1) Si Z es una v.a | $Z \sim N(0,1)$

a) $$E(Z) = \int_{-\infty}^{\infty} z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz = 0.$$

$$E(Z) = \int_{-\infty}^{\infty} z f(z) \, dz$$ por definición de esperanza continua

Si $Z \sim N(0,1) = \Phi_{z_0} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_0} e^{-z^2/2} \, dz$

Por lo tanto

$$E(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} \, dz = 0$$

Por lo cuanto que
sigue en la otra hoja

$\left.\begin{array}{l}\text{MAE}\\ \text{RMSE}\end{array}\right\}$ MAPE

Calculamos la integral.

$$\int z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = \frac{1}{\sqrt{2\pi}} \int z e^{-z^2/2} dz.$$

$$(e^{\varphi})' = e^{\varphi} \varphi' \Rightarrow \left[e^{-z^2/2}\right]' = e^{-z^2/2} \cdot -z.$$

$$\Rightarrow \int z e^{-z^2/2} dz = -e^{-z^2/2}.$$

$$\Rightarrow -e^{-z^2/2}\Big|_{-\infty}^{\infty} = \underbrace{-e^{-\infty}}_{0} + \underbrace{e^{-\infty}}_{0} \quad \therefore E(z) = 0$$

b)

$$Var(z) = E(z^2) = \int z^2 \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0$$

$$Var(z) = E\left[\left(z - E(z)\right)^2\right] = E(z^2) - \left[E(z)\right]^2$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-z^2/2} dz$$

Integración por partes.

$$\int u\,dv = uv - \int v\,du$$

$u = z \Rightarrow du = dz.$

$dv = z\,e^{-z^2/2} dz. \Rightarrow v = -e^{-z^2/2}.$

$$\int u\,dv = \int z.\left(z e^{-z^2/2}\right) dz = z.-e^{-z^2/2} \int -e^{-z^2/2} dz.$$

Integración por partes I L+E

$$\int x^2 \ln(x)\, dx$$

Es una multiplicación ⇒ candidata a integrar x partes

→ son iguales

$$\int u\, dv = uv - \int v\, du$$

$u = \ln(x) \quad du = 1/x$

$dv = x^2\, dx \Rightarrow v = x^3/3$

$$\int \ln(x)\, x^2\, dx = \ln(x)\, \frac{x^3}{3} - \int \frac{x^3}{3}\, \frac{1}{x}\, dx$$

$$= \ln(x)\, x^3/3 - \frac{1}{3}\int x^2\, dx$$

$$= \ln(x)\, x^3/3 - \frac{1}{3}\, \frac{x^3}{3}$$

$$= \ln(x)\, \frac{x^3}{3} - \frac{x^3}{9} = \frac{x^3}{3}\left(\ln(x) - \frac{1}{3}\right)$$

$\boxed{U\, dv = uv - \int v\, du}$

Un dia vi una vaca sin cola vestida de uniforme

$$\int x e^{x} \, dx$$

$$\int t^2 \cos t \, dt$$

$$\int x^3 \ln(2x) \, dx.$$