hydrodynamic calculations are starting from the Glauber¹ model initial con-
96 ditions. The collective expansion of a fluid ends at the freeze-out hypersurface.
97 That surface is usually defined as a constant temperature surface, or equivalently
98 as a cut-off in local energy density. The freeze-out is assumed to occur at the
99 temperature $T = 140$ MeV.

100 2.2 Statistical hadronization

101 Statistical description of heavy ion collision has been successfully used
102 to describe quantitatively *soft* physics, i.e. the regime with the transverse
103 momentum not exceeding 2 GeV. The basic assumption of the statistical
104 approach of evolution of the quark-gluon plasma is that at some point of the
105 space-time evolution of the fireball, the thermal equilibrium is reached. When
106 the system is in the thermal equilibrium the local phase-space densities of

¹The Glauber Model is used to calculate “geometrical” parameters of a collision like an impact parameter, number of participating nucleons or number of binary collisions.

particles follow the Fermi-Dirac or Bose-Einstein statistical distributions. At the end of the plasma expansion, the freeze-out occurs. The freeze-out model incorporated in the THERMINATOR model assumes, that chemical and thermal freeze-out occur at the same time.

One of the crucial elements of the statistical approach is the complete inclusion of hadronic resonances. This is because at the rather high temperature of the freeze-out ≈ 140 - 160 MeV, the resonances contribute very significantly to the observables [3].

2.2.1 Cooper-Frye formalism

The result of the hydrodynamic calculations is the freeze-out hypersurface Σ^μ . A three-dimensional element of the surface is defined as [4]

$$d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial \alpha} \frac{\partial x^\beta}{\partial \beta} \frac{\partial x^\gamma}{\partial \gamma} d\alpha d\beta d\gamma, \quad (2.5)$$

where $\epsilon_{\mu\alpha\beta\gamma}$ is the Levi-Civita tensor and the variables $\alpha, \beta, \gamma \in \{1, 2, 3\}$ are used to parametrize the three-dimensional freeze-out hypersurface in the Minkowski four-dimensional space. The Levi-Civita tensor is equal to 1 when the indices form an even permutation (eg. ϵ_{0123}), to -1 when the permutation is odd (eg. ϵ_{2134}) and has a value of 0 if any index is repeated. Therefore [4],

$$d\Sigma_0 = \begin{vmatrix} \frac{\partial x}{\partial \alpha} & \frac{\partial x}{\partial \beta} & \frac{\partial x}{\partial \gamma} \\ \frac{\partial y}{\partial \alpha} & \frac{\partial y}{\partial \beta} & \frac{\partial y}{\partial \gamma} \\ \frac{\partial z}{\partial \alpha} & \frac{\partial z}{\partial \beta} & \frac{\partial z}{\partial \gamma} \end{vmatrix} d\alpha d\beta d\gamma \quad (2.6)$$

and the remaining components are obtained by cyclic permutations of t, x, y and z .

One can obtain the number of hadrons produced on the hypersurface Σ^μ from the Cooper-Frye formalism. The following integral yields the total number of created particles [4]:

$$N = (2s + 1) \int \frac{d^3p}{(2\pi)^3 E_p} \int d\Sigma_\mu(x) p^\mu f(x, p), \quad (2.7)$$

where

$$f(p \cdot u) = \left\{ \exp \left[\frac{p_\mu u^\mu - (B\mu_B + I_3\mu_{I_3} + S\mu_S + C\mu_C)}{T} \right] \pm 1 \right\}^{-1} \quad (2.8)$$

is the phase-space distribution for particles (for stable ones and resonances). For the Fermi-Dirac distribution in the 2.8 there is a plus sign and for Bose-Einstein statistics minus sign respectively. The thermodynamic quantities in $f(\cdot)$ are T - temperature, μ_B - baryon chemical potential, μ_{I_3} - isospin chemical potential, μ_S - strange chemical potential, μ_C - charmed chemical potential and the s is a spin

135 of a particle. One can simply derive from equation 2.7, the dependence of the
 136 momentum density [9]:

$$E \frac{dN}{d^3p} = \int f(x, p) p^\mu d\Sigma_\mu. \quad (2.9)$$

137 The equations presented above are directly used in the THERMINATOR to generate
 138 the hadrons with the Monte-Carlo method.

139 **Chapter 3**

140 **Particle interferometry**

141 **3.1 HBT interferometry**

142 **3.2 Intensity interferometry in heavy ion collisions**

143 **3.2.1 Theoretical approach**

144 **Two particle wave function**

145 **Source function**

146 **Theoretical correlation function**

147 **Spherical harmonics decomposition of correlation function**

148 **3.2.2 Experimental approach**

149 **3.3 Scaling of femtoscopic radii**

150 **Chapter 4**

151 **Results**

152 **4.1 Identical particles correlations**

153 **4.2 Results of the fit**

154 **4.3 Discussion of results**

¹⁵⁵ **Chapter 5**

¹⁵⁶ **Summary**

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