Get these slides ⇨

# Zero-knowledge-proofs - part 1

Engineering workshop - Dec 2025 (Ireland)

Pawel Jakubas

# Plan of the tutorial

# Plan of the tutorial

Let’s get a little deeper than usual and understand what main building blocks of ZKP looks like

This tutorial will focus on

# Plan of the tutorial

Let’s get a little deeper than usual and understand what main building blocks of ZKP looks like

This tutorial will focus on

1. sketching the landscape of what we want to understand during 3-4 parts

2.

# Plan of the tutorial

Let's get a little deeper than usual and understand what main building blocks of ZKP looks like

This tutorial will focus on

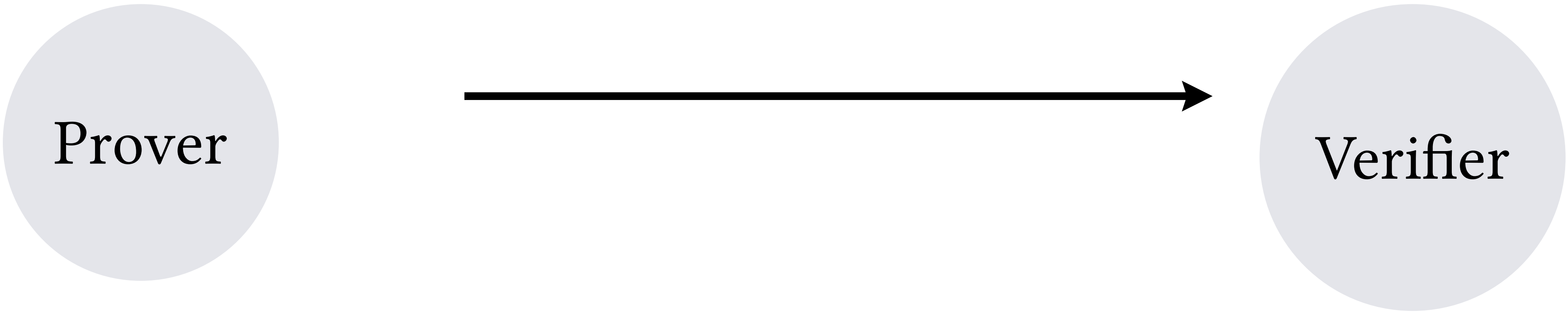
1. sketching the landscape of what we want to understand during 3-4 parts

2. cover the first part in some detail **elliptic curves**

# Verifiable computing vs ZKP (1)

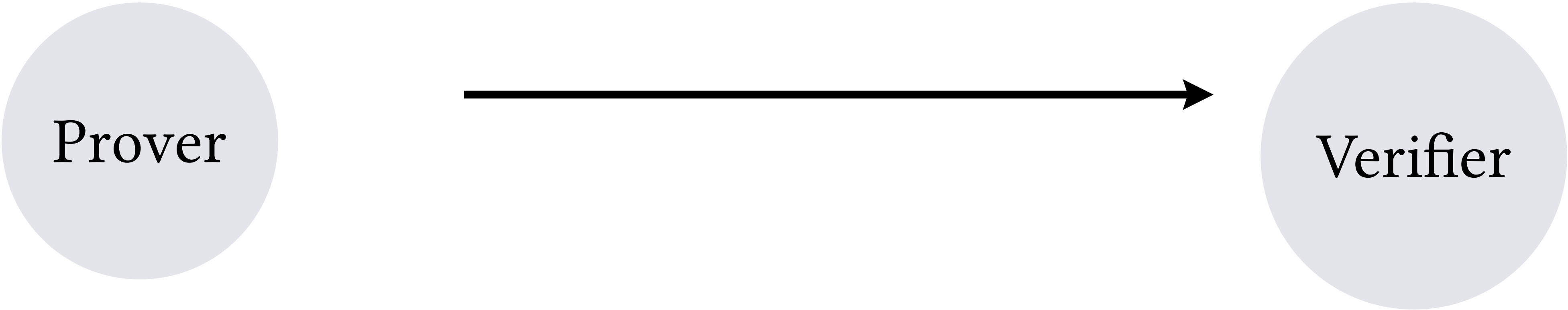
# Verifiable computing vs ZKP (1)

There is **asymmetry** built into those systems. It is much easy to get public key from secret. But not the other way



# Verifiable computing vs ZKP (1)

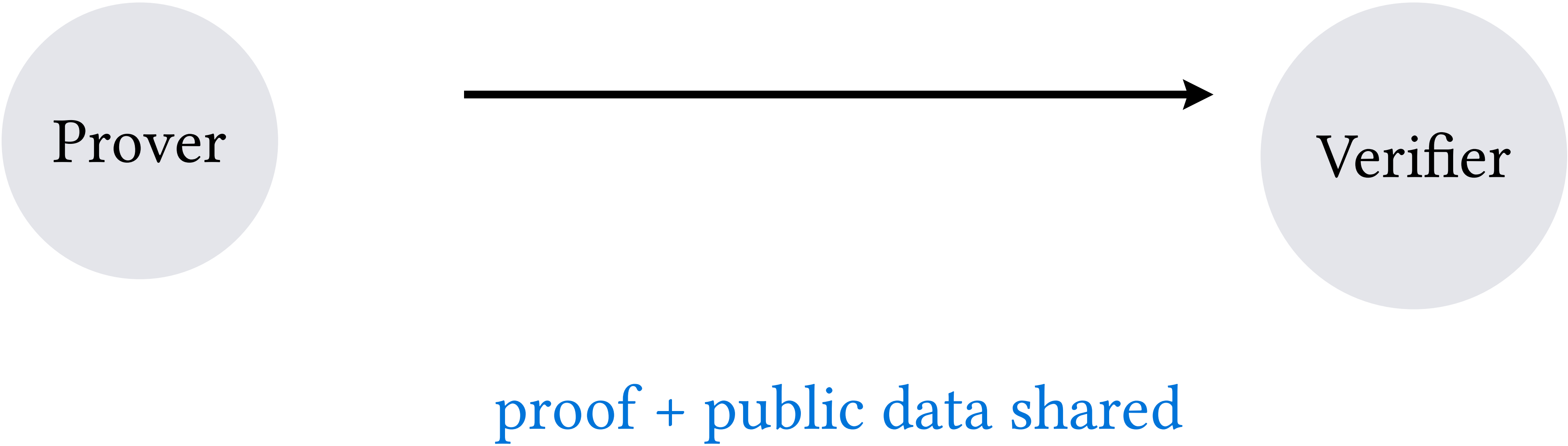
There is **asymmetry** built into those systems. It is much easy to get public key from secret. But not the other way




secret -> (easy) -> public

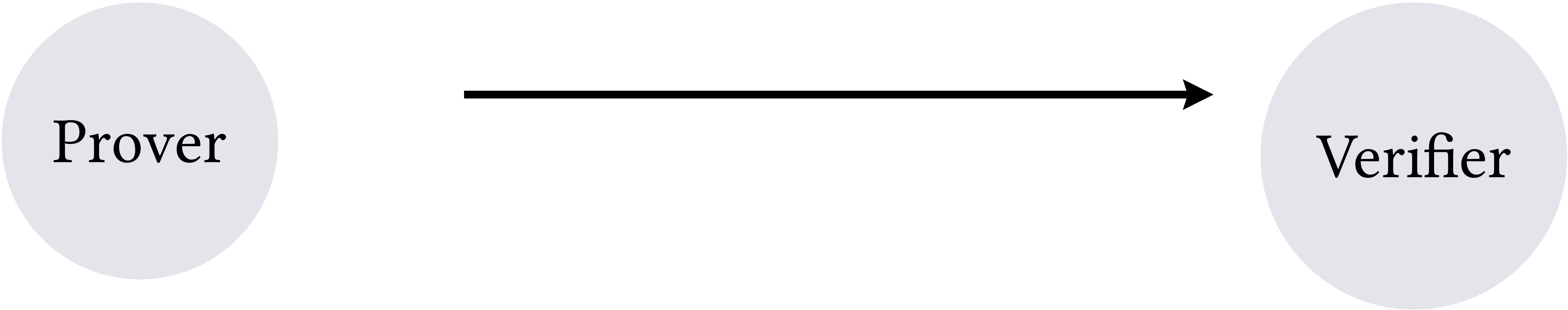
# Verifiable computing vs ZKP (1)

There is **asymmetry** built into those systems. It is much easy to get public key from secret. But not the other way



# Verifiable computing vs ZKP (1)

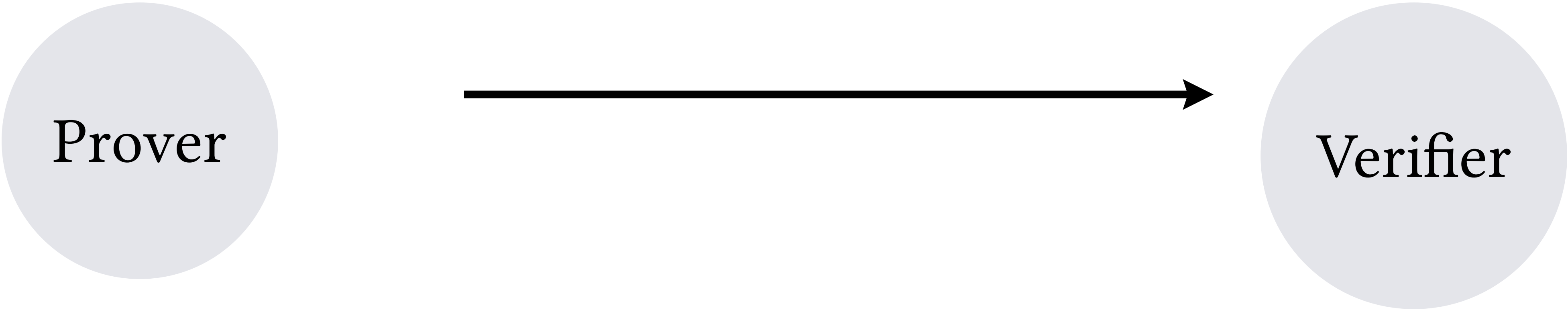
There is **asymmetry** built into those systems. It is much easy to get public key from secret. But not the other way




public -> (very hard) -> secret

# Verifiable computing vs ZKP (1)

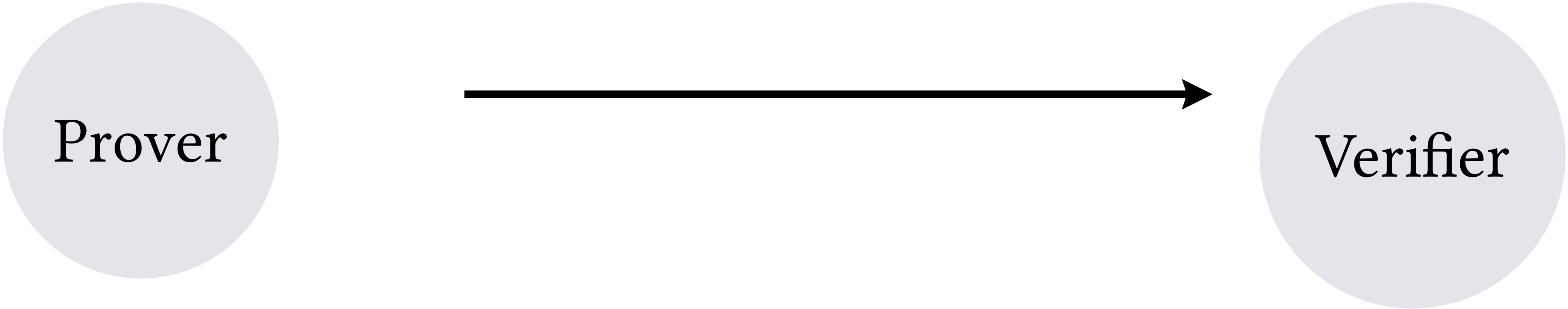
There is **asymmetry** built into those systems. It is much easy to get public key from secret. But not the other way




proof verified using public data

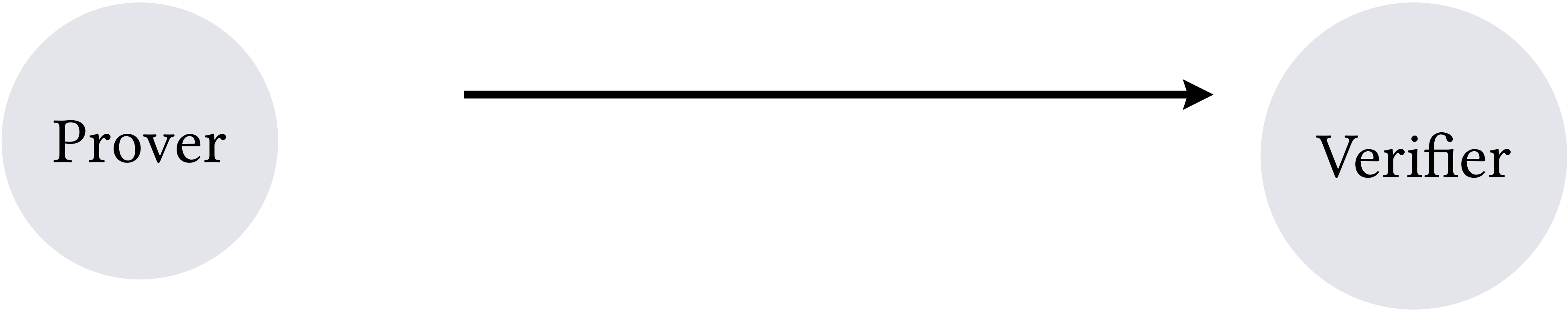
# Verifiable computing vs ZKP (2)

There is **asymmetry** built into those systems. It is much quicker to verify than prove something.



# Verifiable computing vs ZKP (2)

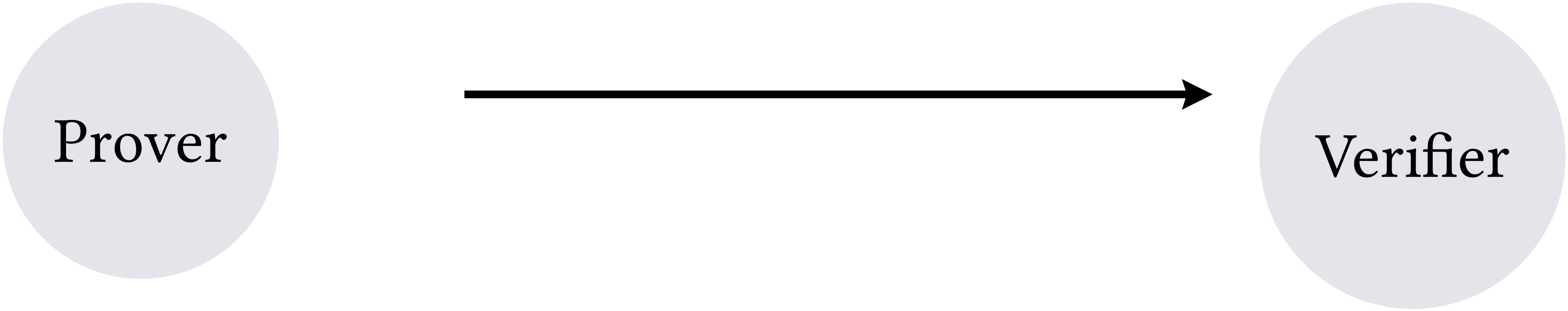
There is **asymmetry** built into those systems. It is much quicker to verify than prove something.




O(n) off-chain

# Verifiable computing vs ZKP (2)

There is **asymmetry** built into those systems. It is much quicker to verify than prove something.

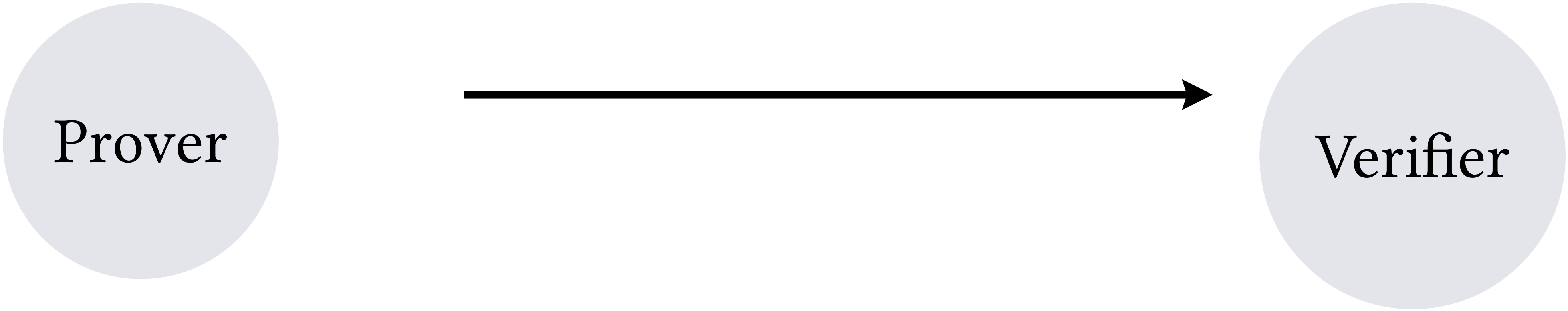



O(n) off-chain

O(log n) on-chain

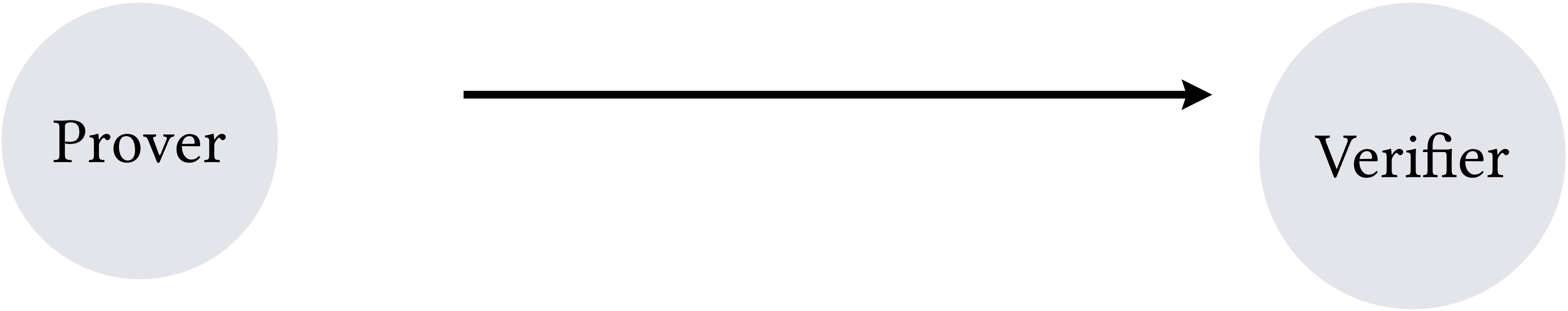
# ZKP (3)

Data sent to verifier is compressed, and can be hidden



# ZKP (3)

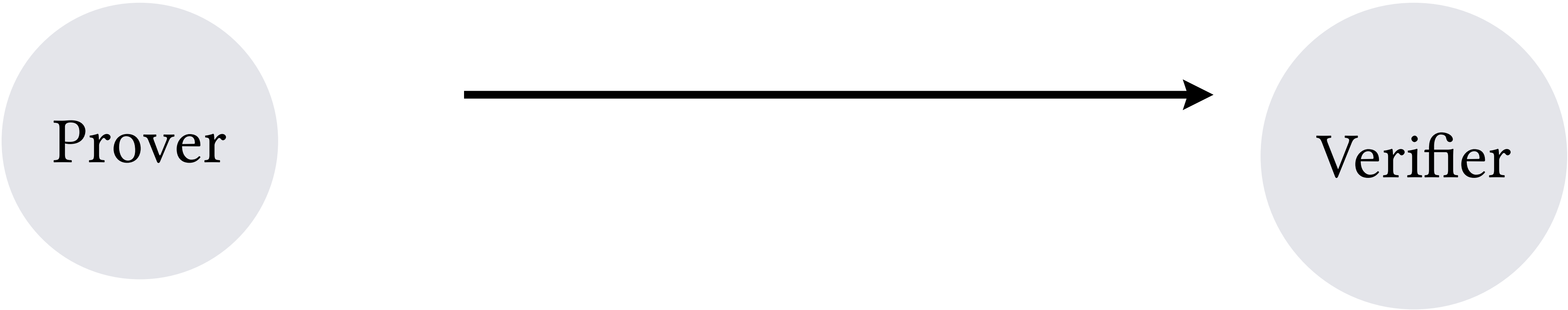
Data sent to verifier is compressed, and can be hidden




size: n

# ZKP (3)

Data sent to verifier is compressed, and can be hidden

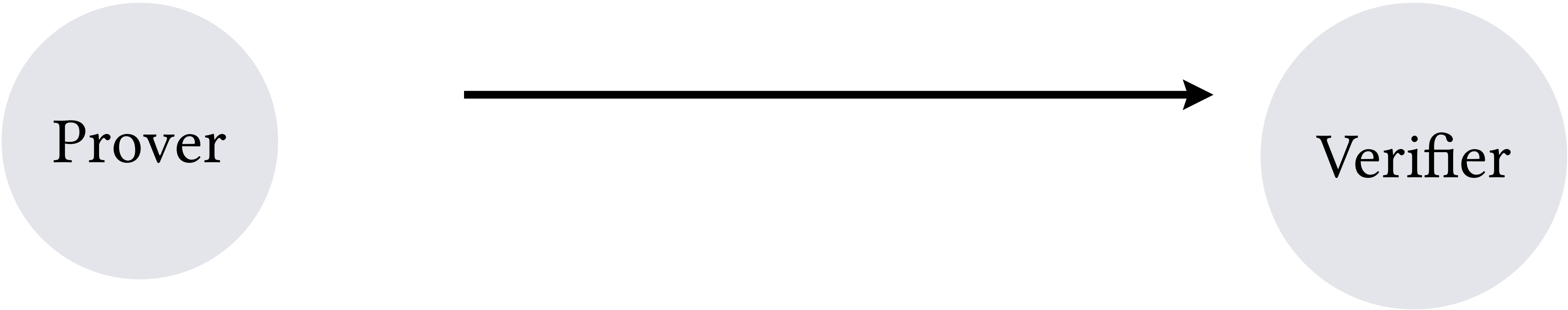



size: n

size: at least log n

# ZKP (3)

Data sent to verifier is compressed, and can be hidden




size: n

size: at least log n

size: at least log n

# Modular arithmetics (1)

It is about integers.

# Modular arithmetics (1)

It is about integers.

Let’s assume we arithmetics **mod 8**. It means the possible values are 0,1,2,3,4,5,6,7. if we move below or above we need to wrap up.

# Modular arithmetics (1)

$$3 + 3 \bmod 8 = 6 \bmod 8$$

$$10 \bmod 8 = 2 \bmod 8$$

$$5 + 5 \bmod 8 = 2 \bmod 8$$

$$5 \cdot 5 \bmod 8 = 25 \bmod 8 = (3 \cdot 8 + 1) \bmod 8 = 1 \bmod 8$$

# Modular arithmetics (1)

$$3 + 3 \bmod 8 = 6 \bmod 8$$

$$10 \bmod 8 = 2 \bmod 8$$

$$5 + 5 \bmod 8 = 2 \bmod 8$$

$$5 \cdot 5 \bmod 8 = 25 \bmod 8 = (3 \cdot 8 + 1) \bmod 8 = 1 \bmod 8$$

congruent groups

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Let's solve the eq in mod 8:

$$9(2x + 7) - 6 \equiv 2x + 6$$

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Let's solve the eq in mod 8:

$$9(2x + 7) - 6 \equiv 2x + 6$$

$$19^{*}2x + 19^{*}7 - 6 \equiv 2x + 6$$

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Let's solve the eq in mod 8:

$9(2x + 7) - 6 \equiv 2x + 6$

$19 * 2x + 19 * 7 - 6 \equiv 2x + 6$

$38x + 133 - 6 \equiv 2x + 6$ # 133 mod 8 = 5

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Let's solve the eq in mod 8:

$9(2x + 7) - 6 \equiv 2x + 6$

$19*2x + 19*7 - 6 \equiv 2x + 6$

$38x + 133 - 6 \equiv 2x + 6$ # 133 mod 8 = 5

$6x + 5 - 6 \equiv 2x + 6$

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Let's solve the eq in mod 8:

$9(2x + 7) - 6 \equiv 2x + 6$

$19*2x + 19*7 - 6 \equiv 2x + 6$

$38x + 133 - 6 \equiv 2x + 6$ # 133 mod 8 = 5

$6x + 5 - 6 \equiv 2x + 6$

$6x + 5 - 6 + 6 \equiv 2x + 6 + 6$

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Let's solve the eq in mod 8:

$9(2x + 7) - 6 \equiv 2x + 6$

$19*2x + 19*7 - 6 \equiv 2x + 6$

$38x + 133 - 6 \equiv 2x + 6$ # 133 mod 8 = 5

$6x + 5 - 6 \equiv 2x + 6$

$6x + 5 - 6 + 6 \equiv 2x + 6 + 6$

$6x + 5 \equiv 2x + 4$ # 12 mod 8 = 4

$6x - 2x + 5 - 5 \equiv 2x - 2x + 4 - 5$

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Let's solve the eq in mod 8:

$9(2x + 7) - 6 \equiv 2x + 6$

$19*2x + 19*7 - 6 \equiv 2x + 6$

$38x + 133 - 6 \equiv 2x + 6$ # 133 mod 8 = 5

$6x + 5 - 6 \equiv 2x + 6$

$6x + 5 - 6 + 6 \equiv 2x + 6 + 6$

$6x + 5 \equiv 2x + 4$ # 12 mod 8 = 4

$6x - 2x + 5 - 5 \equiv 2x - 2x + 4 - 5$

# Modular arithmetics (2)

addition mod 8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 |
| 2 | 3 | 4 | 5 | 6 | 7 | 0 | 1 |
| 3 | 4 | 5 | 6 | 7 | 0 | 1 | 2 |
| 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 0 | 1 | 2 | 3 | 4 |
| 6 | 7 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

multiplication mod 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 4 | 6 | 0 | 2 | 4 | 6 |
| 3 | 6 | 1 | 4 | 7 | 2 | 5 |
| 4 | 0 | 4 | 0 | 4 | 0 | 4 |
| 5 | 2 | 7 | 4 | 1 | 6 | 3 |
| 6 | 4 | 2 | 0 | 6 | 4 | 2 |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Let's solve the eq in mod 8:

$9(2x + 7) - 6 \equiv 2x + 6$

$19*2x + 19*7 - 6 \equiv 2x + 6$

$38x + 133 - 6 \equiv 2x + 6$ # 133 mod 8 = 5

$6x + 5 - 6 \equiv 2x + 6$

$6x + 5 - 6 + 6 \equiv 2x + 6 + 6$

$6x + 5 \equiv 2x + 4$ # 12 mod 8 = 4

$6x - 2x + 5 - 5 \equiv 2x - 2x + 4 - 5$

$4x \equiv 7$ # -1 mod 8 = 7

Now we do **NOT have multiplication inverse** for 4, ie. we cannot divide by 4 in modulo 8, ie. solve this equation We have only multiplication inverse for 1 which is 1; 3 which is 3; 5 which is 5, and 7 which is 7.

# That's it! More to come in the future

Get in touch 👋

⇧My notes on GitHub