Fundamentos de análisis y diseño de algoritmos

Ecuaciones de recurrencia

Recurrencias

Método de iteración

Método maestro*

Método de sustitución

Recurrencias

Método de iteración

Expandir la recurrencia y expresarla como una suma de términos que dependen de n y de las condiciones iniciales

Recurrencias

$T(n) = n + 3T(n/4)$, $T(1)= \Theta(1)$ y n par

Expandir la recurrencia 2 veces

$$T\left(\frac{n}{4}\right) = \frac{n}{4} + 3T\left(\frac{n}{4^2}\right)$$

$$T(n) = n + 3\left(\frac{n}{4} + 3T\left(\frac{n}{4^2}\right)\right)$$

$$T\left(\frac{n}{4^2}\right) = \frac{n}{4^2} + 3T\left(\frac{n}{4^3}\right)$$

$$T(n) = n + 3\left(\frac{n}{4} + 3\left(\frac{n}{4^2} + 3T\left(\frac{n}{4^3}\right)\right)\right)$$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ \ + 3T(n/64)\,))$

$n + 3{*}n/4 + 3^2{*}n/4^2\ \ + 3^3T(n/4^3)$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ + 3T(n/64)\,))$

$n + 3*n/4 + 3^2*n/4^2\ + 3^3T(n/4^3)$

$T(1) = \Theta(1)$

$T(1) = c$

¿Cuándo se detienen las iteraciones?

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16 \;+ 3T(n/64)\,))$

$n + 3{*}n/4 + 3^2{*}n/4^2 \;+ 3^3T(n/4^3)$

$\frac{n}{4^x} = 1$

¿Cuándo se detienen las iteraciones?

Cuando se llega a T(1)

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$ $\quad 4^2$

$n + 3\,(\,n/4 + 3(n/16 \;+ 3T(n/64)\,))$

$n + 3{*}n/4 + 3^2{*}n/4^2 \;+ 3^3T(n/4^3)$

It = 1

It = 2

It = 3

¿Cuándo se detienen las iteraciones?

Cuando se llega a T(1), esto es, cuando $(n/4^i)=1$

$$\boxed{\frac{n}{4^i} = 1} \qquad n = 4^i \qquad \boxed{\log_4(n) = i}$$

$$T(n) = n + 3T\left(\frac{n}{4^1}\right) \quad i=1 \quad T(1)=\Theta(1) \quad i = \log_4(n)$$

$$T(n) = n + 3\left(\frac{n}{4} + 3T\left(\frac{n}{4^2}\right)\right) \quad i=2$$

if n=1
return —
else
rec

$$T(n) = n + \frac{3n}{4} + 3^2 T\left(\frac{n}{4^2}\right)$$

$$i = 3$$

$$T(n) = n + \frac{3}{4}n + 3^2\left(\frac{n}{4^2} + 3T\left(\frac{n}{4^3}\right)\right)$$

$$T(n) = n + \frac{3}{4}n + \frac{3^2}{4^2}n + 3^3 T\left(\frac{n}{4^3}\right)$$

$$T(n) = \frac{3^0}{4^0}n + \frac{3}{4}n + \frac{3^2}{4^2}n + \frac{3^3}{4^3}n + \dots + 3^i T\left(\frac{n}{4^i}\right)$$

$$T(n) = \frac{3^0}{4^0}n + \frac{3}{4}n + \frac{3^2}{4^2}n + \frac{3^3}{4^3}n + \dots + 3^{\log_4(n)} \times T(1)$$

$$\log_4(3) = \frac{\log_{10}(3)}{\log_{10}(4)} \qquad b^{\log_c(a)} = a^{\log_c(b)}$$

$$2^{\log_2(5)} \rightarrow 5^{\log_2(2)} = 5^1$$

$$T(n) = \frac{3^0}{4^0}n + \frac{3}{4}n + \frac{3^2}{4^2}n + \frac{3^3}{4^3}n + \dots + n^{\log_4(3)} \times C$$

$$\frac{3^{\log_4(n)-1}}{4^{\log_4(n)-1}} \times n$$

$$T(n) = n\sum_{i=0}^{\log_4(n)-1}\left(\frac{3}{4}\right)^i + cn^{\log_4(3)}$$

$$\sum_{i=0}^{n} \overset{1}{a}\, \underset{\frac{3}{4}}{r}^i = \frac{ar^{n+1} - a}{r - 1}$$

$$T(n) = n\left(\frac{\left(\frac{3}{4}\right)^{\log_4(n)} - 1}{3/4 - 1}\right) + cn^{\log_4(3)}$$

$$T(n) = n\left(\frac{n^{\log_4\left(\frac{3}{4}\right)} - 1}{\boxed{3/4 - 1}}\right) + cn^{\log_4(3)}$$

$$T(n) = \frac{n^{\overset{-0.2}{\log_4\left(\frac{3}{4}\right)} + 1} - n}{3/4 - 1} + cn^{\log_4(3)}$$

$$\boxed{T(n) = \underset{c}{n^{0.8}} - \boxed{n} + cn^{0.78}} \rightarrow O(n)$$

$$c < 0$$

$$\boxed{T(n) \le c \times n}$$

Recurrencias

$T(1) = \Theta(1)$

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ + 3T(n/64)\,))$

$n + 3*n/4 + 3^2*n/4^2\ + 3^3T(n/4^3)$

$n + 3*n/4 + 3^2*n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}T(1)$

¿Cuándo se detienen las iteraciones?

Cuando se llega a T(1), esto es, cuando $(n/4^i)=1$

$$T(n) = 3T(n/4) + n \qquad i=1 \quad T(1) = \Theta(1) = c$$

$$T(n) = 3\left(3T(n/4^2) + \frac{n}{4}\right) + n \qquad i=2$$

$$T(n) = 3\left(3\left(3T(n/4^3) + \frac{n}{4^2}\right) + \frac{n}{4}\right) + n \qquad i=3$$

$$T(n) = 3^3 T\left(\frac{n}{4^3}\right) + \frac{3^2 n}{4^2} + \frac{3^1}{4^1} n + \frac{3^0}{4^0} n$$

$$T(1) = 3^i T\left(\frac{n}{4^i}\right) \qquad 1 = \frac{n}{4^i} \quad n = 4^i \quad i = \log_4(n)$$

$$T(n) = \underbrace{\frac{3^0}{4^0} n + \frac{3^1}{4^1} n + \left(\frac{3}{4}\right)^2 n + \left(\frac{3}{4}\right)^3 n + \dots + \left(\frac{3}{4}\right)^{\log_4(n)-1} n}_{} + 3^{\log_4(n)} T(1)$$

$$T(n) = \sum_{i=0}^{\log_4(n)-1} \left(\frac{3}{4}\right)^i n + 3^{\log_4(n)} T(1) \qquad \sum_{k=0}^{n} ar^k = \frac{ar^{n+1} - a}{r-1}$$

$$n\left(\frac{\left(\frac{3}{4}\right)^{\log_4(n)} - 1}{\left(\frac{3}{4}\right) - 1}\right) + 3^{\log_4(n)} T(1)$$

$$n\left(\frac{\left(\frac{3}{4}\right)^{\log_4(n)} - 1}{-\frac{1}{4}}\right) + 3^{\log_4(n)} T(1) \qquad -\frac{1}{4}$$

$$4n - \frac{\left(\frac{3}{4}\right)^{\log_4(n)}}{\frac{1}{4}} + 3^{\log_4(n)} \times c$$

$$4n - \frac{n^{\log_4\left(\frac{3}{4}\right)}}{\frac{1}{4}} + n^{\log_4(3)} \times c$$

$$4n - 4n^{-0.2} + n^{0.8} \times c \rightarrow O(n)$$

Explicación del taller 1

log log int $\rightarrow 2^{127} - 1$

T(n) \ n	10^0	10^1	10^3	10^5	10^{10}	10^{20}	10^{30}
$\log n$							
$\sqrt{n}$							
n							
$n \log n$							
n^2				10 seg			
n^3							
2^n							
$n!$							

n

10^{10}

$10^{9} \rightarrow C1 = \frac{10^{10}}{10^{9}} = 10$ seg

$C2 = \frac{10^{10}}{10^{8}} = 100$ seg

Punto

0 ..... n

2

base 2

$10000 = 16$

Recurrencias

$$T(n) = n + 3T(n/4)$$

$$n + 3\ (\ n/4 + 3T(n/16))$$

$$n + 3\ (\ n/4 + 3(n/16\ \ + 3T(n/64)\ ))$$

$$n + 3*n/4 + 3^2*n/4^2\ \ + 3^3T(n/4^3)$$

$$n + 3*n/4 + 3^2*n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}T(1)$$

Después de iterar, se debe tratar de expresar como una sumatoria con forma cerrada conocida

Recurrencias

$T(n) = n + 3T(n/4\rfloor)$

$n + 3\ (\ n/4\rfloor + 3T(n/16\rfloor))$

$n + 3\ (\ n/4\rfloor + 3(n/16\rfloor\ + 3T(n/64\rfloor)\ ))$

$n + 3*n/4\rfloor + 3^2*n/4^2\rfloor\ + 3^3(n/4^3\rfloor) + \ldots + 3^{\log 4n}\Theta(1)$

$\leq\ n + 3n/4 + 3^2n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}\Theta(1)$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ \ + 3T(n/64)\,))$

$n + 3{*}n/4 + 3^2{*}n/4^2\ \ + 3^3(n/4^3) + \ldots + 3^{\log 4n}\Theta(1)$

$\leq\ n + 3n/4 + 3^2n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}\Theta(1)$

$$= \left(\sum_{i=0}^{\log_4 n}\left(\frac{3}{4}\right)^{i} n\right) + 3^{\log_4 n}\Theta(1)$$

$$= n\left(\frac{(3/4)^{(\log_4 n)} - 1}{(3/4) - 1}\right) + n^{\log_4 3} = n * 4(1 - (3/4)^{(\log_4 n)}) + \Theta(n^{\log_4 3})$$

$= O(n)$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1,\ \boxed{T(1) = \Theta(1)}$

$$T(n) = 2\left(2T\left(\frac{n}{2^2}\right)+1\right)+1$$

$$T(n) = 2\left(2\left(2T\left(\frac{n}{2^3}\right)+1\right)+1\right)+1$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 2^2 + 2^1 + 2^0$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 2^2 + 2^1 + 2^0 \qquad i = 3$$

$$T(1) = \Theta(1)$$

$$2^i T\left(\frac{n}{2^i}\right) + 2^{i-1} + \dots + 2^0$$

$$\frac{n}{2^i} = 1 \qquad i = \log_2(n)$$

$$2^{\log_2(n)} T(1) + \underbrace{2^{\log_2(n)-1} + 2^{\log_2(n)-2} + \dots + 2^1 + 2^0}$$

$$T(n) = \sum_{i=0}^{\log_2(n)-1} 2^i + cn$$

$$T(n) = \frac{2^{\log_2(n)} - 1}{2 - 1} + cn$$

$$T(n) = n - 1 + cn \longrightarrow O(n)$$

$\boxed{T(n) = 5T(n/4) + n}$, con T(1) = O(n)?

$$T(n) = 5\left(5T\left(\frac{n}{4^2}\right) + \frac{n}{4}\right) + n$$

$$T(n) = 5^2 T\left(\frac{n}{4^2}\right) + \frac{5}{4}n + n$$

$$T(n) = 5^2\left(5T\left(\frac{n}{4^3}\right) + \frac{n}{4^2}\right) + \frac{5}{4}n + \frac{5^0}{4^0}n$$

$$T(n) = 5^3 T\left(\frac{n}{4^3}\right) + \frac{5^2}{4^2}n + \frac{5}{4}n + \frac{5^0}{4^0}n$$

$$T(n) = 5^i T\left(\frac{n}{4^i}\right) + \left(\frac{5}{4}\right)^{i-1} n + \left(\frac{5}{4}\right)^{i-2} + \cdots + \left(\frac{5}{4}\right)^0 n$$

$T(1)$

$$\frac{n}{4^i} = 1 \qquad i = \log_4(n)$$

$$T(n) = 5^{\log_4(n)} T(1) + \left(\frac{5}{4}\right)^{\log_4(n)-1} \times n + \left(\frac{5}{4}\right)^{\log_4(n)-2} n + \cdots + \left(\frac{5}{4}\right)^0 n$$

$$T(n) = 5^{\log_4(n)} \times cn + \sum_{i=0}^{\log_4(n)-1} \left(\frac{5}{4}\right)^i \times n$$

$$T(n) = n^{\log_4(5)} \times cn + n\left(\frac{\left(\frac{5}{4}\right)^{\log_4(n)} - 1}{\frac{5}{4} - \frac{4}{4}}\right)$$

$O(n)$

$$T(n) = n^{1.16} \times n \times c + n\left(\frac{n^{\log_4\left(\frac{5}{4}\right)} - 1}{\frac{1}{4}}\right)$$

$$T(n) = n^{2.16} \times c + n\left(\frac{n^{0.16} - 1}{\frac{1}{4}}\right) \qquad O(n^3)$$

$$T(n) = n^{2.16} \times c + \frac{1}{4} n^{1.16} - \frac{1}{4} n \rightarrow O(n^{2.16})$$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1$, $T(1)= \Theta(1)$

$T(n) = 2T(n/2) + n$, $T(1)= \Theta(1)$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1,\ T(1)= \Theta(1)$

$T(n) = 2T(n/2) + n,\ T(1)= \Theta(1)$

$T(n) = T(n/2) + 1,\ T(1)= \Theta(1)$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1$, $T(1)= \Theta(1)$

$T(n) = 2T(n/2) + n$, $T(1)= \Theta(1)$

$T(n) = T(n/2) + 1$, $T(1)= \Theta(1)$

Demuestre que $T(n) = T(n/2\rfloor) + n$, es $\Omega(n\log n)$

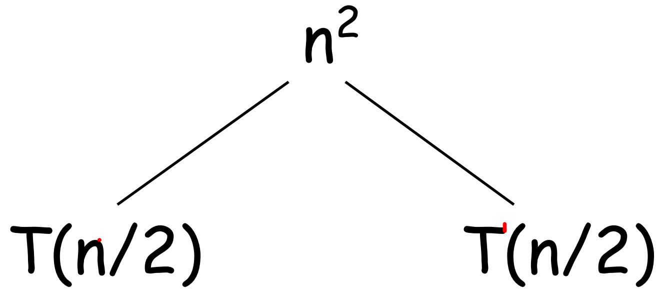
Recurrencias

Iteración con árboles de recursión

$T(1) = 1$

$T(n) = 2T(n/2) + n^2$

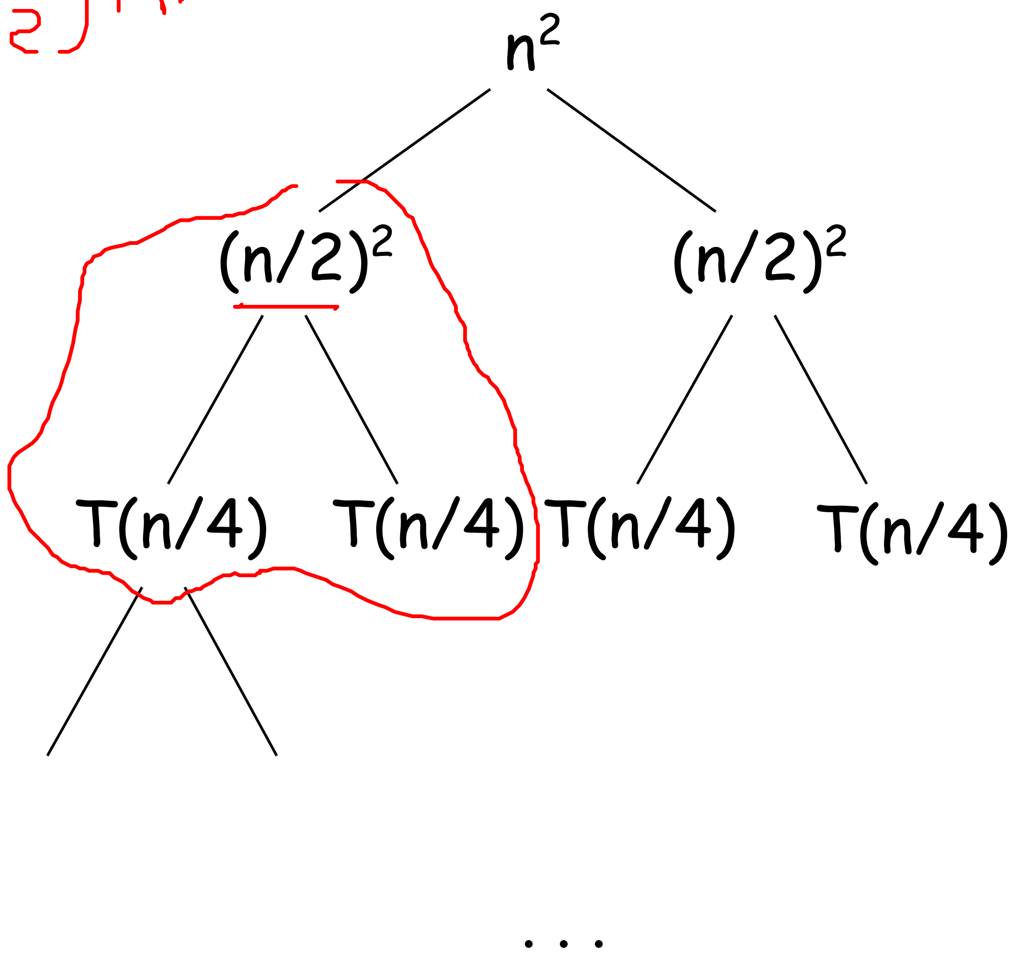
Recurrencias



$$T\left(\frac{n}{2}\right) = \underline{2T\left(\frac{n}{2^2}\right)} + \underline{\left(\frac{n}{2}\right)^2}$$

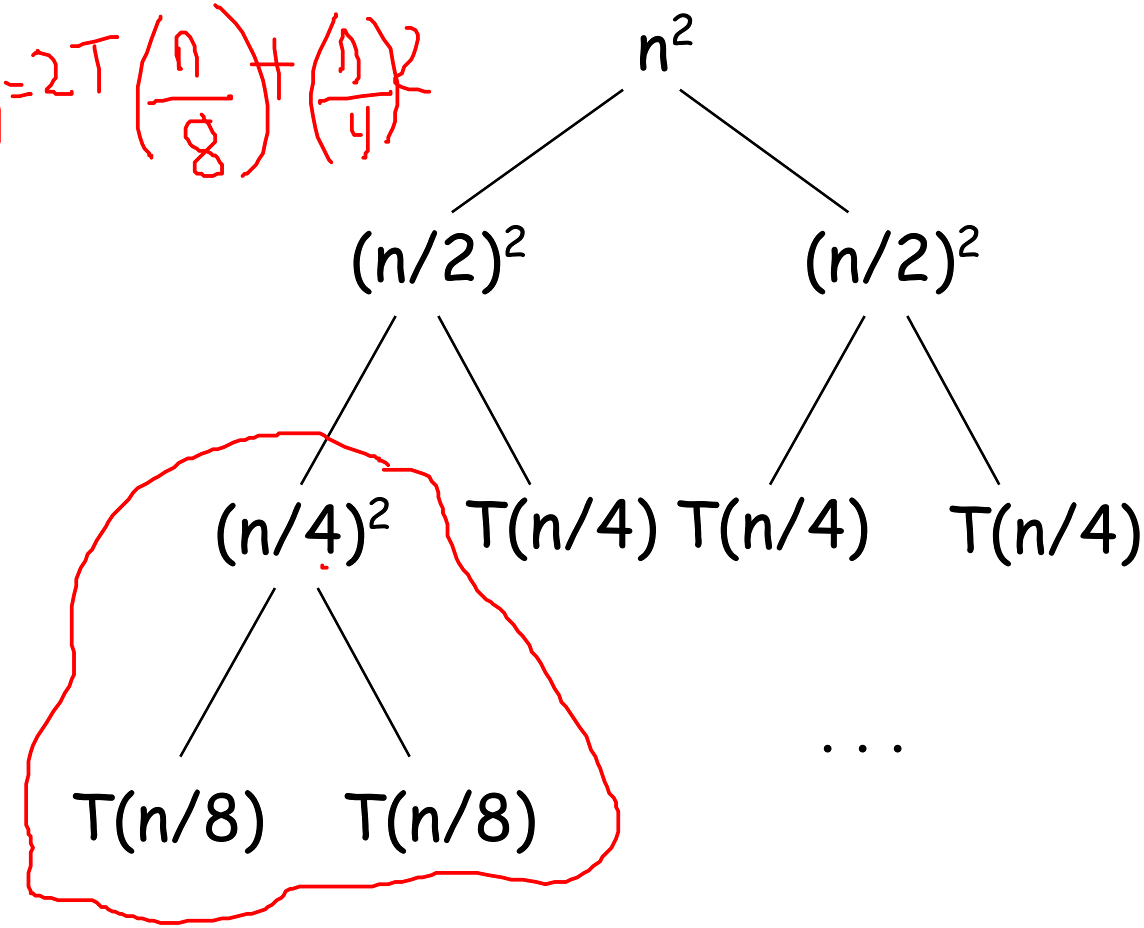
Recurrencias

$T(n) = 2T\left(\frac{n}{2}\right) + n^2$

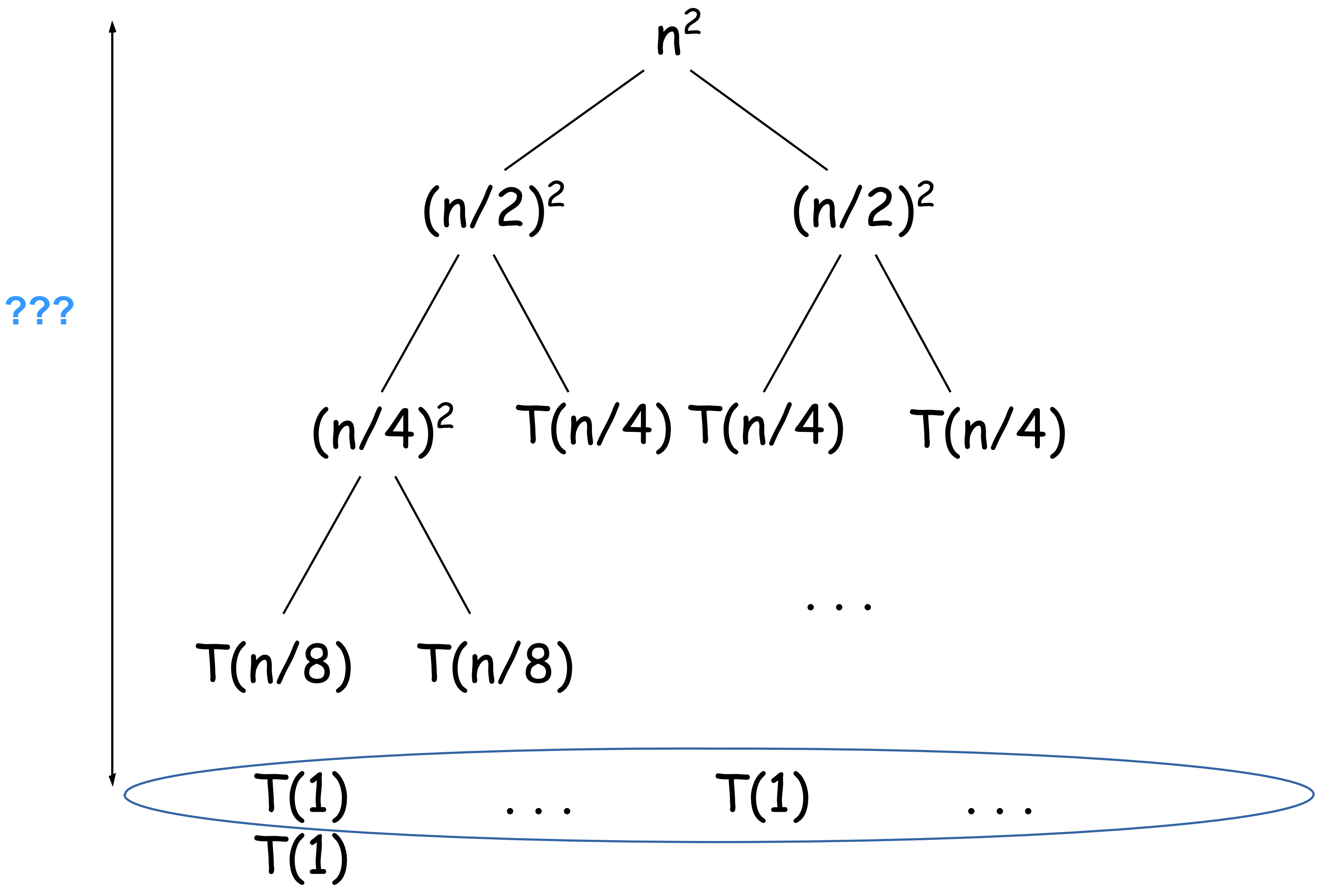


Recurrencias

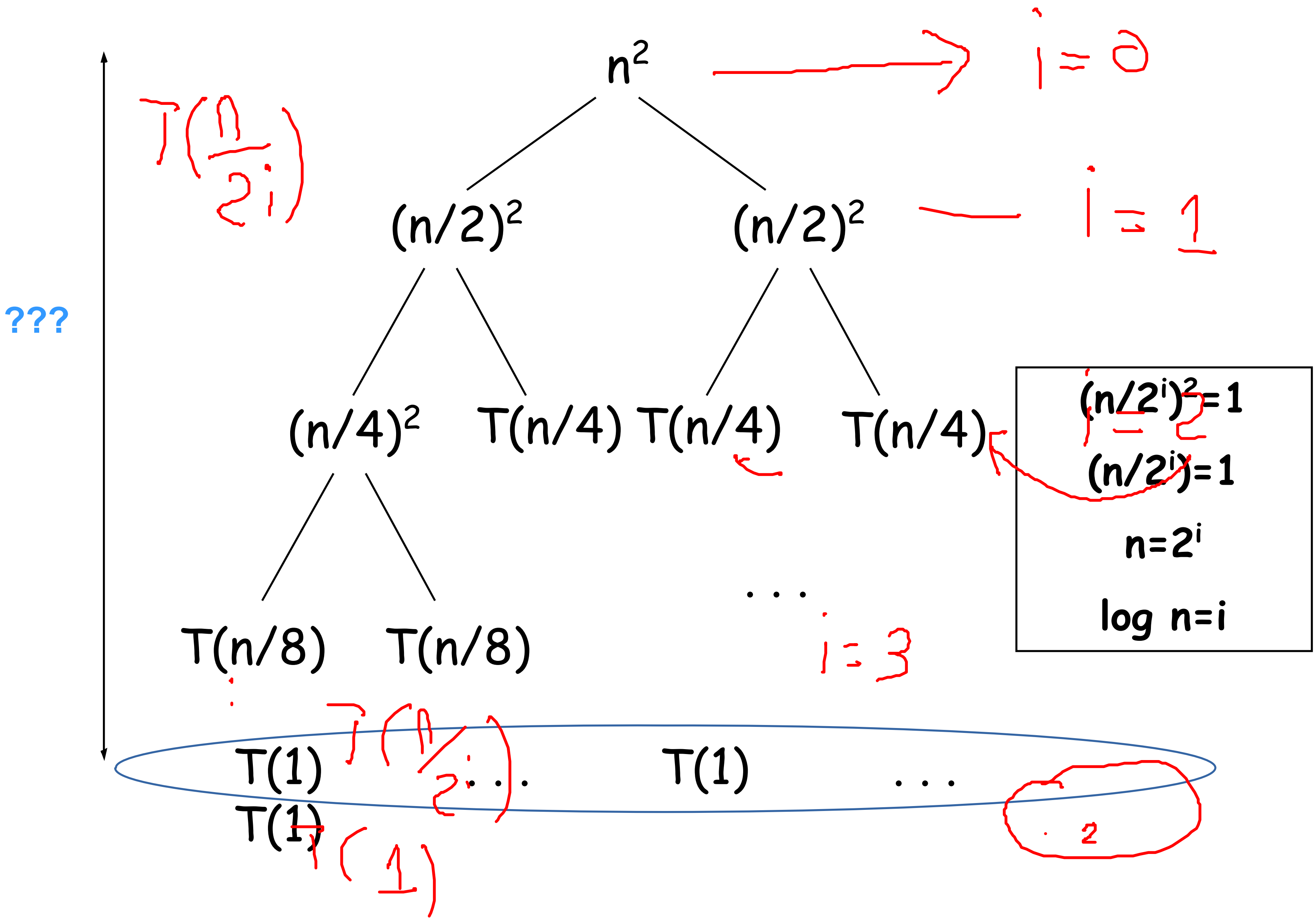
$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2$



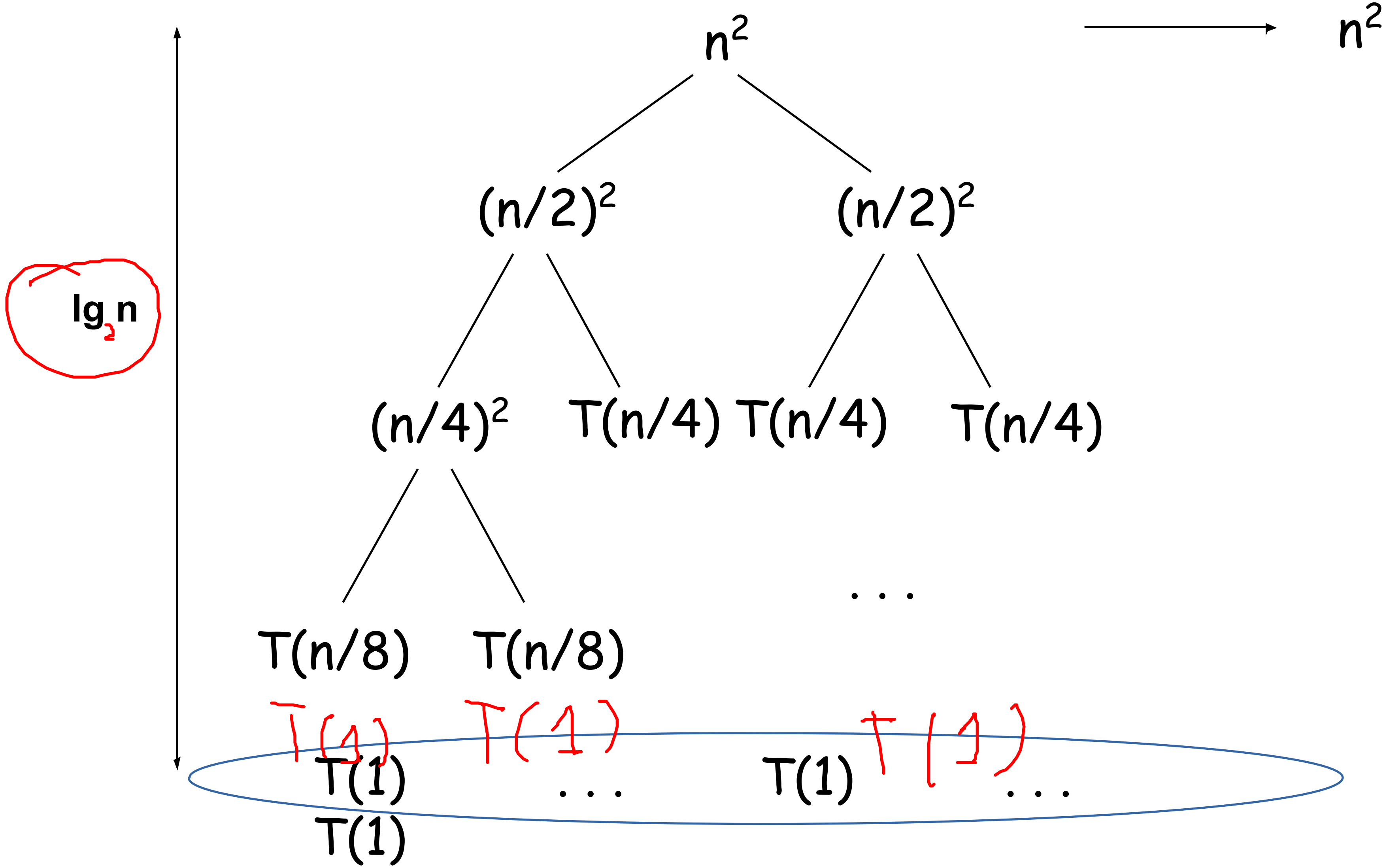
Recurrencias



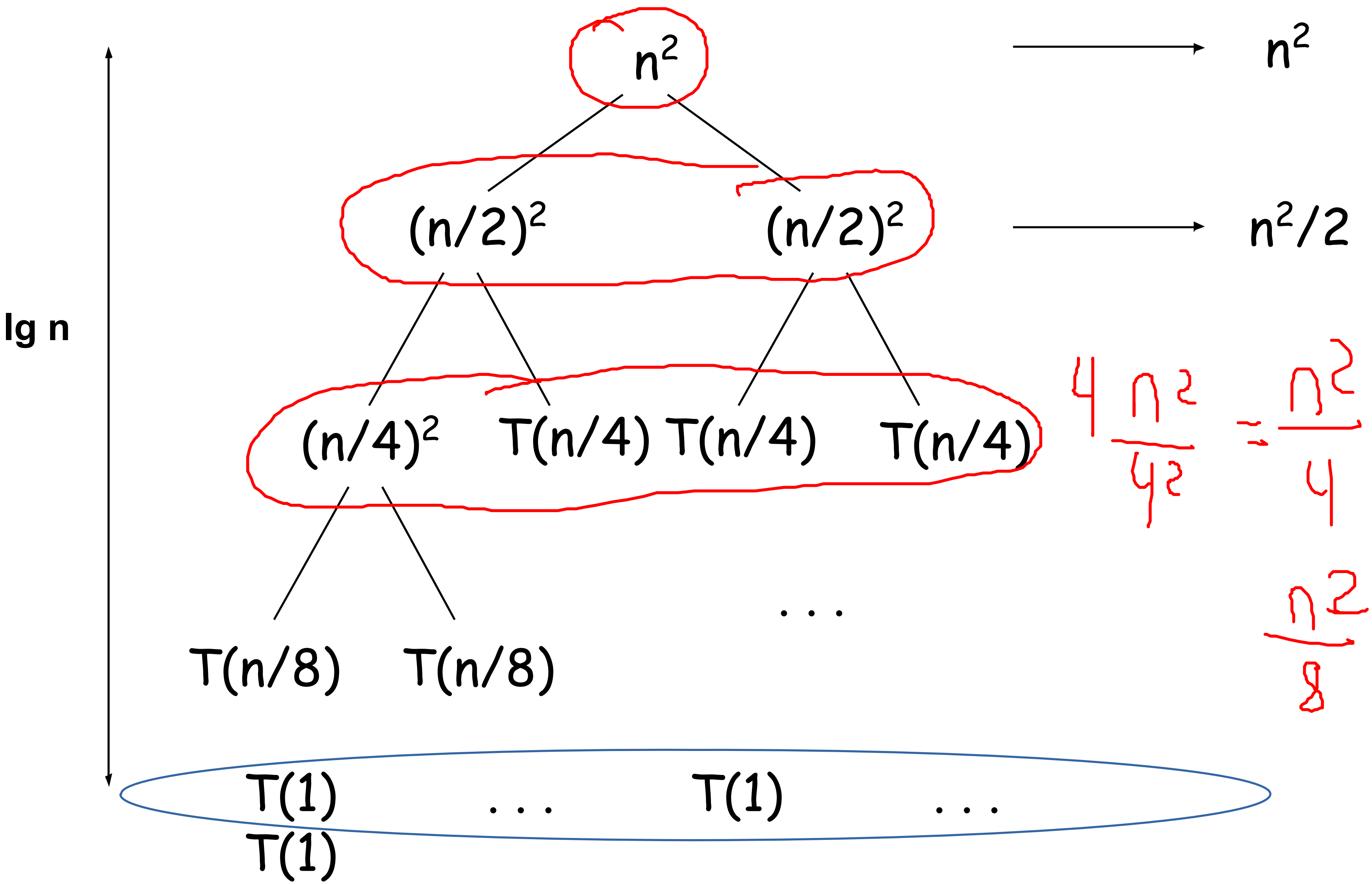
Recurrencias



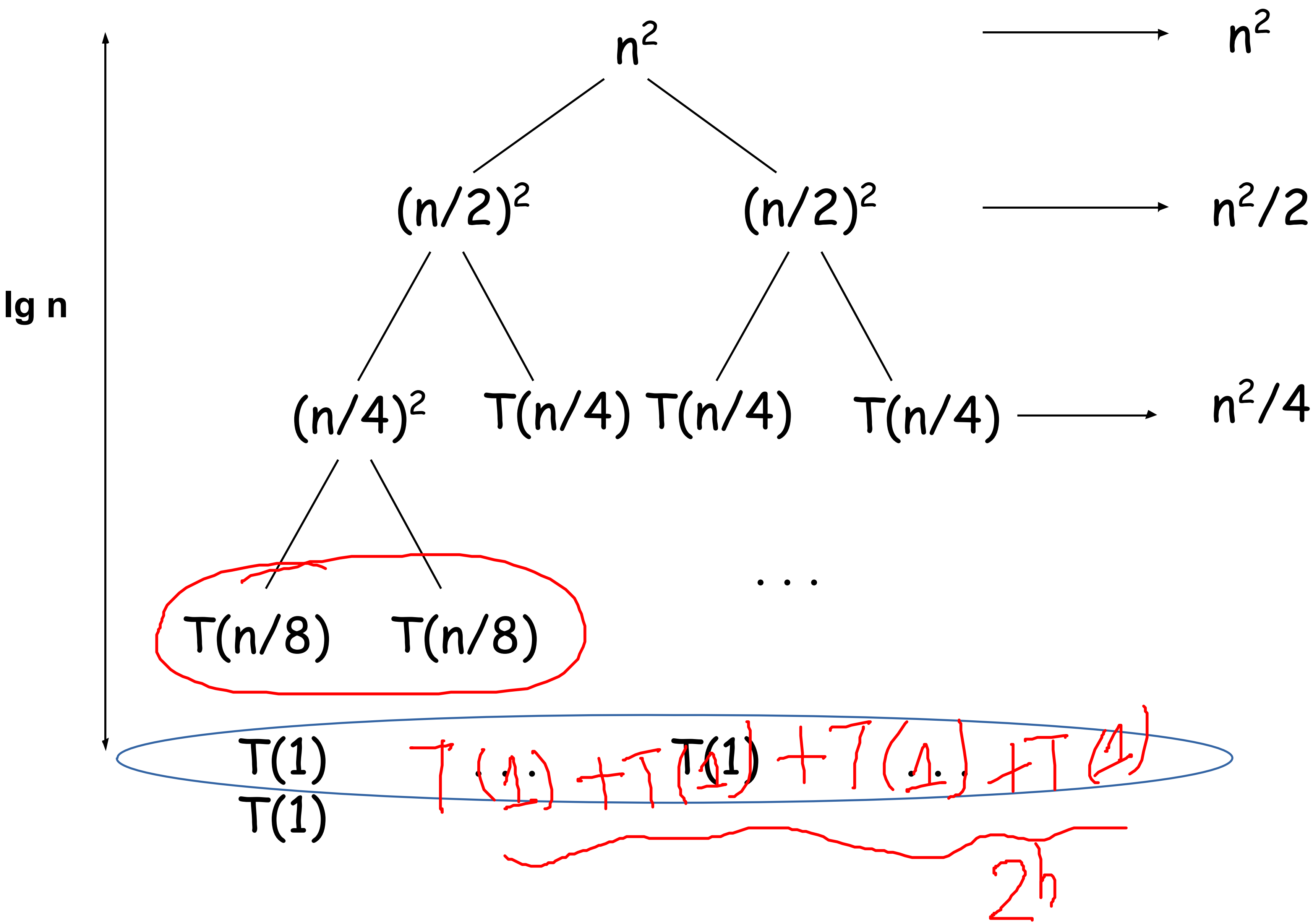
Recurrencias



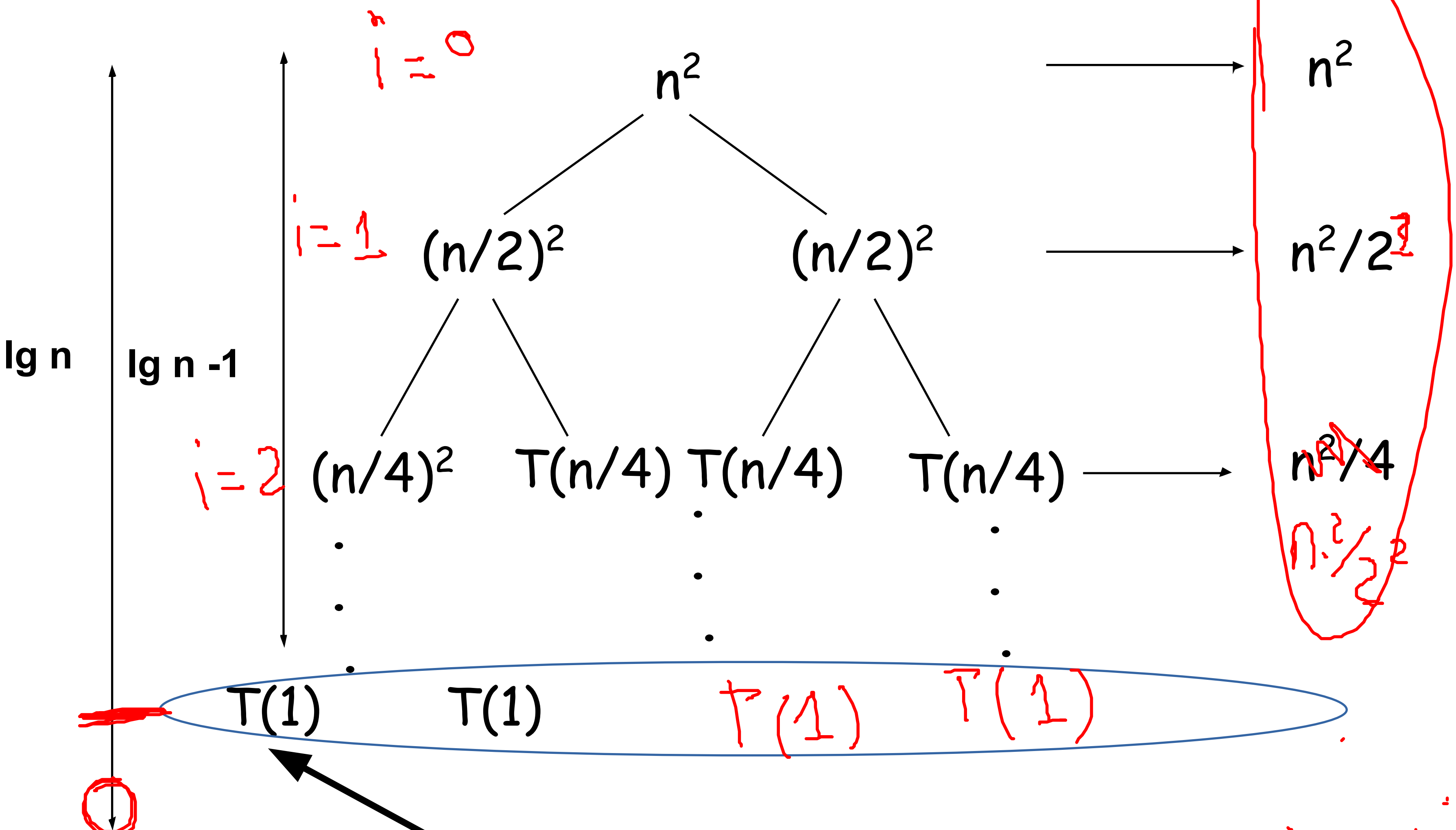
Recurrencias



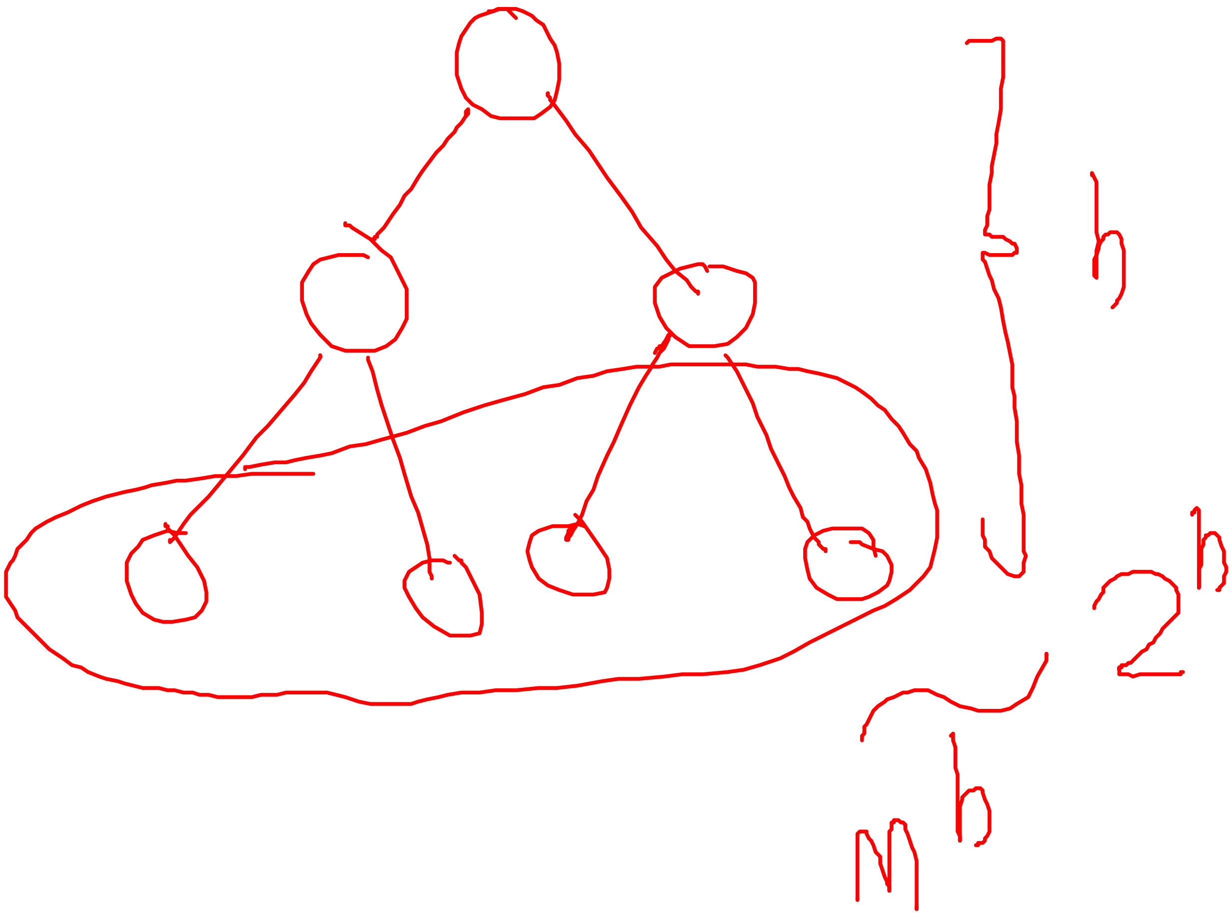
Recurrencias



Recurrencias

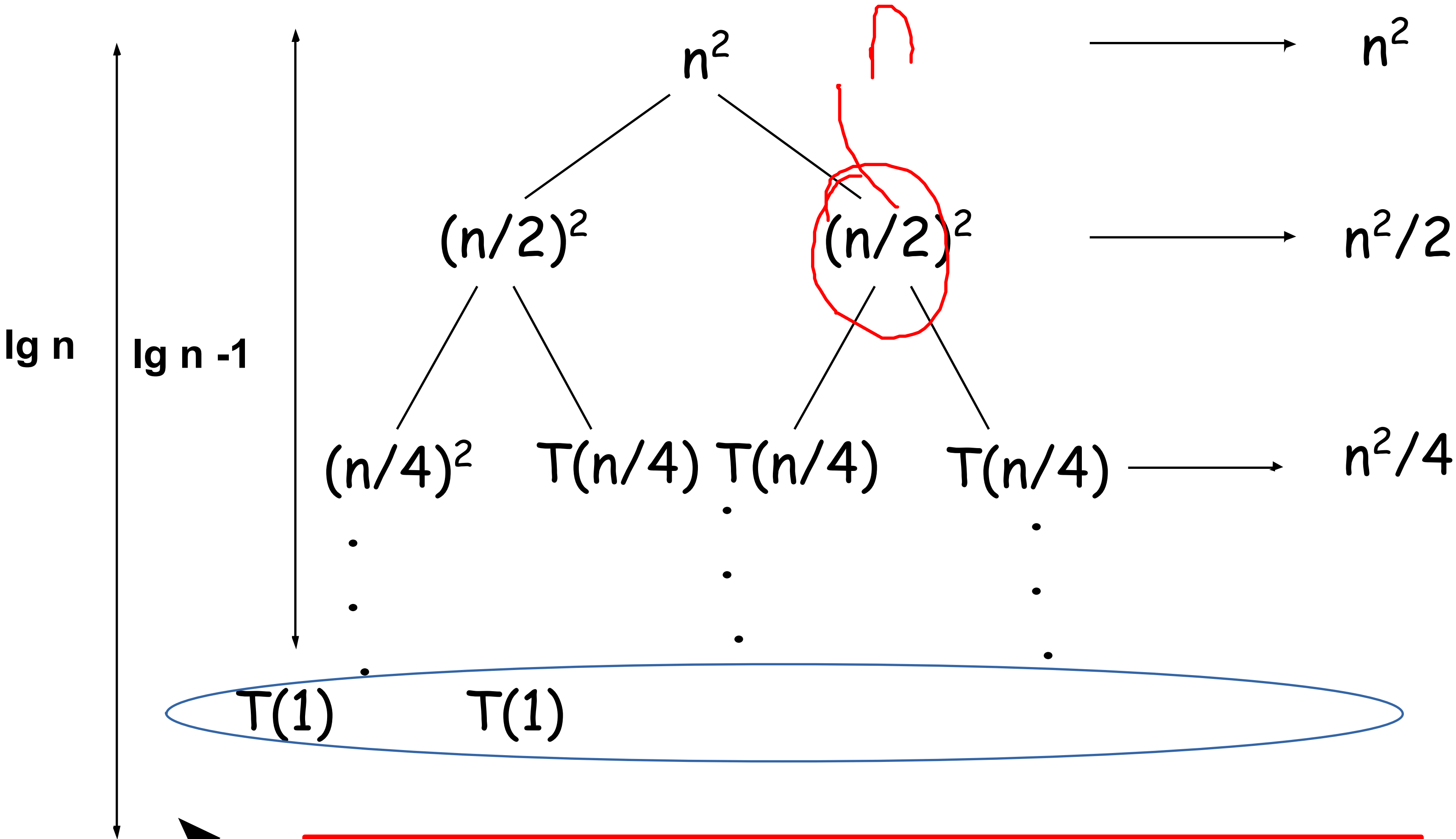


¿Cuantas hojas T(1) tenemos?



h
2^h
N^h

Recurrencias



Si recuerda en un árbol m-ario se tienen máximo m^h. En este caso al ser arbol binario m=2, tenemos $2^{\log(n)}$ hojas. Por lo tanto se

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2$$

n^2 $\quad i=0$

$T\left(\frac{n}{2}\right) \quad T\left(\frac{n}{2}\right)$ $\quad i=1$

n^2

$\left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2$

$T\left(\frac{n}{2^2}\right) \quad T\left(\frac{n}{2^2}\right) \quad T\left(\frac{n}{2^2}\right) \quad T\left(\frac{n}{2^2}\right)$

$n^2 \longrightarrow n^2 \quad i=0$

$\left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \longrightarrow \frac{n^2}{2^1}$

$\left(\frac{n}{2^2}\right)^2 \quad \left(\frac{n}{2^2}\right)^2$ $\quad \frac{n^2}{4} \to \frac{n^2}{2^2}$

$\frac{n^2}{2^3}$

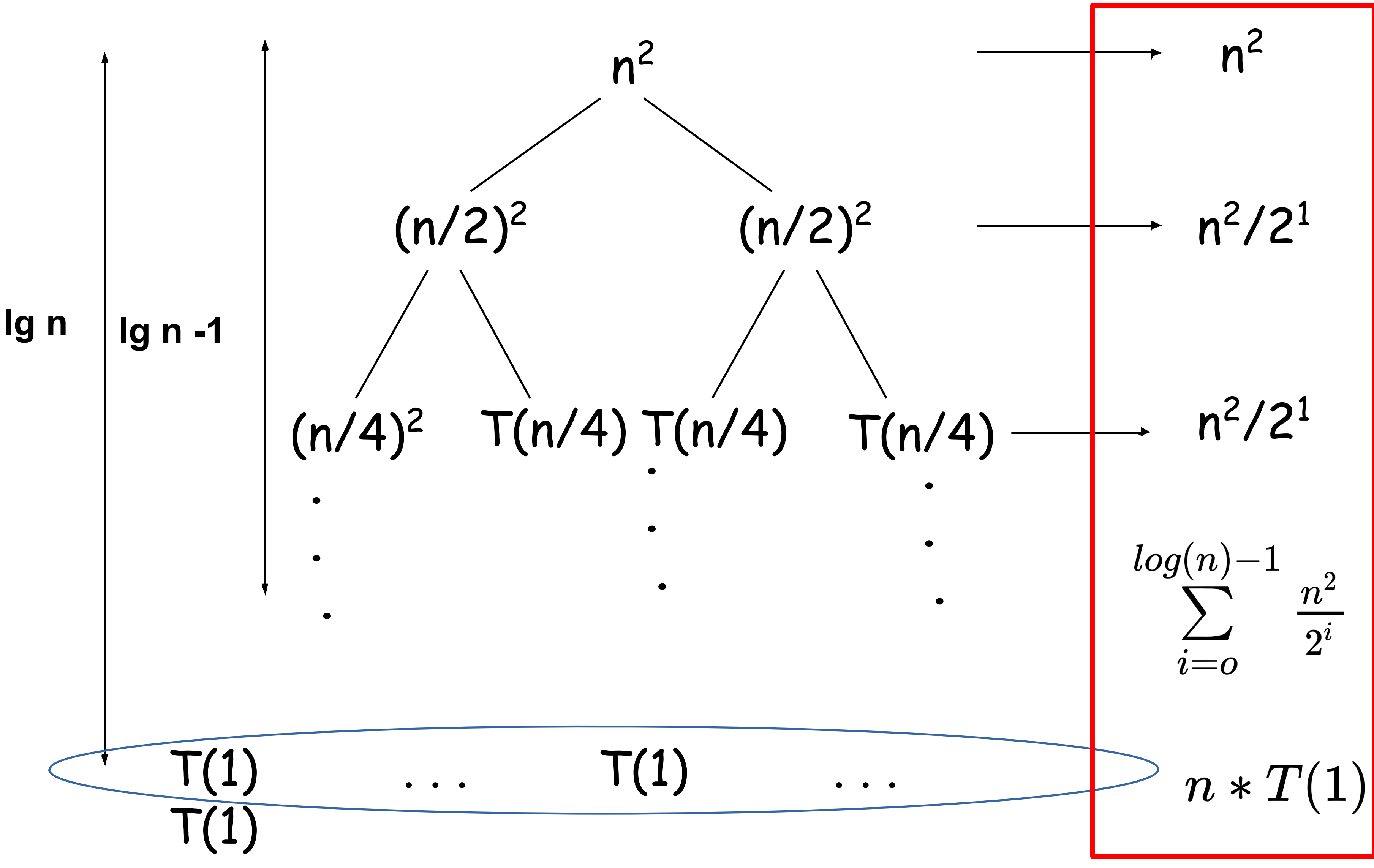
$T\left(\frac{n}{2^i}\right)$

$T(1)$

$\frac{n}{2^i} = 1 \quad i = \log_2(n)$

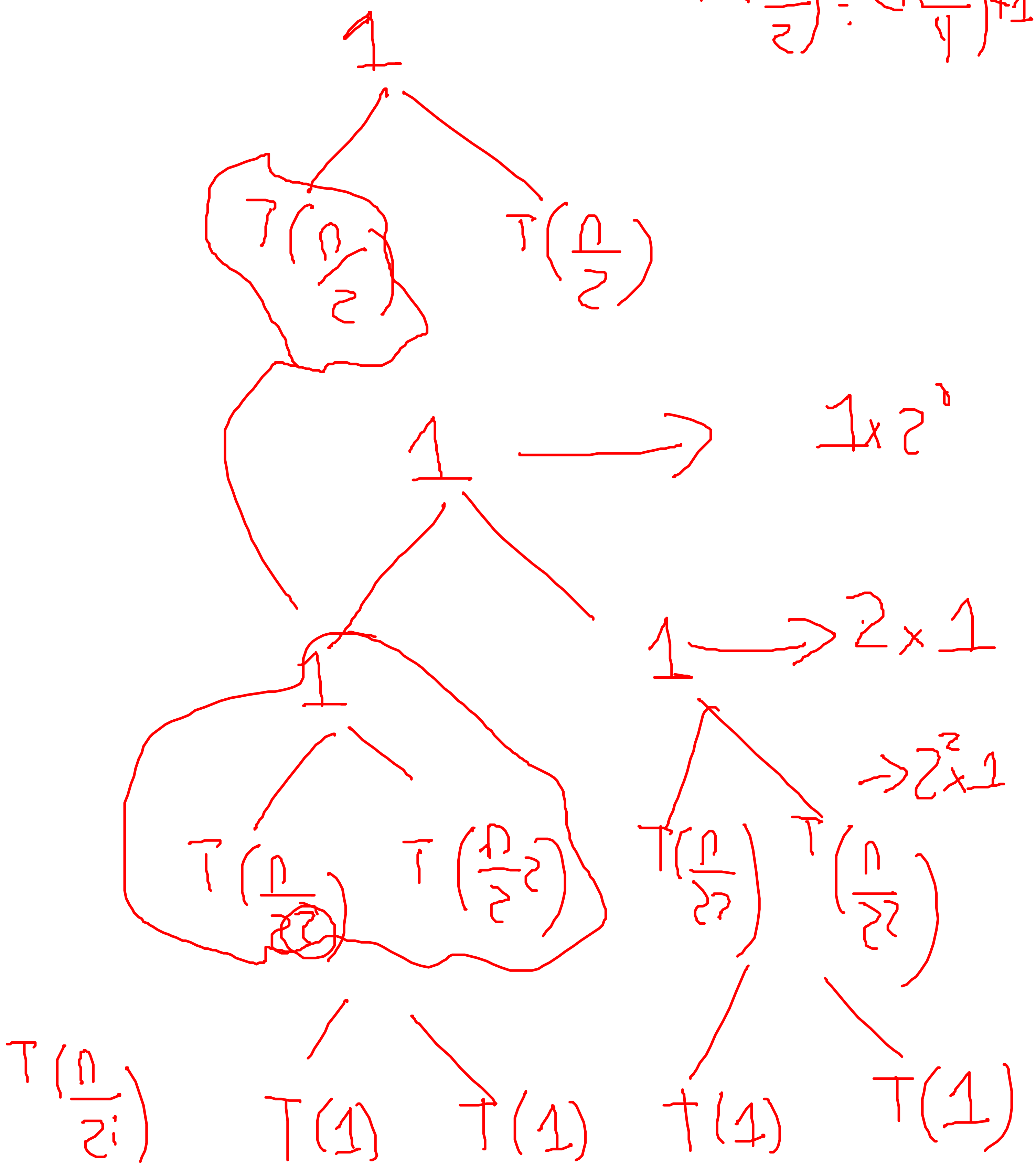
$$\sum_{i=0}^{\log_2(n)-1} \frac{n^2}{2^i} + \frac{n^2}{2^{\log_2(n)}} T(1)$$

Recurrencias



$$T(n) = 2\boxed{T\left(\frac{n}{2}\right)} + 1$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + 1$$



$i = 0$

$i = 1$

$i = 2$

$i = \log_2(n)$

$$2^{\log_2(n)}\, T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i \times 1$$

$$nT(1) + \frac{2^{\log_2(n)} - 1}{2-1} \to n \times C + n - 1$$

$$\approx O(n)$$

Recurrencias

$$T(n) = n * T(1) + \sum_{i=o}^{log(n)-1} \frac{n^2}{2^i}$$

$$T(n) = n * c + n^2 \frac{0.5^{log(n)} - 1}{0.5 - 1}$$

$$T(n) = n * c + n^2 \frac{n^{log(0.5)} - 1}{-0.5}$$

$$T(n) = n * c + n^2 \frac{n^{-1} - 1}{-0.5}$$

$$T(n) = n * c - \frac{n}{0.5} + \frac{n^2}{0.5} = O(n^2)$$

Recurrencias

Resuelva construyendo el árbol

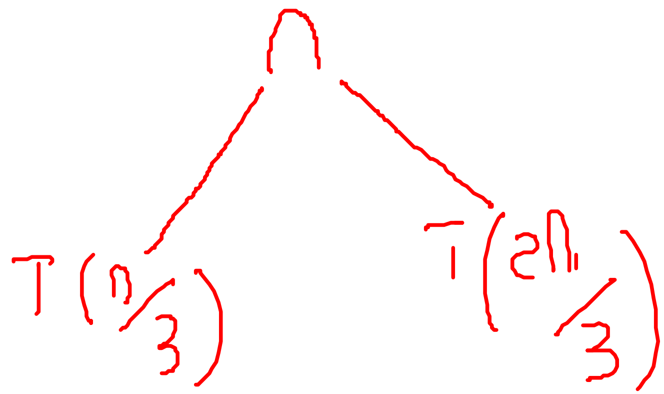
$T(n) = 2T(n/2) + 1,\ T(1) = \Theta(1)$

$T(n) = 2T(n/2) + n,\ T(1) = \Theta(1)$

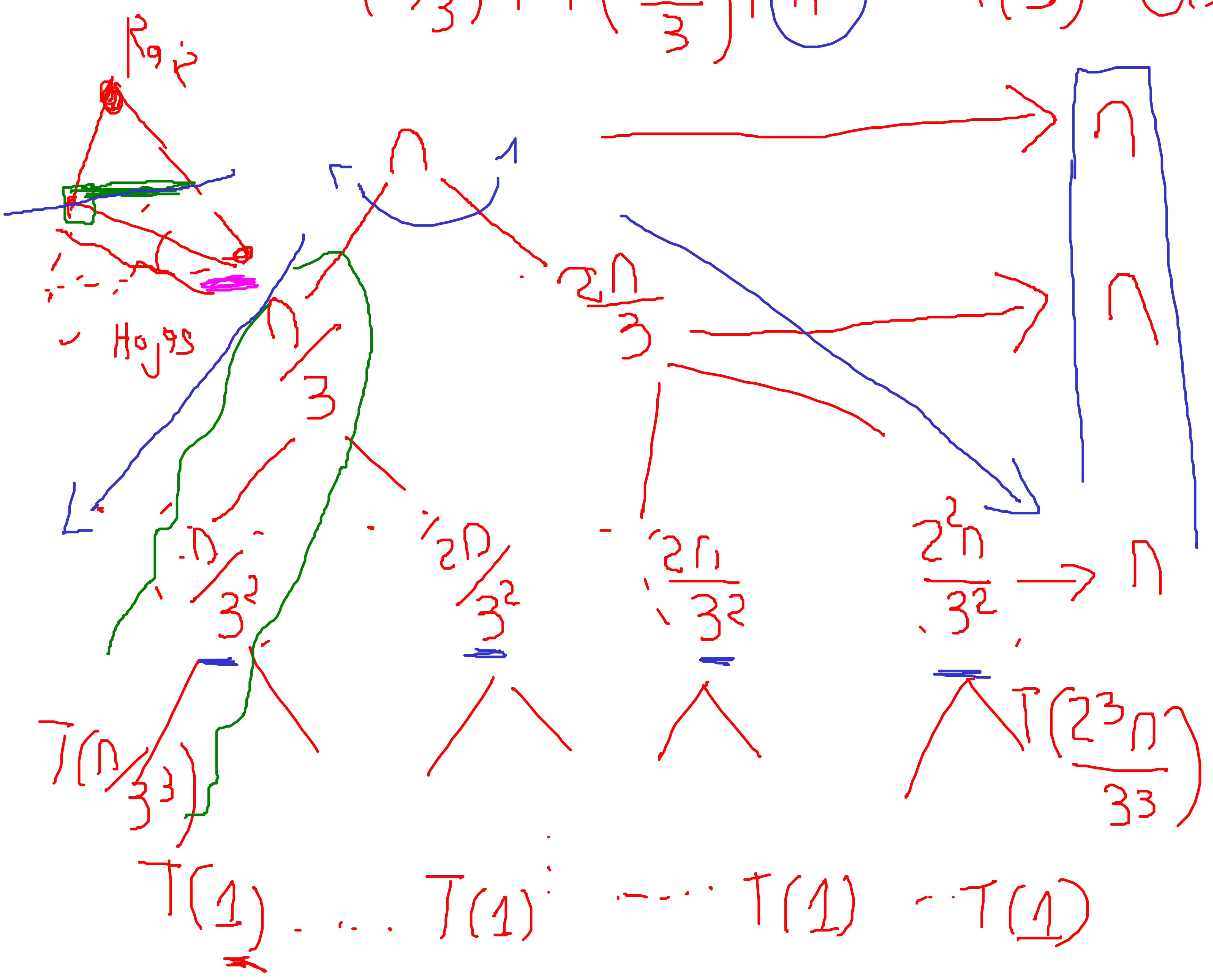
Recurrencias

Resuelva la recurrencia $T(n) = T(n/3) + T(2n/3) + n$

Indique una cota superior y una inferior



$T(n) = T(n/3) + T\left(\frac{2n}{3}\right) + n \qquad T(1) = \Theta(1)$



$T\left(\frac{n}{3^i}\right) = T(1)$

$1 = \frac{n}{3^i} \quad i = \log_3(n) \qquad n=2$

$T(n) = n \times (\log_3(n) - 1) + 2^{\log_3(n)} \times \Theta(1)$

$T(n) = \log_3(n) \times n - n + n^{\cancel{\log_3(2)}} \times C \quad <1$

$T(n) = \Omega(\log_3(n) \times n)$

$T\left(\frac{2^i}{3^i} n\right) = T(1)$

$1 = \left(\frac{2}{3}\right)^i n \qquad \left(\frac{3}{2}\right)^i = n \qquad i = \log_{\frac{3}{2}}(n)$

$T(n) = (\log_{\frac{3}{2}}(n) - 1) \times n + 2^{\log_{\frac{3}{2}}(n)} \times \Theta(1)$

$T(n) = n \log_{\frac{3}{2}}(n) - n + n^{\log_{\frac{3}{2}}(2)} \times \Theta(1)$

$T(n) = n \log_{\frac{3}{2}}(n) - n + n^{1.7} \times C \qquad 1.7 = \frac{\log_{10}(2)}{\log_{10}\left(\frac{3}{2}\right)}$

$O(n^{1.7}) \simeq O(n^2)$

Recurrencias

Método maestro

Permite resolver recurrencias de la forma:

$T(n) = aT(n/b) + f(n)$, donde $a \geq 1$, $b > 1$

Recurrencias

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n)=\Theta(n^{\log_b a})$

Si $f(n)=O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon>0$

2. $T(n)=\Theta(n^{\log_b a}\lg n)$

Si $f(n)=\Theta(n^{\log_b a})$ para algún $\varepsilon>0$

3. $T(n)=\Theta(f(n))$

Si $f(n)=\Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon>0$ si a*f(n/b) ≤c*f(n)

para algun c<1

Recurrencias

Dado T(n) = 9T(n/3) + n

$a = 9 \quad b = 3 \quad f(n) = n$

$$n^{\log_3 9} = n^2 \quad \textbf{Vs} \quad f(n) = n$$

$\log_b(a) = \log_3(9) = 2$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $n = O(n^{2-\varepsilon})$?

$f(n) = O(n^{2-\epsilon}) \qquad n = O(n^{2-\epsilon})$

$n = O(n)$ ✓

$T(n) = \Theta(n^2)$

Recurrencias

Dado T(n) = 9T(n/3) + n

$$n^{\log_3 9} = n^2 \quad \textbf{Vs} \quad f(n) = n$$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $n = O(n^{2-\varepsilon})$?

Si $\varepsilon = 1$ se cumple que $n = O(n)$, por lo tanto, se cumple que:

$$T(n) = \Theta(n^2)$$

Recurrencias

$T(n) = T(2n/3) + 1$

$a = 1 \quad b = \frac{3}{2} \quad F(n) = 1$

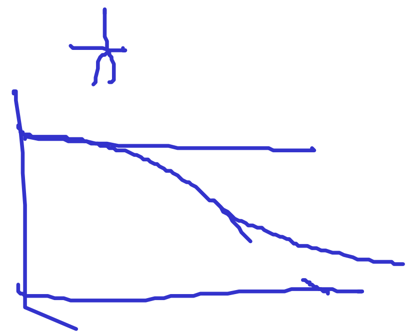
$$n^{\log_{3/2} 1} = n^0 = 1 \quad \textbf{Vs} \quad f(n) = 1$$

$\log_{\frac{3}{2}}(1) = 0$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $1 = O(n^{0-\varepsilon})$?

No existe $\varepsilon > 0$



Recurrencias

T(n) = T(2n/3) + 1

$$n^{\log_{3/2} 1} = n^0 = 1 \quad \text{vs} \quad f(n) = 1$$

Es $f(n) = \Theta(n^{\log_b a})$?

Es $1 = \Theta(1)$?

Si, por lo tanto, se cumple que:

$$T(n) = \Theta(1 * \lg n) = \Theta(\lg n)$$

Recurrencias

T(n) =3 T(n/4) + nlgn

$$n^{\log_4 3} = n^{0.793} \quad \text{vs} \quad f(n) = n\lg n$$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

$n\log(n) = O(n^{0.793-\epsilon})$ X

Es $f(n) = \Theta(n^{\log_b a})$?

$n\log(n) = \Theta(n^{0.793})$

Es $f(n) = \Omega(n^{\log_b a + \varepsilon})$?

$n\log(n) = \Omega(n^{0.793+\epsilon})$

Si, y además, af(n/b)≤cf(n)

$n\log(n) = \Omega(n)$ ✓

3(n/4)lg(n/4) ≤ cnlgn

3(n/4)lgn - 3(n/4)*2 ≤ cnlgn

(3/4)nlgn ≤ cnlgn → c=3/4 y se concluye que $\boxed{T(n) = \Theta(n\lg n)}$

$c < 1$

Recurrencias

$T(n) = 2T(n/2) + n\lg n$

Muestre que no se puede resolver por el método maestro

Recurrencias

Resuelva usando método del maestro

$T(n) = 4T(n/2) + n$

$T(n) = 4T(n/2) + n^2$

$T(n) = 4T(n/2) + n^3$

$aT(n/b) + F(n)$

$- \log_b(a)$

1) $a = 4$

$b = 2$

$F(n) = n$

1) $F(n) = O(n^{\log_b a - \epsilon})$

$n = O(n^{2-\epsilon})$

$n = O(n)$

$\Theta(n^{\log_b a}) = \Theta(n^2)$

$4T(n/2) + n^2$

$a = 4 \quad b = 2$

$F(n) = n^2$

$\log_b(a) = 2$

1)

$F(n)$ es $O(n^{2-\epsilon})$

$\cancel{n^2 \text{ es } O(n^{2-\epsilon})} \quad \epsilon > 0$

2)

$F(n)$ es $\Theta(n^2)$

n^2 es $\Theta(n^2)$

$O(n^2)$

y

$\Omega(n^2)$

$\Theta(n^2 \log(n))$

$$T(n) = 4T(n/2) + n^3$$

1) n^3 es $O(n^{2-\epsilon})$ ✗

2) n^3 es $\Theta(n^2)$ ✗

3) n^3 es $\Omega(n^{2+\epsilon})$ ✓

n^3 es $\Omega(n^3)$

$a f(n/b) \le c \cdot f(n) \quad c < 1 \qquad 4\left(\frac{n}{2}\right)^3 \le c \cdot n^3$

$$\frac{\cancel{n^3}}{2} \le c \cdot \cancel{n^3}$$

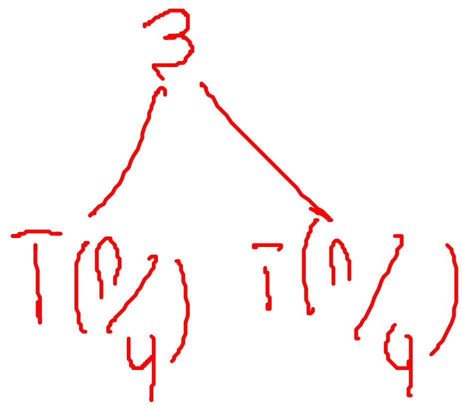
$$c \ge \frac{1}{2} \ ✓$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n^3)$$

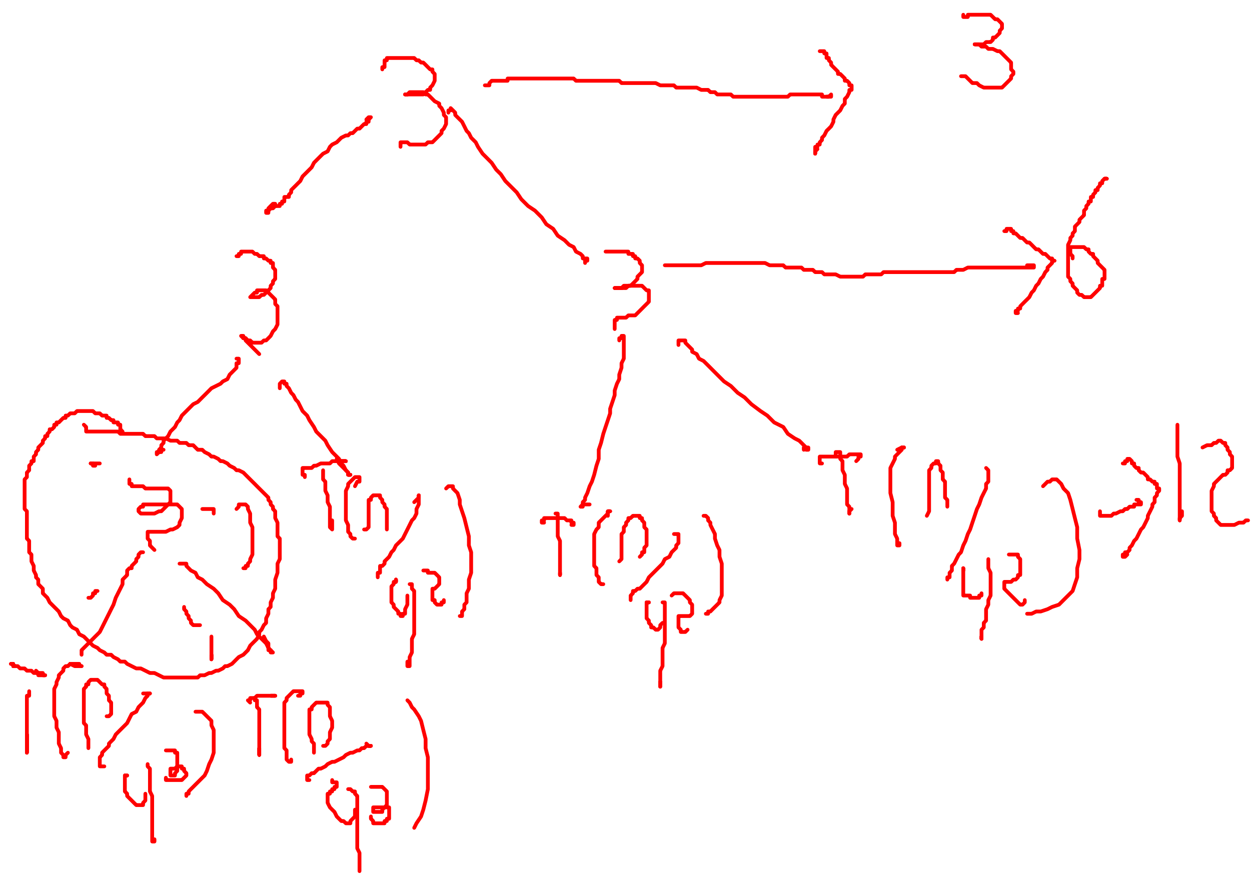
$$T(n) = 2T\left(\frac{n}{4}\right) + 3$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{4^2}\right) + 3$$



$$\frac{n}{4^i} = 1$$

$$i = \log_4(n)$$



$\log_4(n)$ $T(1) = ?$

5 → 5

$T(n/4)$ $T(n/4)$ $T(n/4)$ $T(n/4)$ 5 → 25

1) T(n) = 5T(n/4) + 5

$T(n/4) = 5T(n/4^2) + 5$

$T(n/4^2)$

$T\left(\frac{n}{4^i}\right)$

$\log_6(n)$

2) T(n) = 8T(n/6) + 15n

$T(n/6) = 8T(n/6^2) + \frac{15n}{6}$

$15n$ → $15n$

$T(n/6)$ $\frac{15n}{6}$ → $\frac{8 \times 15n}{6}$

8

$T\left(\frac{n}{6^i}\right)$ 8 hijas $T(n/6^2)$

3) T(n) = T(n-1) + n

n

$n-1$

$T(n-2) \sim T(n-i)$ $n-i=1$

$i = n-1$