Segundo examen parcial Análisis y diseño de algoritmos

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1. Dado el siguiente algoritmo:

def algorithm (n):

$$a = 0$$
 $s = 0$

while $a <= n$:

 $s += 2*n$ (0,0) \rightarrow (1,2n) \rightarrow (2,2n+2n) \rightarrow (3,6n)

return s

Indicar

- $a)~(5~\mathrm{puntos})$ Forma del estado, estado inicial
- (α, S) (0, 0)b) (15 puntos) Transformación de estado y estado final $(9,5) \longrightarrow (9+1,5+21)$
- c) (20 puntos) Invariante de ciclo y su demostración
- 2. Dada la función $f(n) = n^2 + 2n$ indicar:
 - a) (10 puntos) Explicar claramente si f(n) es O(n)
 - b) (10 puntos) Explicar claramente si f(n) es $\Theta(n^2)$
- 3. Resolver por método de expansión o de árboles:

a) (20 puntos)
$$T(n) = 4T(\frac{n}{8}) + n$$
, $T(1) = 1$

b) (20 puntos)
$$T(n) = 3T(\frac{n}{6}(+4, T(1) = 1))$$

Mostrar claramente cada uno de los pasos realizados

Ayudas

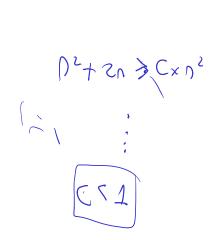
Sumatorias

3) Transforms
$$q = \frac{q+1}{2}$$
 $(q+1)$ $(q+1)$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=0}^{n} ar^k = \frac{\Im ar^{(n+1)} - a}{r-1} \text{ Si } r \neq 1$$

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5)
$$U_{5} + SU \approx C \times U_{5}$$
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J2 (n2)

$$\begin{array}{c} k \\ 1 \end{array} \qquad T(n) = 4T\left(\frac{n}{8}\right) + n \qquad T(1) = 1$$

2)
$$T(0) = 4(4T(\frac{9}{8}) + \frac{9}{8}) + 0$$

$$\frac{4}{8} = \frac{1}{2}$$

3)
$$T(n) = 4 \left(4 \left(4 \left(\frac{1}{83} \right) + \frac{1}{8^2} \right) + \frac{1}{8} \right) + 1$$

$$T(n) = 4 \left(\frac{1}{83} \right) + \frac{1}{8^2} \left(\frac{1}{8} \right)^{\frac{1}{8}} + \frac{1}{8^2} \right)$$

$$T(n) = 4 \left(\frac{1}{8^3} \right) + \frac{1}{8^2} \left(\frac{1}{8} \right)^{\frac{1}{8}} + \frac{1}{8^2} \right)$$

$$T(n) = 6 \left(\frac{1}{8} \right)^{\frac{1}{8}} + \frac{1}{8^2} \left(\frac{1}{2} \right)^{\frac{1}{8}}$$

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$$K = \log_8(n)$$

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1
$$T(n) = 3T(\frac{n}{\sigma}) + 4$$

$$7(n) = 3\left(3 + \left(\frac{n}{\sigma^2}\right) + 4\right) + 4$$

3)
$$T(n) = 3\left(3\left(\frac{3}{6}\right) + 4\right) + 4$$

 $P(n) = 3^{3} + \left(\frac{9}{6}\right) + 3^{2} + 4 + 3 + 4 + 3^{6} + 4$
 $K = \log_{6}(n)$

$$T(a) = 3^{k} + \left(\frac{1}{6^{k}}\right) + \sum_{i=0}^{k-1} 4x3^{k}$$

$$T(n) = 3^{10} J_{10}(n) + \sum_{i=0}^{100} 4x3^{i}$$

$$T(n) = n \frac{\log_{10}(3)}{3 - 1} + \frac{4x3}{3 - 1}$$

$$T(n) = \int_{0}^{\infty} g(3) + 2 \left(\int_{0}^{\infty} g(3) - 1 \right)$$