Methods and Tools for Industrial Automation
Third Project

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1 Mixed Integer Linear Programming Model

Sets We call M the set containing only the retailers. M' is the set in which the supplier is also included. T is the set containing the days in which the simulation is run.

In our example:

$$
\begin{aligned}
M &= \{1, 2, \ldots, 6\} \\
M' &= \{1, 2, \ldots, 6, 7\} \\
T &= \{1, 2, \ldots, 10\}
\end{aligned}
$$

Parameters

$$
\begin{aligned}
c_{ij} &= \text{cost to reach node } j \in M' \text{ from node } i \in M' \\
r_{st} &= \text{demand for supplier } s \in M \text{ in day } t \in T \\
C &= \text{max capacity of truck } (39kL) \\
\hat{I}_s &= \text{initial inventory for supplier } s \in M
\end{aligned}
$$

Variables

$$
\begin{aligned}
x_{st} &= \text{quantity } (\geq 0) \text{ shipped to retailer } s \in M \text{ at time } t \in T \\
I_{st} &= \text{inventory } (\geq 0) \text{ of retailer } s \in M \text{ at time } t \in T \\
z_{i,t} &= \begin{cases} 1 & \text{if node } i \in M' \text{ is visited at time } t \in T \\ 0 & \text{otherwise} \end{cases} \\
y_{ij}^t &= \begin{cases} 1 & \text{if the arc } (i, j) \text{ is traveled at time } t \\ 0 & \text{otherwise} \end{cases} \\
& \quad\; i \in M \qquad j \in \{j | j \in M \wedge j < i\} \qquad t \in T \\
y_{i7}^t &\in \{0, 1, 2\} \quad i \in M \quad t \in T
\end{aligned}
$$

Objective function

$$\min \left[\left(\frac{0.03}{365} \sum_{s \in M} \sum_{t \in T} I_{st} \right) + \left(\sum_{i \in M'} \sum_{j \in M', j<i} \sum_{t \in T} c_{ij} y_{ij}^t \right) \right]$$

Constraints

C1 Inventory definition at retailers

$$I_{st} = I_{st-1} + x_{st-1} - r_{st-1} \quad s \in M, t \in T$$

C2 Truck constraints

$$\sum_{s \in M} x_{st} \leq C \quad t \in T$$

C3 A node can be visited only once

$$\sum_{j \in M', j<i} y_{ij}^t + \sum_{j \in M', j>i} y_{ji}^t = 2z_{it} \quad i \in M', t \in T$$

C4 Constraint to force binary on y_{ij}^t with $j \neq 0$

$$y_{ij}^t \leq 1 \quad i \in M, j \in M, t \in T$$

C5 Constraint to avoid transportation without a trip

$$x_{it} \leq C \sum_{j \in M'} y_{ij}^t \quad i \in M, t \in T$$

C6 Only 3 nodes can be visited for each day (1 supplier + 2 retailers)

$$\sum_{i \in M'} z_{it} \leq 3 \quad t \in T$$

C7 The graph is undirected so the matrix has to be lower triangular

$$y_{ij}^t = 0 \quad i \in M, j \in M, j \geq i, t \in T$$

C8 Constraint to set the initial inventory

$$I_{s0} = \hat{I}_s \quad s \in M$$