Sumatorias dobles

$$\sum_{i=1}^{n}\sum_{j=1}^{n} ij = \sum_{i=1}^{n} i\left(\sum_{j=1}^{n} j\right) = \sum_{i=1}^{n} i\left(\boxed{\frac{n(n+1)}{2}}\right)$$

$$\frac{n(n+1)}{2}\sum_{i=1}^{n} i = \left(\frac{n(n+1)}{2}\right)^2 \qquad \left(\frac{10(11)}{2}\right)^2 = 55^2$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} ij = \sum_{i=1}^{n} i(\underbrace{1+2+3+4+\ldots n}_{\frac{n(n+1)}{2}})$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(i+j) = \sum_{i=1}^{n}\sum_{j=1}^{n} i + \sum_{i=1}^{n}\sum_{j=1}^{n} j$$

$$\sum_{i=1}^{n} ni + \sum_{i=1}^{n}\frac{n(n+1)}{2}$$

$$\frac{n^2(n+1)}{2} + \frac{n^2(n+1)}{2} = n^2(n+1)$$

$$(10)^2 11 = 1100$$

```
for(int i = 1; i<=n; i++){
        for(int j=1; j<=n; j++){
                salida = i + j;
        }
}
```

$$\sum_{i=1}^{n}\sum_{j=1}^{n} i^{j}$$

0^0 $\qquad$ $1^{1000} = 1$

$$\sum_{j=1}^{n} 1 = n$$

$i \backslash j$	1	2	3	4	...	n	
1	1^1 +	1^2 +	1^3 +	1^4 +		1^n	$\sum_{j=1}^{n} 1^j = n$
2	2^1	2^2	2^3	2^4		2^n	$\sum_{j=1}^{n} 2^j = \frac{2^{n+1}-1}{2-1} - 1$
3	3^1	3^2	3^3	3^4		3^n	$\sum_{j=1}^{n} 3^j = \frac{3^{n+1}-1}{3-1} - 1$
4	4^1	4^2	4^3	4^4		4^n	$\sum_{j=1}^{n} 4^j = \frac{4^{n+1}-1}{4-1} - 1$
⋮							
n	n^1	n^2	n^3	n^4		n^n	$\sum_{j=1}^{n} n^j = \frac{n^{n+1}-1}{n-1} - 1$
	$\sum_{i=1}^{n} i$	$\sum_{i=1}^{n} i^2$	$\sum_{i=1}^{n} i^3$	$\sum_{i=1}^{n} i^4$		$\sum_{i=1}^{n} i^n$	

$$n + \sum_{i=2}^{n}\left(\frac{i^{n+1}-1}{i-1} - 1\right)$$

$$n + \sum_{i=2}^{n} \frac{i^{n+1}}{i-1} - \sum_{i=2}^{n} \frac{1}{i-1} - \sum_{i=2}^{n} 1$$

$$\sum_{j=1}^{n} r^{j} = \underbrace{\frac{r^{n+1}-1}{r-1}}_{\sum_{j=0}^{n} r^j} - 1 \qquad r \neq 1$$

$$- r^0$$

$$\sum_{r=0}^{n} a r^{j} = \frac{r^{n+1} - a}{r-1}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} j^2 i$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} (i+j)^2$$

1) $$\sum_{i=1}^{n} i\left(\sum_{j=1}^{n} j^2\right)$$

$$\sum_{i=1}^{n} i\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$\left(\frac{n(n+1)}{2}\right)\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$\frac{10(11)}{2}\left(\frac{10(11)(21)}{6}\right) = 21175$$

$$\overbrace{\sum_{i=1}^{n}\sum_{j=1}^{n}}^{\text{doble for}}\left(i^2 + 2ij + j^2\right)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} ij + \sum_{i=1}^{n}\sum_{j=1}^{n} j^2$$

$$\sum_{i=1}^{n} n i^2 + 2\sum_{i=1}^{n} i\left(\frac{n(n+1)}{2}\right) + \boxed{\sum_{i=1}^{n}\left(\frac{n(n+1)(2n+1)}{6}\right)}$$

$$2\left(\frac{n^2(n+1)(2n+1)}{6}\right) + 2\left(\frac{n(n+1)}{2}\right)^2$$

$$\frac{n^2(n+1)(2n+1)}{3} + \frac{(n(n+1))^2}{2}$$

$$\frac{10^2(11)(21)}{3} + \frac{((10)(11))^2}{2} = 13750$$

$$\sum_{i=100}^{n}\sum_{j=-300}^{n} ij$$

$$\sum_{i=-400}^{n}\sum_{j=800}^{n} (i+j)$$

$$\sum_{i=-400}^{n}\sum_{j=800}^{n}(i+j) \qquad \sum_{i=-400}^{n}\sum_{j=800}^{n} i + \sum_{i=-400}^{n}\sum_{j=800}^{n} j$$

$$\sum_{i=-400}^{n} i\left(\sum_{j=800}^{n}1\right) + \sum_{i=-400}^{n}\left(\sum_{j=800}^{n} j\right)$$

$$\sum_{i=-400}^{n} i\left(\sum_{j=1}^{n}1 - \sum_{j=1}^{799}1\right) + \sum_{i=-400}^{n}\left(\sum_{j=1}^{n} j - \sum_{j=1}^{799} j\right)$$

$$\sum_{i=-400}^{n} i(n-799) + \sum_{i=-400}^{n}\left(\frac{n(n+1)}{2} - \frac{799(800)}{2}\right)$$

$$(n-799)\sum_{i=-400}^{n} i + \left(\frac{n(n+1)}{2} - 799(400)\right)\sum_{i=-400}^{n}1$$

$$\sum_{i=-400} i \qquad \sum_{i=-400} j \qquad \sum_{i=-400} 2j$$

$$(n-799)\left(-400-399-\cdots-1+0+1+2+\cdots n\right) \qquad +$$

$$n+401$$

$$\left(\frac{n(n+1)}{2} - 799(400)\right)\left(1+1+1+\cdots 1+1+1+1+1+\cdots+1\right)$$

-400 -399 -398 ... -1 0 1 2 n ← indice

$$(n-799)\left(-\sum_{i=1}^{400} i + \sum_{i=1}^{n} i\right) + \left(\frac{n(n+1)}{2} - 799(400)\right)(n+401)$$

$$(n-799)\left(-\frac{400(401)}{2} + \frac{n(n+1)}{2}\right) + \left(\frac{n(n+1)}{2} - 799(400)\right)(n+401)$$

$n=1000$ 337921200

$$\sum_{i=100}^{n}\sum_{j=-300}^{n} ij = \sum_{i=100}^{n} i\left(\sum_{j=-300}^{n} j\right)$$

$$\sum_{i=100}^{n} i\left(-300-299-298-\cdots-1+\underline{0+1+2+3\cdots+n}\right)$$

$$\sum_{i=100}^{n} i\left(\underline{-300-299-298-\cdots-1}+0+\sum_{j=1}^{n} j\right)$$

$$\sum_{i=100}^{n} i\left(-\sum_{j=1}^{300} j+\sum_{j=1}^{n} j\right) = \sum_{i=100}^{n} i\left(-\frac{300(301)}{2}+\frac{n(n+1)}{2}\right)$$

$$\left(-\frac{300(301)}{2}+\frac{n(n+1)}{2}\right)\underbrace{\sum_{i=100}^{n} i} = \left(-\frac{300(301)}{2}+\frac{n(n+1)}{2}\right)\left(\underbrace{\sum_{i=1}^{n} i}-\sum_{i=1}^{99} i\right)$$

$$\left(-\frac{300(301)}{2}+\frac{n(n+1)}{2}\right)\left(\frac{(n)(n+1)}{2}-\frac{99(100)}{2}\right)$$

$$\left(-\frac{300(301)}{2}+\frac{500(501)}{2}\right)\left(\frac{500(501)}{2}-\frac{99(100)}{2}\right) = \underline{9636030000}$$

```java
for(long i =1; i<=n; i++){
  for(long j=i; j<=n; j++){
    System.out.println(i+" "+j);
  }
}
```

1

$$\sum_{i=1}^{n}\sum_{j=i}^{n} 1$$

i \ j	1	2	3		n	
1	1	+ 1	+ 1	+ …	1	$\sum_{j=1}^{n} 1$ = n
2		1	+ 1	+ …	1	$\sum_{j=2}^{n} 1 = \sum_{j=1}^{n} 1 - 1$
3			1	+ …	1	$\sum_{j=3}^{n} 1 = \sum_{j=1}^{n} 1 - 1 - 1$
⋮						
n					1	$\sum_{j=n}^{n} 1 = \sum_{j=1}^{n} 1 - 1 - 1 \ldots 1$

$n \times n$

$i-1$

$n-1$

$$n^2 - \sum_{i=2}^{n}(i-1)$$

$$n^2 - \left(\sum_{i=1}^{n}(i-1) - \sum_{i=1}^{2}(i-1)\right)$$

$$n^2 - \left(\frac{n(n+1)}{2} - n\right) - \left(\frac{(2)(3)}{2} - 2\right)$$

$$n^2 - \left(\frac{n(n+1)}{2} - n\right) - 1 = \boxed{n^2 - \frac{n^2+n}{2} - n - 1}$$

```java
for(long i =1; i<=n; i+=2){
  for(long j=1; j<=n; j++){
    System.out.println(i+" "+j);
  }
}
```

$\frac{n}{2}$

1, 3, 5, 7, 9, ... n

$\boxed{it} \to 0 \quad 1 \quad 2 \quad 3 \quad 4 \ldots$

$$\sum_{k=0}^{\frac{n-1}{2}} \sum_{j=1}^{n} (2k+1)$$

iterod

$i = 2k+1$

$n = 2k+1 \qquad k = \frac{n-1}{2}$

$\frac{n^2}{2}$