Lógica preposicional

¿Que es una preposición?
R/ Es una oración cuya valor es verdadero o falso.

- El gran colombiano apoya el paro --> Falso
- ¿Cuanto será mi nota final de discretas? ---> Número (no es una preposición?

Conectores -- Conjunción (Y)
-- Disyunción (O)
-- Negación (NO)
-- Implicación Sí .... entonces

Doble implicación
Sí y sólo si
-- ⊗
o ... o

x	y	$x \wedge y$	$x \vee y$	$\neg x$	$x \to y$	$x \leftrightarrow y$	$x \oplus y$
V	V	V	V	F	V	V	F
V	F	F	V	F	F	F	V
F	V	F	V	V	V	F	V
F	F	F	F	V	V	V	F

$x \leftrightarrow y \therefore x \to y \wedge y \to x$ $\quad x \to y \quad \neg x \vee y$

Si gano habrán menos impuestos y más salario. Demostremos que esto es falso

yo gano → p
habra menos impuesto → q
subira el salario → r
$p \to q \wedge r$

p = V q = V r = F
$V \to V \wedge F \quad V \to F \quad \therefore F$

$$p \to q \equiv \neg(p \wedge \neg q)$$

p	q	$p \to q$	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
V	V	V	F	F	V
V	F	F	V	V	F
F	V	V	F	F	V
F	F	V	V	F	V

$$p \to q \equiv \neg(p \wedge \neg q)$$

$$\neg p \vee q \equiv \neg p \vee q$$

ana es amiga sonia

es amiga — proposición

$P(x, y)$ x es amigus de y

$P(ana, sonia)$

$P(mauricio, jaime)$

$\forall x\, P(x, juan) \quad x \in S$

$\exists x\, P(jorge, x)$

$\nexists y\, P(jerrmias, y)$

$P(x, y) \quad x, y \in \{a, b\}$

x	y	
a	a	V
a	b	F
b	a	F
b	b	V

$\forall x \forall y\, P(x, y) \equiv F$

$\forall x \exists y\, P(x, y)$ $x=a$ $y=a$ $x=b$ $y=b$

$\exists y \forall x\, P(x, y) \equiv F$

$\exists x \exists y\, P(x, y) \equiv V$

$\exists x \forall y\, P(x, y) \equiv F$

$P(x,y,z)$ $D=\{0,1\}$

x	y	z	$P(x,y,z)$
1	1	1	V
1	1	0	F
1	0	1	V
1	0	0	V
0	1	1	F
0	1	0	F
0	0	1	V
0	0	0	F

$\forall x \forall y \forall z\ P(x,y,z) = F$

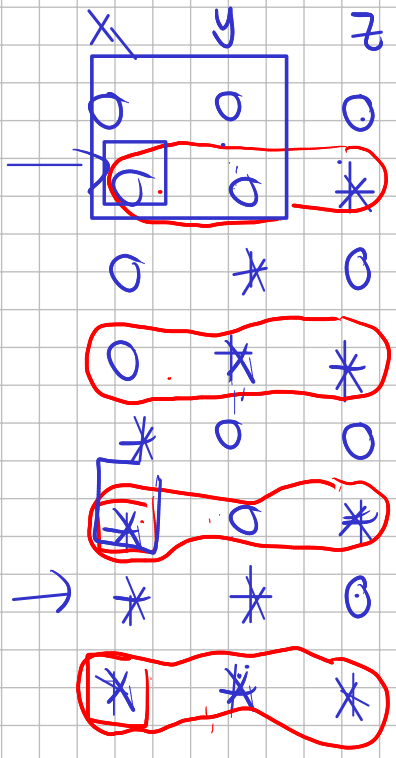
$\exists x \forall y \forall z\ P(x,y,z) = F$

$\exists y \forall x \forall z\ P(x,y,z) = F$

$\exists x \exists y \forall z\ P(x,y,z) = V$

$\forall x \exists y \exists z\ P(x,y,z) = V$

$P(x,y,z)$ $D \in \{o, *\}$



V
V
F
F
F
V
V
F

$\exists x \forall y\ P(x,y,*) = F$

$\forall y \exists x\ P(x,y,*) = F$

$\forall z \exists y \exists x\ P(x,y,z) = V$

$\exists x \exists y \forall z\ P(x,y,z) = V$

$X \in \{$productos cafeteria$\}$

$y \in \{$estudiantes$\}$

$P(x,y)$ a y le gusta X

$\exists x \forall y\ P(x,y)$ X = empanadas

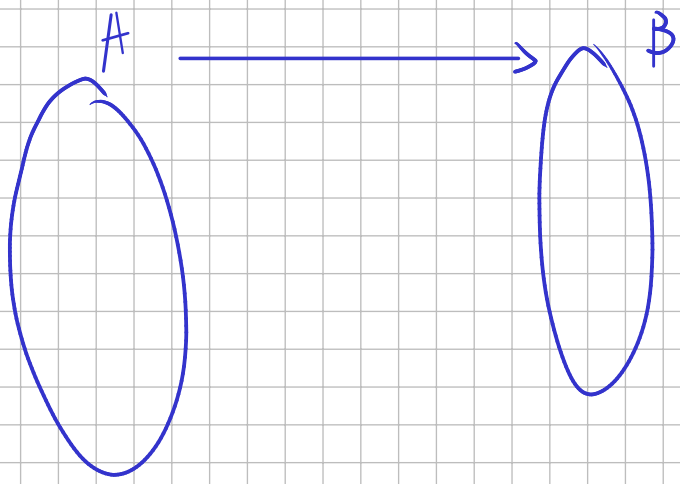
$\exists y \forall x\ P(x,y)$ y = Rivas

$\forall x \exists y\ P(x,y)$ V $\forall y \exists x\ P(x,y)$

$\forall x \forall y\ P(x,y)$ F V

$\exists x \exists y\ P(x,y)$ V

Funciones y conjuntos



$\emptyset$ $\quad \{\}$ $\quad \{\emptyset\}$

Conjunto vacío

$|\emptyset| = 0$ $\quad |\{\emptyset\}| = 1$ $\quad |\{\{\{\}\}\}| = 1$

$|\{\emptyset, \{\}\}| = 1$ $\quad \{1, 1, \emptyset, 2\} = 3$

Unión $\quad \{x \mid x \in A \vee x \in B\}$

$|A \cup B| = |A| + |B| - |A \cap B|$

Intersección $\quad \{x \mid x \in A \wedge x \in B\}$

$A - B \quad \{x \mid x \in A \wedge x \notin B\}$

$\overline{A} = \{x \mid x \notin A\} = \{x \mid x \in U \wedge x \notin A\}$

A	B	A ∪ B	A ∩ B	A − B	B − A	A ⊕ B
1	1	1	1	0	0	0
1	0	1	0	1	0	1
0	1	1	0	0	1	1
0	0	0	0	0	0	0

Producto potencia $A = \{a, b, c\}$ $2^{|A|}$

$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

producto cartesiano

$A = \{a, b\}$ $B = \{d, e\}$ $A \times B$

$\{(a,d), (a,e), (b,d), (b,e)\}$ $|A| \times |B|$

$B \times A$

$\{(d,a)(d,b)(e,a)(e,b)\}$

$|P(A \cup B)|$ $A = \{a, b, c\}$ $B = \{b, d, e\}$ 2^5

$|P(A \times B)|$ 2^9

$A \cup U = U$ $A \cap U = A$ $\bar{A} \cup A = U$

$\bar{A} \cap A = \emptyset$ $A - B$

$$\overline{(A \cup B) \cup C} \equiv \overline{((A \cup B) \cap \overline{C})}$$

$$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$$

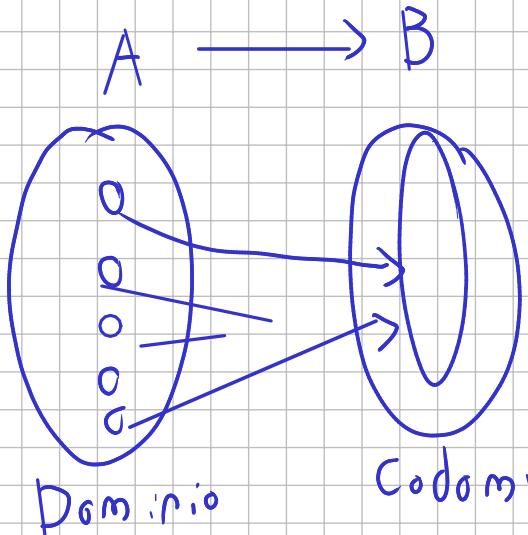
$$\{x \mid x \in \overline{(A \cup B) \cup C}\} \equiv \{x \mid x \in \overline{((A \cup B) \cap \overline{C})}\}$$

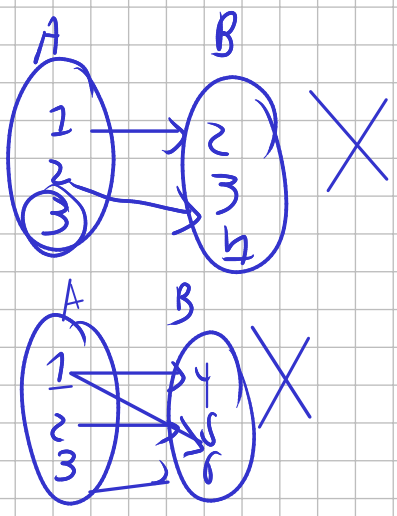
$$\{x \mid x \in \overline{(A \cup B)} \vee x \in C\} \equiv \{x \mid x \in \overline{(A \cup B)} \vee x \in C\}$$

$$\{x \mid (x \in \overline{A} \wedge x \in \overline{B}) \vee x \in C\} \equiv \{x \mid (x \in \overline{A} \wedge x \in \overline{B}) \vee x \in C\}$$

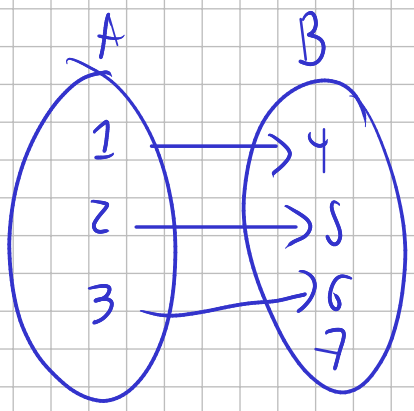
A	B	C	$\overline{(A \cup B)}$	$\overline{(A \cup B)} \cup C$	$\overline{C}$	$A \cup B$	$\cap$	$\overline{((A \cup B) \cap \overline{C})}$
1	1	1	0	1	0	1	0	1
1	1	0	0	0	1	1	1	0
1	0	1	0	1	0	1	0	1
1	0	0	0	0	1	1	1	0
0	1	1	0	1	0	1	0	1
0	1	0	0	0	1	1	1	0
0	0	1	1	1	0	0	0	1
0	0	0	1	1	1	0	0	1

Funciones

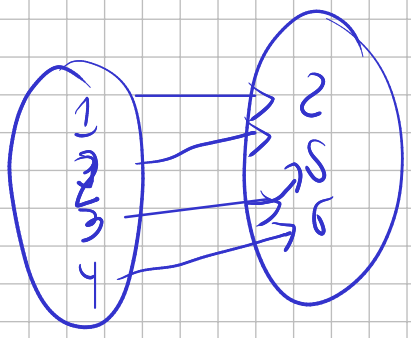




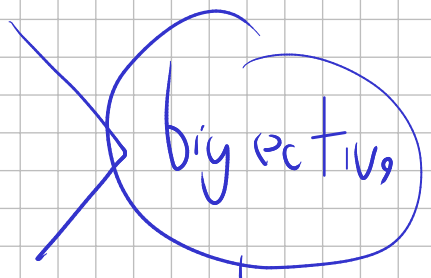
Rango $\subseteq$ Codominio



Inyectivo



Sobreyectivo



invertible

$$F: A \to B$$

$$F^{-1}\ B \to A$$

Sucesión → Sec numeros

$\{1, 1, 2, 3, 5, 8, 13, \ldots\}$ Suc Fibonacci

Aritmeticos

$\overset{a_0}{\{3}\ \underset{3}{6}\ \underset{3}{9}\ \underset{3}{12}\ \underset{3}{15}\ \underset{3}{18}\ldots\}$ $a_n = t + d_n$

Geométricos

$a_n = 3 + 3 \times n$

$\overset{a_0}{\{2}, 4, 8, 16, 32, 64, 128, \ldots\}$

$\times 2\ \times 2\ \times 2\ \times 2\ \times 2\ \times 2$

$a_n = 2^n \times a_0 \rightarrow a_n = t r^n$

$$1+2+3+4+5+\dots+100$$

$1+100=101$
$2+99=101$
$3+98=101$
$\vdots$
50 veces

101×50

5050

$$\frac{n(n+1)}{2}$$

101

$$\sum_{i=1}^{100} i \quad \text{vs} \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+4^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

n^2+1^2
$(n-1)^2+2^2$ $\quad (n-2)^2+3^2$
$n^2-2n+1+2^2$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=0}^{n} ar^i = \frac{ar^{n+1} - a}{r-1} \quad / \quad r = 1$$

$$\sum_{i=100}^{50700} i^2 \Rightarrow \sum_{i=1}^{50700} i^2 - \sum_{i=1}^{99} i^2$$

$$\frac{50700(50701)(101401)}{6} - \frac{99(100)(199)}{6}$$

$$\sum_{i=-300}^{80541} i = \sum_{i=1}^{80541} i + (0 - 1 - 2 - \cdots - 300)$$

$$-(0 + 1 + 2 + 3 + \cdots + 300)$$

$$-\left(0 + \sum_{i=1}^{300} i\right)$$

$$\frac{80541(80542)}{2} - \frac{300(301)}{2} \quad :)$$

$$\sum_{i=-700}^{10^6} i^2 \rightarrow \boxed{\sum_{i=1}^{10^6} i^2} + (0)^2 + (-1)^2 + (-2)^2 + \cdots + (-700)^2$$

$$0^2 + \sum_{i=1}^{700} i^2$$

$$\sum_{i=-999}^{10^8} 2^i \rightarrow \sum_{i=0}^{10^8} 2^i + 2^{-1} + 2^{-2} + 2^{-3} + \cdots + 2^{-999}$$

$$\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots + \left(\frac{1}{2}\right)^{999}$$

$$2^{-3} = \frac{1}{2^3} \rightarrow \left(\frac{1}{2}\right)^3$$

$$\sum_{i=0}^{999} \left(\frac{1}{2}\right)^i - \left(\frac{1}{2}\right)^0$$

$$\sum_{i=9}^{106} \sum_{j=4}^{200} i \times j$$

$$\sum_{i=9}^{100} i \sum_{j=4}^{200} j$$

$$\sum_{i=3}^{100} \sum_{j=10}^{200} i^j$$

i	3	4	5	6	7	$\cdots$	100
j 10	3^{10}	4^{10}	5^{10}	6^{10}	7^{10}		100^{10}
11	3^{11}	4^{11}					
12	3^{12}						
13	13^{13}						
$\vdots$							
200	3^{200}						100^{200}

$$\therefore \Rightarrow \sum_{k=10}^{200} 3^k + \sum_{k=10}^{200} 4^k + \cdots \sum_{k=10}^{200} 100^k$$

$$\sum_{k=0}^{200} 3^k - \sum_{k=0}^{9} 3^k + \cdots + \sum_{k=0}^{200} 100^k - \sum_{k=0}^{9} 100^k$$

$$\frac{3^{201}-1}{2} - \frac{3^{10}-1}{2} + \cdots + \frac{100^{201}-1}{2} - \frac{100^{10}-1}{2}$$