Riemann Sums

Approximating the definite integral

Calculus

What is a Riemann sum?

- Approximates $\int_a^b f(x)\,dx$
- Divide $[a, b]$ into n subintervals
- Build rectangles; sum their areas

$$S_n = \sum_{i=1}^{n} f(x_i^*)\,\Delta x, \quad \Delta x = \frac{b-a}{n}$$

Left, Right, Midpoint

Sample point x_i^*

- **Left:** $x_i^* = x_{i-1}$
- **Right:** $x_i^* = x_i$
- **Midpoint:** $x_i^* = \frac{x_{i-1}+x_i}{2}$

Different choices give different approximations; all tend to the same limit as $n \to \infty$.

Limit = Integral

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

As n increases, the Riemann sum gets closer to the true area under the curve.

Example: $f(x) = x^2$ on $[0, 2]$

- $n = 4$, $\Delta x = \frac{1}{2}$
- Left sum: $S_4 \approx 1.75$
- Exact: $\int_0^2 x^2\,dx = \frac{8}{3} \approx 2.667$

Try larger n (e.g. in the notebook or Flask app) to see the approximation improve.

Summary

- Riemann sum = sum of rectangle areas
- Left/right/midpoint = different sample points
- Limit as $n \to \infty$ equals the definite integral