

$$T(n) = 8T(n/2) + \frac{n^2}{2}$$

$$T(1) = 4$$

$$i=1 \quad T(n/2) = 8T(n/2^2) + \frac{1}{2} \left(\frac{n}{2}\right)^2$$

$$i=3 \quad T(n) = 8^3 T(n/2^3) + \frac{8^2}{2} \left(\frac{n}{2^2}\right)^2 + \frac{8}{2} \left(\frac{n}{2}\right)^2$$

$$i=2 \quad T(n) = 8^2 T(n/2^2) + \frac{8}{2} \left(\frac{n}{2}\right)^2$$

$$T(n) = 8^i T\left(\frac{n}{2^i}\right) + \frac{8^{i-1}}{2} \left(\frac{n}{2^{i-1}}\right)^2 + \frac{8^{i-2}}{2} \left(\frac{n}{2^{i-2}}\right)^2 + \dots$$

$$T(n/2^2) = 8T(n/2^3) + \frac{1}{2} \left(\frac{n}{2^2}\right)^2$$

$$\frac{n}{2^i} = 1 \quad i = \log_2(n)$$

$$T(n) = \sum_{p=0}^i \frac{8^p}{2} \left(\frac{n}{2^p}\right)^2 = \sum_{p=0}^{\log_2(n)} \frac{8^p}{2} \left(\frac{n}{2^p}\right)^2$$

$$T(n) = \sum_{p=0}^{\log_2(n)} \frac{8^p}{2} \frac{n}{2^p} \frac{n}{2^p} = \frac{n^2}{2} \sum_{p=0}^{\log_2(n)} \frac{8^p}{2^{2p}} = \frac{n^2}{2} \sum_{p=0}^{\log_2(n)} \frac{8^p}{4^p} = \frac{n^2}{2} \sum_{p=0}^{\log_2(n)} \left(\frac{8}{4}\right)^p$$

$$T(n) = \frac{n^2}{2} \sum_{p=0}^{\log_2(n)} 2^p = \left(\frac{n^2}{2}\right) \frac{2^{\log_2(n)+1} - 1}{2 - 1} = \left(\frac{n^2}{2}\right) \frac{2^{\log_2(n)} \times 2 - 1}{2 - 1} = \left(\frac{n^2}{2}\right) \frac{2n - 1}{1} = n^3 - n^2$$

$$O(n^3)$$

$$a=8 \quad b=2 \quad F(n) = \frac{n^2}{2}$$

$$\log_b a = \log_2 8 = 3$$

$$1) \frac{n^2}{2} \text{ is } O(n^{3-\epsilon}) \quad \checkmark$$

$$\epsilon = 1$$

$$O(n^{\log_b a}) = O(n^3)$$