Relative error is given by the formula below:

$$\text{Relative error} = \left|\frac{\text{true value} - \text{approximate value}}{\text{true value}}\right|$$

Comment

Step 2 of 4

Consider the function:

$$f(x) = \frac{1}{\left(1-3x^2\right)^2}$$

Calculate the derivative of this function and get the equation:

$$f'(x) = \frac{6x}{\left(1-3x^2\right)^2}$$

Substitute $x = 0.577$ in the derivative:

$$\begin{aligned} f'(x) &= \frac{6\times 0.577}{\left(1-3\times 0.577^2\right)^2} \\ &= \frac{3.462}{\left(1-3\times 0.332929\right)^2} \quad \text{...... (1)} \\ &= \frac{3.462}{\left(1-0.998787\right)^2} \\ &= 2352911 \end{aligned}$$

There is no difficulty in to get the solution as the denominator does not come out to be zero in spite of being very close to it.

Comment

Step 3 of 4

Now, consider the 3-digit chopping case. Reduce the equation (1) into 3-digit arithmetic:

$$\begin{aligned} &= \frac{3.46}{\left(1-3\times 0.332\right)^2} \\ &= \frac{3.46}{\left(.004\right)^2} \\ &= \frac{3.46}{0.000004} \\ &= 216250 \end{aligned}$$

Hence, the solution in this case will come out as 216250. Now, the percentage of relative error can be calculated as:

$$\begin{aligned} \varepsilon_t &= \left|\frac{2352911 - 216250}{2352911}\right| \\ &= 90.8\% \end{aligned}$$

Comment

Step 4 of 4

Now, consider the 4-digit chopping case. Reduce the equation (1) into 4-digit arithmetic:

$$\begin{aligned} &= \frac{3.462}{\left(1-0.9987\right)^2} \\ &= \frac{3.462}{\left(.0013\right)^2} \end{aligned}$$

3.462

Step 1 of 2

Consider the following equation,

$$x^2 - 5000.002x + 10 = 0$$

Consider the roots of quadratic equation $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Compute the roots of the equation from the formula below

$$x = \frac{5000.002 \pm \sqrt{(5000.002)^2 - 4 \times 10}}{2}$$

So, $x_1 = 5000$ & $x_2 = 0.002$

Chop to 5 digits

$$\begin{aligned} x &= \frac{5000.0 \pm \sqrt{5000.0^2 - 4 \times 10}}{2} \\ &= \frac{5000.0 \pm \sqrt{25000000 - 4 \times 10}}{2} \\ &= \frac{5000.0 \pm \sqrt{24999960}}{2} \\ &= \frac{5000.0 \pm 4999.9}{2} \end{aligned}$$

Solve for two different roots:

$$\begin{aligned} \frac{5000.0 \pm 4999.9}{2} &= \frac{9999.9}{2} \text{ and } \frac{0.1}{2} \\ &= 4999.9 \text{ and } 0.05 \end{aligned}$$

Hence the relative percent error of the first root 4999.9 is

$$= \left|\frac{5000 - 4999.9}{5000}\right| \times 100\%$$

$$= \boxed{0.002\%}$$

And the relative percent error of the second root 0.05 is

$$= \left|\frac{0.002 - 0.05}{0.002}\right| \times 100\%$$

$$= \boxed{2400\%}$$

Comment

Step 2 of 2

Compute the roots again from the formula below:

$$\begin{aligned} \frac{-2c}{b \pm \sqrt{b^2 - 4ac}} &= \frac{-2(10)}{-5000.0 \pm \sqrt{25000000 - 4 \times 10}} \\ &= \frac{-20}{-5000.0 \pm \sqrt{24999960}} \\ &= \frac{-20}{-5000.0 \pm 4999.9} \end{aligned}$$

Solve for two different roots:

$$\begin{aligned} \frac{-20}{-5000.0 \pm 4999.9} &= \frac{-20}{-0.1} \& \frac{-20}{-9999.9} \\ &= 200 \ \& \ 0.00200002 \end{aligned}$$

Hence, the relative percent error of the first root 200 is

Step 1 of 2

Consider the polynomial:

$$y = x^3 - 7x^2 + 8x - 0.35$$

Put the value of $x = 1.37$ in the equation:

$$\begin{aligned} y &= 1.37^3 - 7\times1.37^2 + 8\times1.37 - 0.35 \\ &= 2.571353 - 7\times1.8769 + 8\times1.37 - 0.35 \\ &= 0.043053 \end{aligned}$$

Apply 3-digit Chopping:

$$\begin{aligned} &= 2.57 - 7\times1.87 + 8\times1.37 - 0.35 \\ &= 2.57 - 13.0 + 10.9 - 0.35 \\ &= 0.12 \end{aligned}$$

Calculate the percent relative error as following:

$$\frac{0.043053 - 0.12}{0.043053}\times100\% = 178.7\%$$

Hence, the percent relative error is $\boxed{178.7\%}$.

Comments (1)

Step 2 of 2

Consider the function:

$$y = ((x-7)x+8)x - 0.35$$

Put the value of $x = 1.37$ in the equation

$$\begin{aligned} y &= ((1.37-7)1.37+8)1.37 - 0.35 \\ &= (-7.7131+8)1.37 - 0.35 \\ &= (0.2869)1.37 - 0.35 \\ &= 0.043053 \end{aligned}$$

Calculate the function with 3 digit chopping

$$\begin{aligned} &= ((1.37-7)1.37+8)1.37 - 0.35 \\ &= (-7.71+8)1.37 - 0.35 \\ &= 0.047 \end{aligned}$$

Calculate the percent relative error as following

$$\begin{aligned} &= \frac{0.043053 - 0.047}{0.043053}\times100\% \\ &= 9.2\% \end{aligned}$$

Hence, the percent relative error is $\boxed{9.2\%}$. The error is reduced by great extent.

Comment