MMDS - EVRP

Stefano Lavaggi, Matteo Cardano

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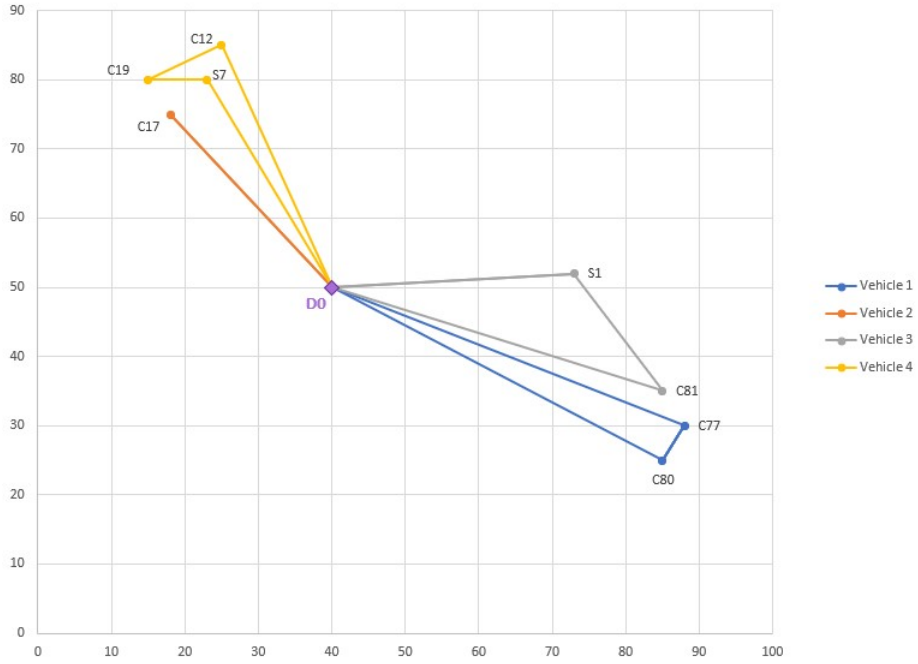

1 Introduction

A set of K EVs must serve the delivery requests of a set N of customers:

- EVs have a battery capacity Q (kW).
- EVs can travel at 3 different speeds (km/h); each velocity has an associated battery consumption rate for km (vcr in [kW/km])
- Battery consumption depends also on the loaded cargo (kg), expressed by lcr $[(kW/(kg \cdot km)]$. The battery consumption on an arc (i, j) is given as follows, where D_{ij} is the distance, vcr_s is the vehicle consumption rate

associated with the chosen speed of the vehicle on the arc and $load_{ij}$ is the load on the vehicle on the arc.

$$BatteryConsumption_{ij} = D_{ij} \cdot (vcr_s + lcr \cdot load_{ij})$$

Each node, including depot has the following data:

- x and y coordinate
- Demand (zero for depot and stations)
- Time window given be ReadyTime and DueDate (given in minutes from 0)
- ServiceTime (zero for depot and stations)

2 Formulation

Variables:

- Links used by vehicles:

$$x_{ijk} \in \{0, 1\} \quad \forall i, j \in, i \neq j, k \in K$$

- Start time of the service at the node:

$$w_{ik} \geq 0 \quad \forall i \in N, k \in K$$

- Load in input at the node:

$$L_i \leq load_{ik} \leq C \quad \forall k \in K, i \in N$$

- State of charge (SOC) at arrival at node:

$$0 \leq z_{ik} \leq Q \quad \forall k \in K, i \in V$$

Parameters:

- Matrix of the distance between node (arc from node i to node j):

$$dist_{ij}$$

- Velocity of the vehicle:

$$speeds$$

- Velocity consumption rate [kW/km]:

$$vcr$$

- Recharging rate [min/kW]:

$$G$$

- Capacity of the battery [kW]:

$$Q$$

- Minimum time distance between services startTimes in nodes

$$M_{ij} = \max\{0, dueDate_i + serviceTime_i + travelTime_{i,j} - readyTime_j\}$$

$$\forall (i,j) \in A$$

Objective function:

$$\min \sum_{i,j \in A} \sum_{k \in K} load_{jk} \cdot lcr \cdot dist_{ij} + \sum_{i,j \in A} \sum_{k \in K} dist_{ij} \cdot vcr \cdot x_{ijk} + \sum_{i,j \in A} \sum_{k \in K} dist_{ij} \cdot x_{ijk}$$

All customers must be visited:

$$\sum_{k \in K} \sum_{i \in V/n+1} x_{ijk} = 1 \quad \forall j \in N, i \neq j$$

Every station (and their clones) can be visited at most once:

$$\sum_{k \in K} \sum_{i \in V/n+1} x_{ijk} \leq 1 \quad \forall j \in F', i \neq j$$

Flow conservation:

$$\sum_{j \in V/0} x_{ijk} = \sum_{j \in V/n+1} x_{jik} \quad \forall j \in N \cup F', i \neq j, k \in K$$

Every vehicle can leave depot at most once:

$$\sum_{j \in N \cup F'} x_{0jk} \leq 1 \quad \forall k \in K$$

Every exited vehicle must return to the depot:

$$\sum_{j \in N \cup F'} x_{ijk} - \sum_{i \in N \cup F'} x_{i,v-1,k} = 0 \quad \forall k \in K$$

Vehicle capacity:

$$\sum_{i \in V j \in N \cup F', i \neq j} demand_j \cdot x_{ijk} \leq C \quad \forall k \in K$$

Subtour Elimination and capacity constrains:

$$load_{jk} - load_{ik} + C \cdot x_{ijk} \leq C - demand_i \cdot x_{ijk}$$

$$\forall (i,j) \in A, k \in K, demand_i + demand_j \leq C$$

Arrival time at nodes from customers:

$$w_{jk} \geq w_{ik} + serviceTime_i + dist_{ij}/speeds - M_{ij} \cdot (1 - x_{ijk})$$

$$\forall i \in N0, j \in V, i \neq j, j \neq 0, k \in K$$

Arrival time at nodes from stations:

$$w_{jk} \geq w_{ik} + G \cdot (Q - z_{ik}) + dist_{ij}/speeds - M_{ij} \cdot (1 - x_{ijk})$$

$$\forall i \in F', j \in V, i \neq j, j \neq 0, k \in K$$

Time window lower bound:

$$w_{ik} \geq readyTime_i \quad \forall i \in V, k \in K$$

Time window Upper bound:

$$w_{ik} \leq dueDate_i \quad \forall i \in V, k \in K$$

Battery lower capacity:

$$z_{ik} \geq 0 \quad \forall i \in V, k \in K$$

Battery upper capacity:

$$z_{ik} \leq Q \quad \forall i \in V, k \in K$$

Starting battery capacity:

$$z_{0k} = Q \quad \forall k \in K$$

Battery constrains:

- from depot or customer to customer

$$z_{ik} - z_{jk} \geq dist_{ij} \cdot vcr \cdot x_{ijk} + load_{jk} \cdot lcr \cdot dist_{ij} - B \cdot (1 - x_{ijk})$$

$$\forall i \in N0, j \in N, i \neq j, k \in K$$

- from depot or customer to station

$$z_{ik} \geq dist_{ij} \cdot vcr \cdot x_{ijk} + load_{jk} \cdot lcr \cdot dist_{ij} - B \cdot (1 - x_{ijk})$$

$$\forall i \in N0, j \in F', k \in K$$

- from station to customer

$$Q - z_{jk} \geq dist_{ij} \cdot vcr \cdot x_{ijk} + load_{jk} \cdot lcr \cdot dist_{ij} - B \cdot (1 - xijk)$$

$$\forall i \in F', j \in N, k \in K$$

- from station to depot (final depot n-1)

$$Q - z_{v-1,k} \geq dist_{i,v-1} \cdot vcr \cdot x_{i,v-1,k} + load_{v-1,k} \cdot lcr \cdot dist_{i,v-1} - B \cdot (1 - x_{i,v-1,k})$$

$$\forall i \in F', k \in K$$

- customer to depot (final depot n-1)

$$z_{ik} - z_{v-1,k} \geq dist_{i,v-1} \cdot vcr \cdot x_{i,v-1,k} + load_{v-1,k} \cdot lcr \cdot dist_{i,v-1} - B \cdot (1 - x_{i,v-1,k})$$

$$\forall i \in N, k \in K$$