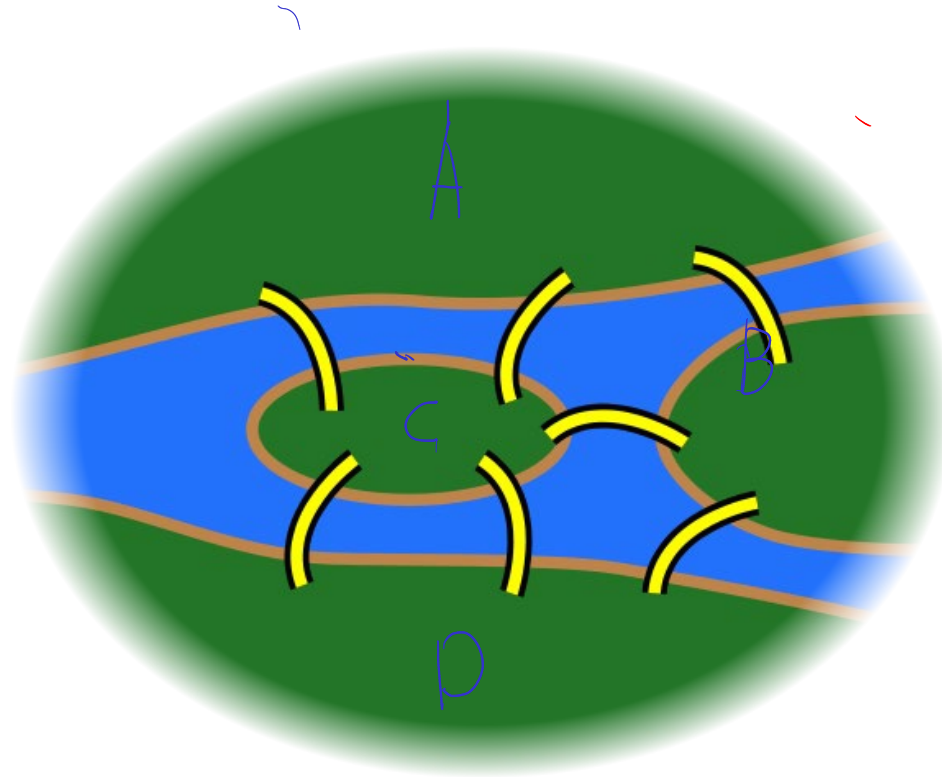


Estructuras de datos

Grafos

- Grafos
- Teorema de *Handshaking*
- Grafos completos
- Matriz de adyacencia
- Algoritmo de Warshall

Grafos



El problema de los puentes de Königsberg (Euler)

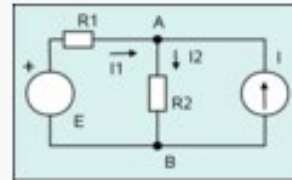
Grafos



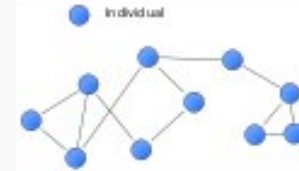
Plano de
estaciones del
metro



Plano de
autopistas



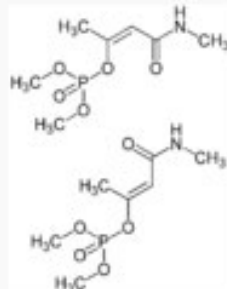
Circuitos
eléctricos



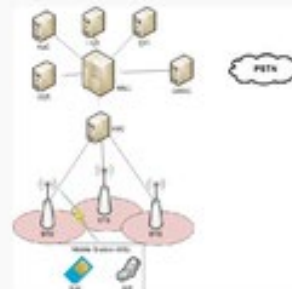
Sociograma de
una red social



Topología de red
de computadores



Isómeros



Redes de
telefonía móvil



Elemento	Eliminado	Eliminado	Eliminado	Eliminado
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50

Draws de
eliminación directa

Grafos

Grafos

Un grafo G es un par ordenado $G=(V,E)$, donde:

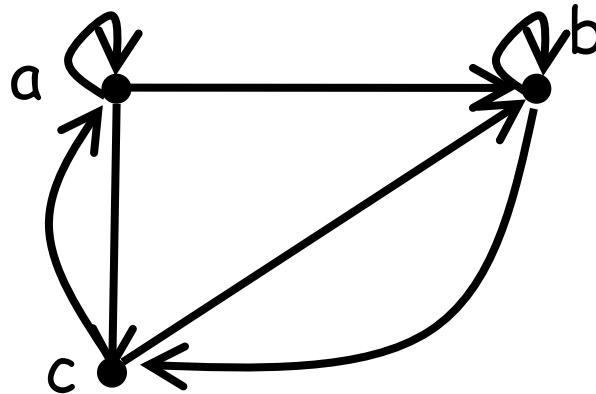
- V es un conjunto de vértices o nodos
- E es un conjunto de aristas que relacionan los nodos

Grafos

Grafos

Un grafo G es un par ordenado $G=(V,E)$, donde:

- V es un conjunto de vértices o nodos
- E es un conjunto de aristas que relacionan los nodos

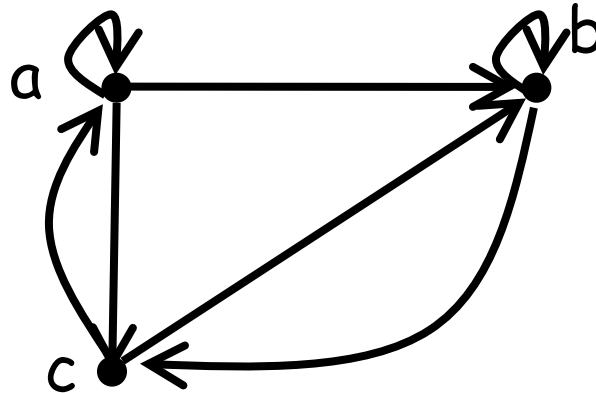


Grafos

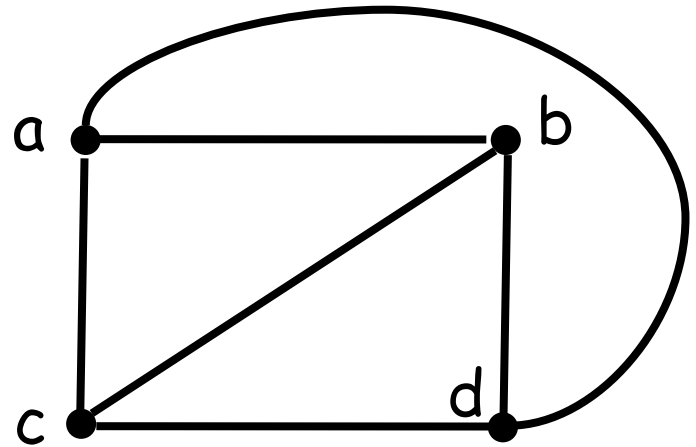
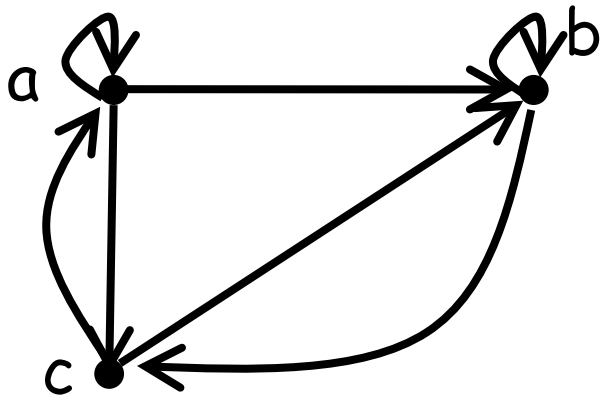
Grafos

Un grafo G es un par ordenado $G=(V,E)$, donde:

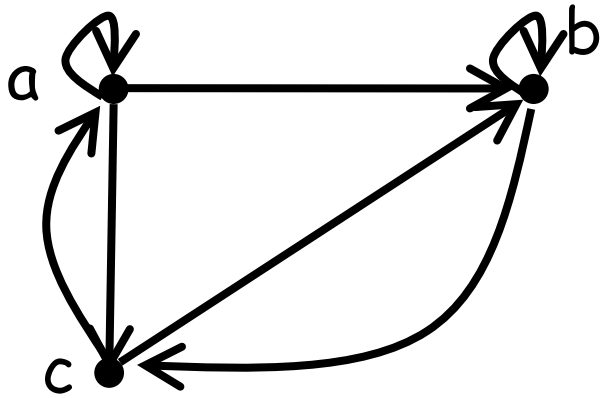
- $V=\{a,b,c\}$
- $E=\{(a,a),(a,b),(a,c),(b,b),(b,c),(c,a),(c,b)\}$



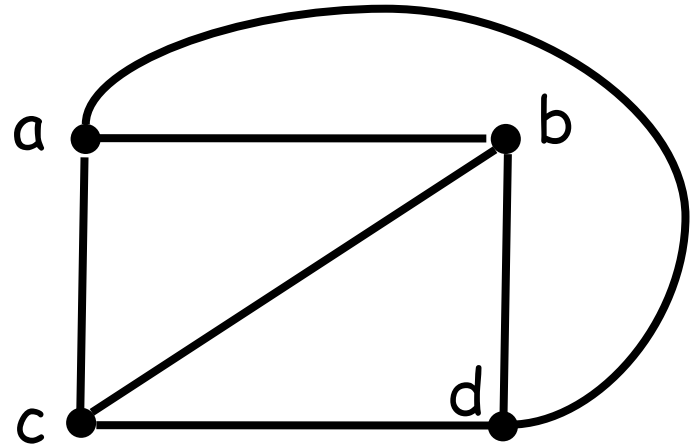
Grafos



Grafos

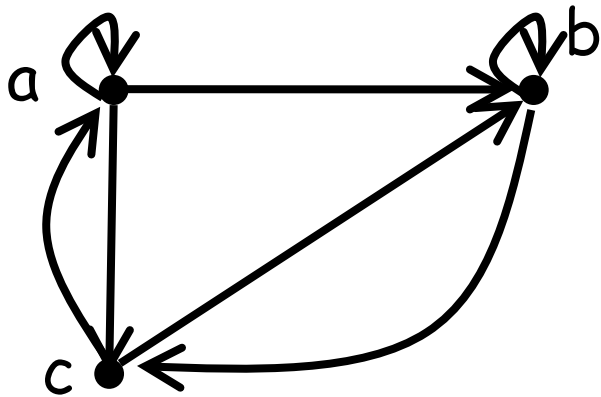


Grafo dirigido



Grafo no dirigido

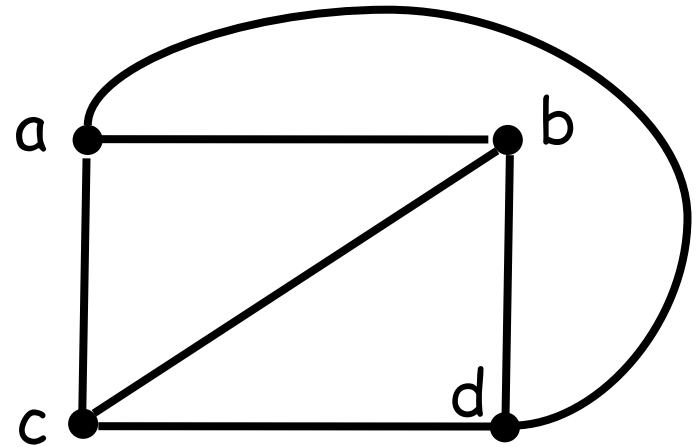
Grafos



Grafo dirigido

$$V=\{a,b,c\}$$

$$E=\{(a,a),(a,b),(a,c),(b,b),(b,c),(c,a),(c,b)\}$$

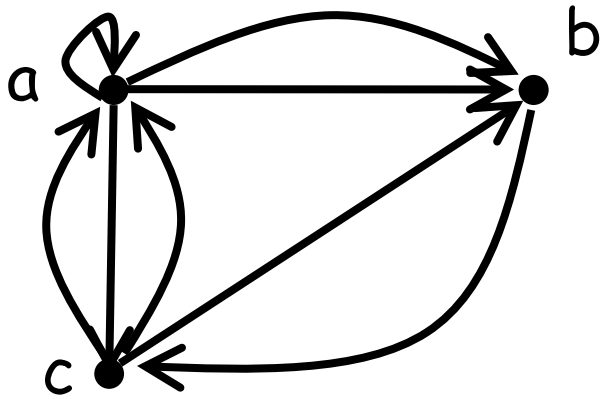


Grafo no dirigido

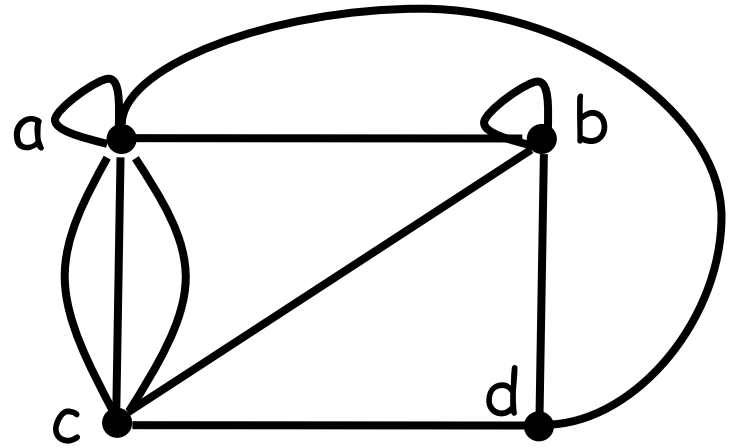
$$V=\{a,b,c\}$$

$$E=\{\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$$

Grafos

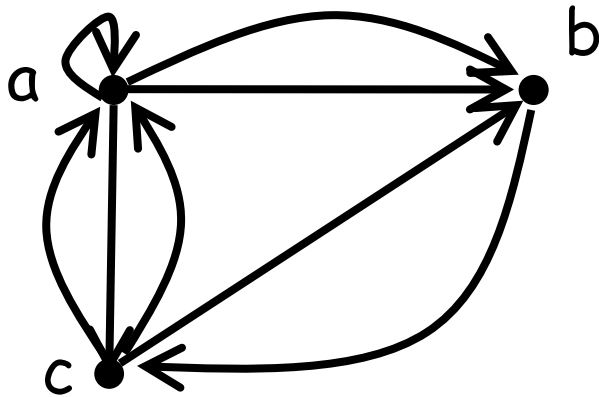


Grafo dirigido



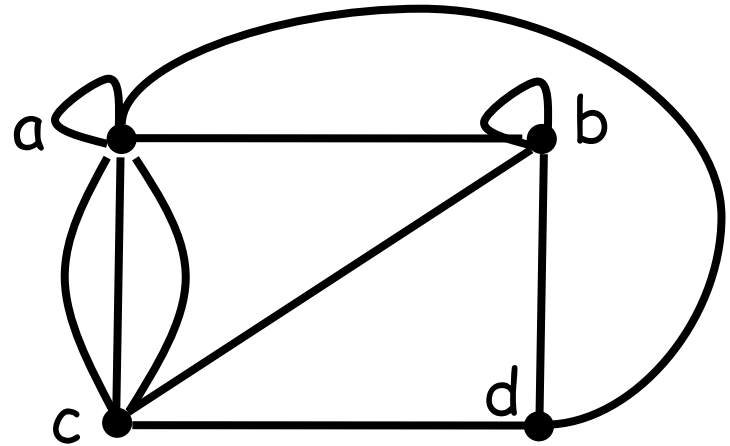
Grafo no dirigido

Grafos



Grafo dirigido

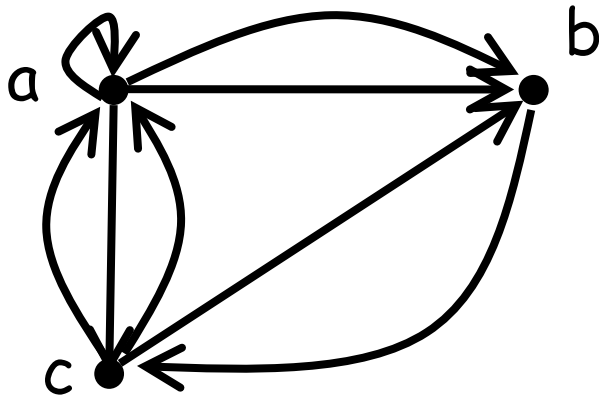
A las aristas (a,a) , (c,c) se les conoce como **bucles**



Grafo no dirigido

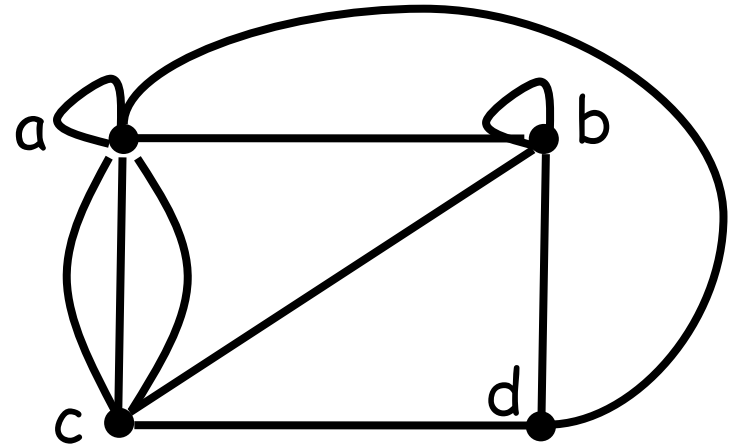
A las aristas $\{a,a\}$, $\{b,b\}$ se les conoce como **bucles**

Grafos



Grafo dirigido

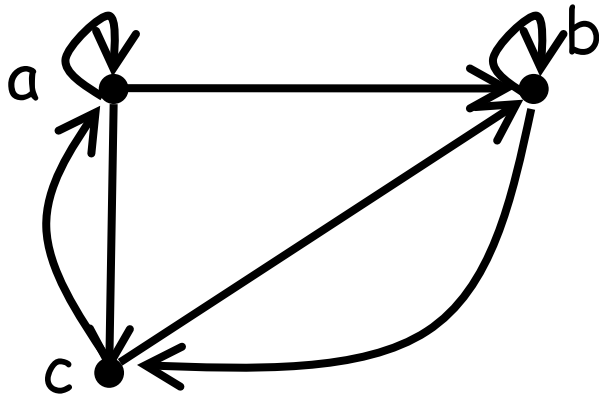
A las dos aristas que van de c hacia a se les conoce como **aristas paralelas**



Grafo no dirigido

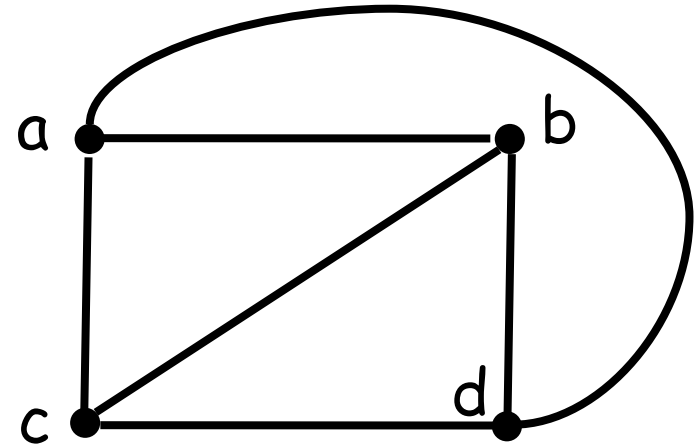
A las tres aristas que relacionan los nodos a y c se les conoce como **aristas paralelas**

Grafos



Grafo dirigido

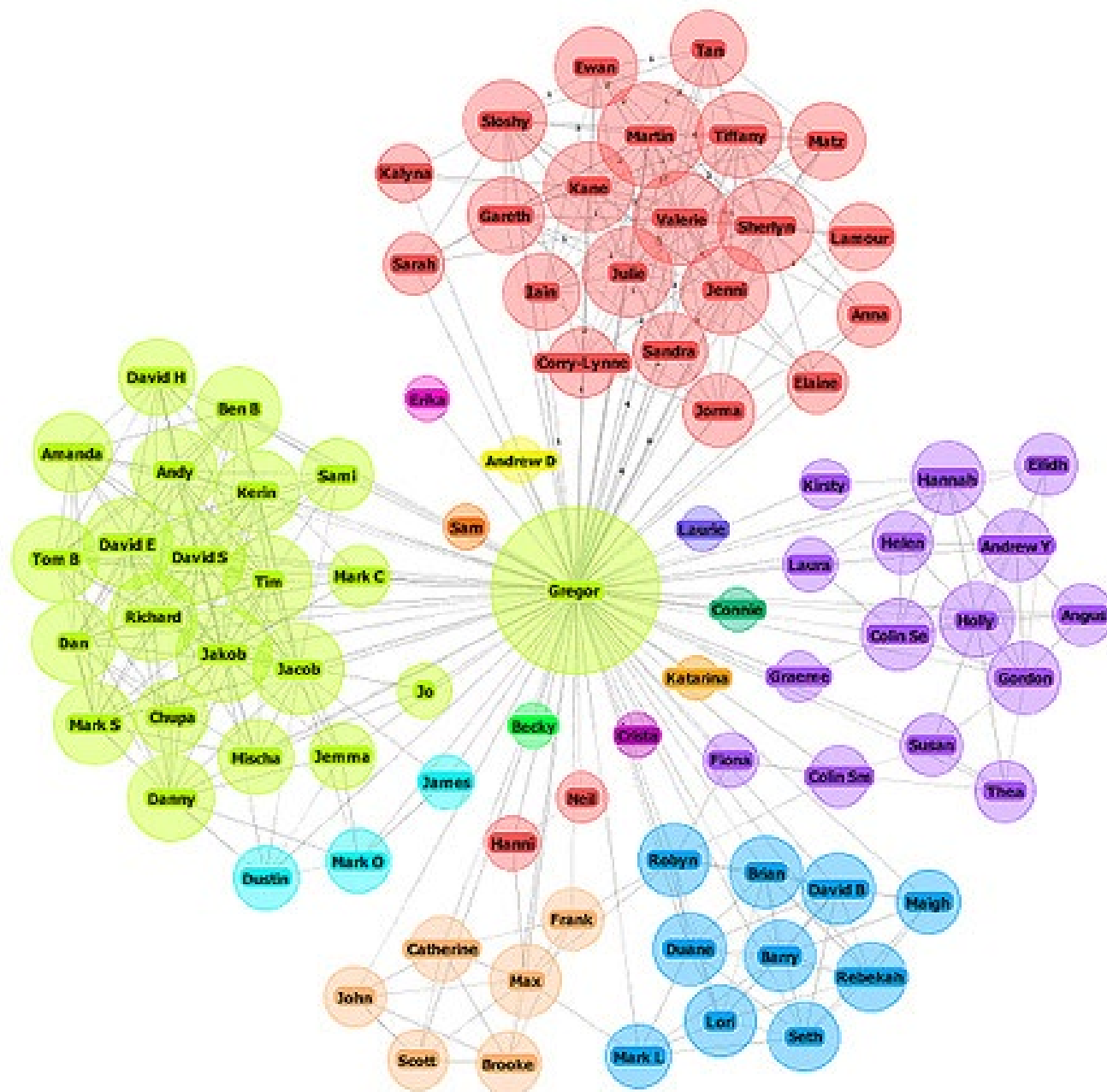
A las aristas (a,a) , (b,b) y (c,c) se les conoce como **bucles**



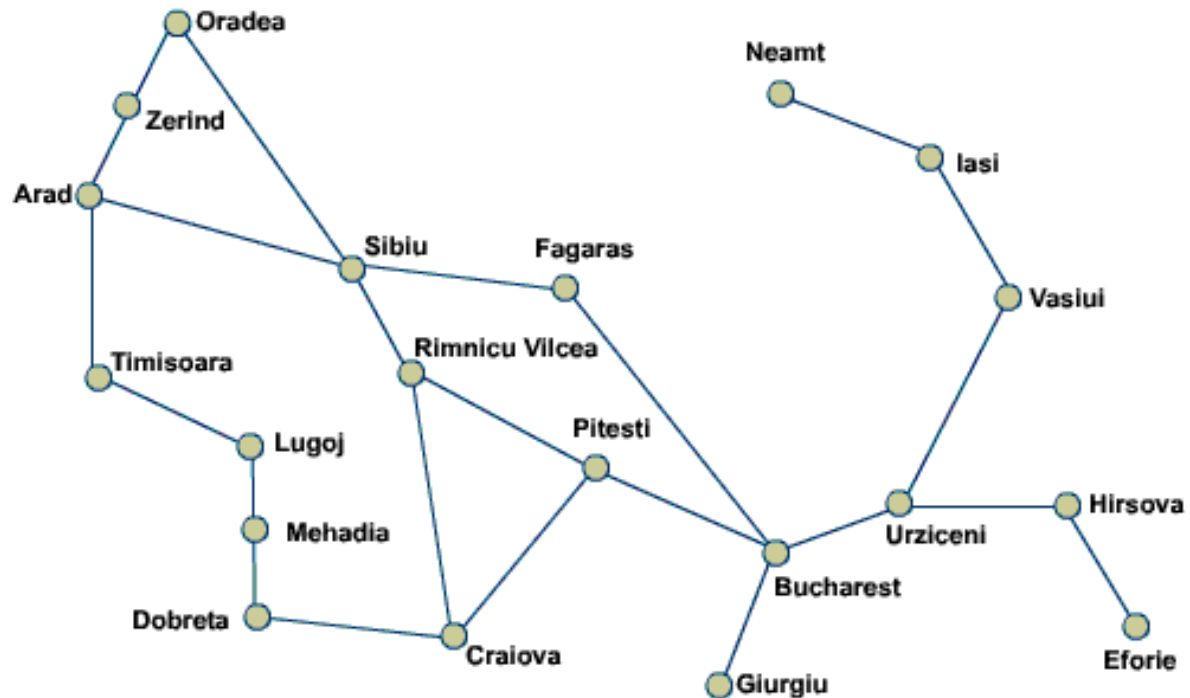
Grafo no dirigido

$$V=\{a,b,c\}$$

$$E=\{\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$$

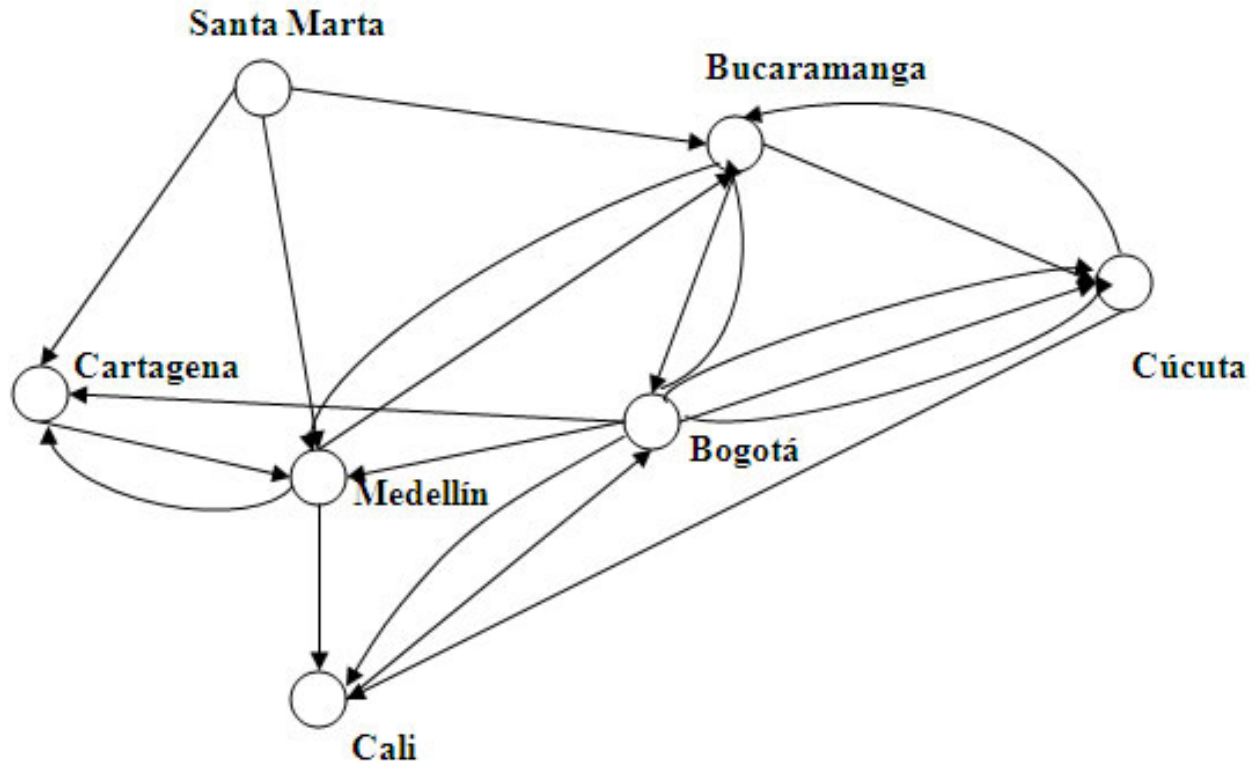


Grafos



Mapa simplificado de Rumania

Grafos

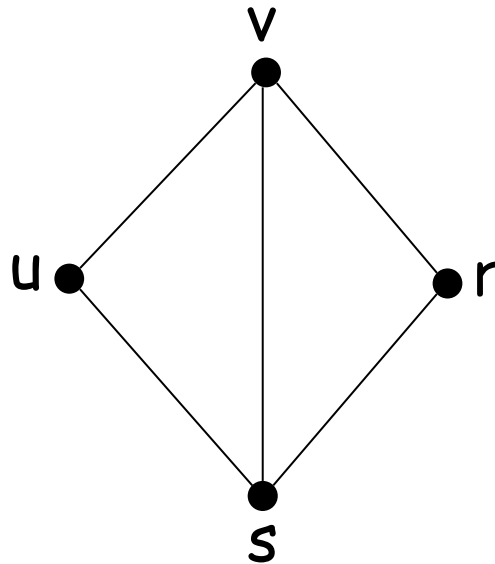


Grafo de una red de tráfico aéreo

Grafos

Grafo simple

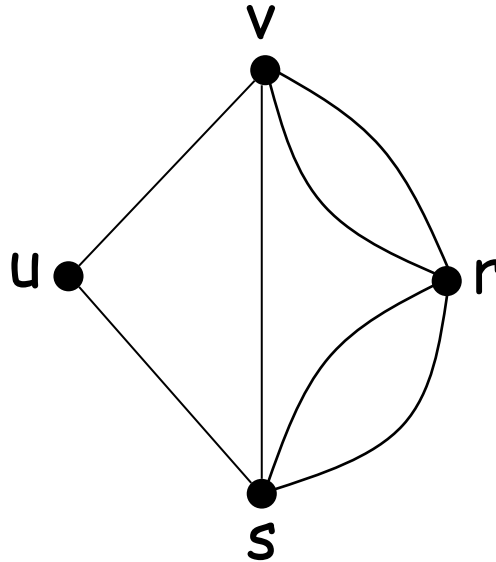
Un **grafo simple** $G=(V,E)$ es un grafo sin aristas paralelas ni bucles



Grafos

Multigrafo

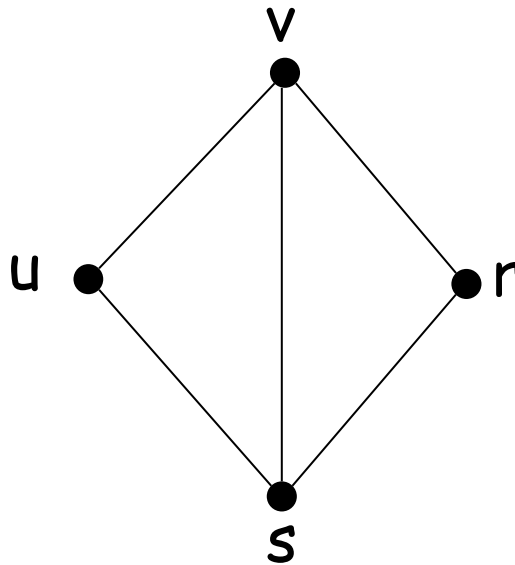
Un multigrafo $G=(V,E)$ es un grafo con aristas paralelas



Grafos

Adyacencia

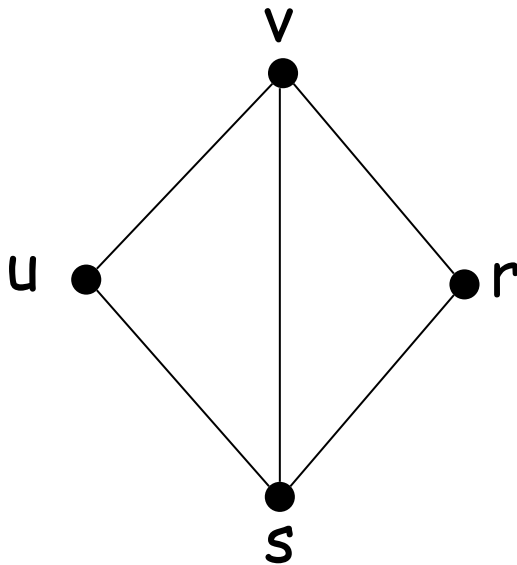
Dos vértices u y v de un grafo no dirigido G son **adyacentes** si $e=\{u,v\}$ es una arista de G . Se dice que e es incidente con los vértices u y v



Grafos

Grado de un vértice

El grado de un vértice v de un grafo no dirigido es el número de aristas incidentes con él y se denota por $\delta(v)$



$$\delta(u)=2$$

$$\delta(v)=3$$

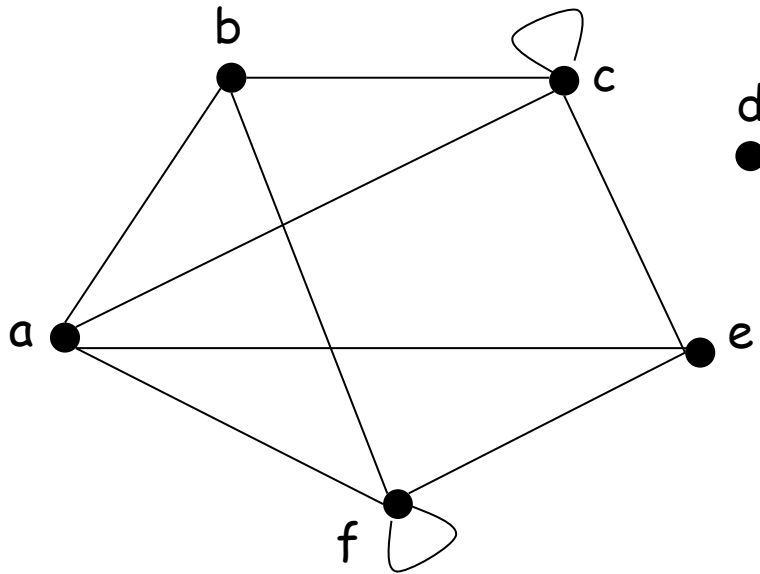
$$\delta(r)=2$$

$$\delta(s)=3$$

Grafos

Grado de un vértice

El grado de un vértice v de un grafo no dirigido es el número de aristas incidentes con él y se denota por $\delta(v)$



$$\delta(a)=?$$

$$\delta(b)=?$$

$$\delta(c)=?$$

$$\delta(d)=?$$

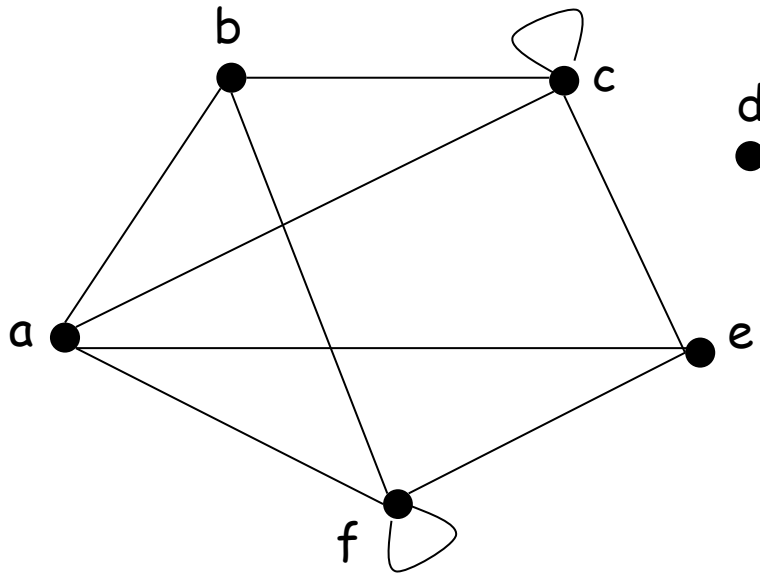
$$\delta(e)=?$$

$$\delta(f)=?$$

Grafos

Grado de un vértice

El grado de un vértice v de un grafo no dirigido es el número de aristas incidentes con él y se denota por $\delta(v)$



$$\delta(a)=4$$

$$\delta(b)=3$$

$$\delta(c)=5$$

$$\delta(d)=0$$

$$\delta(e)=3$$

$$\delta(f)=5$$

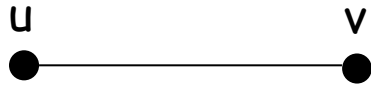
Grafos

Teorema de Handshaking

Sea $G=(V,E)$ un grafo no dirigido con e aristas. Se tiene que:

$$2e = \sum_{v \in V} \delta(v)$$

Grafos

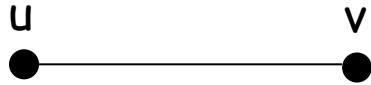


$$e=1$$

$$\delta(u)=1$$

$$\delta(v)=1$$

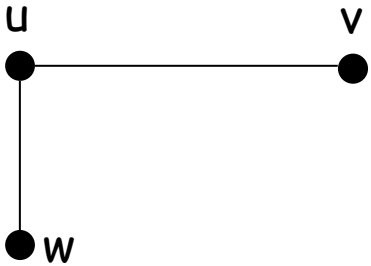
Grafos



$$e=1$$

$$\delta(u)=1$$

$$\delta(v)=1$$



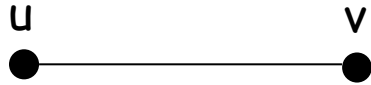
$$e=2$$

$$\delta(u)=2$$

$$\delta(v)=1$$

$$\delta(w)=1$$

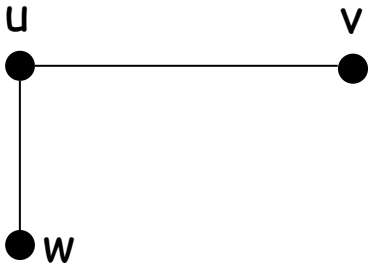
Grafos



$$e=1$$

$$\delta(u)=1$$

$$\delta(v)=1$$

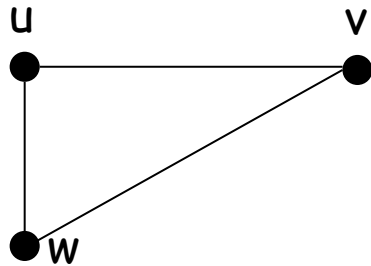


$$e=2$$

$$\delta(u)=2$$

$$\delta(v)=1$$

$$\delta(w)=1$$



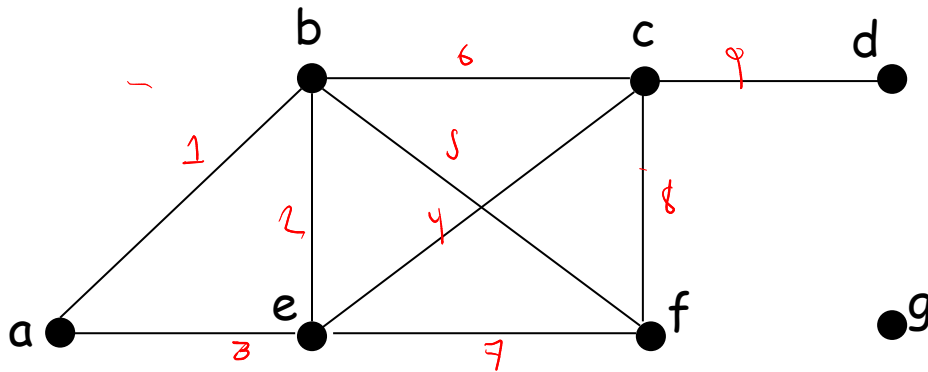
$$e=3$$

$$\delta(u)=2$$

$$\delta(v)=2$$

$$\delta(w)=2$$

Grafos



e=?

$\delta(a)=?$ 2

$\delta(b)=?$ 4

$\delta(c)=?$ 4

$\delta(d)=?$ 1

$\delta(e)=?$ 4

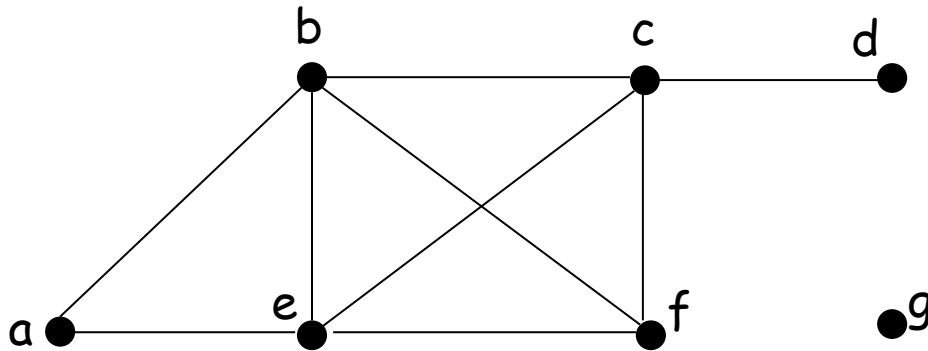
$\delta(f)=?$ 3

$\delta(g)=?$ 0

18

$$e = \frac{18}{2} = 9$$

Grafos



$$e=9$$

$$\delta(a)=2$$

$$\delta(b)=4$$

$$\delta(c)=4$$

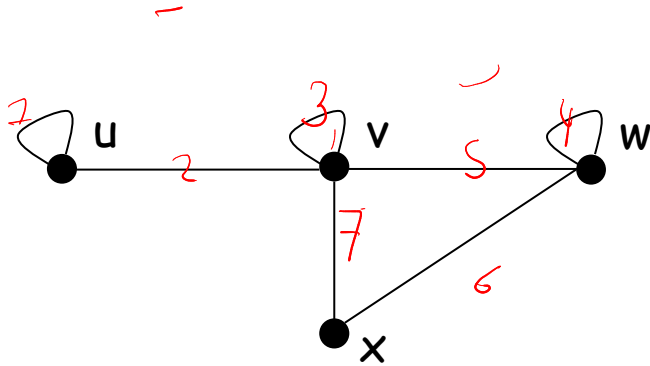
$$\delta(d)=1$$

$$\delta(e)=4$$

$$\delta(f)=3$$

$$\delta(g)=0$$

Grafos



$e=?$

$\delta(u)=?$ 3

$\delta(v)=?$ 5

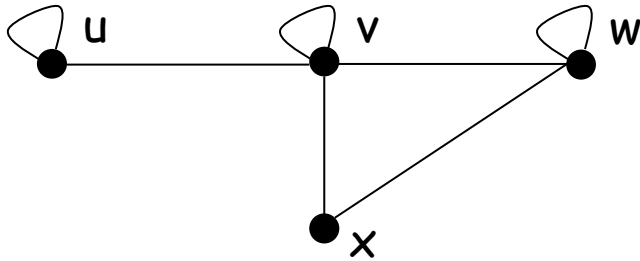
$\delta(w)=?$ 4

$\delta(x)=?$ 2

14

$$e = \frac{14}{2} = 7$$

Grafos



$$e=7$$

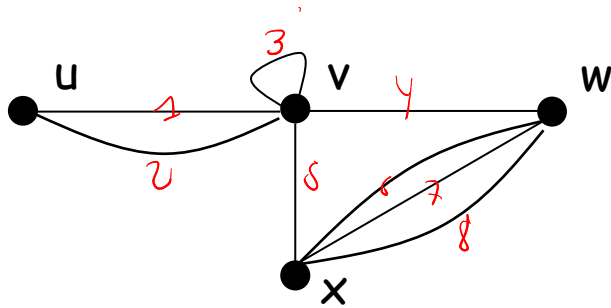
$$\delta(u)=3$$

$$\delta(v)=5$$

$$\delta(w)=4$$

$$\delta(x)=2$$

Grafos



$e=?$

$\delta(u)=? 2$

$\delta(v)=? 8$

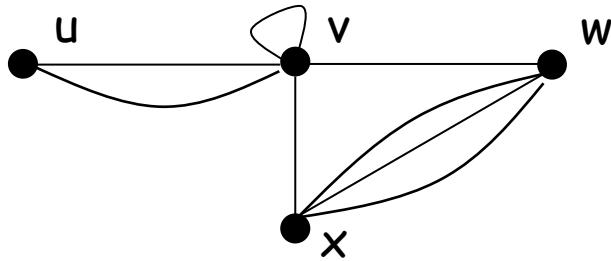
$\delta(w)=? 4$

$\delta(x)=? 4$

16

$$e = \frac{16}{2} = 8$$

Grafos



$$e=8$$

$$\delta(u)=2$$

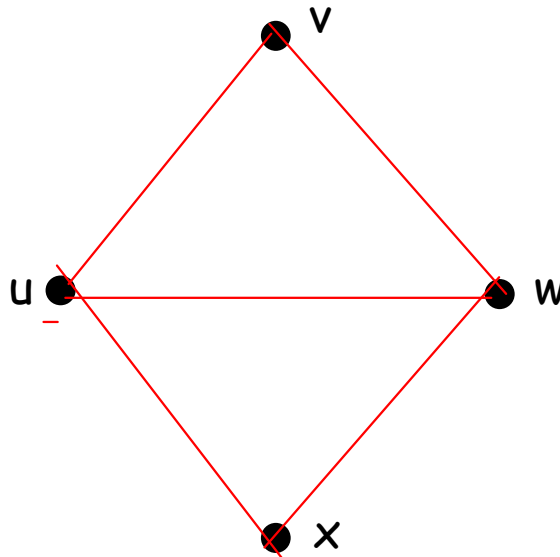
$$\delta(v)=6$$

$$\delta(w)=4$$

$$\delta(x)=4$$

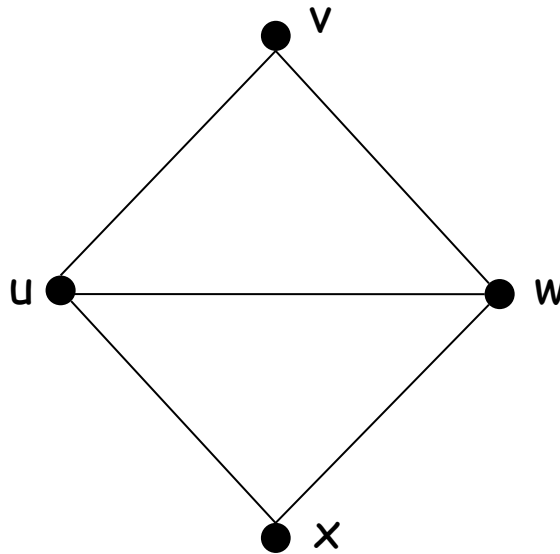
Grafos

Dibuje un grafo no dirigido con 4 vértices (u,v,w,x) cuyos grados sean $\delta(u)=3$, $\delta(v)=2$, $\delta(w)=3$, $\delta(x)=2$



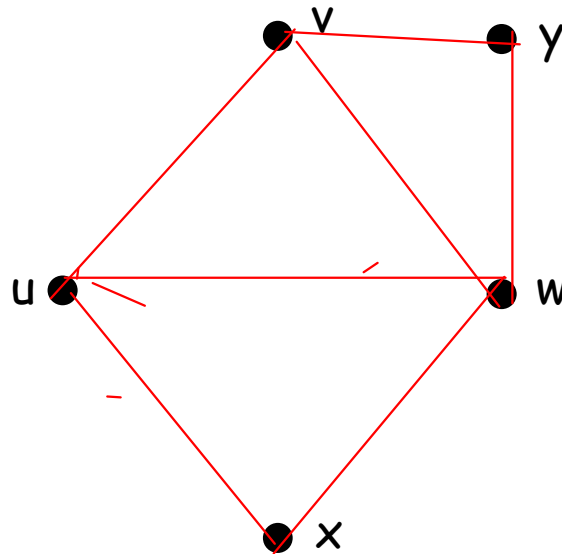
Grafos

Dibuje un grafo no dirigido con 4 vértices (u, v, w, x) cuyos grados sean $\delta(u)=3$, $\delta(v)=2$, $\delta(w)=3$, $\delta(x)=2$



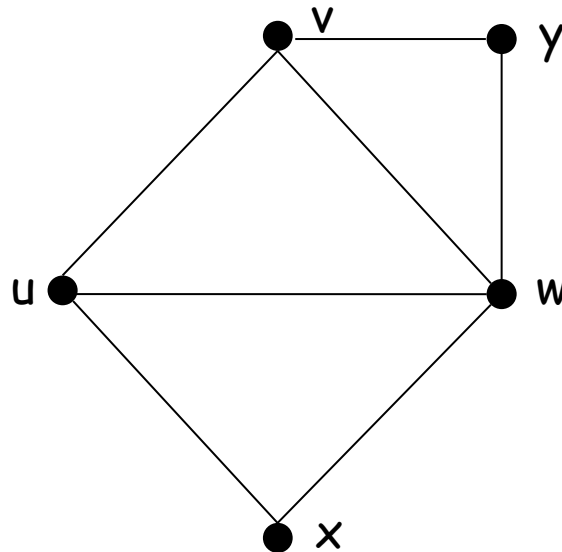
Grafos

Dibuje un grafo no dirigido con 4 vértices (u, v, w, x) cuyos grados sean $\delta(u)=3$, $\delta(v)=3$, $\delta(w)=4$, $\delta(x)=2$, $\delta(y)=2$

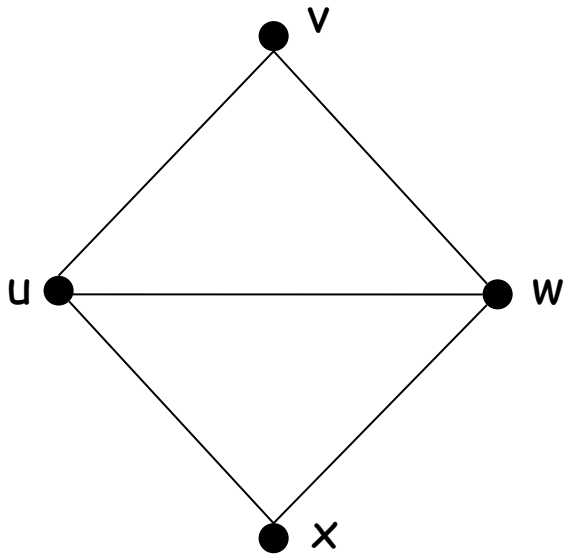


Grafos

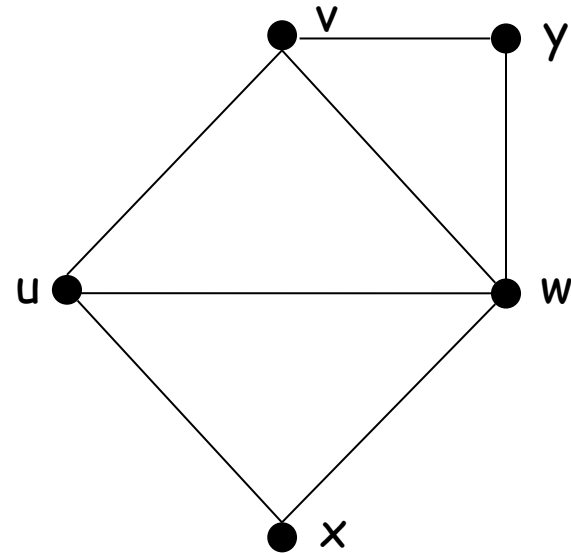
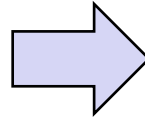
Dibuje un grafo no dirigido con 4 vértices (u, v, w, x) cuyos grados sean $\delta(u)=3$, $\delta(v)=3$, $\delta(w)=4$, $\delta(x)=2$, $\delta(y)=2$



Grafos

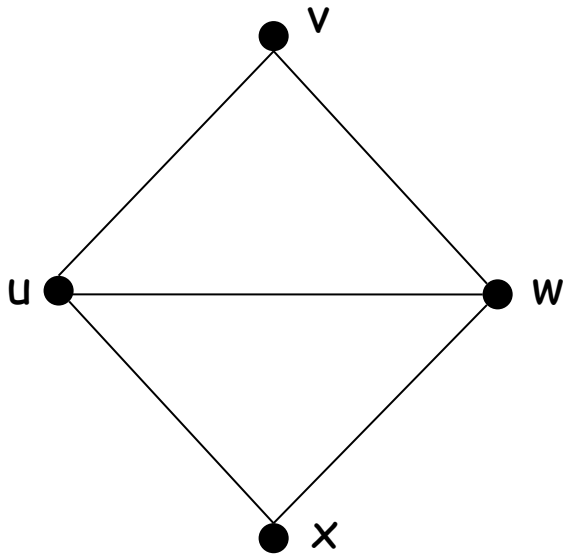


$$\delta(u)=3, \delta(v)=2, \delta(w)=3, \delta(x)=2$$

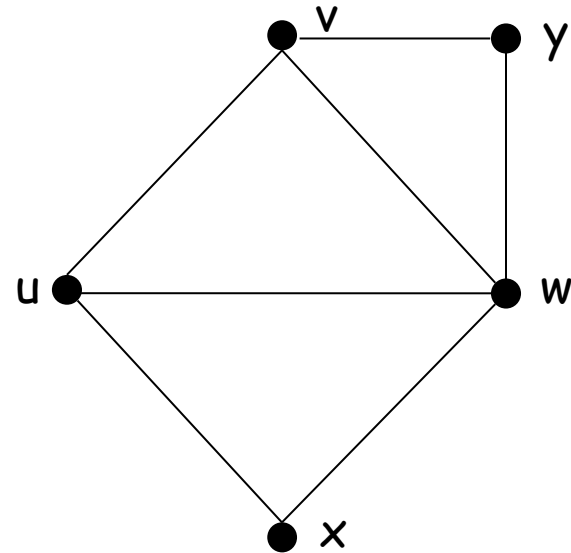
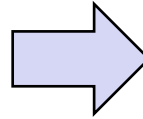


$$\delta(u)=3, \delta(v)=3, \delta(w)=4, \delta(x)=2, \delta(y)=2$$

Grafos



$$\delta(u)=3, \delta(v)=2, \delta(w)=3, \delta(x)=2$$



$$\delta(u)=3, \delta(v)=3, \delta(w)=4, \delta(x)=2, \delta(y)=2$$

¿Cuántos nodos son de grado **impar**?

Grafos

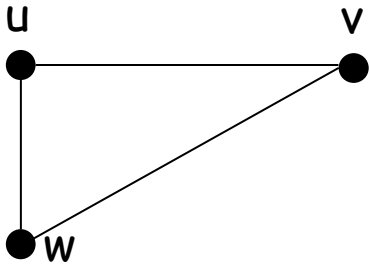
Teorema

Todo grafo no dirigido tiene un número par de vértices de grado impar

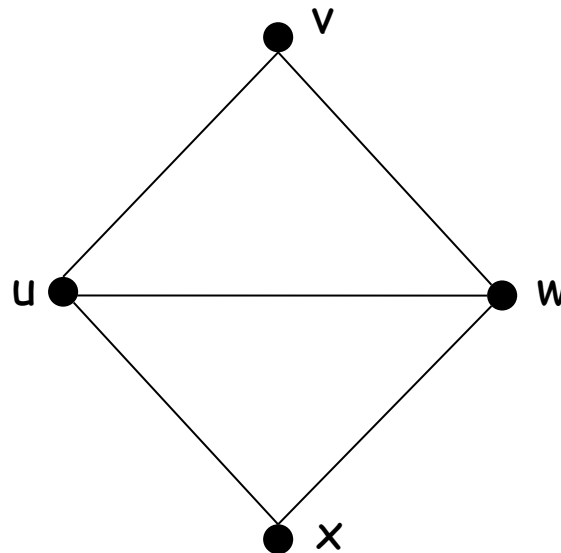
Grafos

Teorema

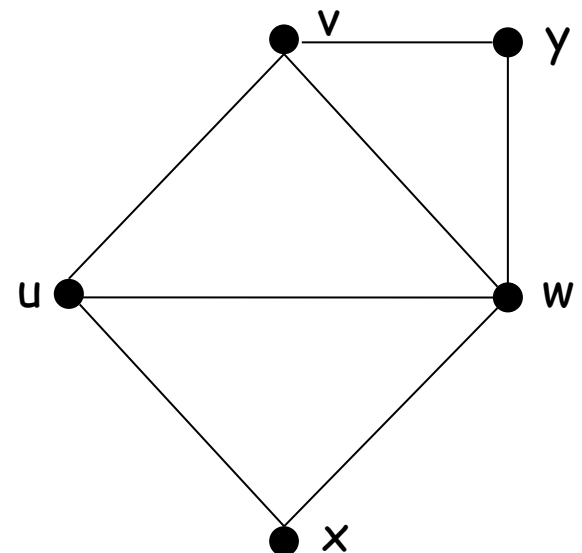
Todo grafo no dirigido tiene un número par de vértices de grado impar



$$\delta(u)=2, \delta(v)=2, \delta(w)=2$$



$$\delta(u)=3, \delta(v)=2, \\ \delta(w)=3, \delta(x)=2$$

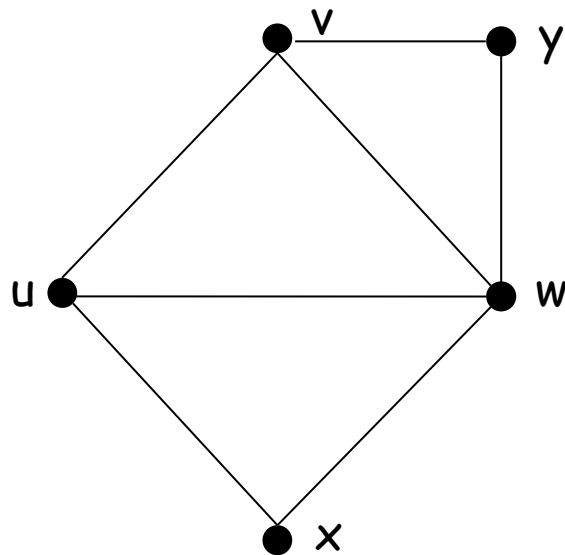


$$\delta(u)=3, \delta(v)=3, \delta(w)=4, \\ \delta(x)=2, \delta(y)=2$$

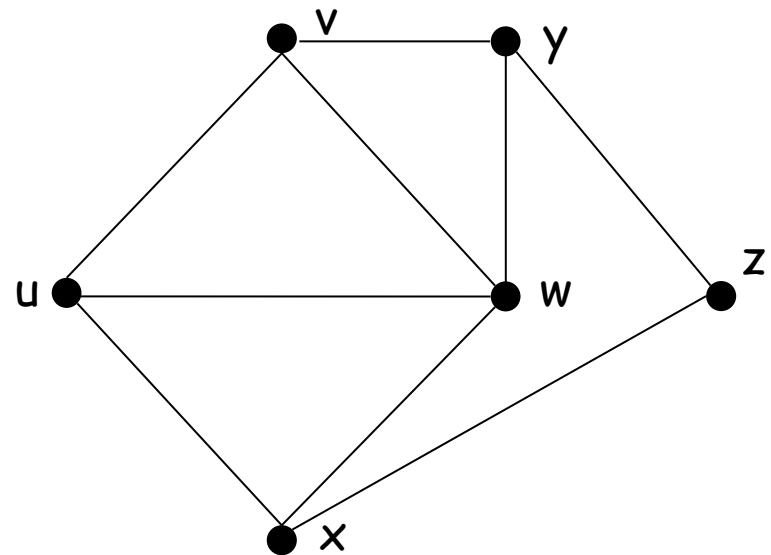
Grafos

Teorema

Todo grafo no dirigido tiene un número par de vértices de grado impar



$$\delta(u)=3, \delta(v)=3, \delta(w)=4, \\ \delta(x)=2, \delta(y)=2$$



$$\delta(u)=3, \delta(v)=3, \delta(w)=4, \\ \delta(x)=3, \delta(y)=3, \delta(z)=2$$

Grafos

Indique si existen grafos no dirigidos de 5 vértices con los siguientes grados:

- 3, 3, 3, 3, 2
- 1, 2, 3, 4, 4
- 0, 1, 2, 2, 3
- 1, 2, 3, 4, 5
- 3, 4, 3, 4, 3
- 1, 1, 1, 1, 1

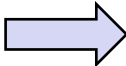
Grafos

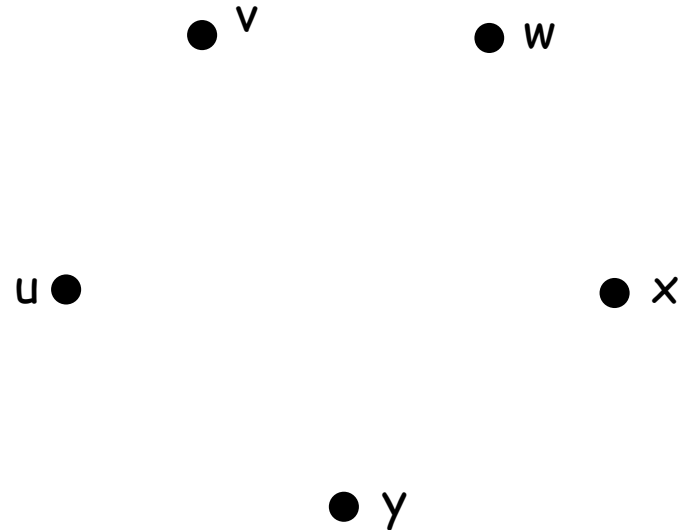
Indique si existen grafos no dirigidos de 5 vértices con los siguientes grados:

- 3, 3, 3, 3, 2 SI
- 1, 2, 3, 4, 4 SI
- 0, 1, 2, 2, 3 SI
- 1, 2, 3, 4, 5 NO
- 3, 4, 3, 4, 3 NO
- 1, 1, 1, 1, 1 NO

Grafos

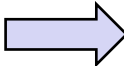
Indique si existen grafos no dirigidos de 5 vértices con los siguientes grados:

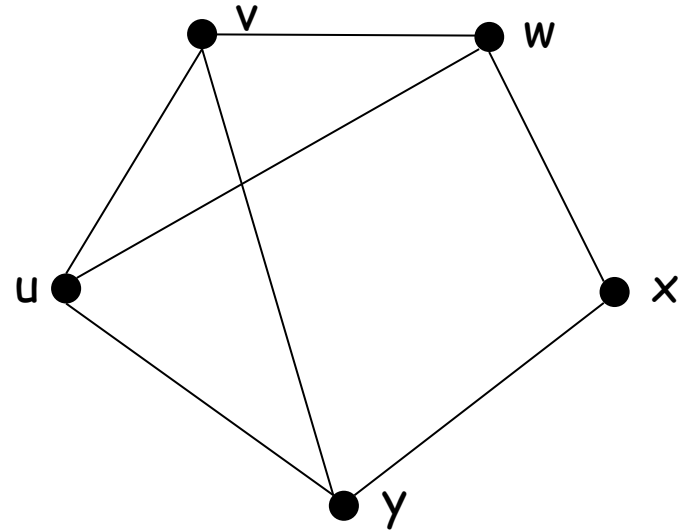
- 3, 3, 3, 3, 2 SI 
- 1, 2, 3, 4, 4 SI
- 0, 1, 2, 2, 3 SI
- 1, 2, 3, 4, 5 NO
- 3, 4, 3, 4, 3 NO
- 1, 1, 1, 1, 1 NO



Grafos

Indique si existen grafos no dirigidos de 5 vértices con los siguientes grados:

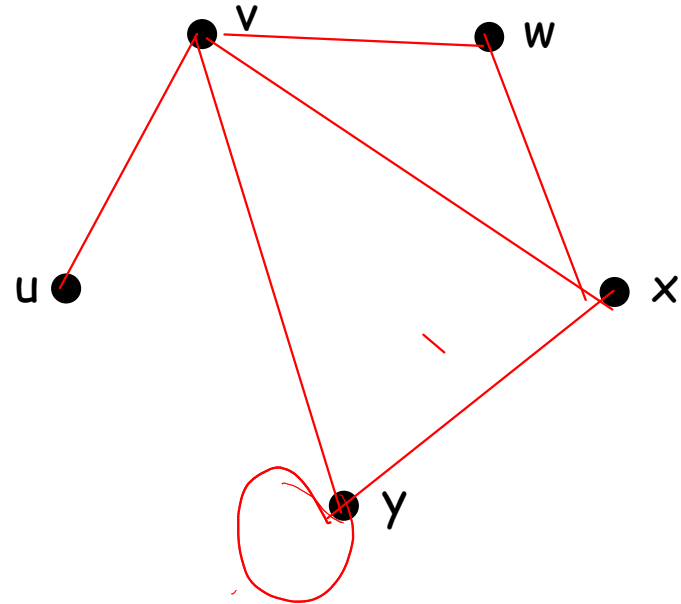
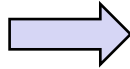
- 3, 3, 3, 3, 2 SI 
- 1, 2, 3, 4, 4 SI
- 0, 1, 2, 2, 3 SI
- 1, 2, 3, 4, 5 NO
- 3, 4, 3, 4, 3 NO
- 1, 1, 1, 1, 1 NO



Grafos

Indique si existen grafos no dirigidos de 5 vértices con los siguientes grados:

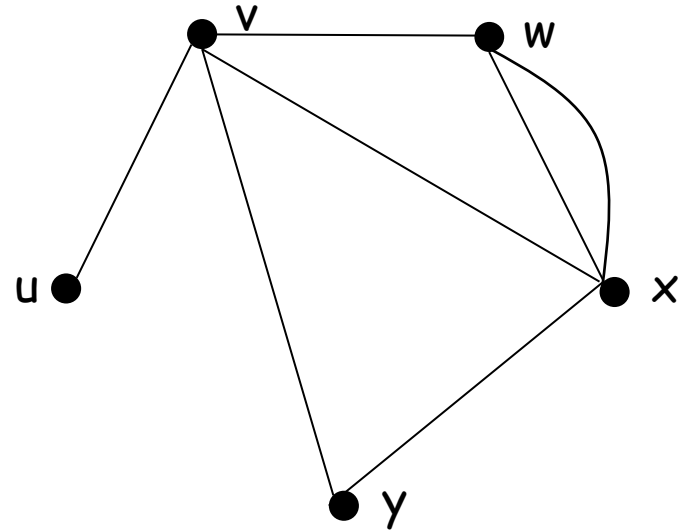
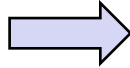
- 3, 3, 3, 3, 2 SI
- 1, 2, 3, 4, 4 SI
- 0, 1, 2, 2, 3 SI
- 1, 2, 3, 4, 5 NO
- 3, 4, 3, 4, 3 NO
- 1, 1, 1, 1, 1 NO



Grafos

Indique si existen grafos no dirigidos de 5 vértices con los siguientes grados:

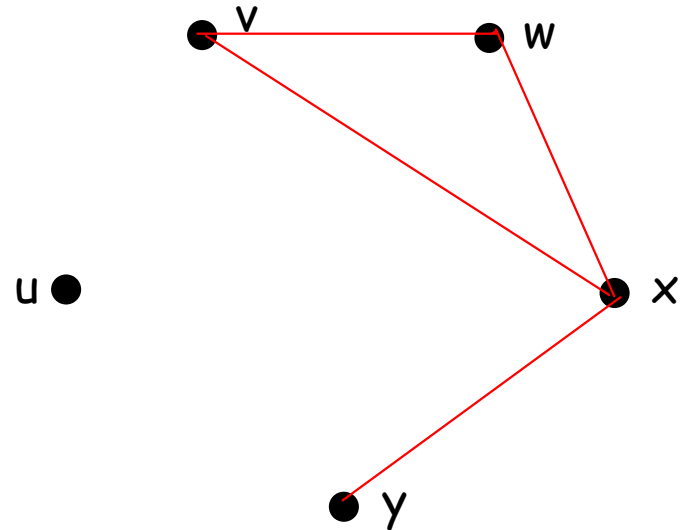
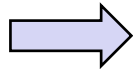
- 3, 3, 3, 3, 2 SI
- 1, 2, 3, 4, 4 SI
- 0, 1, 2, 2, 3 SI
- 1, 2, 3, 4, 5 NO
- 3, 4, 3, 4, 3 NO
- 1, 1, 1, 1, 1 NO



Grafos

Indique si existen grafos no dirigidos de 5 vértices con los siguientes grados:

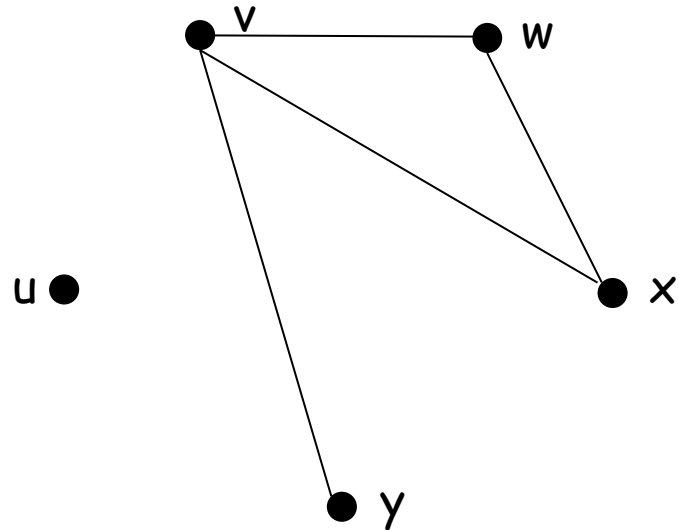
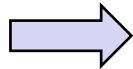
- 3, 3, 3, 3, 2 SI
- 1, 2, 3, 4, 4 SI
- 0, 1, 2, 2, 3 SI
- 1, 2, 3, 4, 5 NO
- 3, 4, 3, 4, 3 NO
- 1, 1, 1, 1, 1 NO



Grafos

Indique si existen grafos no dirigidos de 5 vértices con los siguientes grados:

- 3, 3, 3, 3, 2 SI
- 1, 2, 3, 4, 4 SI
- 0, 1, 2, 2, 3 SI
- 1, 2, 3, 4, 5 NO
- 3, 4, 3, 4, 3 NO
- 1, 1, 1, 1, 1 NO



Grafos

Indique la cantidad de aristas de un grafo si los grados de sus vértices son:

- 3, 3, 3, 3, 2 Sí $E = 7$
- 1, 2, 3, 4, 4 Sí $E = 4$
- 0, 1, 2, 2, 3 Sí $E = 4$
- 4, 5, 5, 2, 2 Sí $E = 9$
- 1, 3, 2, 2, 2, 2, 4 Sí $E = 8$

Grafos

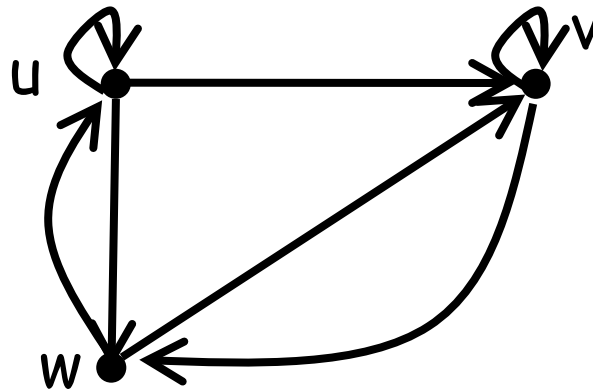
Indique la cantidad de aristas de un grafo si los grados de sus vértices son:

- 3, 3, 3, 3, 2 (7)
- 1, 2, 3, 4, 4 (7)
- 0, 1, 2, 2, 3 (4)
- 4, 5, 5, 2, 2 (9)
- 1, 3, 2, 2, 2, 2, 4 (8)

Grafos

Grafos dirigidos

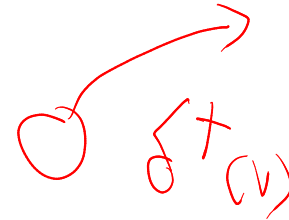
Dada una arista (u,v) en un grafo dirigido que inicia en u y termina en v , se dice que u es el **vértice inicial** y v es el **vértice final**



Grafos

Grado en un grafo dirigido

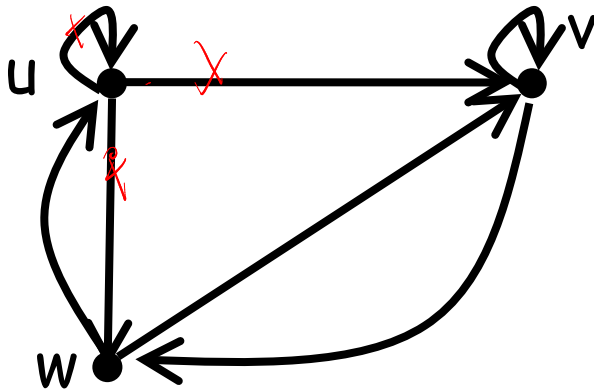
El **grado de entrada** de un vértice v , denotado como $\delta^-(v)$ es el número de aristas que llegan a v . El **grado de salida**, denotado como $\delta^+(v)$ es el número de aristas que salen de v



Grafos

Grado en un grafo dirigido

El **grado de entrada** de un vértice v , denotado como $\delta^-(v)$ es el número de aristas que llegan a v . El **grado de salida**, denotado como $\delta^+(v)$ es el número de aristas que salen de v

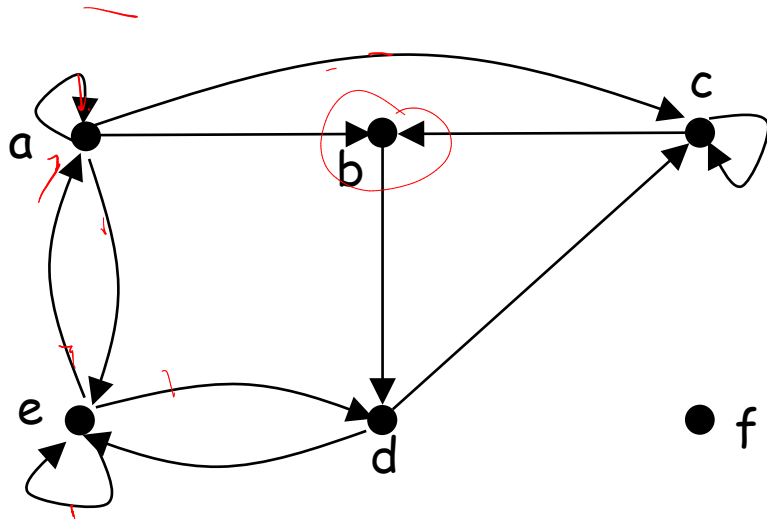


$$\delta^-(u)=2, \delta^+(u)=3$$

$$\delta^-(v)=3, \delta^+(v)=2$$

$$\delta^-(w)=2, \delta^+(w)=2$$

Grafos



$\delta^-(a)=?, \delta^+(a)=?$

$\delta^-(b)=?, \delta^+(b)=?$

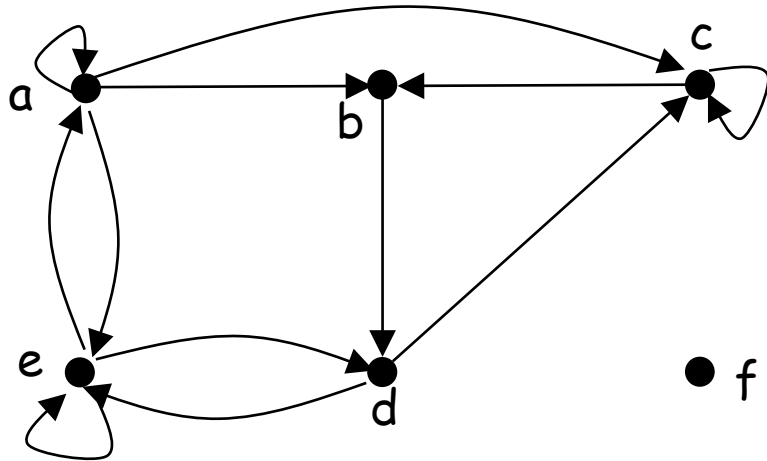
$\delta^-(c)=?, \delta^+(c)=?$

$\delta^-(d)=?, \delta^+(d)=?$

$\delta^-(e)=?, \delta^+(e)=?$

$\delta^-(f)=?, \delta^+(f)=?$

Grafos



$$\delta^-(a)=2, \delta^+(a)=4$$

$$\delta^-(b)=2, \delta^+(b)=1$$

$$\delta^-(c)=3, \delta^+(c)=2$$

$$\delta^-(d)=2, \delta^+(d)=2$$

$$\delta^-(e)=3, \delta^+(e)=3$$

$$\delta^-(f)=0, \delta^+(f)=0$$

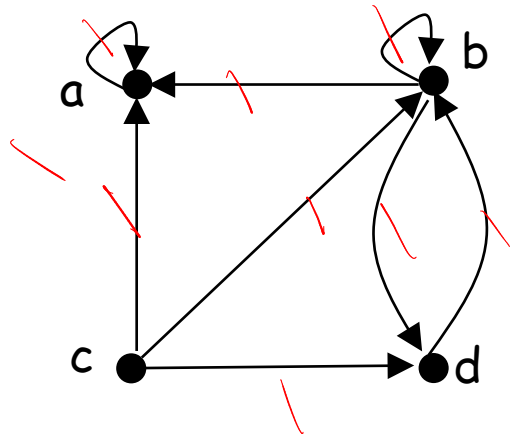
Grafos

Teorema

Sea $G=(V,E)$ un grado dirigido, se cumple que:

$$\sum_{v \in V} \delta^-(v) = \sum_{v \in V} \delta^+(v) = |E|$$

Grafos



$$3 = \delta^-(a)=?, \delta^+(a)=?$$

$$3 = \delta^-(b)=?, \delta^+(b)=?$$

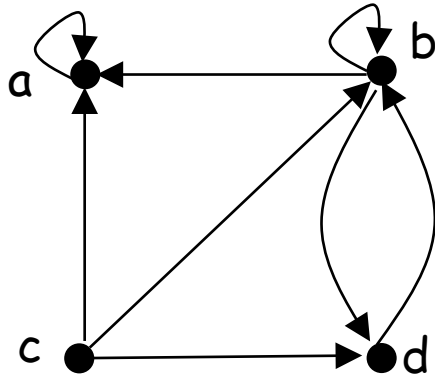
$$0 = \delta^-(c)=?, \delta^+(c)=?$$

$$2 = \delta^-(d)=?, \delta^+(d)=?$$

8

8

Grafos



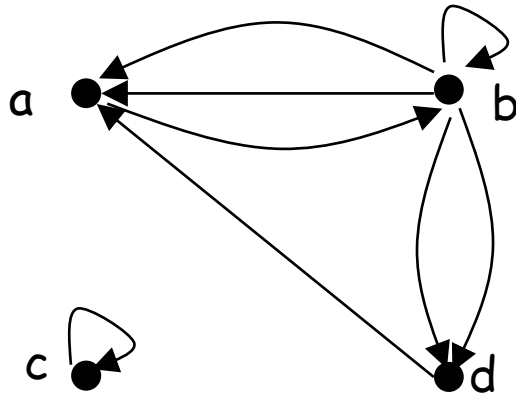
$$\delta^-(a)=3, \delta^+(a)=1$$

$$\delta^-(b)=3, \delta^+(b)=3$$

$$\delta^-(c)=0, \delta^+(c)=3$$

$$\delta^-(d)=2, \delta^+(d)=1$$

Grafos



$$3 = \delta^-(a)=?, \delta^+(a)=? \quad 1$$

$$2 = \delta^-(b)=?, \delta^+(b)=? \quad 5$$

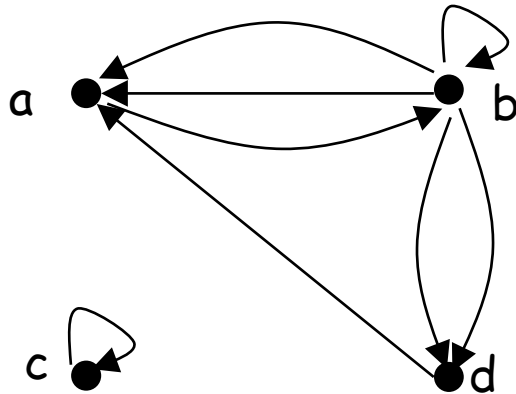
$$1 = \delta^-(c)=?, \delta^+(c)=? \quad 1$$

$$2 = \delta^-(d)=?, \delta^+(d)=? \quad 1$$

8

8

Grafos



$$\delta^-(a)=3, \delta^+(a)=1$$

$$\delta^-(b)=2, \delta^+(b)=5$$

$$\delta^-(c)=1, \delta^+(c)=1$$

$$\delta^-(d)=2, \delta^+(d)=1$$

Grafos

Grafo completo

El **grafo completo** de n vértices, se denota por K_n , es el grafo simple que contiene exactamente una arista entre cada par de vértices distintos

Grafos

Grafo completo

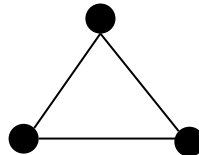
El **grafo completo** de n vértices, se denota por K_n , es el grafo simple que contiene exactamente una arista entre cada par de vértices distintos



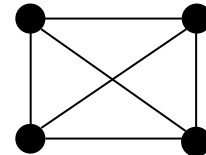
K_1



K_2



K_3

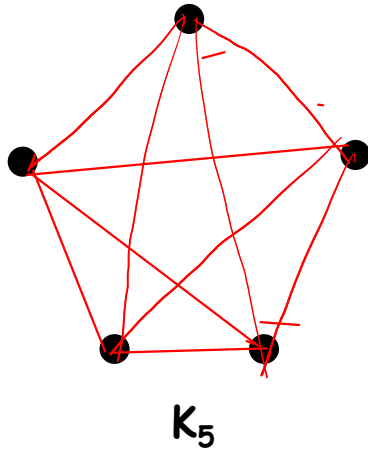


K_4

Grafos

Grafo completo

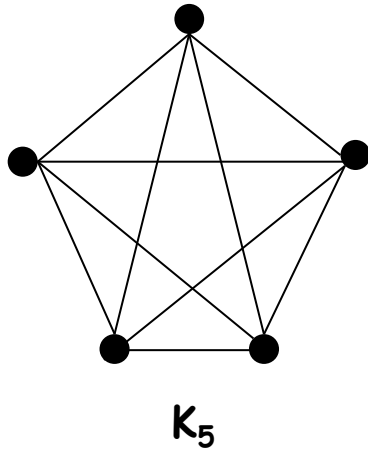
El **grafo completo** de n vértices, se denota por K_n , es el grafo simple que contiene exactamente una arista entre cada par de vértices distintos



Grafos

Grafo completo

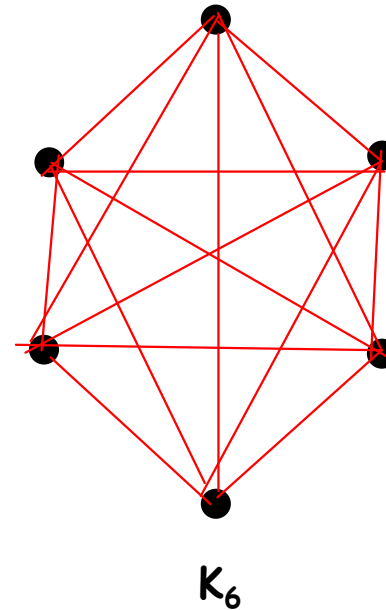
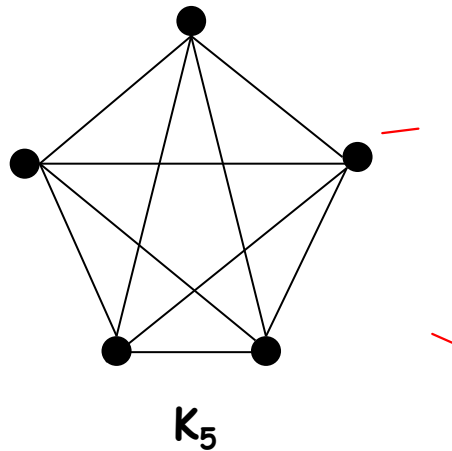
El **grafo completo** de n vértices, se denota por K_n , es el grafo simple que contiene exactamente una arista entre cada par de vértices distintos



Grafos

Grafo completo

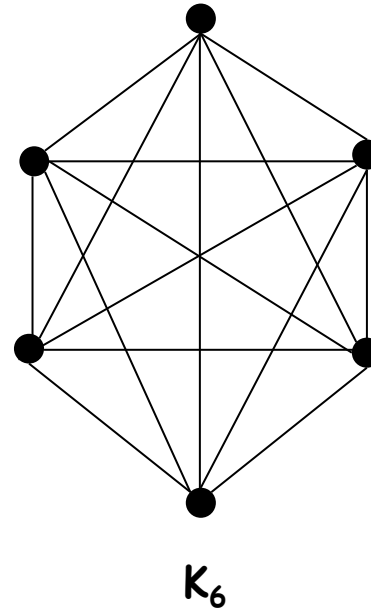
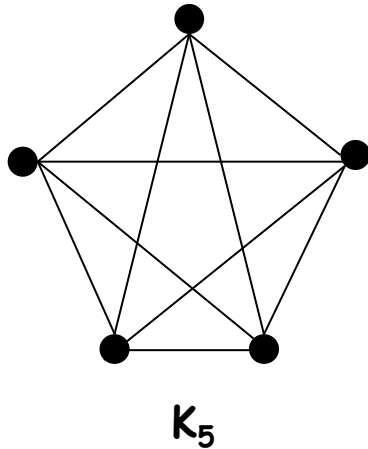
El **grafo completo** de n vértices, se denota por K_n , es el grafo simple que contiene exactamente una arista entre cada par de vértices distintos



Grafos

Grafo completo

El **grafo completo** de n vértices, se denota por K_n , es el grafo simple que contiene exactamente una arista entre cada par de vértices distintos

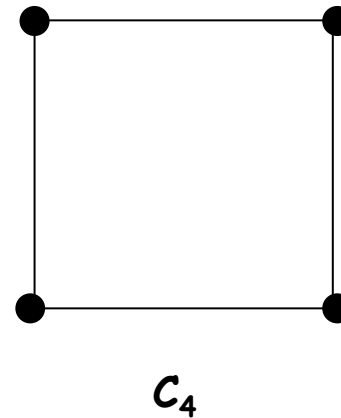
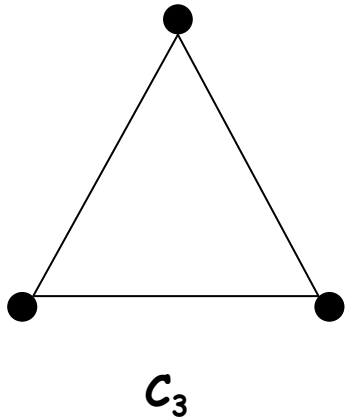


Grafos

Ciclo completo

El ciclo C_n ($n \geq 3$), consta de n vértices v_1, v_2, \dots, v_n y las aristas:

$$\{v_1, v_2\}, \{v_2, v_3\}, \dots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$$

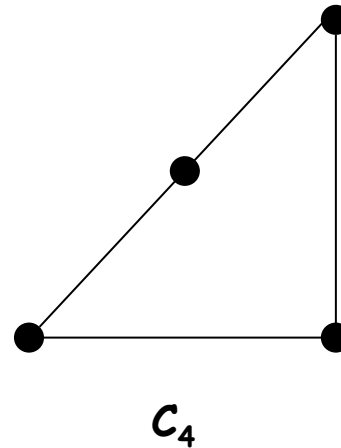
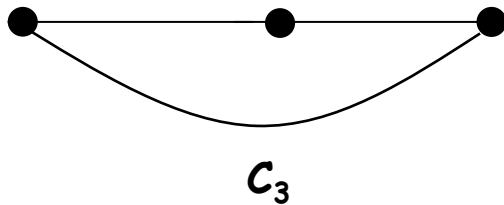


Grafos

Ciclo completo

El ciclo C_n ($n \geq 3$), consta de n vértices v_1, v_2, \dots, v_n y las aristas:

$$\{v_1, v_2\}, \{v_2, v_3\}, \dots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$$

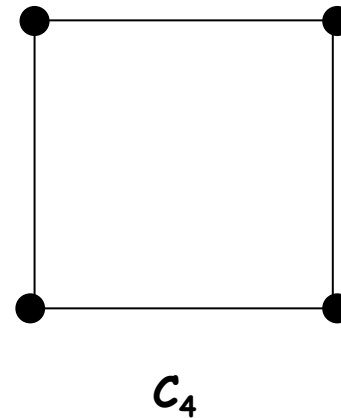
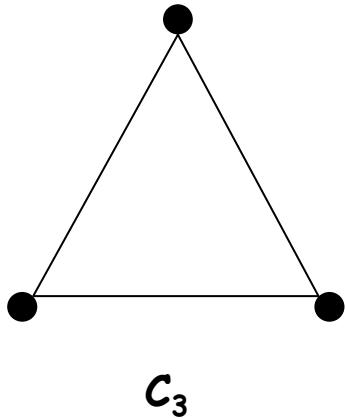


Grafos

Ciclo completo

El ciclo C_n ($n \geq 3$), consta de n vértices v_1, v_2, \dots, v_n y las aristas:

$$\{v_1, v_2\}, \{v_2, v_3\}, \dots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$$

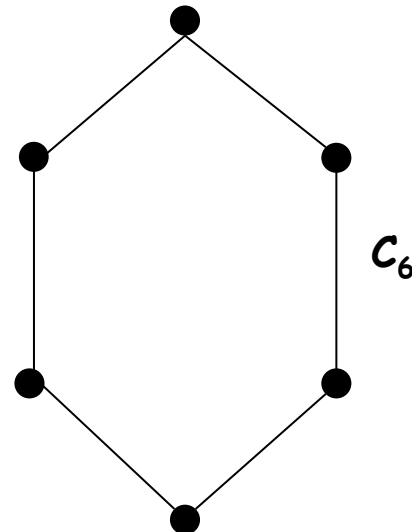
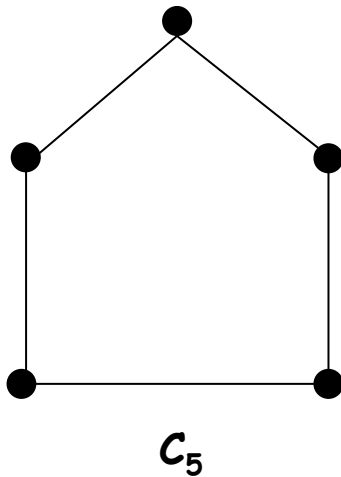


Grafos

Ciclo completo

El ciclo C_n ($n \geq 3$), consta de n vértices v_1, v_2, \dots, v_n y las aristas:

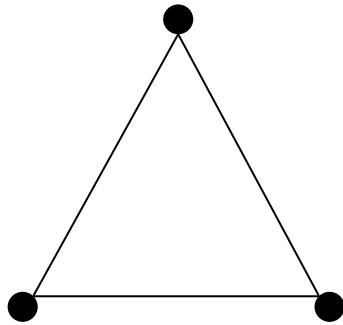
$$\{v_1, v_2\}, \{v_2, v_3\}, \dots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$$



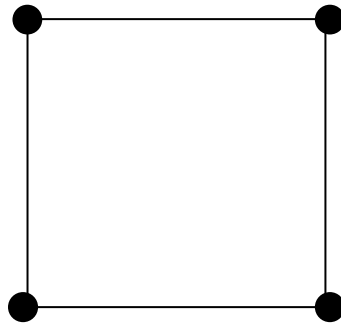
Grafos

Rueda

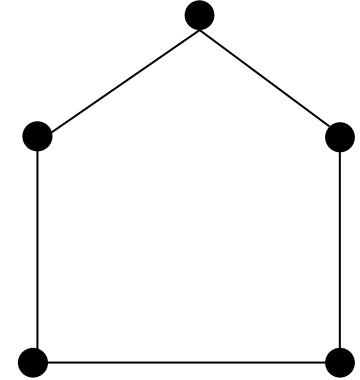
Una **rueda** W_n se obtiene al añadir un vértice al ciclo C_n que se conecta con cada uno de los n vértices del ciclo



C_3



C_4

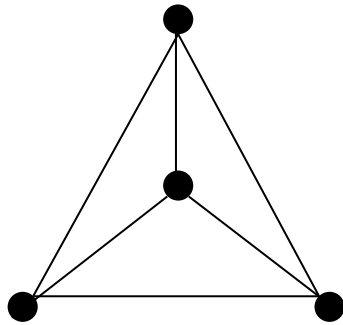


C_5

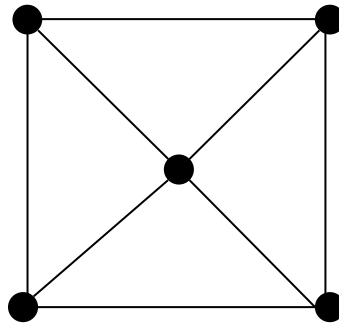
Grafos

Rueda

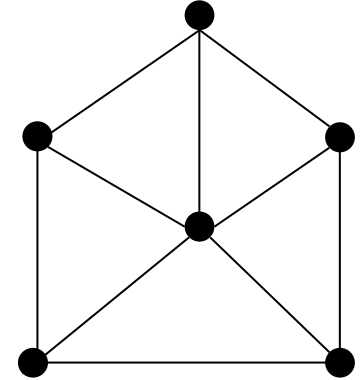
Una **rueda** W_n se obtiene al añadir un vértice al ciclo C_n que se conecta con cada uno de los n vértices del ciclo



W_3



W_4



W_5

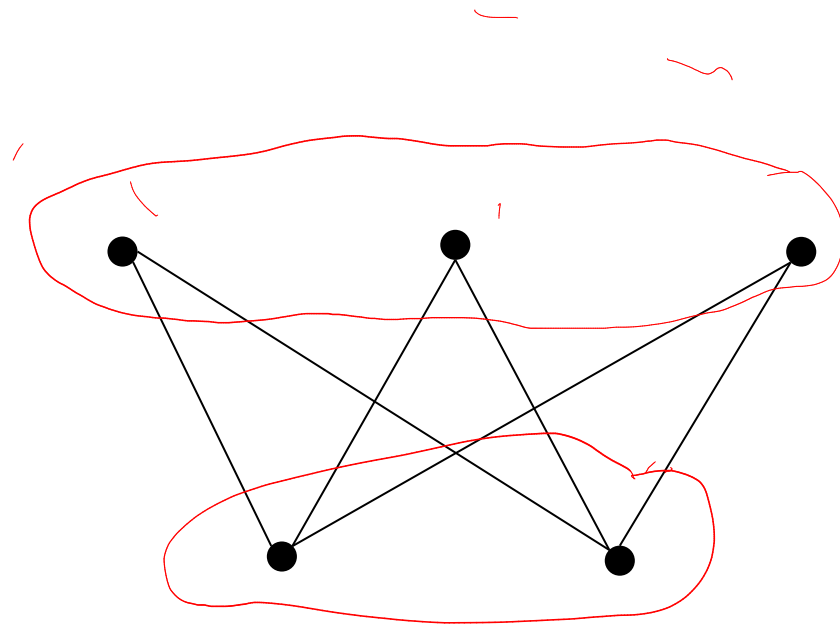
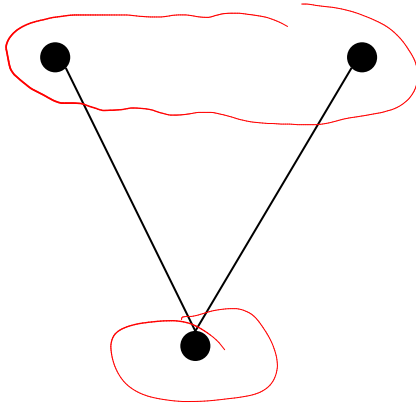
Grafos

Grafo bipartito

Un grafo simple $G=(V,E)$ es **bipartito** si su conjunto de vértices V se puede dividir en dos conjuntos disjuntos V_1 y V_2 tales que cada arista del grafo conecte un vértice de V_1 con un vértice de V_2 (de manera que no haya ninguna arista que conecte entre sí dos vértices de V_1 ni tampoco dos vértices de V_2)

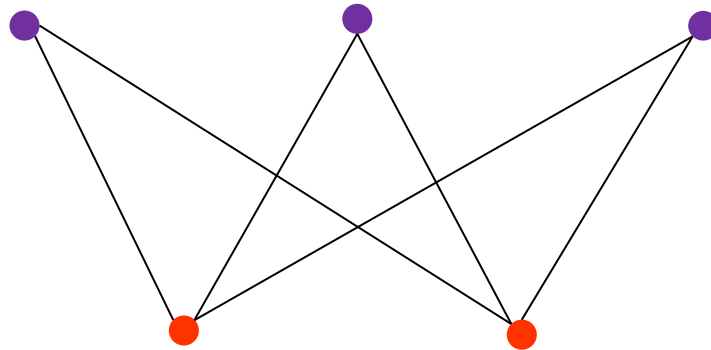
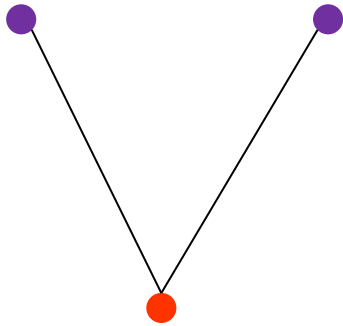
Grafos

Grafo bipartito



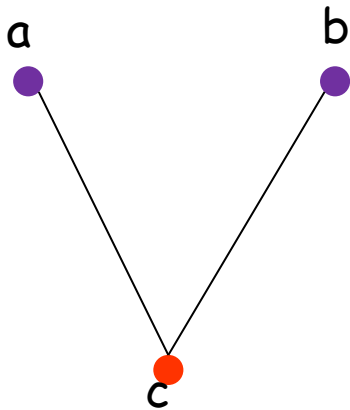
Grafos

Grafo bipartito



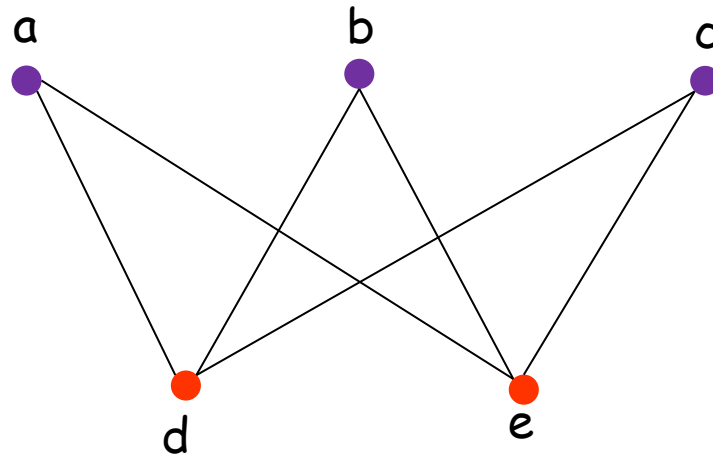
Grafos

Grafo bipartito



$$V_1 = \{a, b\}$$

$$V_2 = \{c\}$$

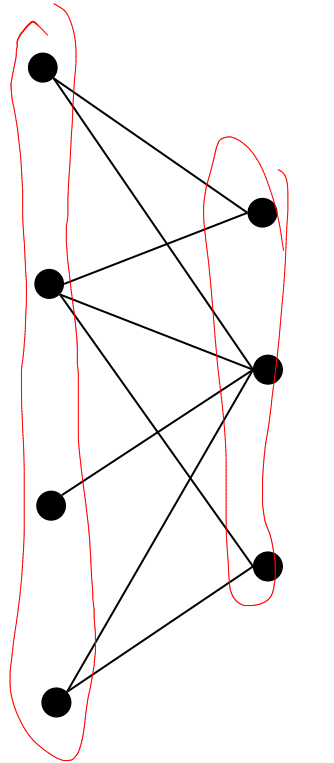


$$V_1 = \{a, b, c\}$$

$$V_2 = \{d, e\}$$

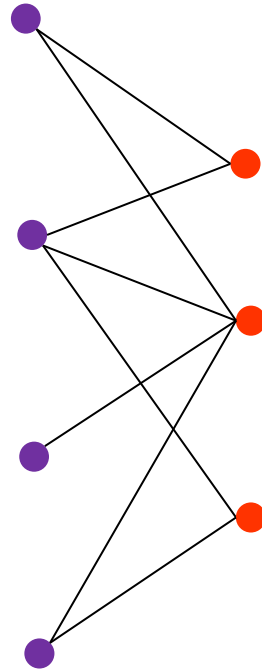
Grafos

Grafo bipartito



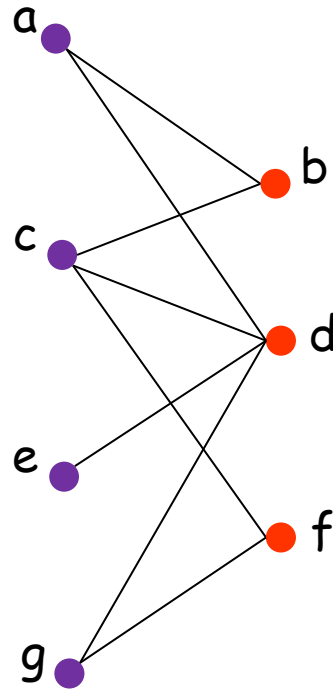
Grafos

Grafo bipartito



Grafos

Grafo bipartito

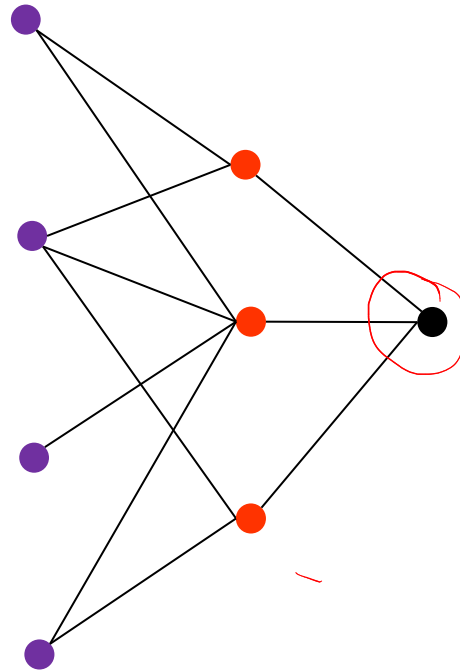


$$V_1 = \{a, c, e, g\}$$

$$V_2 = \{b, d, f\}$$

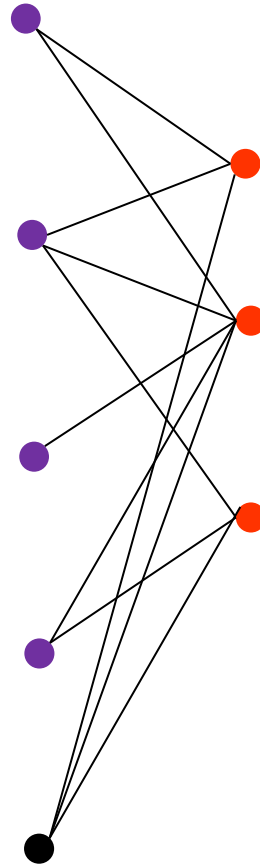
Grafos

Grafo bipartito



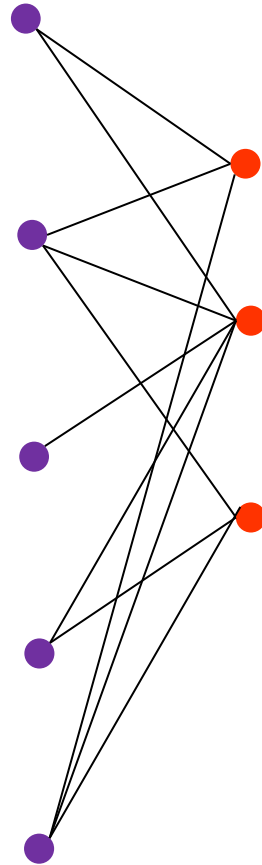
Grafos

Grafo bipartito



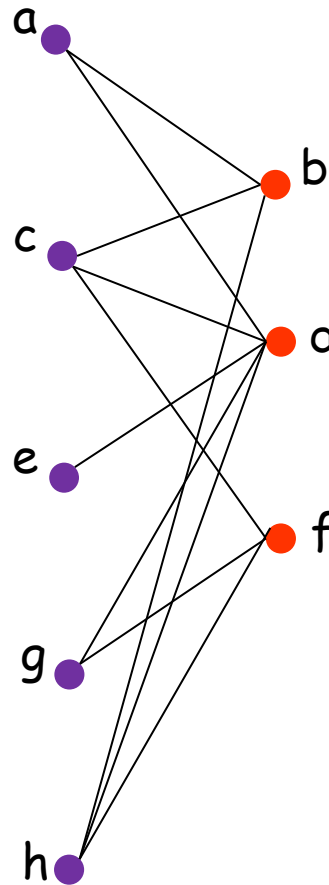
Grafos

Grafo bipartito



Grafos

Grafo bipartito

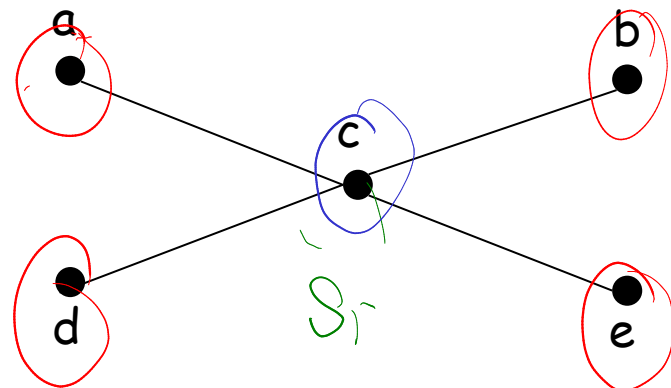
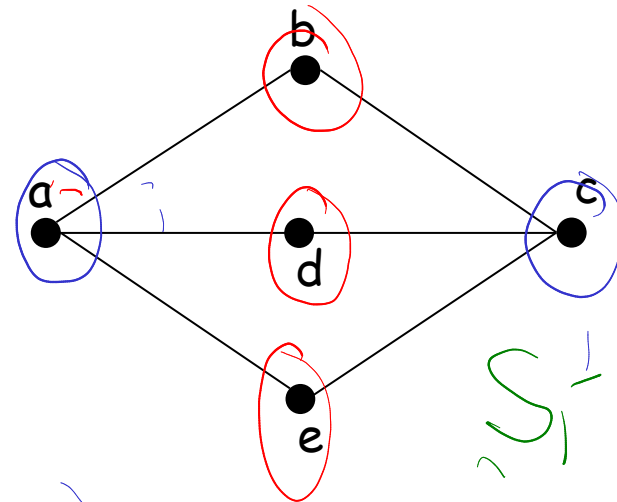
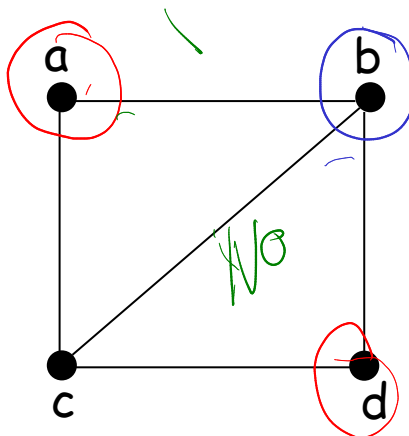
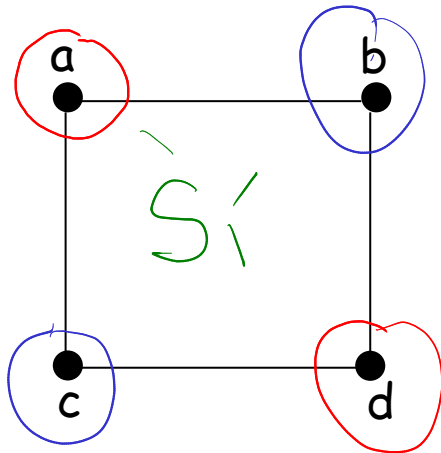


$$V_1 = \{a, c, e, g, h\}$$

$$V_2 = \{b, d, f\}$$

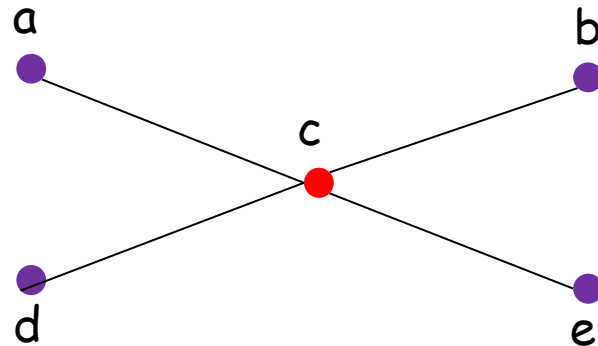
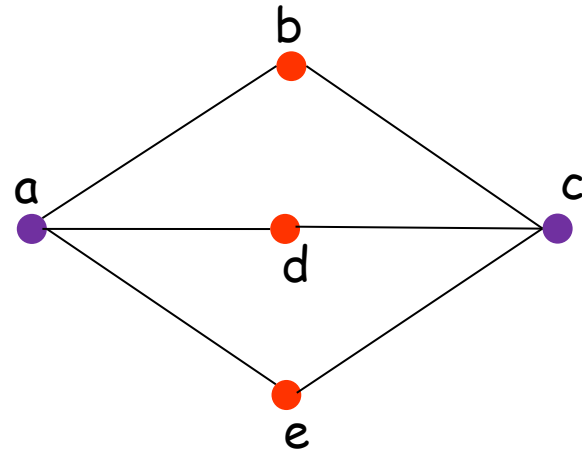
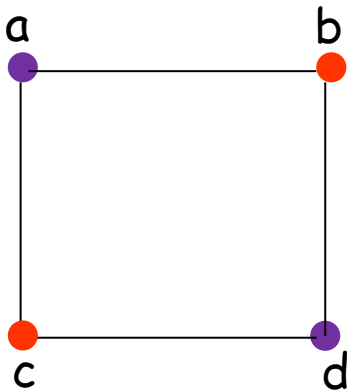
Grafos

Indique cuáles de los siguientes grafos son bipartitos.
Muestre V_1 y V_2 para los que sean bipartitos



Grafos

Indique cuáles de los siguientes grafos son bipartitos.
Muestre V_1 y V_2 para los que sean bipartitos



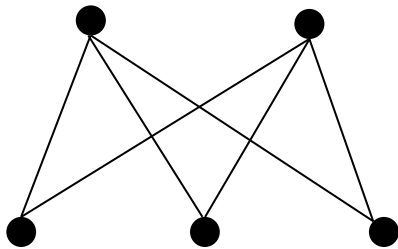
Grafos

Grafo bipartito completo

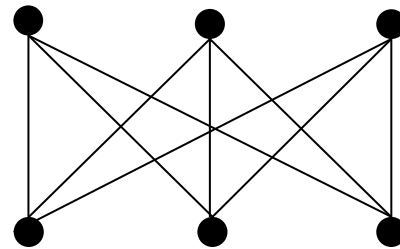
El **grafo bipartito completo** $K_{m,n}$ es el grafo cuyo conjunto de vértices está formado por dos subconjuntos de m y n vértices tal que hay una arista entre dos vértices si, y solo si, un vértice está en el primer subconjunto y el otro vértices está en el segundo subconjunto

Grafos

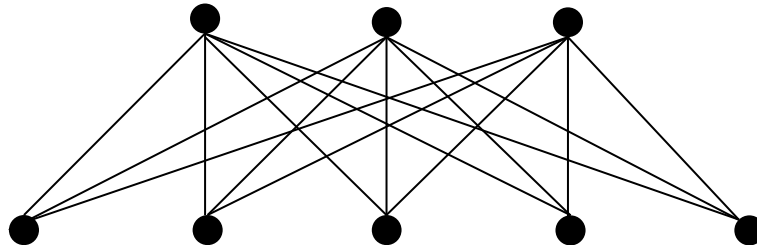
Grafo bipartito completo $K_{n,m}$



$K_{2,3}$



$K_{3,3}$

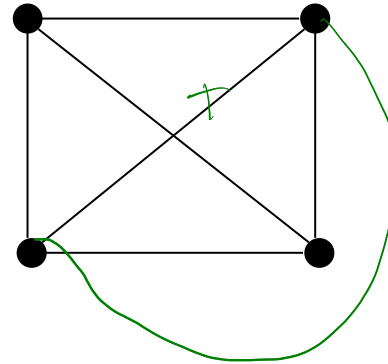
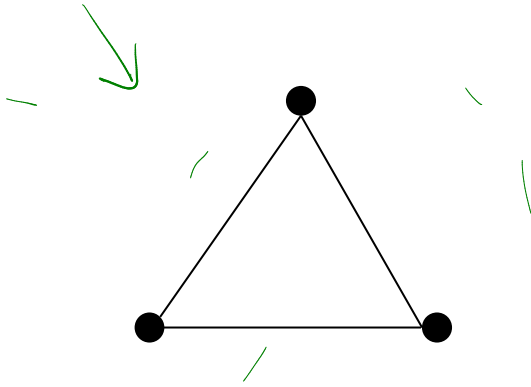


$K_{3,5}$

Grafos

Grafo plano

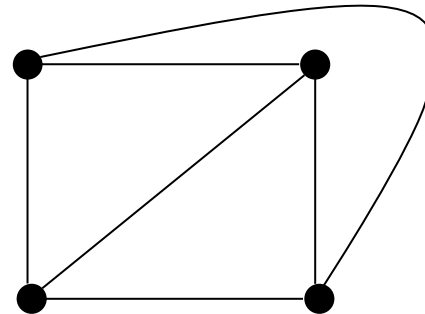
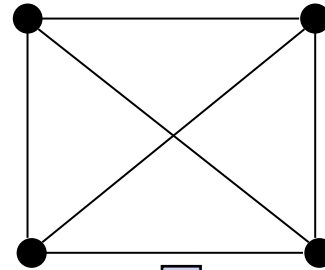
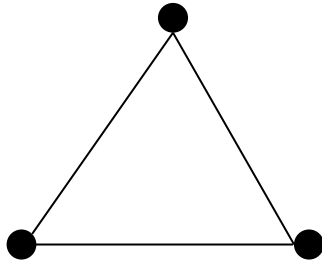
Un grafo (o multigrafo) G es **plano** si se puede dibujar en el plano de modo que sus aristas no se crucen



Grafos

Grafo plano

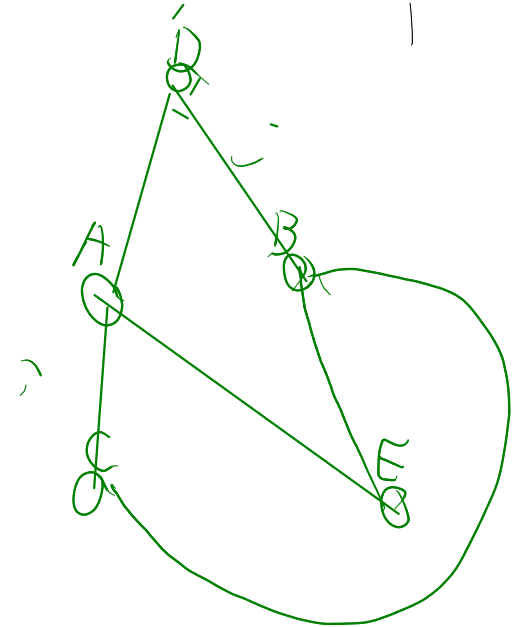
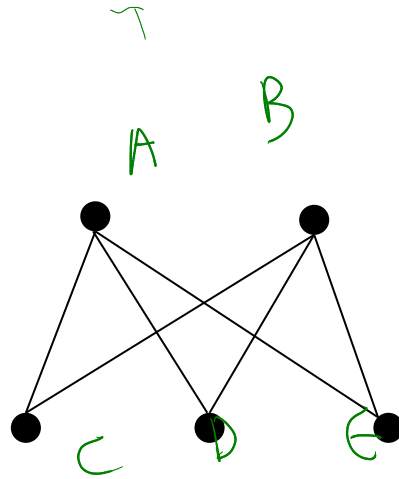
Un grafo (o multigrafo) G es **plano** si se puede dibujar en el plano de modo que sus aristas no se crucen



Grafos

Grafo plano

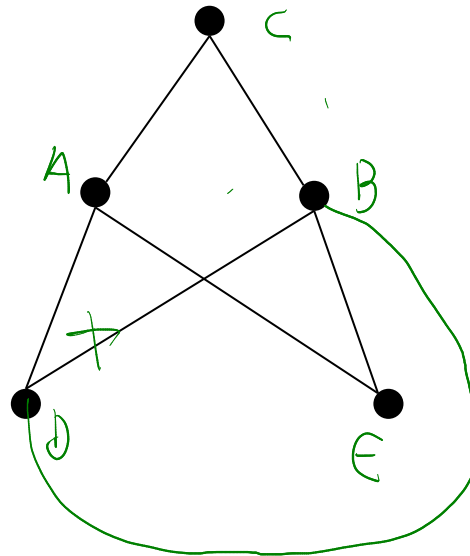
Un grafo (o multigrafo) G es **plano** si se puede dibujar en el plano de modo que sus aristas no se crucen



Grafos

Grafo plano

Un grafo (o multigrafo) G es **plano** si se puede dibujar en el plano de modo que sus aristas no se crucen

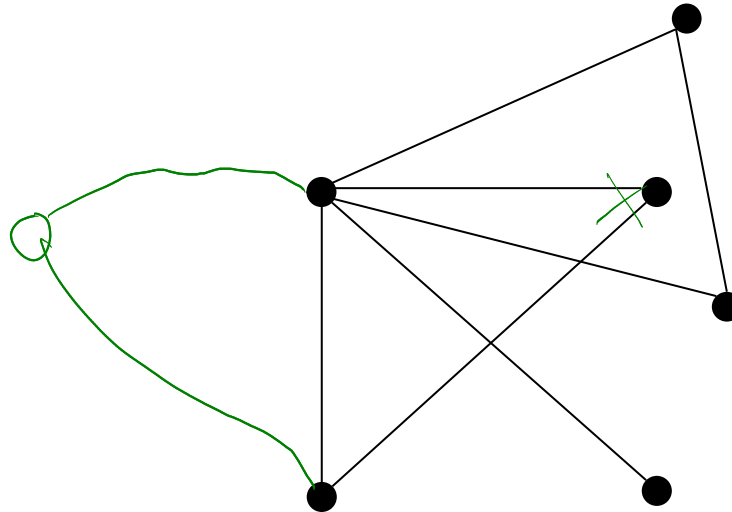


El grafo ~~no~~
es plano

Grafos

Grafo plano

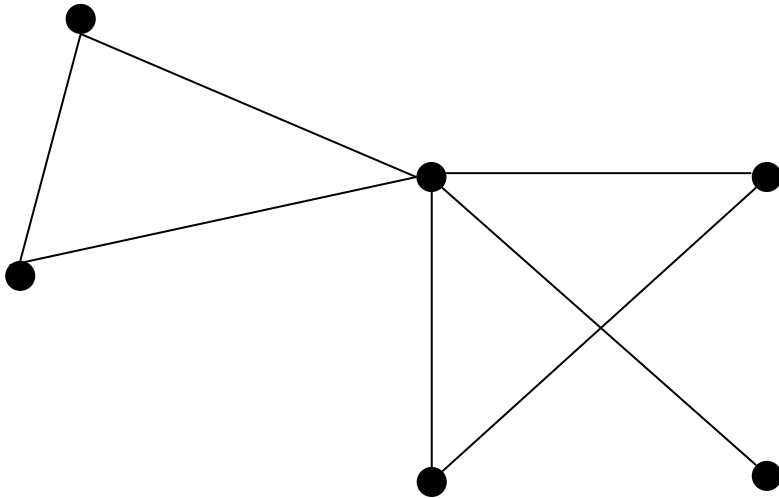
Un grafo (o multigrafo) G es **plano** si se puede dibujar en el plano de modo que sus aristas no se crucen



Grafos

Grafo plano

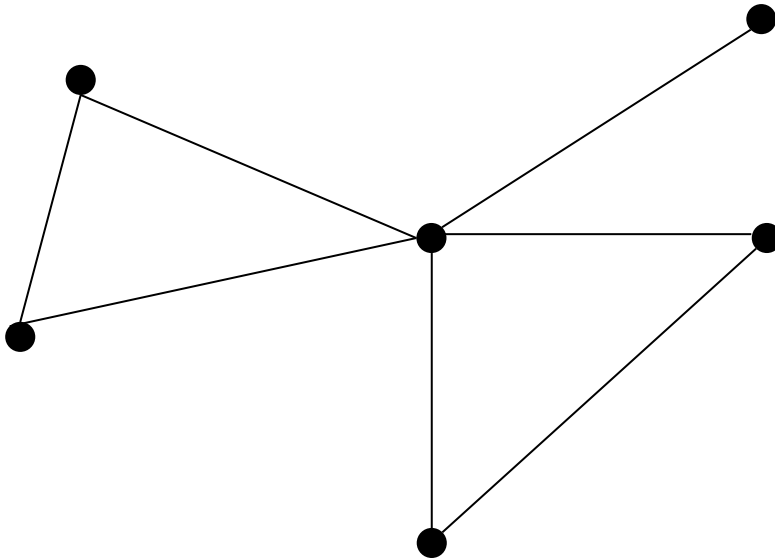
Un grafo (o multigrafo) G es **plano** si se puede dibujar en el plano de modo que sus aristas no se crucen



Grafos

Grafo plano

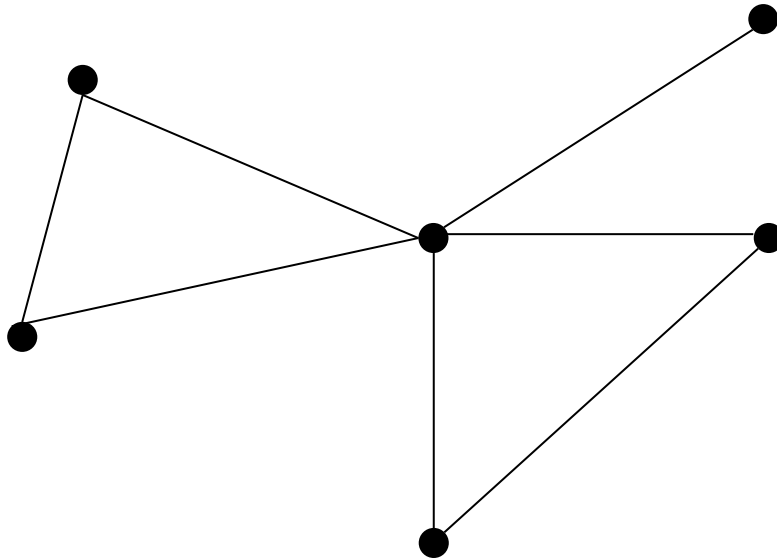
Un grafo (o multigrafo) G es **plano** si se puede dibujar en el plano de modo que sus aristas no se crucen



Grafos

Grafo plano

Un grafo (o multigrafo) G es **plano** si se puede dibujar en el plano de modo que sus aristas no se crucen



El grafo es
plano

Grafos

Teorema

Se G un grafo simple con e aristas y v vértices, entonces el número de regiones de una representación plana de G es $r = e - v + 2$

menos

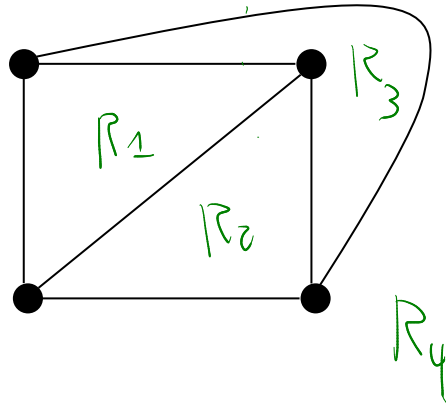
Teorema de rechazo: Es un teorema que rechaza que alguna información (Que es plano) sin embargo si pasa el teorema NO QUIERE DECIR QUE SEA PLANO.

Grafos

Teorema

Se G un grafo simple con e aristas y v vértices, entonces el **número de regiones** de una representación plana de G es $r=e-v+2$

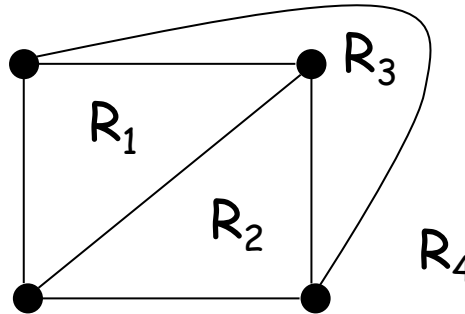
$$r = 6 - 4 + 2 = 4$$



Grafos

Teorema

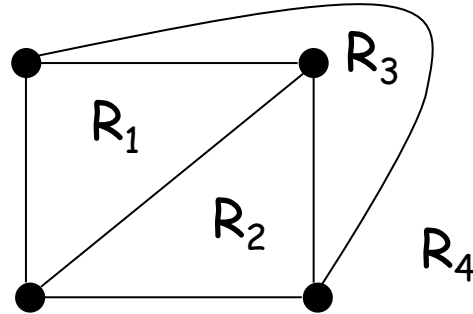
Se G un grafo simple con e aristas y v vértices, entonces el **número de regiones** de una representación plana de G es $r=e-v+2$



Grafos

Teorema

Se G un grafo simple con e aristas y v vértices, entonces el **número de regiones** de una representación plana de G es $r=e-v+2$



$$e=6$$

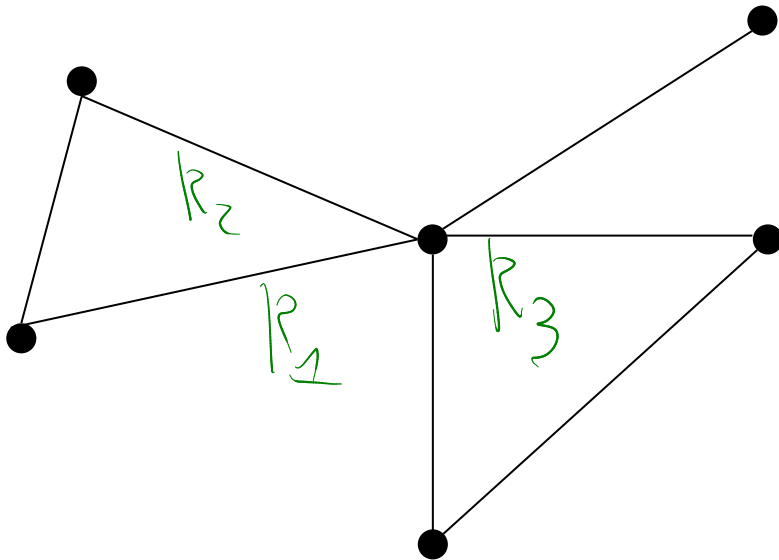
$$v=4$$

$$r=6-4+2=4$$

Grafos

Teorema

Se G un grafo simple con e aristas y v vértices, entonces el **número de regiones** de una representación plana de G es $r = e - v + 2$



$e=?$

$v=?$

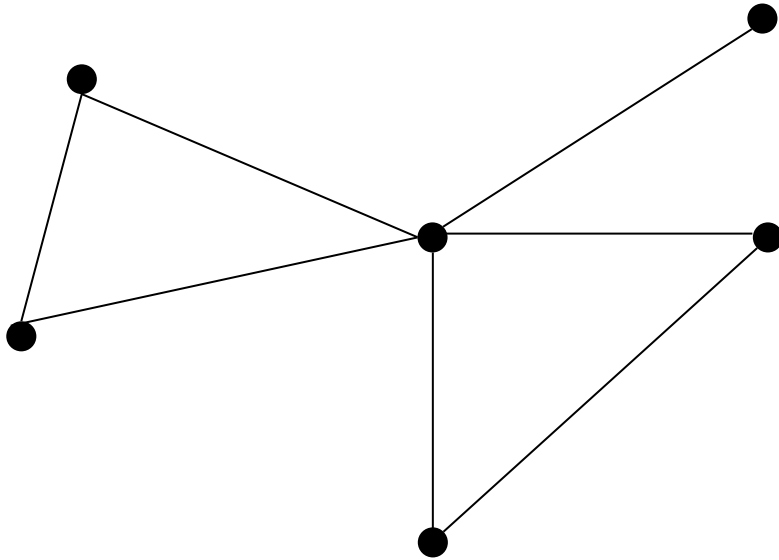
$r=?$

7
6
3

Grafos

Teorema

Sea G un grafo simple con e aristas y v vértices, entonces el **número de regiones** de una representación plana de G es $r=e-v+2$



$$e=7$$

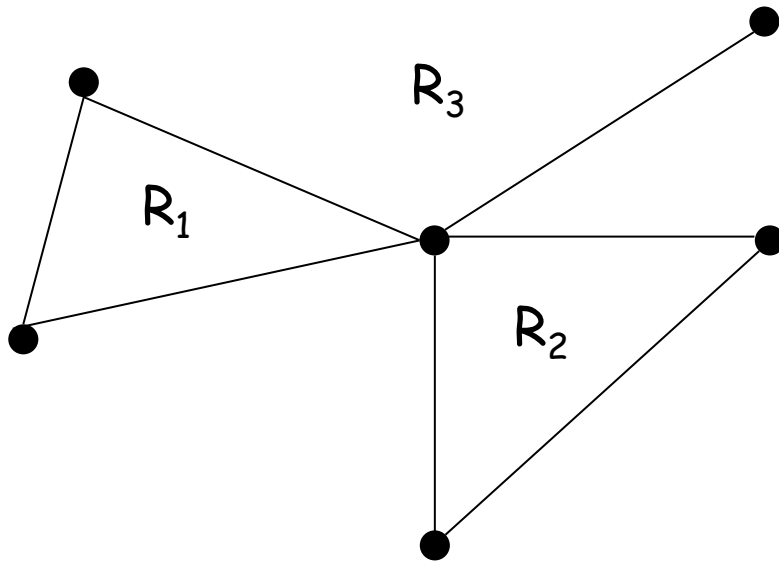
$$v=6$$

$$r=7-6+2=3$$

Grafos

Teorema

Sea G un grafo simple con e aristas y v vértices, entonces el **número de regiones** de una representación plana de G es $r=e-v+2$



$$e=7$$

$$v=6$$

$$r=7-6+2=3$$

Grafos

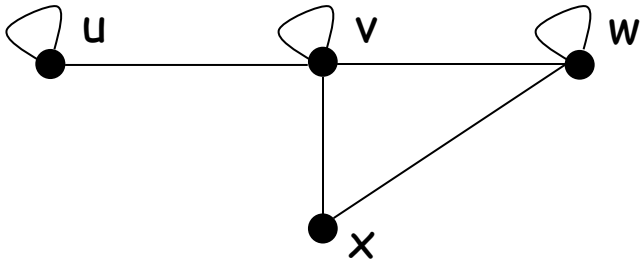
Matriz de adyacencia

Sea $G=(V,E)$ un grafo con n vértices, la **matriz de adyacencia** es la matriz booleana de $n \times n$ tal que:

$$a_{ij} = \begin{cases} 1 & \text{si } \{v_i, v_j\} \text{ es una arista de } G \\ 0 & \text{en caso contrario} \end{cases}$$

Grafos

Matriz de adyacencia



	u	v	w	x
u	1	1	0	0
v	1	1	1	1
w	0	1	1	1
x	0	1	1	0

Grafos

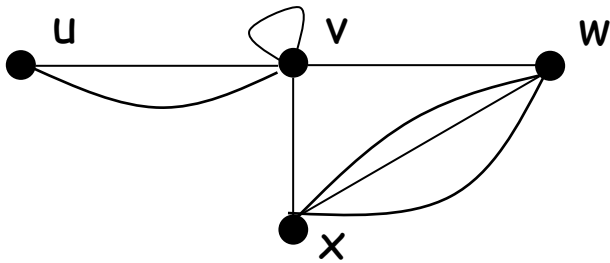
Matriz de adyacencia

La matriz de adyacencia de un grafo con aristas paralelas indica la cantidad de aristas que hay entre cada par de nodos v_i y v_j

Grafos

Matriz de adyacencia

La matriz de adyacencia de un grafo con aristas paralelas indica la cantidad de aristas que hay entre cada par de nodos v_i y v_j



	u	v	w	x
u	0	2	0	0
v	2	1	1	1
w	0	1	0	3
x	0	1	3	0

Grafos

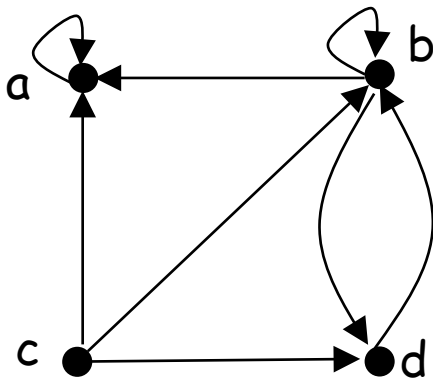
Matriz de adyacencia

La matriz de adyacencia de un **grafo dirigido** $G=(V,E)$ tiene un 1 en la posición (i,j) de la matriz si existe una arista que va de v_i a v_j

Grafos

Matriz de adyacencia

La matriz de adyacencia de un **grafo dirigido** $G=(V,E)$ tiene un 1 en la posición (i,j) de la matriz si existe una arista que va de v_i a v_j



Salida

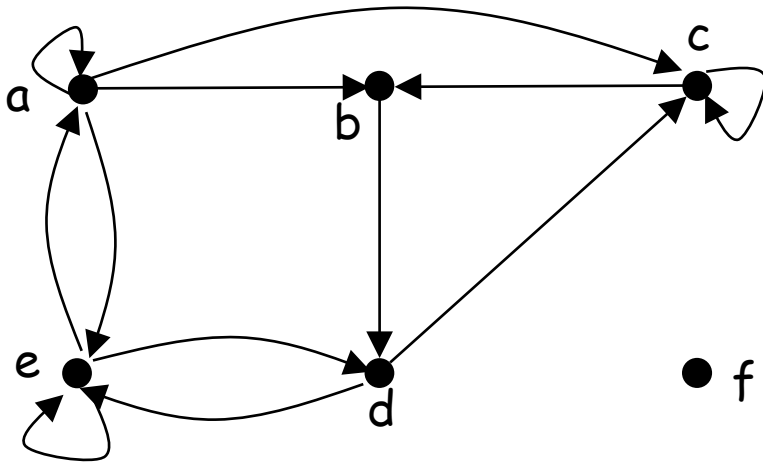
	a	b	c	d
a	1	0	0	0
b	1	1	0	1
c	1	1	0	1
d	0	1	0	0

Entrada

Grafos

Matriz de adyacencia

La matriz de adyacencia de un **grafo dirigido** $G=(V,E)$ tiene un 1 en la posición (i,j) de la matriz si existe una arista que va de v_i a v_j



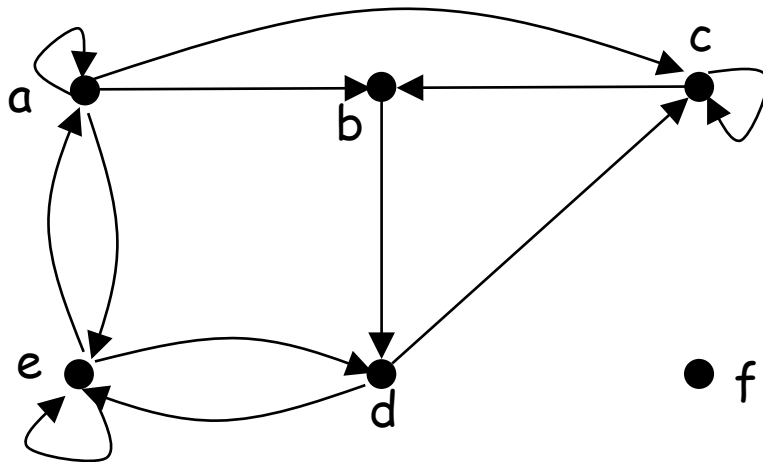
	a	b	c	d	e	f
a	1	1	0	0	1	0
b	0	0	0	1	0	0
c	0	1	1	0	0	0
d	0	0	1	0	1	0
e	1	0	0	1	1	0
f	0	0	0	0	0	0

Grafos



Matriz de adyacencia

La matriz de adyacencia de un **grafo dirigido** $G=(V,E)$ tiene un 1 en la posición (i,j) de la matriz si existe una arista que va de v_i a v_j



	a	b	c	d	e	f
a	1	1	1	0	1	0
b	1	0	0	1	0	0
c	0	1	1	0	0	0
d	0	0	1	0	1	0
e	1	0	0	1	1	0
f	0	0	0	0	0	0

Grafos

Teorema

Sea M_R la matriz de adyacencia de un grafo, se tiene que:

$$M_R \otimes M_R = M_R^2$$

\otimes es el producto booleano y M_R^2 es la matriz que indica si hay caminos de longitud 2 en el grafo

Grafos

Producto booleano de matrices

Dadas dos matrices A y B de órdenes $m \times k$ y $k \times n$, respectivamente, $A \otimes B$ es una matriz de orden $m \times n$ en la que cada elemento c_{ij} se calcula como:

$$c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \dots (a_{in} \wedge b_{nj})$$

Handwritten diagram illustrating the calculation of c_{ij} . It shows two matrices:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

The calculation for c_{ij} is shown as:

$$1 \times 9 + b \times 4 + c \times 7$$

Grafos

Producto booleano de matrices

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} (1 \wedge 1) \vee (0 \wedge 0) & 1 \vee 0 = 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Grafos

Producto booleano de matrices

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} (1 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 1) \vee (0 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \\ (0 \wedge 1) \vee (1 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \\ (1 \wedge 1) \vee (1 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \end{pmatrix}$$

Grafos

Producto booleano de matrices

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} (1 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 1) \vee (0 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \\ (0 \wedge 1) \vee (1 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \\ (1 \wedge 1) \vee (1 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} 1 \vee 0 & 1 \vee 0 & 0 \vee 0 \\ 0 \vee 0 & 0 \vee 1 & 0 \vee 1 \\ 1 \vee 0 & 1 \vee 1 & 0 \vee 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Grafos

Teorema

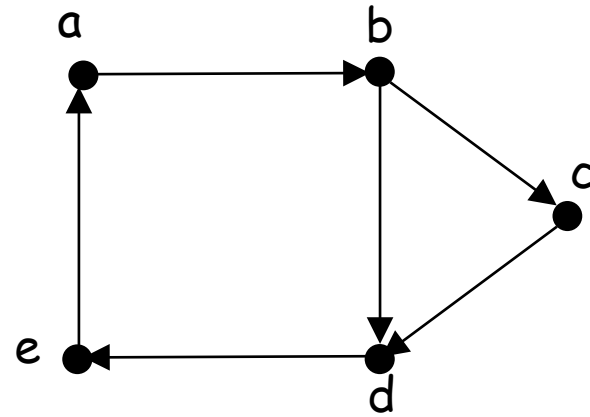
Sea M_R la matriz de adyacencia de un grafo, se tiene que:

$$M_R \otimes M_R = M_R^2$$

\otimes es el producto booleano y M_R^2 es la matriz que indica si hay caminos de longitud 2 en el grafo

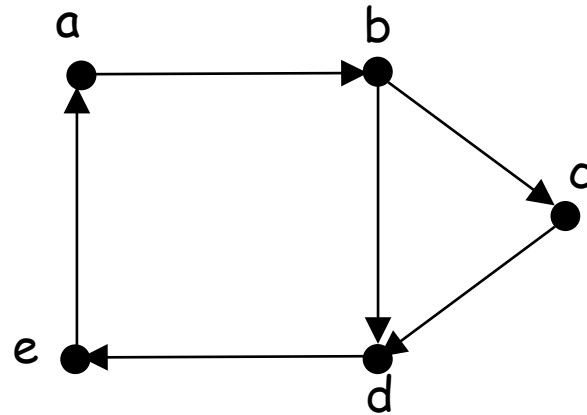
Grafos

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



Grafos

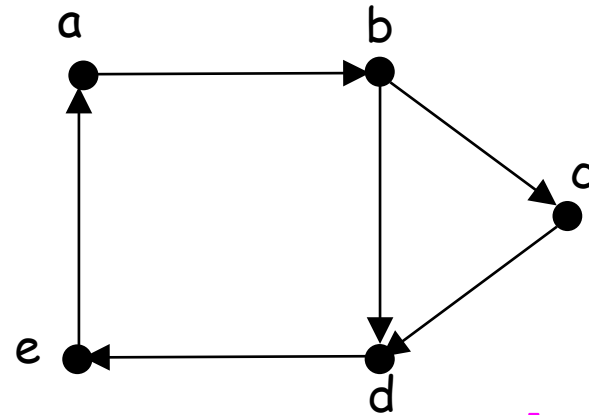
$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



$$M_R^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Grafos

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



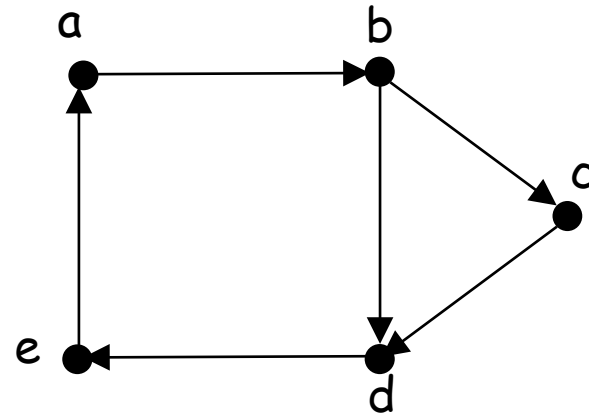
$$M_R^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Handwritten annotations in red and pink:

- Red box around the first row of the first matrix: $[0 \ 1 \ 0 \ 0 \ 0]$
- Pink box around the first column of the second matrix: $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$
- Pink box around the third column of the third matrix: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
- Red box around the first row of the third matrix: $[0 \ 0 \ 1 \ 1 \ 0]$
- Red text on the left: a, b, c, d, e (vertical)
- Red text on the right: a, b, c, d, e (vertical)
- Red text at the top right: a, b, c, d, e (horizontal)

Grafos

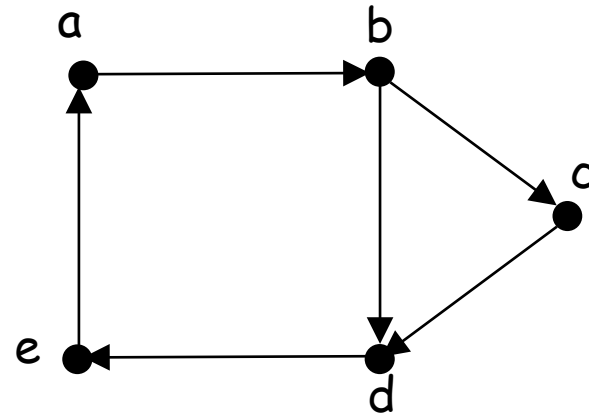
$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



$$M_R^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

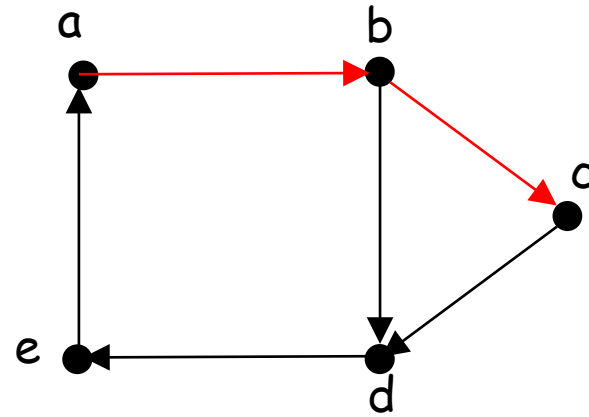


$$M_R^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Indica si hay caminos de longitud 2 en el grafo

Grafos

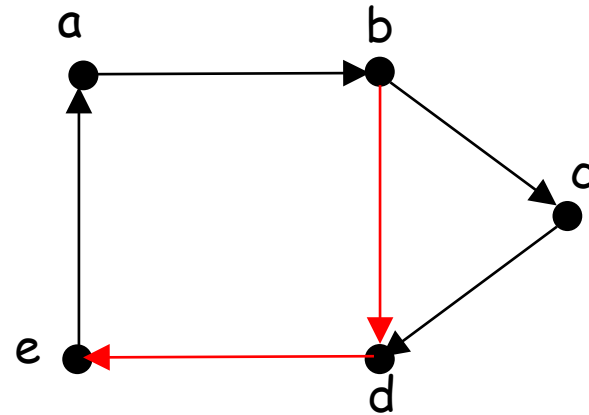
$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



$$M_R^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 0 & \mathbf{1} & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



$$M_R^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & \mathbf{1} \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

Teorema

Sea M_R la matriz de adyacencia de un grafo, se tiene que:

$$M_R \otimes M_R = M_R^2$$

$$M_R \otimes M_R \otimes M_R = M_R^3$$

$$M_R \otimes M_R \otimes M_R \otimes M_R = M_R^4$$

...

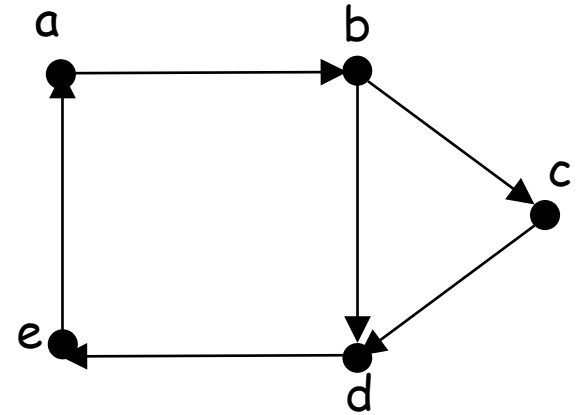
$$M_R \otimes M_R \otimes M_R \otimes \dots \otimes M_R = M_R^n$$

M_R^i es la matriz que indica si hay caminos de longitud i en el grafo

Grafos

$$M_R^3 = M_R^2 \otimes M_R$$

$$= \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

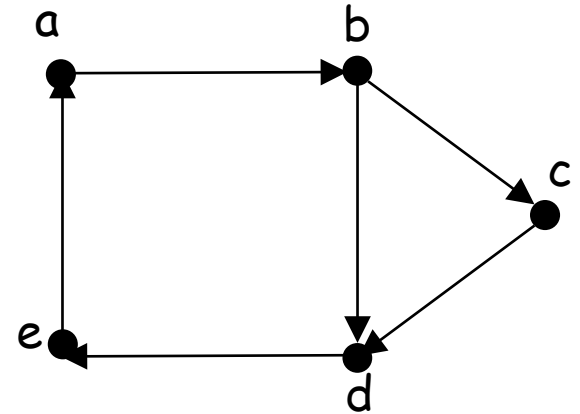


Grafos

$$M_R^3 = M_R^2 \otimes M_R$$

$$= \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{matrix} a & b & c & d & e \\ \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{matrix}$$



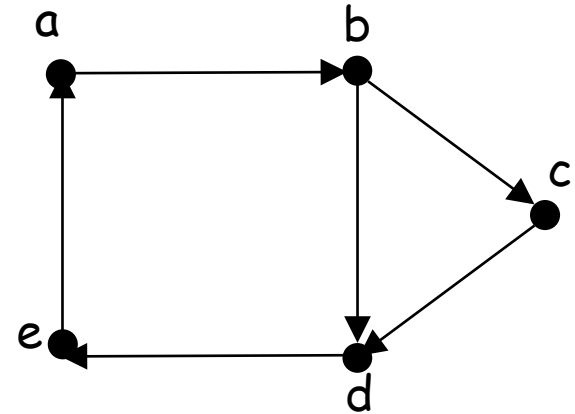
Grafos

$$M_R^3 = M_R^2 \otimes M_R$$

$$= \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{matrix}$$

Indica si hay caminos de longitud 3 en el grafo

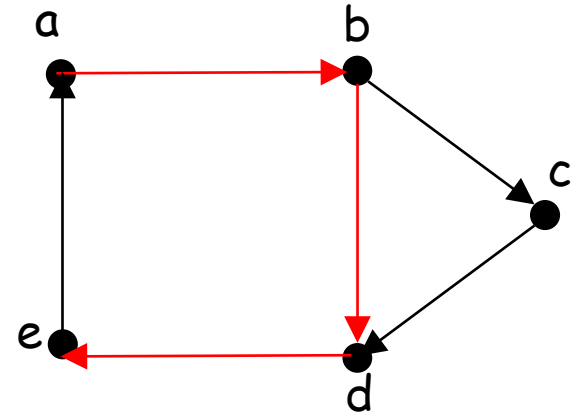


Grafos

$$M_R^3 = M_R^2 \otimes M_R$$

$$= \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 1 & \mathbf{1} \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{matrix}$$

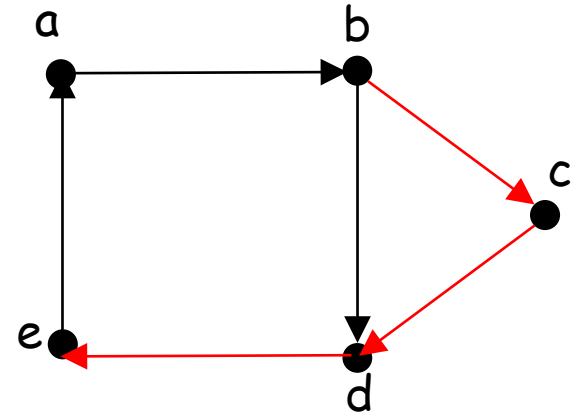


Grafos

$$M_R^3 = M_R^2 \otimes M_R$$

$$= \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & \mathbf{1} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{matrix}$$



Grafos

Matriz de conectividad

Se define como:

$$M_R^\infty = M_R \vee M_R^2 \vee M_R^3 \vee \dots \vee M_R^n$$

$$M_R^\infty = M_R \vee M_R^2 \vee M_R^3 \vee M_R^4 \vee \dots \vee M_R^n$$

$$M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_R^2 = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$M_R^3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Grafos

$$M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_R^2 = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$M_R^3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$M_R \vee M_R^2 \vee M_R^3 = \begin{matrix} & \begin{matrix} a & b & c & d & p \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ p \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{matrix}$$

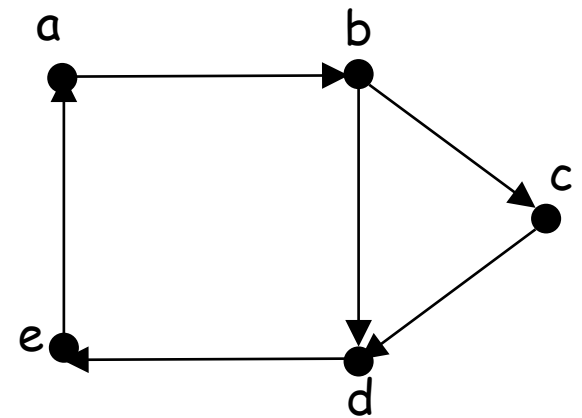
Grafos

$$M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_R^2 = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$M_R^3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

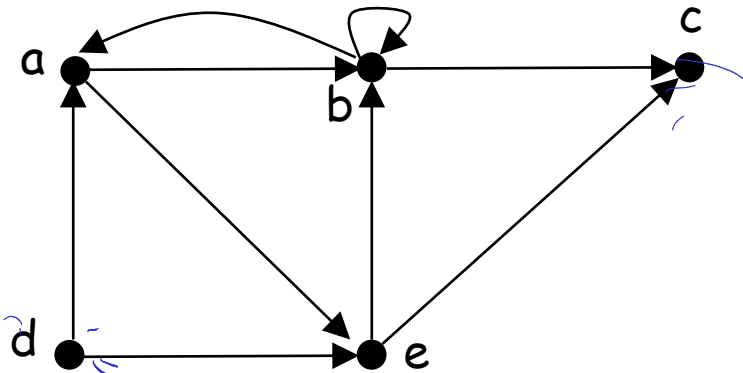
$$M_R \vee M_R^2 \vee M_R^3 = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 1 & 1 & 1 \\ b & 1 & 0 & 1 & 1 & 1 \\ c & 1 & 0 & 0 & 1 & 1 \\ d & 1 & 1 & 0 & 0 & 1 \\ e & 1 & 1 & 1 & 1 & 0 \end{array}$$



Indica si existe un camino de longitud 1, 2, ó 3 en el grafo

Grafos

Mostrar M_R y M_R^2 para el siguiente grafo:



	a	b	c	d	e
a	0	1	0	0	1
b	1	1	1	0	0
c	0	0	0	0	0
d	1	0	0	0	1
e	0	1	1	0	0

	a	b	c	d	e
a	1	1	1	0	0
b	1	1	1	0	1
c	0	0	0	0	0
d	0	1	1	0	1
e	1	1	1	0	0

$b-c$

$b-b-c$ ✓

$b-a$

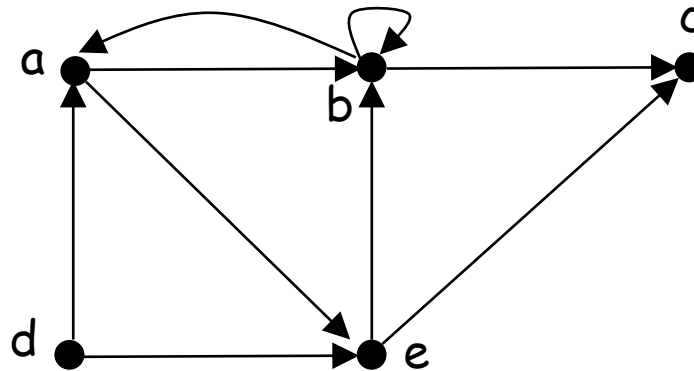
$b-a-e$

$d-d$

$d-e-f$

Grafos

Mostrar M_R y M_R^2 para el siguiente grafo:

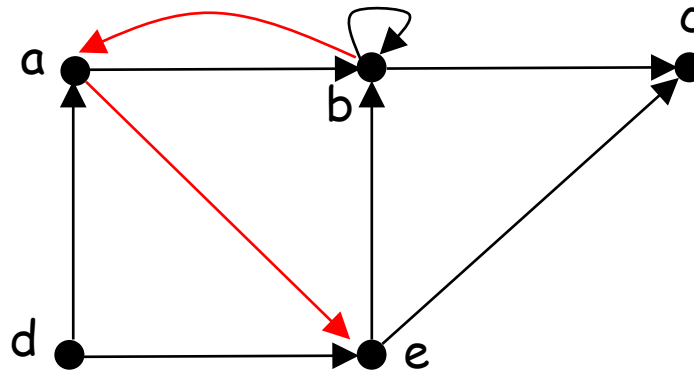


$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$M_R^2 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

Mostrar M_R y M_R^2 para el siguiente grafo:

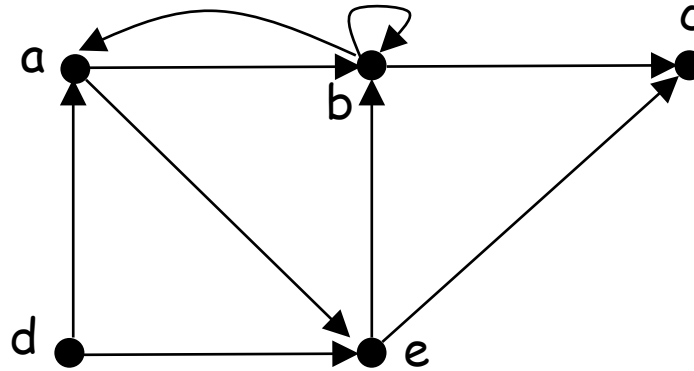


$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$M_R^2 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & \mathbf{1} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

Mostrar M_R y M_R^2 para el siguiente grafo:

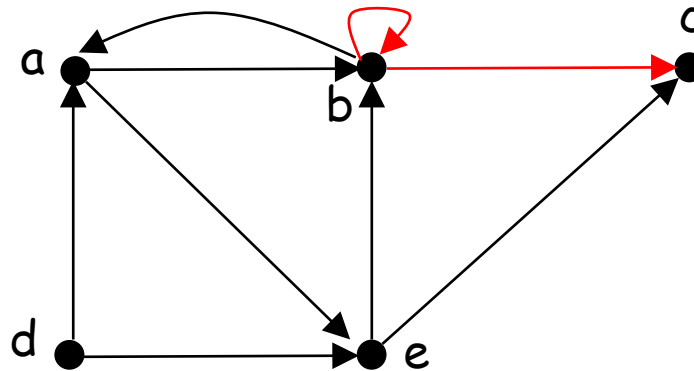


$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$M_R^2 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & \mathbf{1} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

Mostrar M_R y M_R^2 para el siguiente grafo:

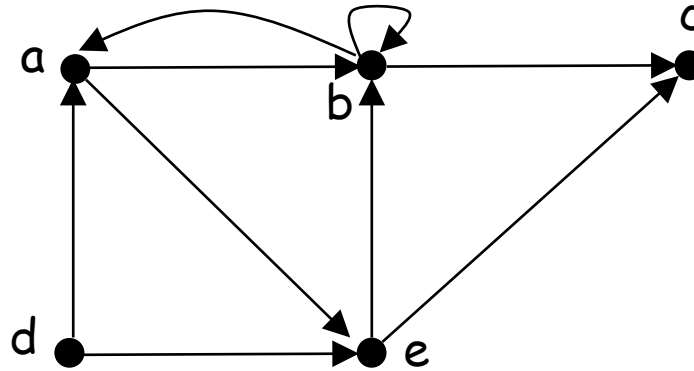


$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$M_R^2 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & \mathbf{1} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

Mostrar M_R^3

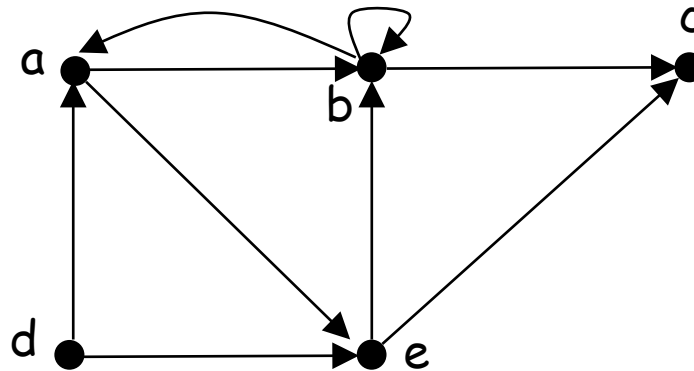


$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$M_R^2 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

Mostrar M_R^3



$d \rightarrow c$

$d \rightarrow a \rightarrow b \rightarrow c$
1 2 3

$d \rightarrow e \rightarrow b \rightarrow c$

$M_R^3 =$

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	0
e	1	1	1	0	1

$e \rightarrow a$

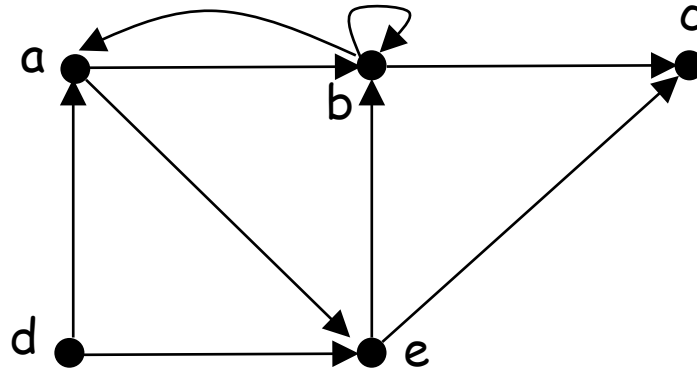
$a \rightarrow b \rightarrow e \rightarrow a$
1 2 3

$e \rightarrow e$

$e \rightarrow b \rightarrow a \rightarrow e$

Grafos

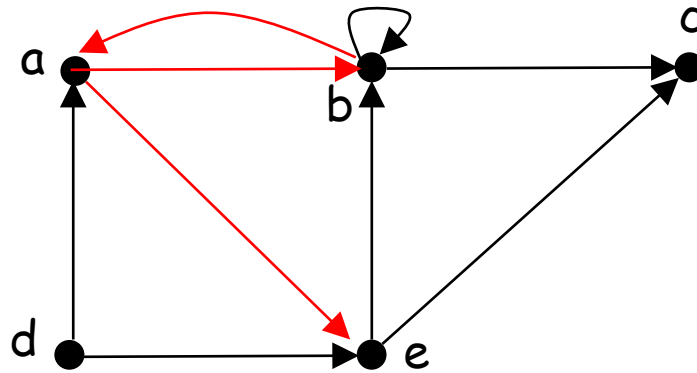
Mostrar M_R^3



$$M_R^3 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & \mathbf{1} \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

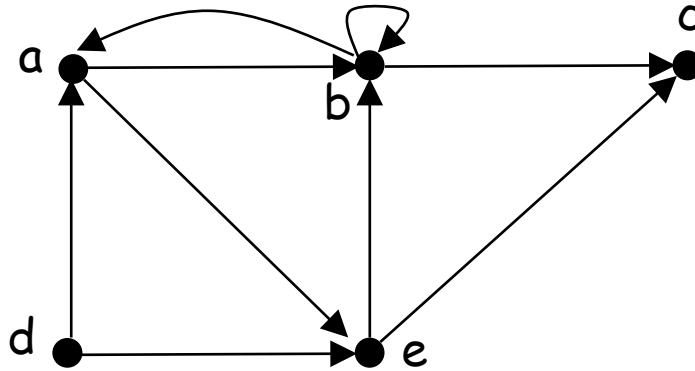
Mostrar M_R^3



$$M_R^3 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & \mathbf{1} \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

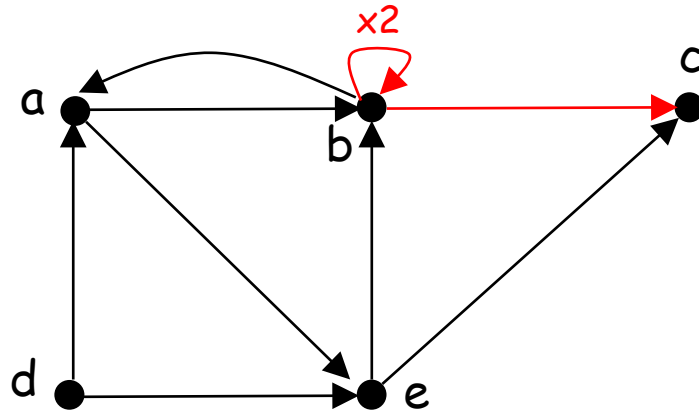
Mostrar M_R^3



$$M_R^3 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & \mathbf{1} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

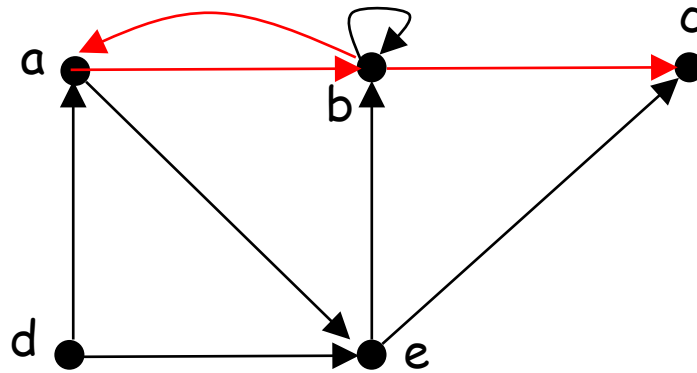
Mostrar M_R^3



$$M_R^3 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & \mathbf{1} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

Mostrar M_R^3

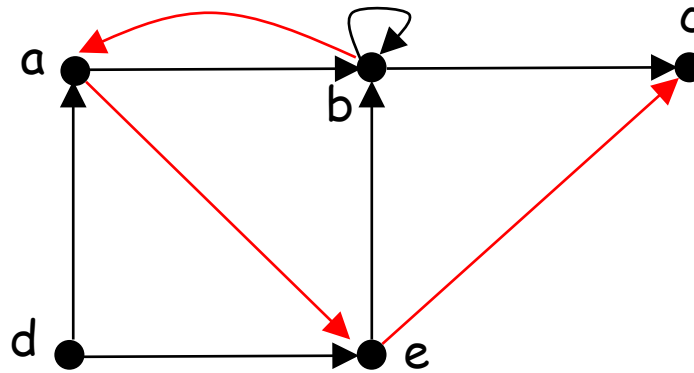


$$M_R^3 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & \mathbf{1} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

Mostrar M_R^3

$a-b$
 $a-b-b-a$
 $\quad 1 \quad 2 \quad 3 \quad 4$



$$M_R^3 = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & \mathbf{1} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

$$M_R^\infty = M_R \vee M_R^2 \vee M_R^3 \vee M_R^4$$

$\underbrace{\hspace{10em}}$
 correctado



Algoritmo de Warshall

Permite conocer la matriz de conectividad de un grafo realizando menos operaciones que con la multiplicación de matrices booleanas

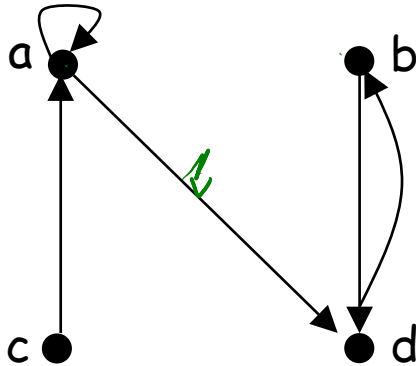
Grafos

Algoritmo de Warshall

$$M_R^\infty = M_R^1 \vee M_R^2 \vee M_R^3 \vee M_R^4 \quad n^3 \quad n^4$$

$4 \times 4 \times 4$

Permite conocer la matriz de conectividad de un grafo realizando menos operaciones que con la multiplicación de matrices booleanas

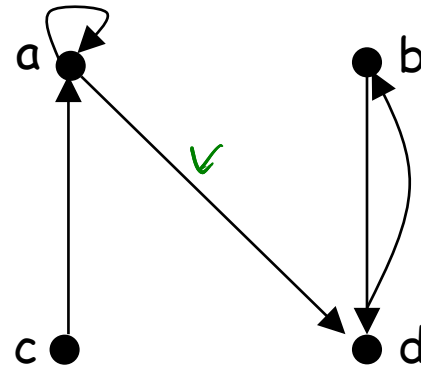


$$M_R^\infty = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix} \end{matrix}$$

Handwritten annotations: Green circles around the first row of the matrix. Green arrows pointing to the first row with labels 1, 1, 0, 1. A green arrow points to the first column with label 1.

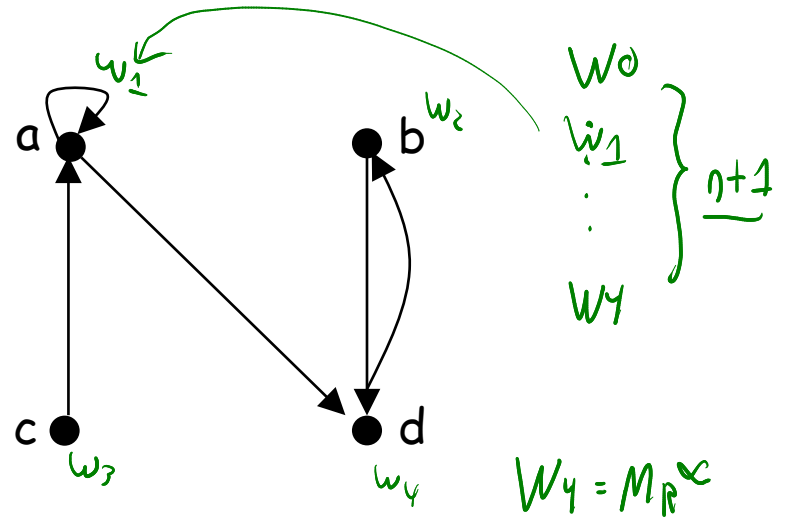
Grafos

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



Grafos

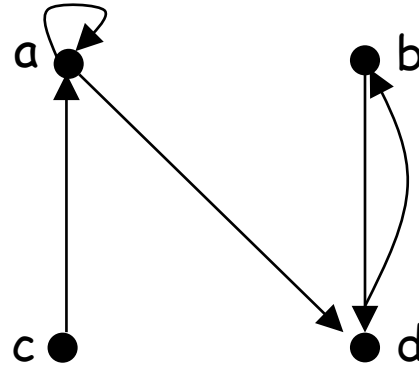
$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



Grafos

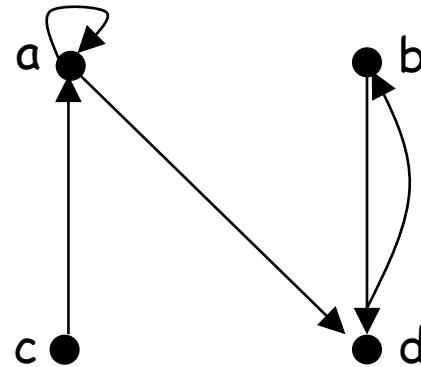
$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_1 (pivote a)



Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

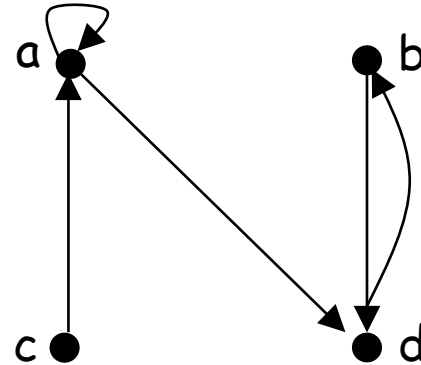


W_1 (pivote a)

$$W_1 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & & & \\ 1 & & & \\ 0 & & & \end{pmatrix} \end{matrix}$$

Grafos

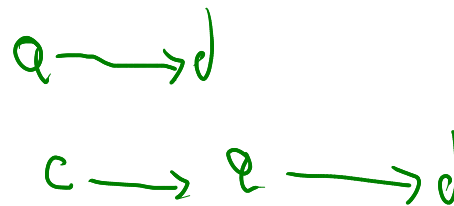
$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

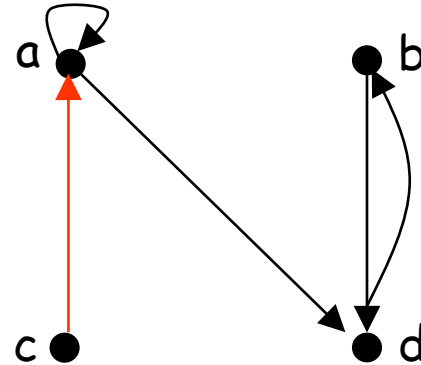
$$W_1 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & & & \\ 1 & & & \\ 0 & & & \end{pmatrix} \end{matrix}$$

The matrix shows the result of row operations with pivot 'a'. The first row is shaded light blue. The element '1' at row c, column d is circled in green, with a dashed arrow pointing from the circled '1' at row c, column a. The element '1' at row a, column d is also circled in green.



Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



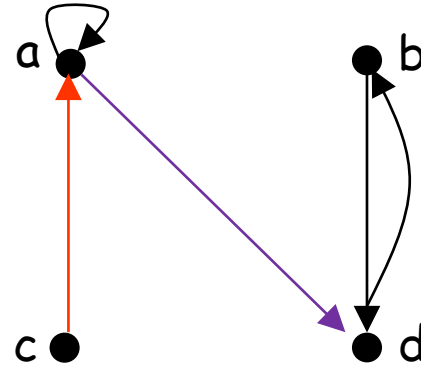
W_1 (pivote a)

$$W_1 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & & & \\ 1 & & & 1 \\ 0 & & & \end{pmatrix} \end{matrix}$$



Grafos

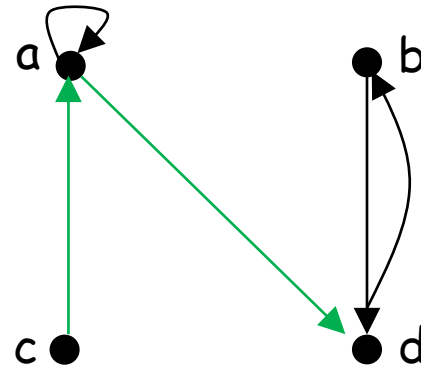
$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



$$W_1 \text{ (pivote a)} = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & & & \\ \color{red}{1} & & & 1 \\ 0 & & & \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



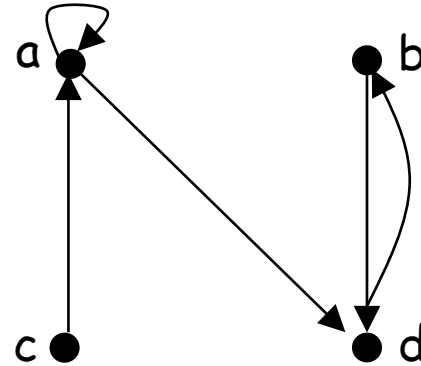
W_1 (pivote a)

$$W_1 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & & & \\ 1 & & & 1 \\ 0 & & & \end{pmatrix} \end{matrix}$$

1 significa que hay
un camino desde c
hasta d

Grafos

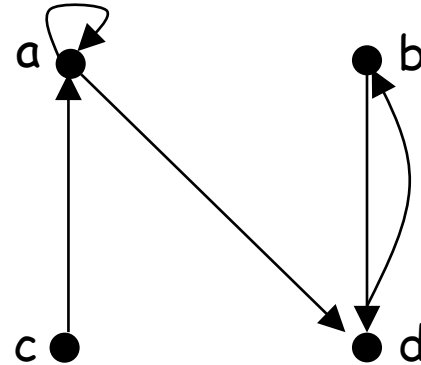
$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



$$W_1 \text{ (pivote a)} = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & & & \\ 1 & & & 1 \\ 0 & & & \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$W_1 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

$b \rightarrow d$ S_1
 $b \rightarrow a \rightarrow d$ X

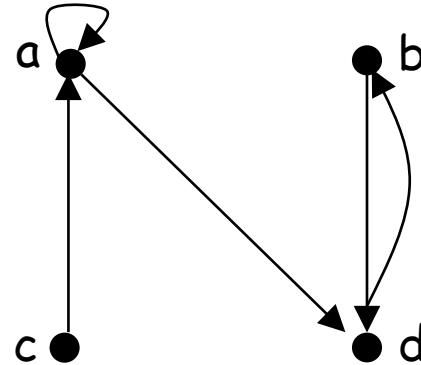
Se completa la matriz W_1 con las demás aristas de W_0

$$W_{ij'}^2 = W_{0ij'} \vee (W_{0ik} \wedge W_{0ik'})$$

$c \rightarrow b$ NO
 $c \rightarrow a \rightarrow b$ NO

Grafos

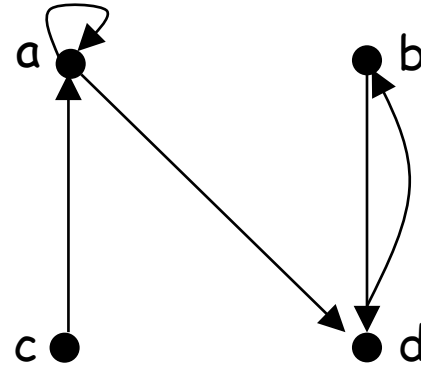
$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



$$W_1 \text{ (pivote a)} = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



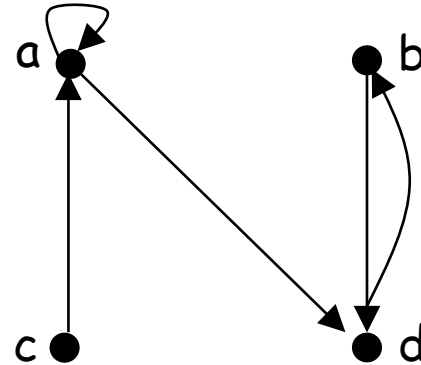
W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

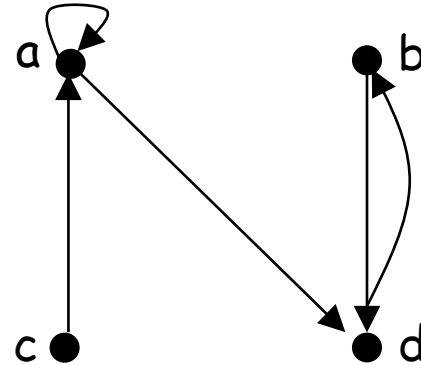


W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

$W_2 =$

$$W_0 = M_R =$$

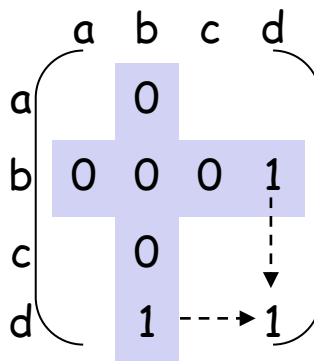


W_1 (pivot a)

	a	b	c	d
a	1	0	0	1
b	0	0	0	1
c	1	0	0	1
d	0	1	0	0

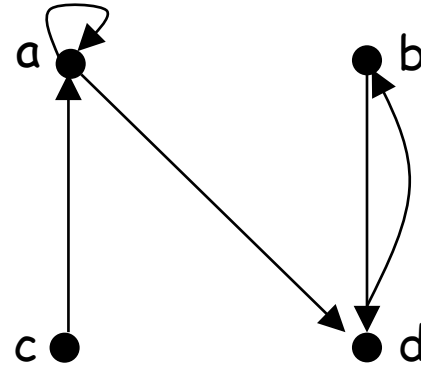
W_2 (pivote b)

$$W_2 =$$



Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

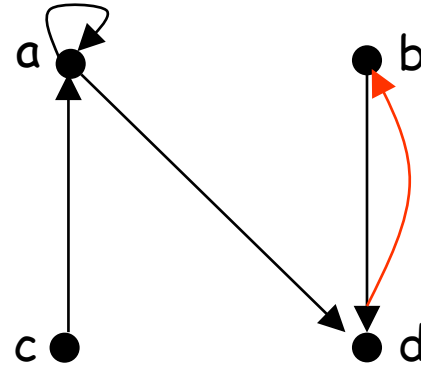
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$W_2 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

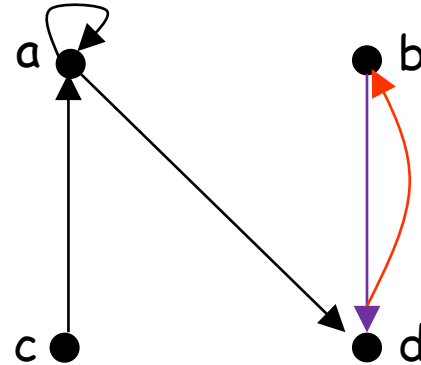
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$W_2 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

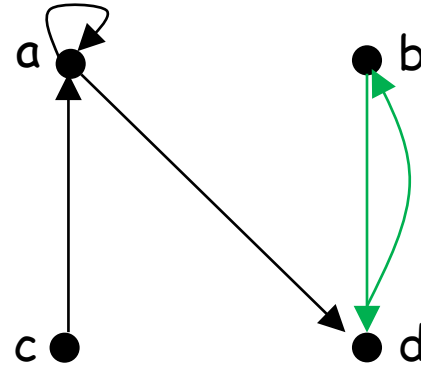
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$W_2 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

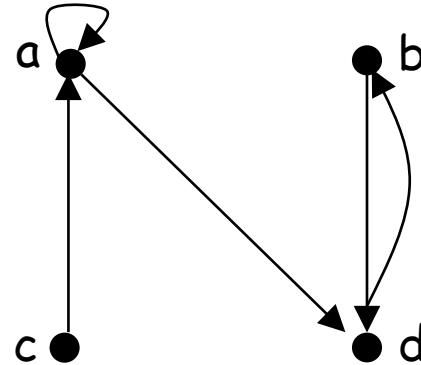
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$W_2 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & & & \\ 0 & 0 & 0 & 1 \\ 0 & & & \\ 1 & & & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

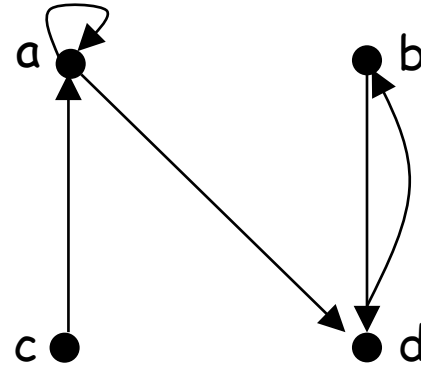
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$W_2 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

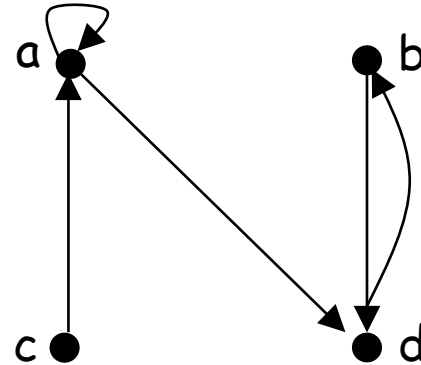
W_2 (pivote b)

$$W_2 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} & 0 & & \\ 0 & 0 & 0 & 1 \\ & 0 & & \\ & 1 & & 1 \end{pmatrix} \end{matrix}$$

Se completa la matriz W_2 con las demás aristas de W_1

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

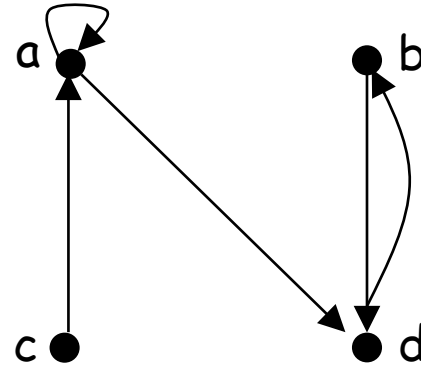
W_2 (pivote b)

$$W_2 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Se completa la matriz W_2 con las demás aristas de W_1

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

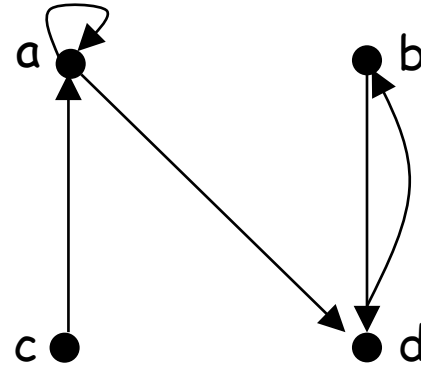
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

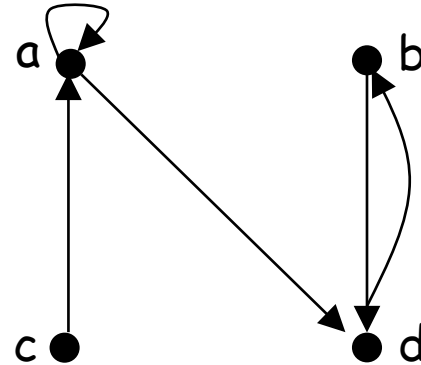
W_3 (pivote c)

$W_3 =$

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

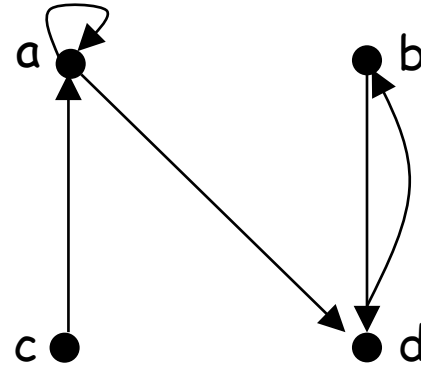
W_3 (pivote c)

$$W_3 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} & & 0 & \\ & & 0 & \\ 1 & 0 & 0 & 1 \\ & & 0 & \end{pmatrix} \end{matrix}$$

No se adicionan aristas!

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

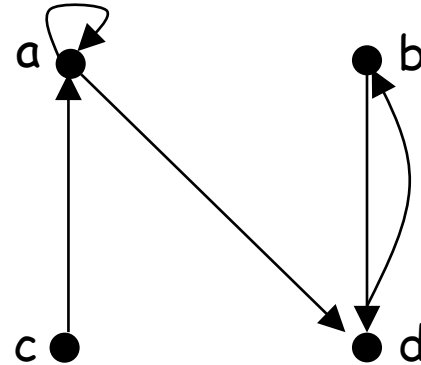
W_3 (pivote c)

$$W_3 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} & & 0 & \\ & & 0 & \\ 1 & 0 & 0 & 1 \\ & & 0 & \end{pmatrix} \end{matrix}$$

Se completa la matriz W_3 con las demás aristas de W_2

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

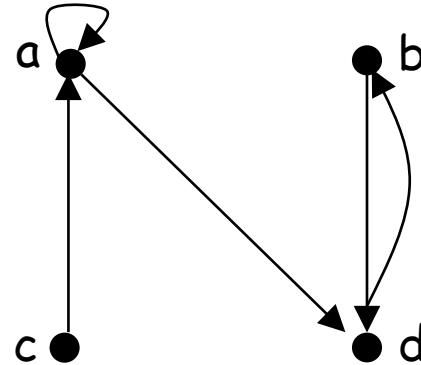
W_3 (pivote c)

$W_3 =$

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

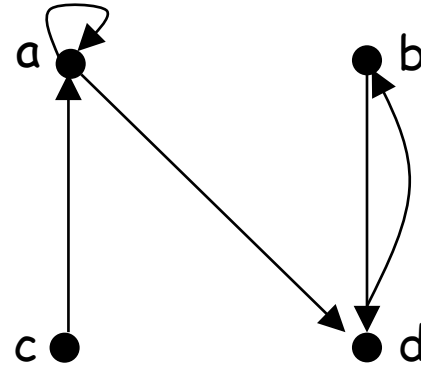
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

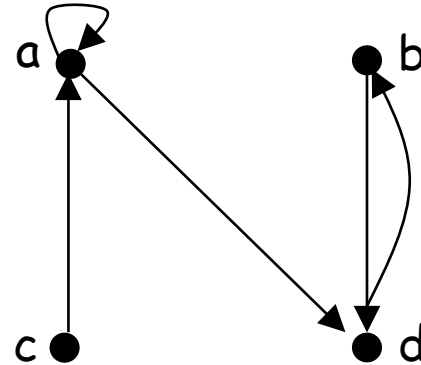
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_4 (pivote d)

$$W_4 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

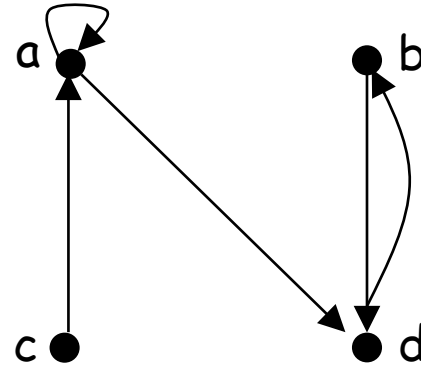
$$W_4 =$$

W_4 (pivote d)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} & 1 & \leftarrow & 1 \\ & & & 1 \\ & & & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

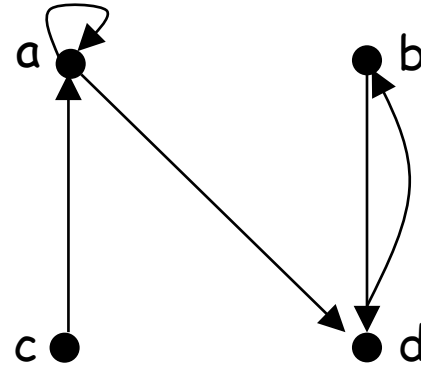
$$W_4 =$$

W_4 (pivote d)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} & 1 & & 1 \\ & 1 & \leftarrow & 1 \\ & & \uparrow & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

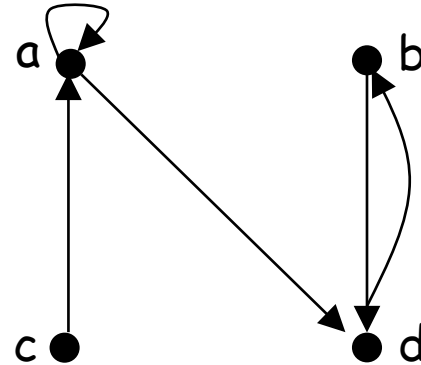
$$W_4 =$$

W_4 (pivote d)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} & 1 & & 1 \\ & 1 & & 1 \\ & 1 & \leftarrow & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

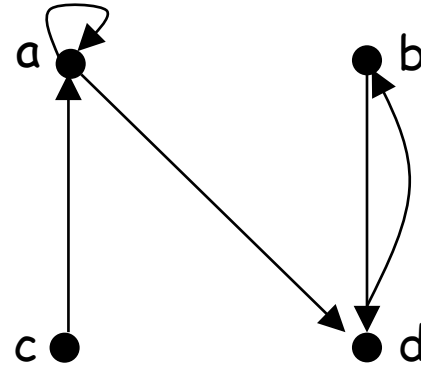
W_4 (pivote d)

$$W_4 = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} & 1 & & 1 \\ & 1 & & 1 \\ & 1 & & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Se completa la matriz W_4 con las demás aristas de W_3

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

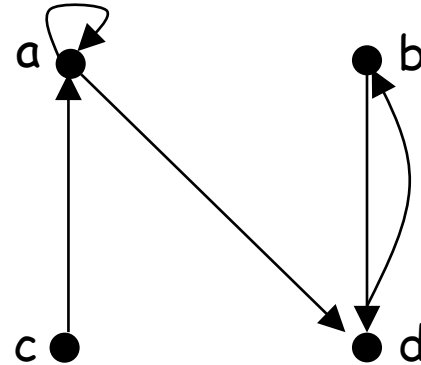
$$W_4 =$$

W_4 (pivote d)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

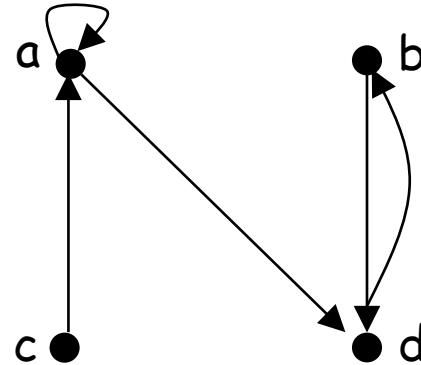
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_4 (pivote d)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

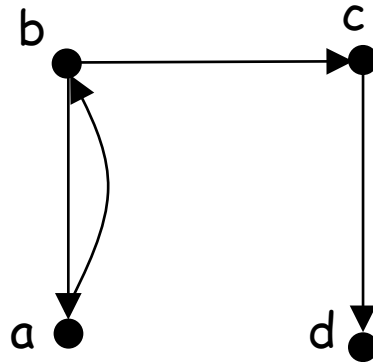
W_4 (pivote d)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

$$= M_R^\infty$$

Grafos

Aplicar el algoritmo de Warshall



$$W_{ij}^k = \begin{cases} M_R & k=0 \\ W_{ij}^{k-1} \vee (W_{ik}^{k-1} \wedge W_{kj}^{k-1}) \end{cases}$$

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix} = W_0$$

$$W^1: \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 1 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

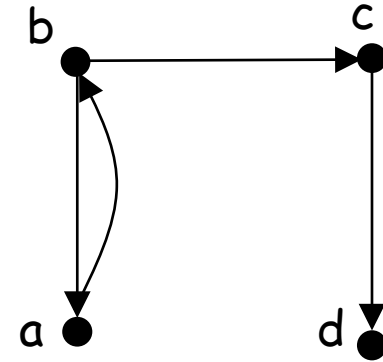
$$W^2: \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 0 \\ b & 1 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

$$W^3: \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 1 & 1 & 1 & 1 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

$$W_4: \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 1 & 1 & 1 & 1 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

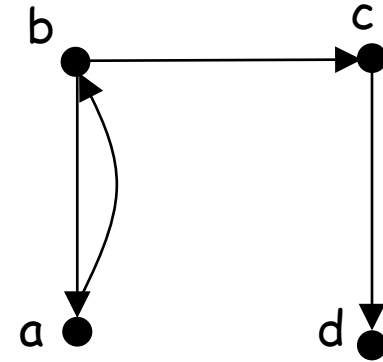


W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

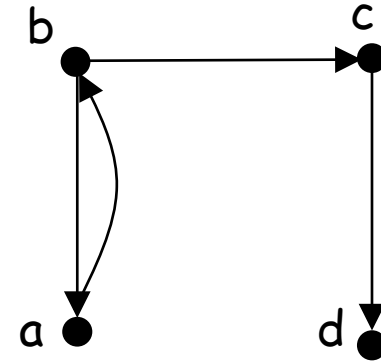
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

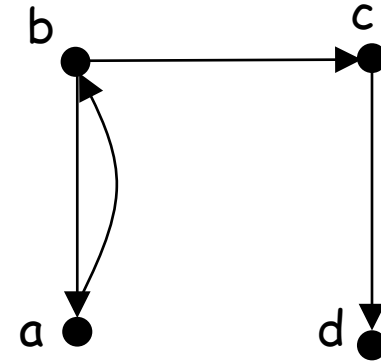
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

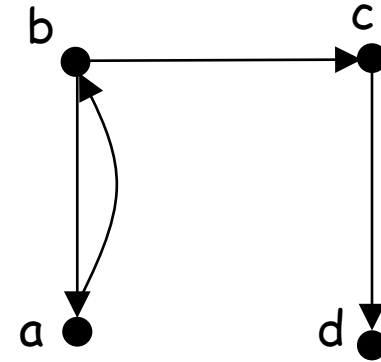
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_4 (pivote d)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$



W_1 (pivote a)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_2 (pivote b)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_3 (pivote c)

$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

W_4 (pivote d)

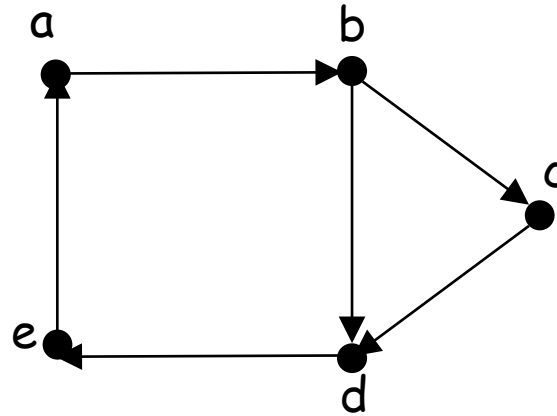
$$\begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$= M_R^\infty$$

Matriz de
conectividad

Grafos

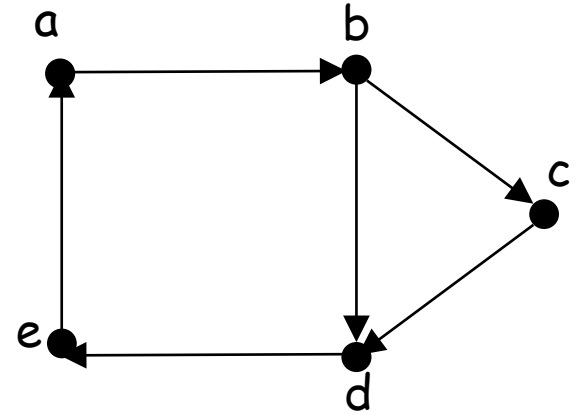
Aplicar el algoritmo de Warshall



$$M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

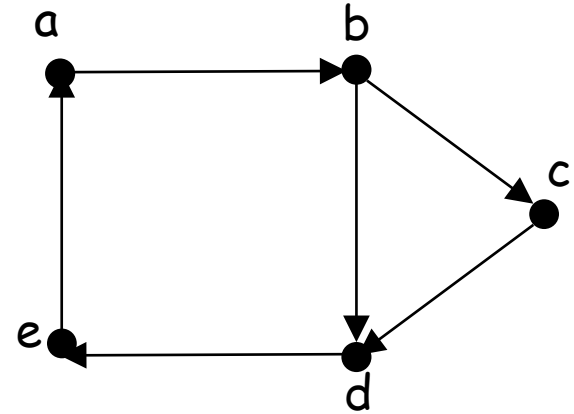


W_1 (pivote a)

	a	b	c	d	e
a	0	1	0	0	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	0	0	0

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

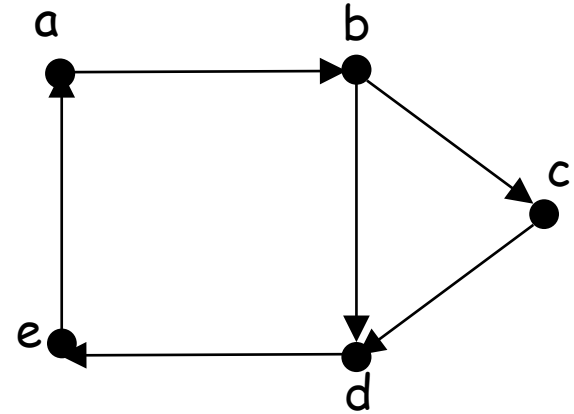
W_2 (pivote b)

$$\begin{matrix} & a & b & c & d & e \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$\begin{matrix} & a & b & c & d & e \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

	a	b	c	d	e
a	0	1	0	0	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	0	0	0

W_2 (pivote b)

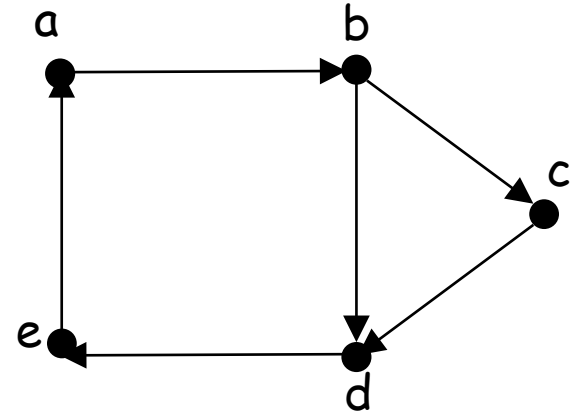
	a	b	c	d	e
a	0	1	1	1	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	1	1	0

W_3 (pivote c)

	a	b	c	d	e
a	0	1	1	1	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	1	1	0

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

	a	b	c	d	e
a	0	1	0	0	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	0	0	0

W_2 (pivote b)

	a	b	c	d	e
a	0	1	1	1	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	1	1	0

W_3 (pivote c)

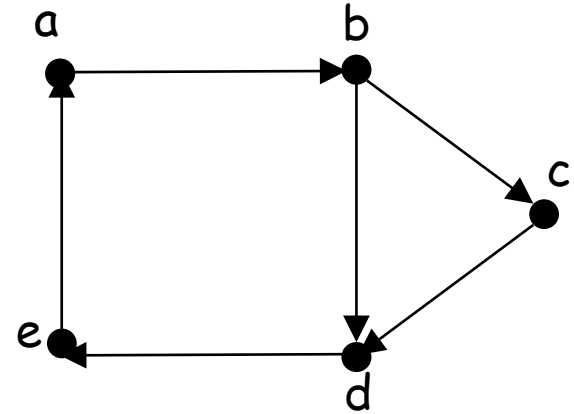
	a	b	c	d	e
a	0	1	1	1	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	1	1	0

W_4 (pivote d)

	a	b	c	d	e
a	0	1	1	1	1
b	0	0	1	1	1
c	0	0	0	1	1
d	0	0	0	0	1
e	1	1	1	1	1

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

	a	b	c	d	e
a	0	1	0	0	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	0	0	0

W_2 (pivote b)

	a	b	c	d	e
a	0	1	1	1	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	1	1	0

W_3 (pivote c)

	a	b	c	d	e
a	0	1	1	1	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	1	1	0

W_4 (pivote d)

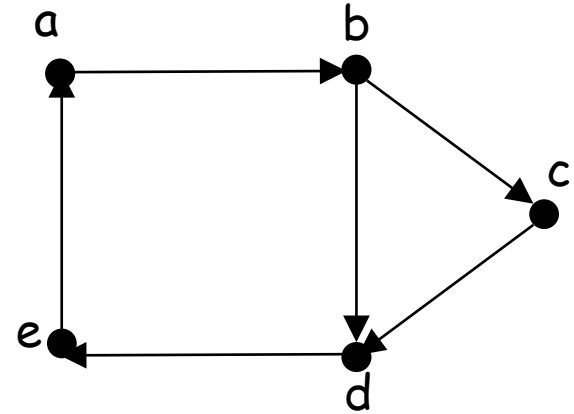
	a	b	c	d	e
a	0	1	1	1	1
b	0	0	1	1	1
c	0	0	0	1	1
d	0	0	0	0	1
e	1	1	1	1	1

W_5 (pivote e)

	a	b	c	d	e
a	1	1	1	1	1
b	1	1	1	1	1
c	1	1	1	1	1
d	1	1	1	1	1
e	1	1	1	1	1

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

	a	b	c	d	e
a	0	1	0	0	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	0	0	0

W_2 (pivote b)

	a	b	c	d	e
a	0	1	1	1	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	1	1	0

W_3 (pivote c)

	a	b	c	d	e
a	0	1	1	1	0
b	0	0	1	1	0
c	0	0	0	1	0
d	0	0	0	0	1
e	1	1	1	1	0

W_4 (pivote d)

	a	b	c	d	e
a	0	1	1	1	1
b	0	0	1	1	1
c	0	0	0	1	1
d	0	0	0	0	1
e	1	1	1	1	1

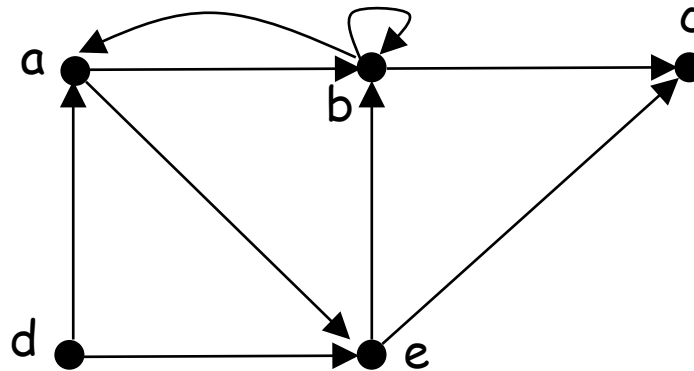
W_5 (pivote e)

	a	b	c	d	e
a	1	1	1	1	1
b	1	1	1	1	1
c	1	1	1	1	1
d	1	1	1	1	1
e	1	1	1	1	1

Matriz de
conectividad

Grafos

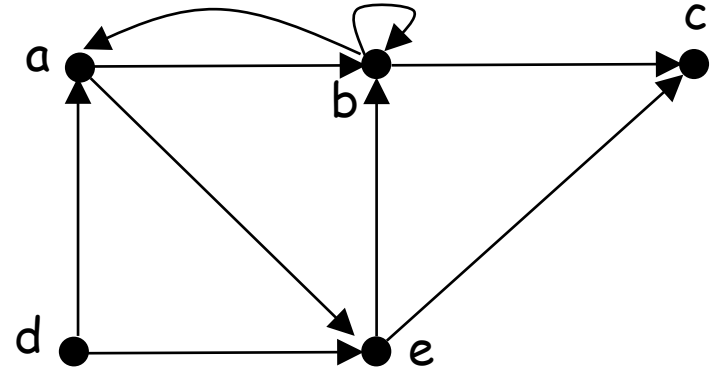
Aplicar el algoritmo de Warshall



$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

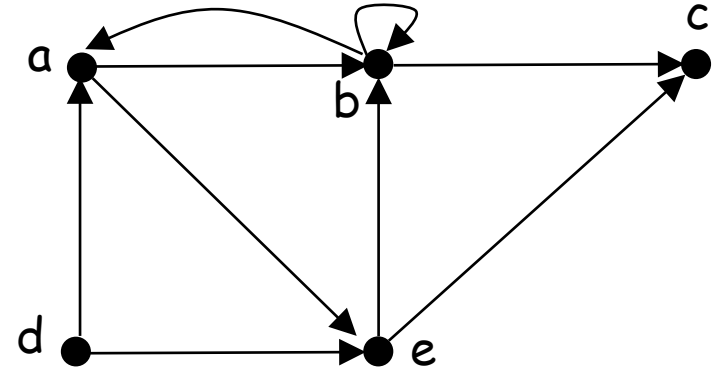


W_1 (pivote a)

	a	b	c	d	e
a	0	1	0	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	0	0	1
e	0	1	1	0	0

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

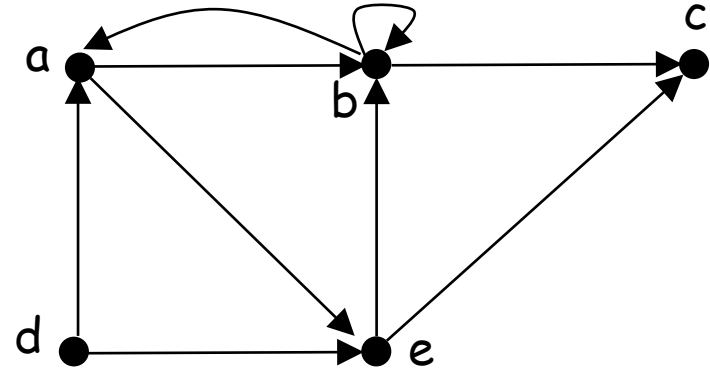
W_2 (pivote b)

	a	b	c	d	e
a	0	1	0	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	0	0	1
e	0	1	1	0	0

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

W_2 (pivote b)

W_3 (pivote c)

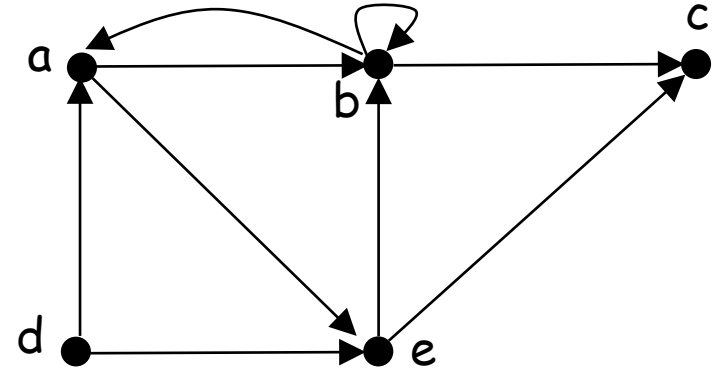
	a	b	c	d	e
a	0	1	0	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	0	0	1
e	0	1	1	0	0

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

	a	b	c	d	e
a	0	1	0	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	0	0	1
e	0	1	1	0	0

W_2 (pivote b)

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

W_3 (pivote c)

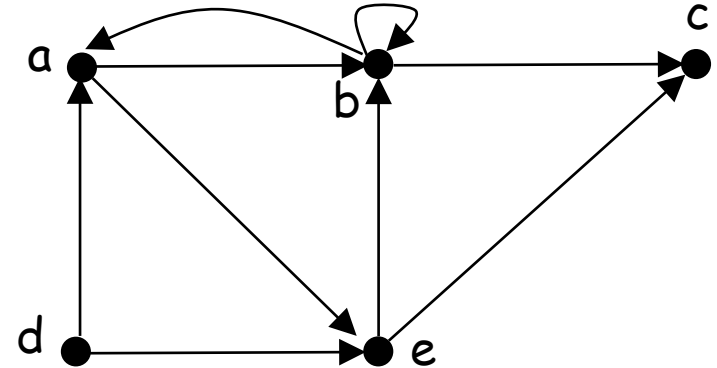
	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

W_4 (pivote d)

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$



W_1 (pivote a)

	a	b	c	d	e
a	0	1	0	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	0	0	1
e	0	1	1	0	0

W_2 (pivote b)

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

W_3 (pivote c)

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

W_4 (pivote d)

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

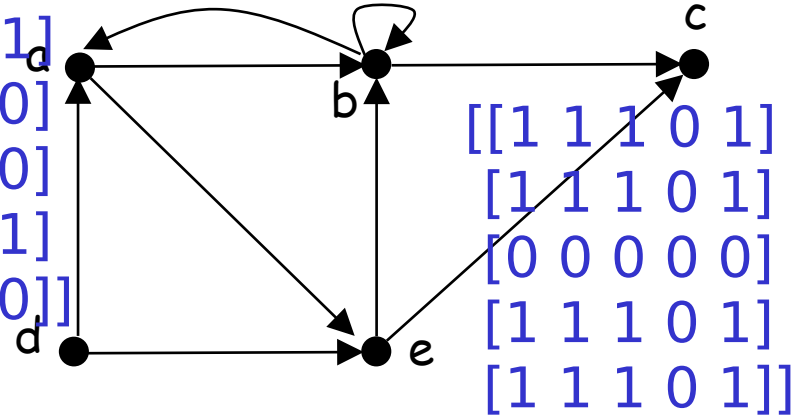
W_5 (pivote e)

	a	b	c	d	e
a	1	1	1	0	1
b	1	1	1	0	1
c	0	0	0	0	0
d	1	1	1	0	1
e	1	1	1	0	1

Grafos

$$W_0 = M_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$



W_1 (pivote a)

W_2 (pivote b)

W_3 (pivote c)

W_4 (pivote d)

W_5 (pivote e)

$$\begin{matrix} & a & b & c & d & e \\ a & 0 & 1 & 0 & 0 & 1 \\ b & 1 & 1 & 1 & 0 & 0 \\ c & 0 & 0 & 0 & 0 & 0 \\ d & 1 & 1 & 0 & 0 & 1 \\ e & 0 & 1 & 1 & 0 & 0 \end{matrix}$$

$$\begin{matrix} & a & b & c & d & e \\ a & 1 & 1 & 1 & 0 & 1 \\ b & 1 & 1 & 1 & 0 & 1 \\ c & 0 & 0 & 0 & 0 & 0 \\ d & 1 & 1 & 1 & 0 & 1 \\ e & 1 & 1 & 1 & 0 & 1 \end{matrix}$$

$$\begin{matrix} & a & b & c & d & e \\ a & 1 & 1 & 1 & 0 & 1 \\ b & 1 & 1 & 1 & 0 & 1 \\ c & 0 & 0 & 0 & 0 & 0 \\ d & 1 & 1 & 1 & 0 & 1 \\ e & 1 & 1 & 1 & 0 & 1 \end{matrix}$$

$$\begin{matrix} & a & b & c & d & e \\ a & 1 & 1 & 1 & 0 & 1 \\ b & 1 & 1 & 1 & 0 & 1 \\ c & 0 & 0 & 0 & 0 & 0 \\ d & 1 & 1 & 1 & 0 & 1 \\ e & 1 & 1 & 1 & 0 & 1 \end{matrix}$$

$$\begin{matrix} & a & b & c & d & e \\ a & 1 & 1 & 1 & 0 & 1 \\ b & 1 & 1 & 1 & 0 & 1 \\ c & 0 & 0 & 0 & 0 & 0 \\ d & 1 & 1 & 1 & 0 & 1 \\ e & 1 & 1 & 1 & 0 & 1 \end{matrix}$$

$$w^0[k, i] \wedge w^0[j, k]$$

Matriz de
conectividad