Methods and Tools for Industrial Automation
Second Project

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1 Mixed Integer Linear Programming Model

Parameters

$$\begin{aligned} n &= \text{number of jobs} \\ m &= \text{number of machines} \\ d_{i,k} &= \text{duration of } J_i \text{ on } M_k \\ M &= \text{upper bound of completion time} \end{aligned}$$

Variables

$$\begin{aligned} x_{k,i,j} &= \begin{cases} 1 & \text{if on } M_k \text{ job } J_i \text{ is executed before } J_j \\ 0 & \text{otherwise} \end{cases} \\ t_{i,k} &= \text{start time of } J_i \text{ on } M_k \\ C_{max} &= \text{max completion time} \end{aligned}$$

Objective function

$$\min C_{max}$$

Constraints

1 Avoid overlapping of jobs in machines

$$\begin{aligned} t_{j,k} &\geq t_{i,k} + d_{i_k} - M(1 - x_{k,i,j}) \\ t_{i,k} &\geq t_{j,k} + d_{j_k} - M(x_{k,i,j}) \end{aligned}$$

for each $i = 1, \ldots, n$ and $j = 1, \ldots, n$ and $k = 1, \ldots, m$ with $i \neq j$

2 Jobs in machines has to be completed in order

$$t_{i,k} + d_{i,k} \leq t_{i,k+1}$$

for each $i = 1, \ldots, n$ and $k = 1, \ldots, m - 1$

3 Completion time

$$C_{max} \geq t_{i,m} + d_{i,m}$$

for each $i = 1, \ldots, n$

4 Positive times

$$t_{i,k} \geq 0$$

for each $i = 1, \ldots, n$ and $k = 1, \ldots, m$

5 Binary variables

$$x_{k,i,j} \in \{0, 1\}$$

for each $i = 1, \ldots, n$ and $j = 1, \ldots, n$ and $k = 1, \ldots, m$