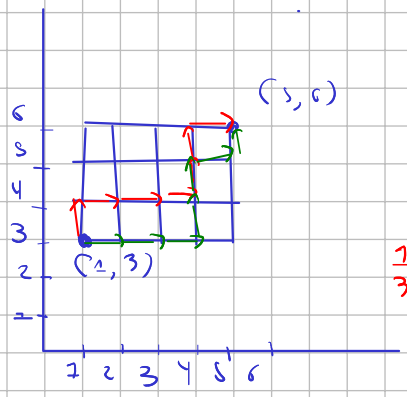
¿Cuántas trayectorias diferentes en el plano xy hay de $(1, 3)$ a $(5, 6)$ si la trayectoria avanza una unidad a la vez yendo una hacia la derecha (D) o una hacia arriba (A)?

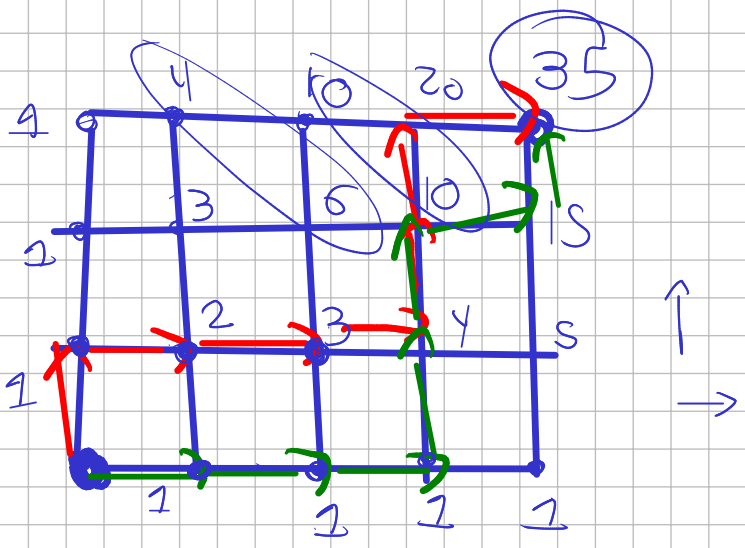


A ↑ 3
D → 4

$7C3 \times 4C4 = 35$
$7C4 \times 3C3 = 35$

D D D A A D A
1 2 3 4 5 6 7

1 A
3 D



How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that a_1 **mod** $5 = a_2$ **mod** 5 and b_1 **mod** $5 = b_2$ **mod** 5?

N

$x \bmod 5 = \{0, 1, 2, 3, 4\}$

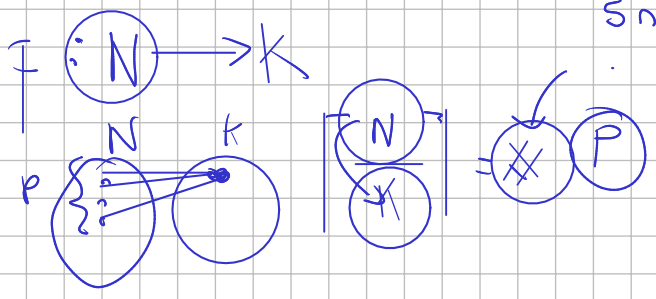
5 numeros

$(0,0)$ ⋮ $(5,5)$ } 25

$(a_1, b_1) = (a_2, b_2)$

$\frac{a_1}{5}$ $\frac{b_1}{5}$

25



$\left\lceil \frac{N}{25} \right\rceil = 2$

$N = (2-1) \times 25 + 1$

$N = 26$

$$T(n) = 3T\left(\frac{n}{3}\right) + 3n + 4 \qquad T(1) = 4$$

$T(n) = cT(n-1) + \dots$ T_k $T(n-k)$

$T(3) = 3T(1) + 3\times 3 + 4$

$T(3) = 12 + 9 + 4 = 25$

$n = 3^k \qquad 3^{k-1}$

$$T(3^k) = 3T\left(\frac{3^k}{3}\right) + 3\times 3^k + 4$$

$T(3^k) = T_k$

$$T_k = 3T_{k-1} + 3\times 3^k + 4$$

R. R no homogénea

$T_k = T_k^{(h)} + T_k^{(p)}$ $\quad r = 3$

$$T_k^{(p)} = B + Ck3^k$$

$r - 3 = 0$

$T_k^{(h)} = A(3)^k$

$B + Ck3^k = 3B + 3C(k-1)3^k + 3\times 3^k + 4$

$k3^k$	$C = C$ ✓
3^k	$0 = -C + 3 \quad C = 3$ ✓
	$B = 3B + 4 \quad B = \frac{4}{-2} = -2$

$T_k = A(3)^k - 2 + 3k3^k$ c+p

$n = 3^k \qquad k = \log_3(n)$

$x^{\log_b a} = a^{\log_b x}$

$\log_b a = \frac{\log_c a}{\log_c b}$ c+p

$T(n) = A(3)^{\log_3(n)} - 2 + 3\log_3(n)\,3^{\log_3(n)}$

$T(n) = A(n)^{\log_3 3} - 2 + 3\log_3(n)\,n^{\log_3(3)}$

$T(n) = An - 2 + 3\log_3(n)\,n \longrightarrow O(n\log(n))$

$4 = A - 2 \qquad A = 6$

$T(n) = 6n - 2 + 3\log_3(n)\times n$

$T(1) = 4$ ✓

$T(3) = 6\times 3 - 2 + 3\log_3(3)\times 3$

$18 - 2 + 3\times 3 =$ (25) :)