Fundamentos de análisis y diseño de algoritmos

Ecuaciones de recurrencia

Recurrencias

Método de iteración

Método maestro*

Método de sustitución

Recurrencias

Método de iteración

Expandir la recurrencia y expresarla como una suma de términos que dependen de n y de las condiciones iniciales

Recurrencias

$T(n) = n + 3T(n/4)$, $T(1)= \Theta(1)$ y n par

Expandir la recurrencia 2 veces

Recurrencias

$T(n) = n + 3\boxed{T(n/4)}$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ + 3T(n/64)\,))$

$n + 3^*n/4 + 3^{2*}n/4^2\ + 3^3T(n/4^3)$

$T(1)$

$T\left(\frac{n}{4}\right) = \boxed{\frac{n}{4} + 3T\left(\frac{n}{4^2}\right)}$

$T\left(\frac{n}{4^2}\right) = \frac{n}{4^2} + 3T\left(\frac{n}{4^3}\right)$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\ (\ n/4 + 3T(n/16))$

$n + 3\ (\ n/4 + 3(n/16\ \ + 3T(n/64)\ ))$

$n + 3^*n/4 + 3^2{}^*n/4^2\ \ + 3^3T(n/4^3)$

¿Cuándo se detienen las iteraciones?

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ + 3T(n/64)\,))$

$n + 3*n/4 + 3^2*n/4^2\ + 3^3T(n/4^3)$

¿Cuándo se detienen las iteraciones?

Cuando se llega a $T(1)$

Recurrencias

$3^i T\left(\frac{n}{4^i}\right)$

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16 \;+ 3T(n/64)\,))$

$\hookrightarrow n + 3{*}n/4 + 3^2{*}n/4^2 + 3^3T(n/4^3)$

$n + 3\frac{n}{4} + n\left(\frac{3}{4}\right)^2 + n\left(\frac{3}{4}\right)^3 + n\left(\frac{3}{4}\right)^4 + \dots + n\left(\frac{3}{4}\right)^{i-1} + 3^i T\left(\frac{n}{4^i}\right)$

¿Cuándo se detienen las iteraciones?

Cuando se llega a T(1), esto es, cuando $(n/4^i)=1$

$4^i = n \qquad i = \log_4(n)$

$\left(\frac{3}{4}\right)^0 n + \left(\frac{3}{4}\right) n + \left(\frac{3}{4}\right)^2 n + \left(\frac{3}{4}\right)^3 n + \dots + \left(\frac{3}{4}\right)^{\log_4(n)-1} n + 3^{\log_4(n)} T(1)$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\ (\ n/4 + 3T(n/16))$

$n + 3\ (\ n/4 + 3(n/16\ \ + 3T(n/64)\ ))$

$n + 3*n/4 + 3^2*n/4^2\ \ + 3^3T(n/4^3)$

$n + 3*n/4 + 3^2*n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}T(1)$

¿Cuándo se detienen las iteraciones?

Cuando se llega a T(1), esto es, cuando $(n/4^i)=1$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ \ + 3T(n/64)\ ))$

$n + 3*n/4 + 3^2*n/4^2\ \ + 3^3T(n/4^3)$

$n + 3*n/4 + 3^2*n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}T(1)$

Después de iterar, se debe tratar de expresar como una sumatoria con forma cerrada conocida

Recurrencias

$$T(n) = n + 3T(n/4\rfloor)$$

$$n + 3\ (n/4\rfloor + 3T(n/16\rfloor))$$

$$n + 3\ (n/4\rfloor + 3(n/16\rfloor\ + 3T(n/64\rfloor)\ ))$$

$$n + 3{*}n/4\rfloor + 3^2{*}n/4^2\rfloor\ + 3^3(n/4^3\rfloor) + \ldots + 3^{\log 4n}\Theta(1)$$

$$\leq\ n + 3n/4 + 3^2n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}\Theta(1)$$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ + 3T(n/64)\,))$

$n + 3*n/4 + 3^2*n/4^2\ + 3^3(n/4^3) + \ldots + 3^{\log 4n}\Theta(1)$

$\leq\ n + 3n/4 + 3^2 n/4^2 + 3^3 n/4^3 + \ldots + 3^{\log 4n}\Theta(1)$

$$= \left(\sum_{i=0}^{\log_4 n}\left(\frac{3}{4}\right)^i n\right) + 3^{\log_4 n}\Theta(1) \qquad n^{\log_4\left(\frac{3}{4}\right)}$$

$$= n\left(\frac{(3/4)^{(\log_4 n)} - 1}{(3/4) - 1}\right) + n^{\log_4 3} = n * 4(1 - (3/4)^{(\log_4 n)}) + \Theta(n^{\log_4 3})$$

$= O(n)$

Recurrencias

Resuelva por el método de iteración

$$T(n) = 2\boxed{T(n/2)} + 1,\ T(1) = \Theta(1)$$

$$T(n) = 2\left(\boxed{2T\left(\frac{n}{2^2}\right)} + 1\right) + 1$$

$$T(n) = 2\left(2\left(\boxed{2T\left(\frac{n}{2^3}\right)} + 1\right) + 1\right) + 1$$

$$T(n) = 2\left(2\left(2\left(2T\left(\frac{n}{2^4}\right) + 1\right) + 1\right) + 1\right) + 1$$

$$T(n) = 2^4 T\left(\frac{n}{2^4}\right) + 2^3 + 2^2 + 2 + 2^0$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + \underline{2^{i-1} + 2^{i-2} + \dots + 2^2 + 2^1 + 2^0}$$

$$T(1) \qquad 1 = \frac{n}{2^i} \qquad i = \log_2(n)$$

$$a^{\log_b(c)} = c^{\log_b(a)}$$

$$2^{\log_2(n)} T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i$$

$$n^{\log_2(2)} \times C + \frac{2^{\log_2(n) - \cancel{1} + 1} - 1}{2-1}$$

$$Cn + n - 1 = O(n)$$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1,\ T(1) = \Theta(1)$

$T(n) = 2T(n/2) + n,\ T(1) = \Theta(1)$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

$$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + 1\right) + 1$$

$$T(n) = 2\left(2\left(2T\left(\frac{n}{2^3}\right) + 1\right) + 1\right) + 1$$

$$T(n) = 2\left(2\left(2\left(2T\left(\frac{n}{2^4}\right) + 1\right) + 1\right) + 1\right) + 1$$

$$T(n) = 2^4 T\left(\frac{n}{2^4}\right) + 2^3 + 2^2 + 2^1 + 2^0$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + 2^{i-1} + 2^{i-2} + \dots + 2^1 + 2^0$$

$$T(1)$$

$$\frac{n}{2^i} = 1 \qquad i = \log_2(n)$$

$$T(n) = 2^{\log_2(n)} T(1) + 2^{\log_2(n)-1} + 2^{\log_2(n)-2} + \dots + 2^1 + 2^0$$

$$\boxed{\sum_{k=0}^{n} ar^i = \frac{ar^{n+1} - a}{r-1}}$$

$$T(n) = n\,T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i$$

$$T(n) = n\,\Theta(1) + \frac{2^{\log_2(n)-1+1} - 1}{2-1}$$

$$T(n) = n\,\Theta(1) + n - 1 \longrightarrow \Theta(1)$$

$$T(n) = 2\,\boxed{T\left(\frac{n}{2}\right)} + n$$

$$2\left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right) + n$$

$$2\left(2\left(2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right) + \frac{n}{2}\right) + n$$

$$2^3 T\left(\frac{n}{2^3}\right) + n + n + n$$

$$2^3 T\left(\frac{n}{2^3}\right) + 3n$$

$$2^2 T\left(\frac{n}{2^2}\right) + n + n$$

$2^i T\left(\frac{n}{2^i}\right) + in$ $\qquad \frac{n}{2^i} = 1 \qquad i = \log_2(n)$

$2^{\log_2(n)} T(1) + \log_2(n)\, n$

$n\,\Theta(1) + \cancel{n\log_2(n)} = O(n\log(n))$

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2 \qquad \Theta(1) = T(1)$$

1) $$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right) + n^2$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2\left(\frac{n}{2}\right)^2 + n^2$$

$$T(n) = 2^2\left(2T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right) + 2\left(\frac{n}{2}\right)^2 + n^2$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 2^2\left(\frac{n}{2^2}\right)^2 + 2\left(\frac{n}{2}\right)^2 + n^2$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + 2^{i-1}\left(\frac{n}{2^{i-1}}\right)^2 + 2^{i-2}\left(\frac{n}{2^{i-2}}\right)^2 + \dots + 2\left(\frac{n}{2}\right)^2 + 2^0\left(\frac{n}{2^0}\right)^2$$

$$i = \log_2(n)$$

$$T(n) = n\,T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i \left(\frac{n}{2^i}\right)^2$$

$$\frac{\cancel{2^i}\, n^2}{(2^i)^{\cancel{2}}}$$

$$T(n) = n\,\Theta(1) + \sum_{i=0}^{\log_2(n)-1} n^2 \left(\frac{1}{2^i}\right)$$

$$T(n) = n\,\Theta(1) + n^2 \sum_{i=0}^{\log_2(n)-1} \underbrace{\left(\frac{1}{2}\right)}_{r}^{i}$$

$$T(n) = n\,\theta(1) + n^2\left(\frac{\left(\frac{1}{2}\right)^{\log_2(n)} - 1}{\frac{1}{2} - 1}\right)$$

$$T(n) = n\,\theta(1) + n^2\left(\frac{n^{\log_2(0.5)} - 1}{-\frac{1}{2}}\right)$$

$$T(n) = n\,\theta(1) + n^2\left(-2\left(n^{-1} - 1\right)\right)$$

$$T(n) = n\,\theta(1) - 2n + 2n^2$$

$$\theta(n^2)$$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1,\ T(1) = \Theta(1)$

$T(n) = 2T(n/2) + n,\ T(1) = \Theta(1)$

$T(n) = T(n/2) + 1,\ T(1) = \Theta(1)$

$$T(n) = 4T\left(\frac{n}{3}\right) + n + 1 \qquad T(1) = 1$$

$$T(n) = 4\left(4T\left(\frac{n}{3^2}\right) + \frac{n}{3} + 1\right) + n + 1$$

$$T(n) = 4^2 T\left(\frac{n}{3^2}\right) + \left(\frac{4}{3}\right)n + 4 + n + 1$$

$$T(n) = 4^2\left(4T\left(\frac{n}{3^3}\right) + \frac{n}{3^2} + 1\right) + \frac{4}{3}n + 4 + n + 1$$

$$T(n) = 4^3 T\left(\frac{n}{3^3}\right) + \frac{4^2 n}{3^2} + 4^2 + \frac{4}{3} n + 4 + \left(\frac{4}{3}\right)^0 n + 4^0$$

$$T(n) = 4^i T\left(\frac{n}{3^i}\right) + \left(\frac{4}{3}\right)^{i-1} n + 4^{i-1} + \left(\frac{4}{3}\right)^{i-2} n + 4^{i-2} \cdots + \left(\frac{4}{3}\right)^0 n + 4^0$$

$$T(1) \qquad 1 = \frac{n}{3^i} \qquad i = \log_3(n)$$

$$T(n) = 4^{\log_3(n)} \times \underbrace{T(1)}_{1} + \sum_{i=0}^{\log_3(n)-1} \left(\left(\frac{4}{3}\right)^i n + 4^i \right)$$

$$T(n) = 1n^{\log_3(4)} + n\left(\frac{\frac{4}{3}^{\log_3(n)} - 1}{4/3 - \cancel{1}\frac{3}{3}}\right) + \left(\frac{4^{\log_3(n)} - 1}{4 - 1}\right)$$

$$T(n) = n^{\log_3(4)} + 3n \times n^{\log_3\left(\frac{4}{3}\right)} - 3n + \frac{n^{\log_3(4)} - 1}{3}$$

$$\Theta\left(n^{\log_3(4)}\right)$$

$$\log_3\left(\frac{4}{3}\right) + 1$$

$$\frac{\log_3(4) - \log_3(3) + 1}{1}$$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1$, $T(1)= \Theta(1)$

$T(n) = 2T(n/2) + n$, $T(1)= \Theta(1)$

$T(n) = T(n/2) + 1$, $T(1)= \Theta(1)$

Demuestre que $T(n) = T(n/2\rfloor) + n$, es $\Omega(n\log n)$

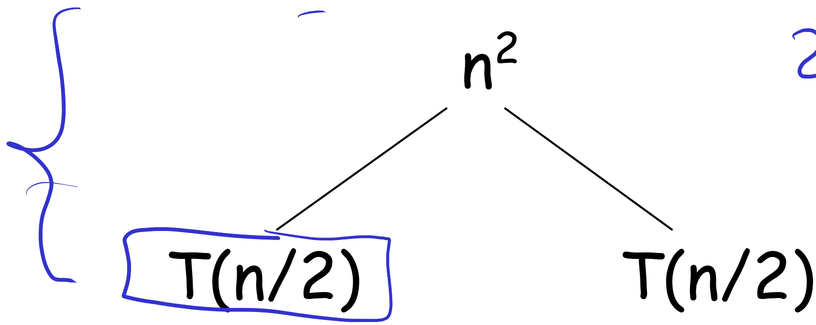
Recurrencias

Iteración con árboles de recursión

$T(1) = 1$

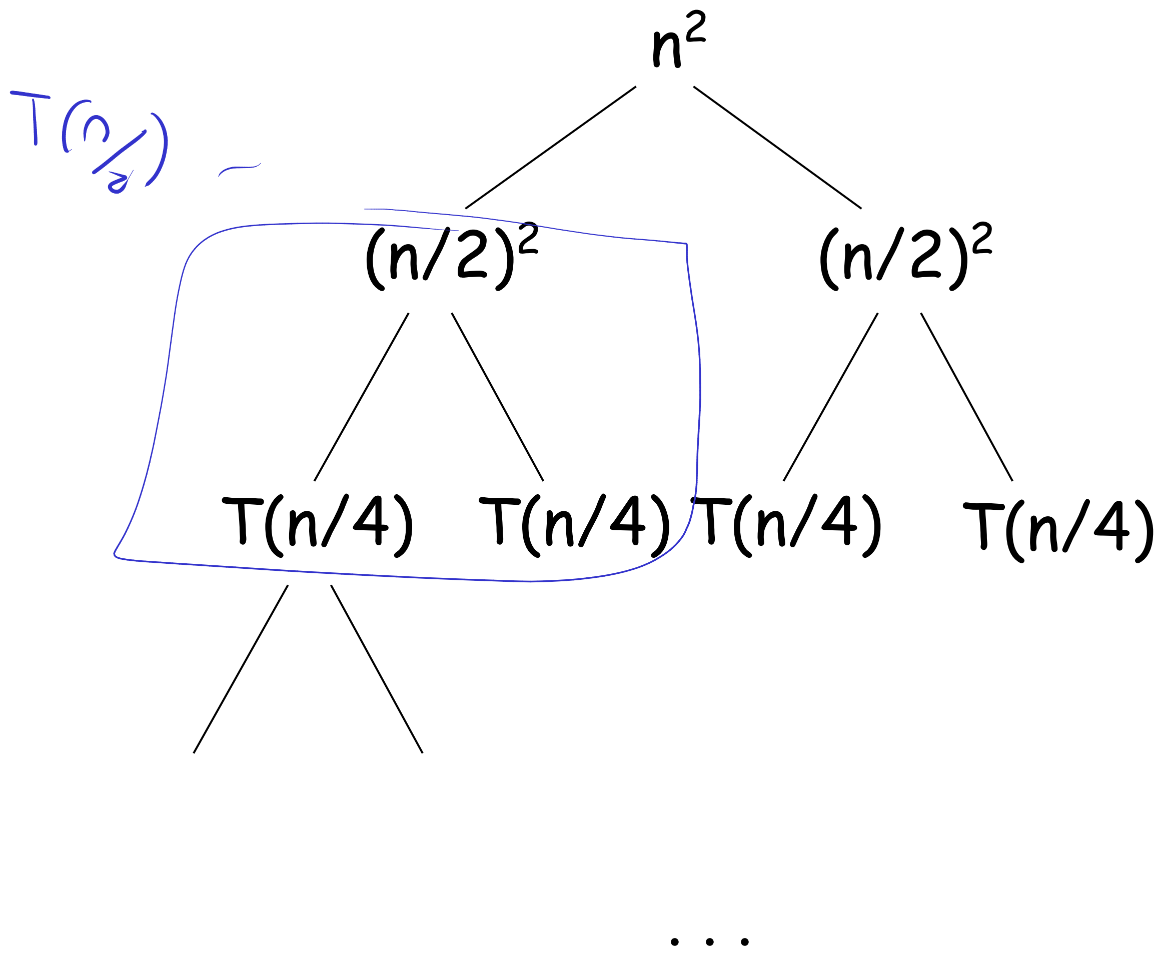
$T(n) = 2T(n/2) + n^2$

Recurrencias



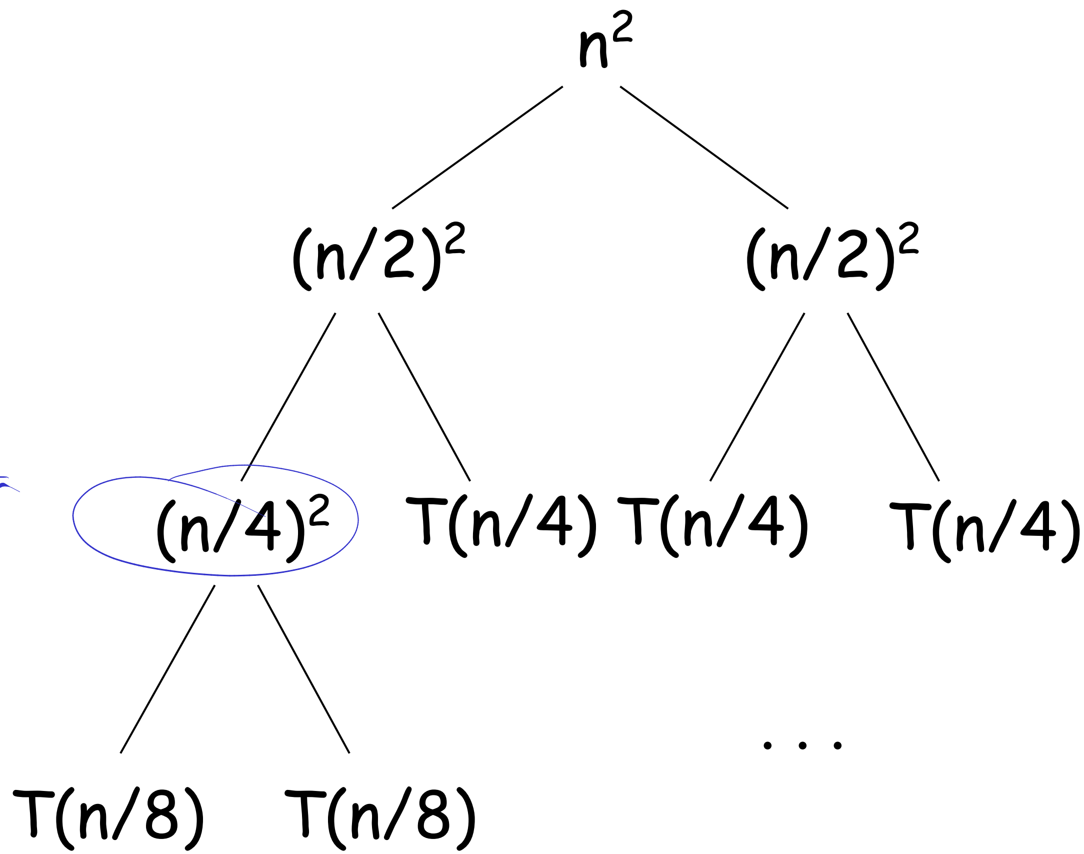
$2T(n/2) + n^2$

Recurrencias



Recurrencias

$4\left(\frac{n}{4}\right)^2$



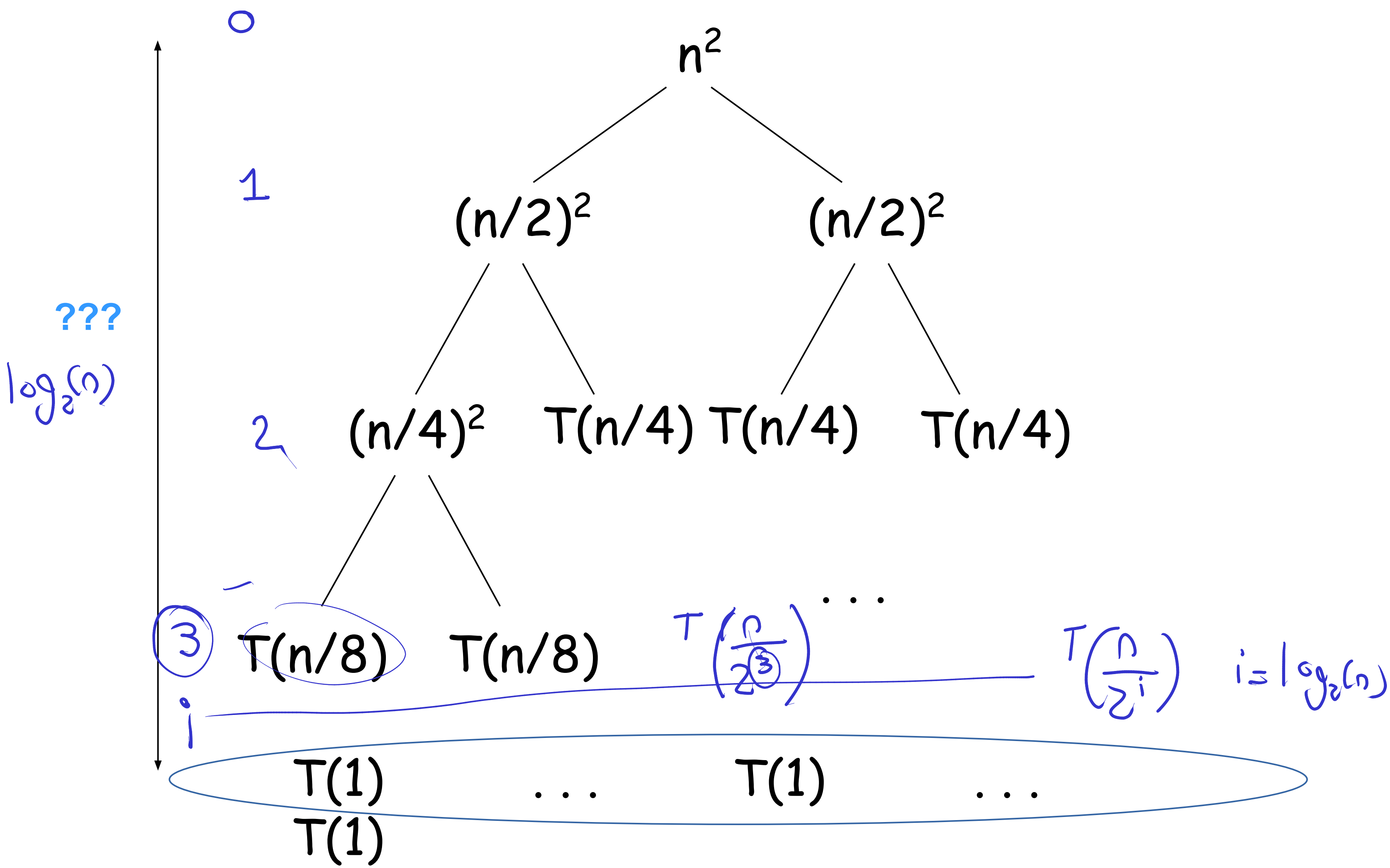
$n^2 = \frac{n^2}{2^0}$

$\frac{n^2}{2} = \frac{n^2}{2^1}$

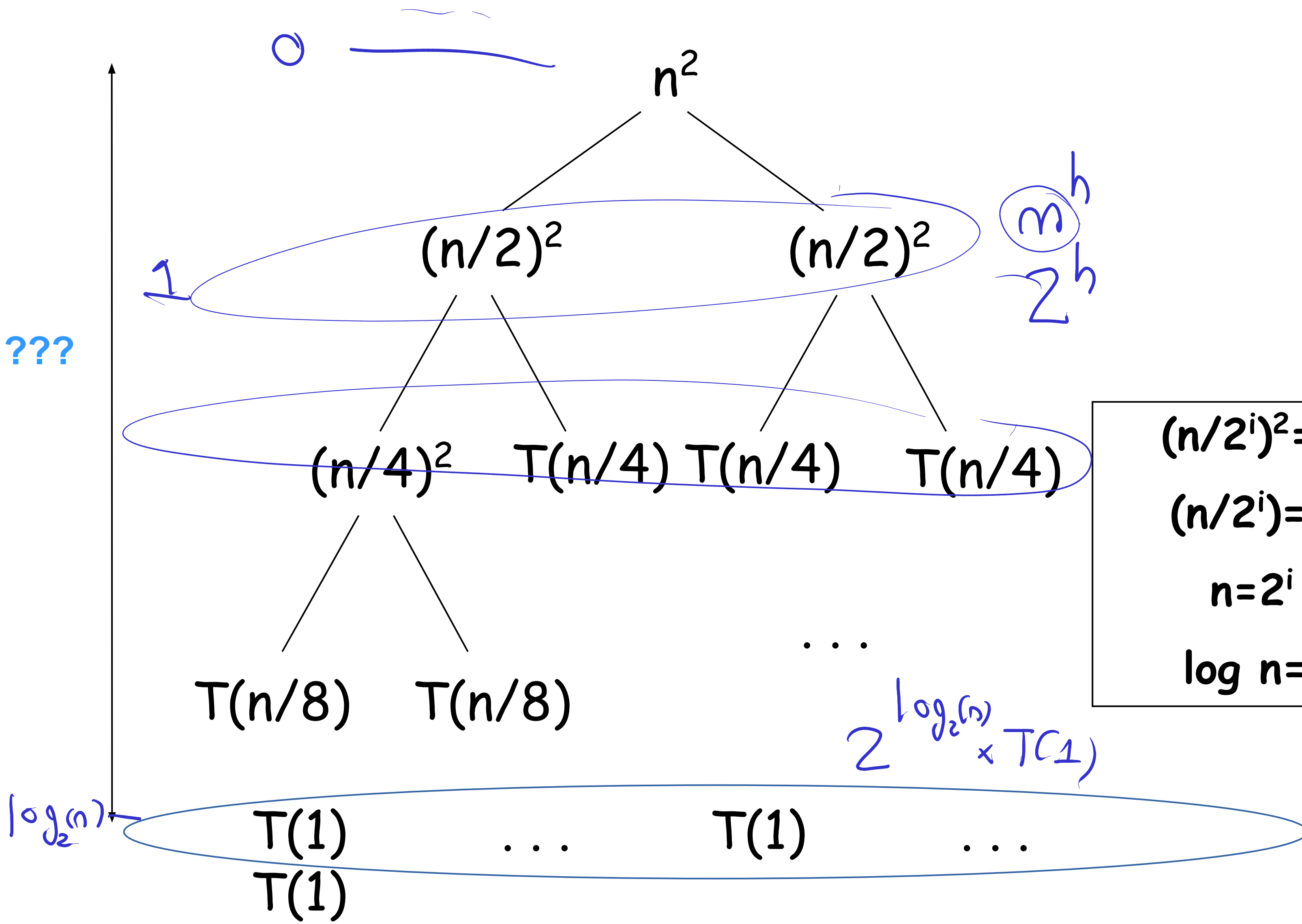
$\frac{n^2}{4} = \frac{n^2}{2^2}$

$\frac{n^2}{8} \quad \frac{n^2}{2^3}$

Recurrencias



Recurrencias



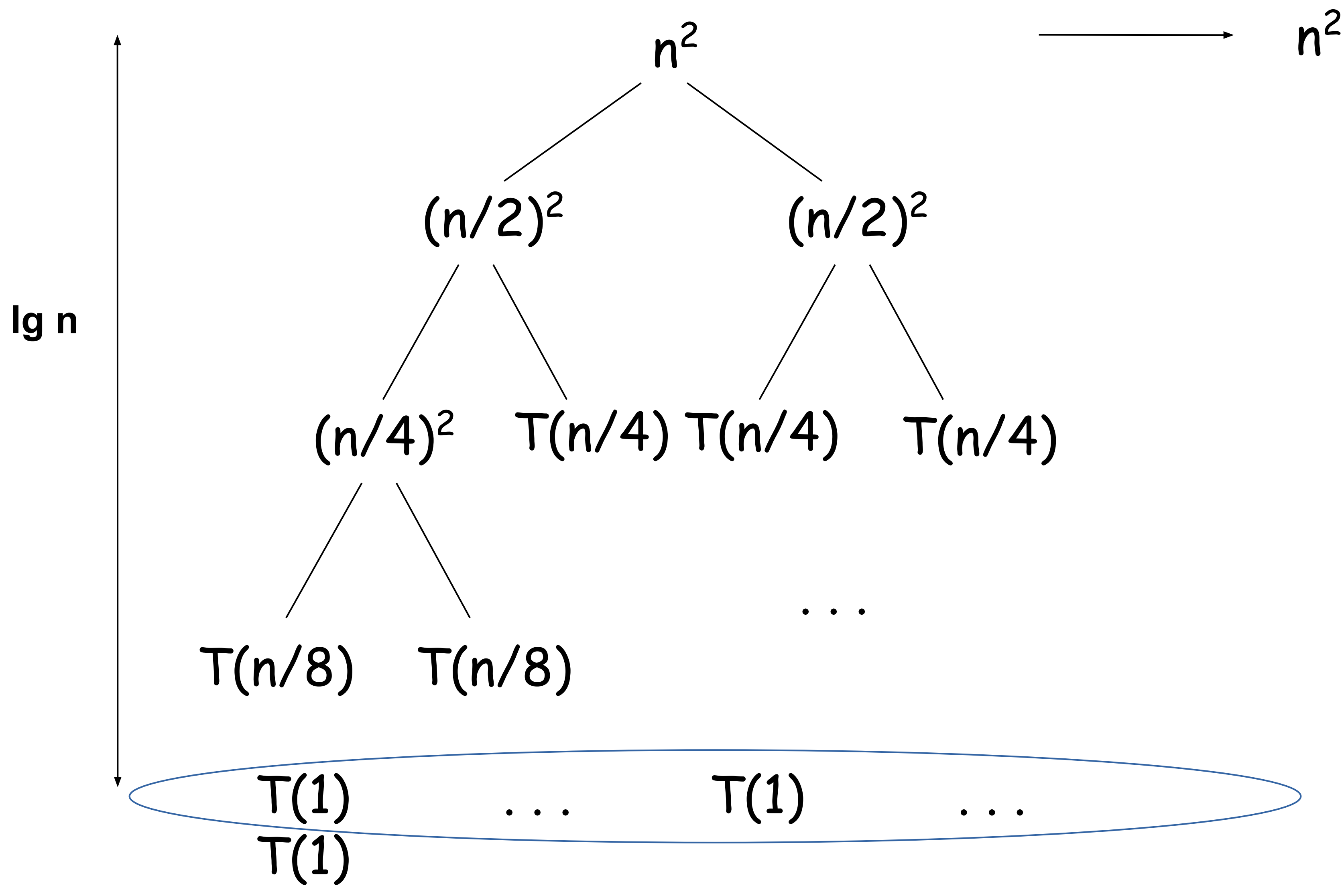
$(n/2^i)^2=1$

$(n/2^i)=1$

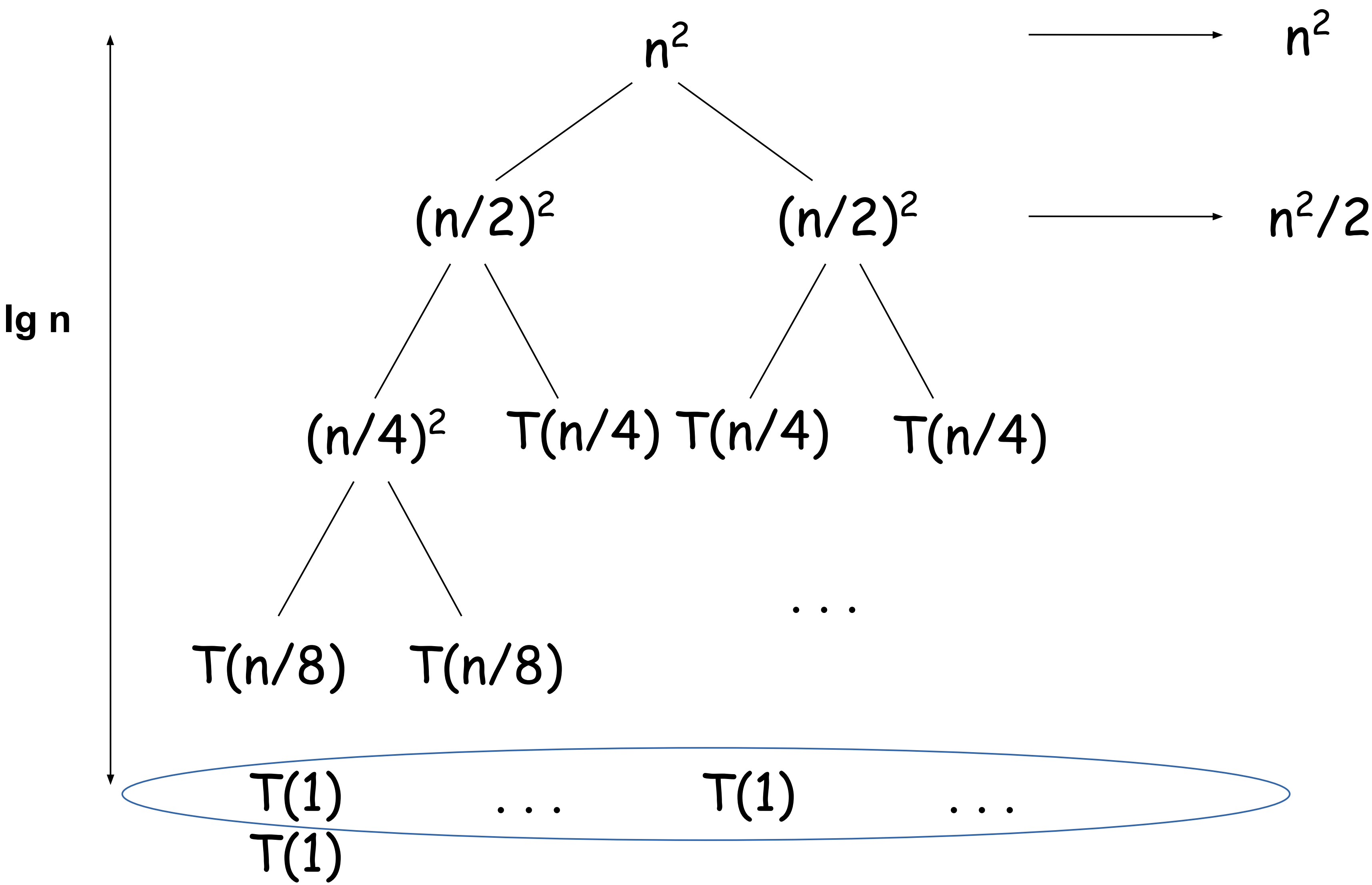
$n=2^i$

$\log n=i$

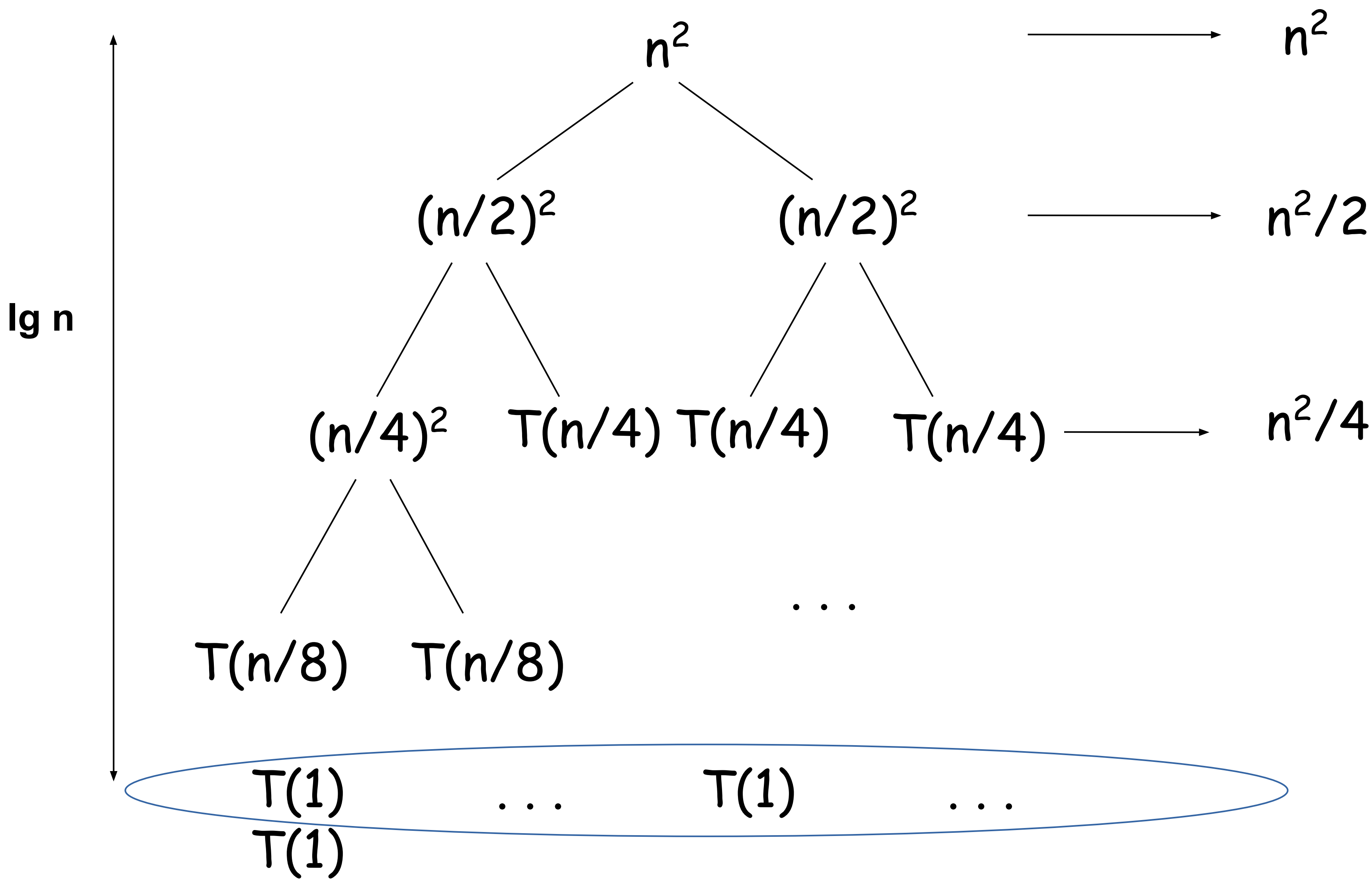
Recurrencias



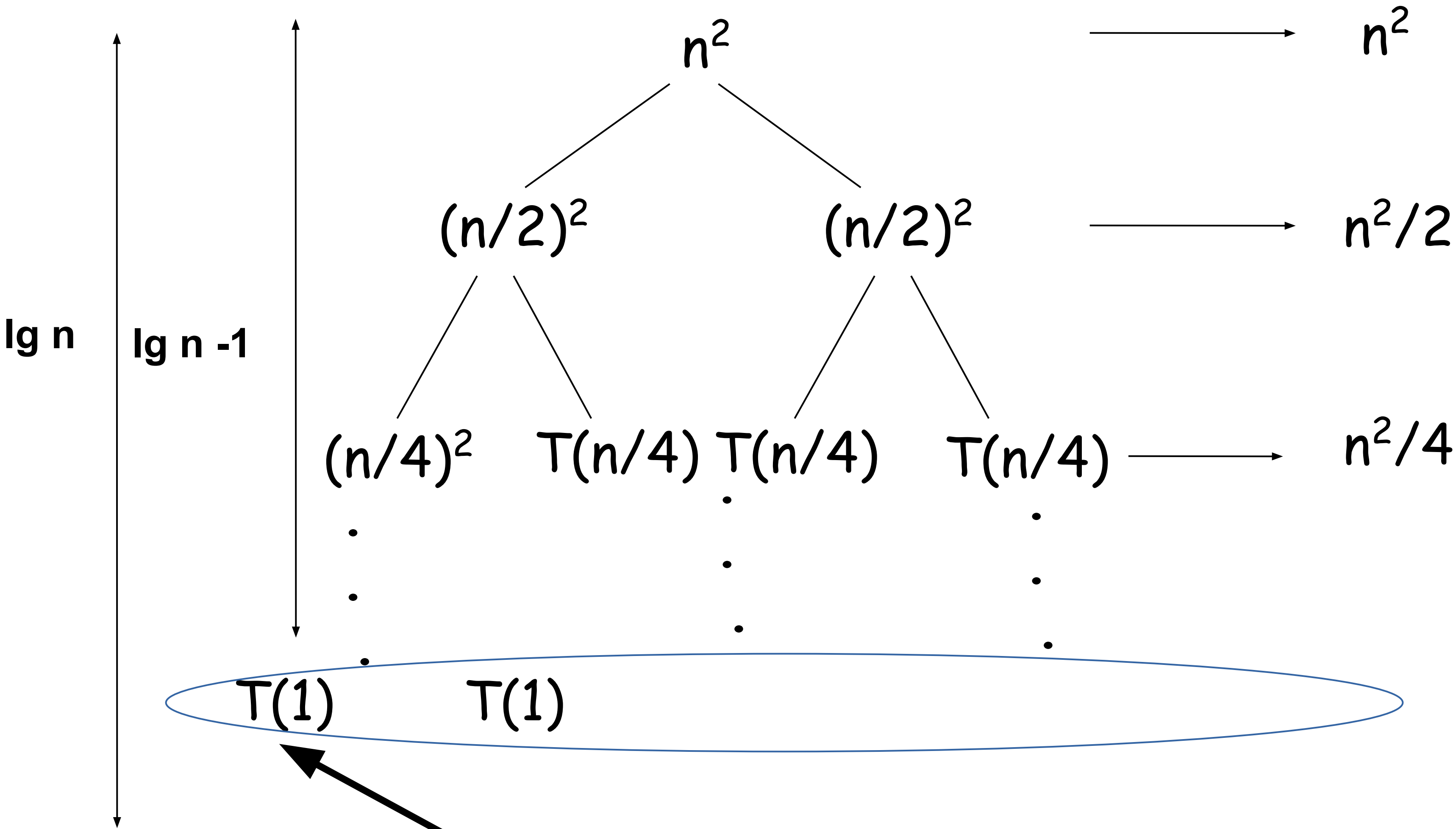
Recurrencias



Recurrencias

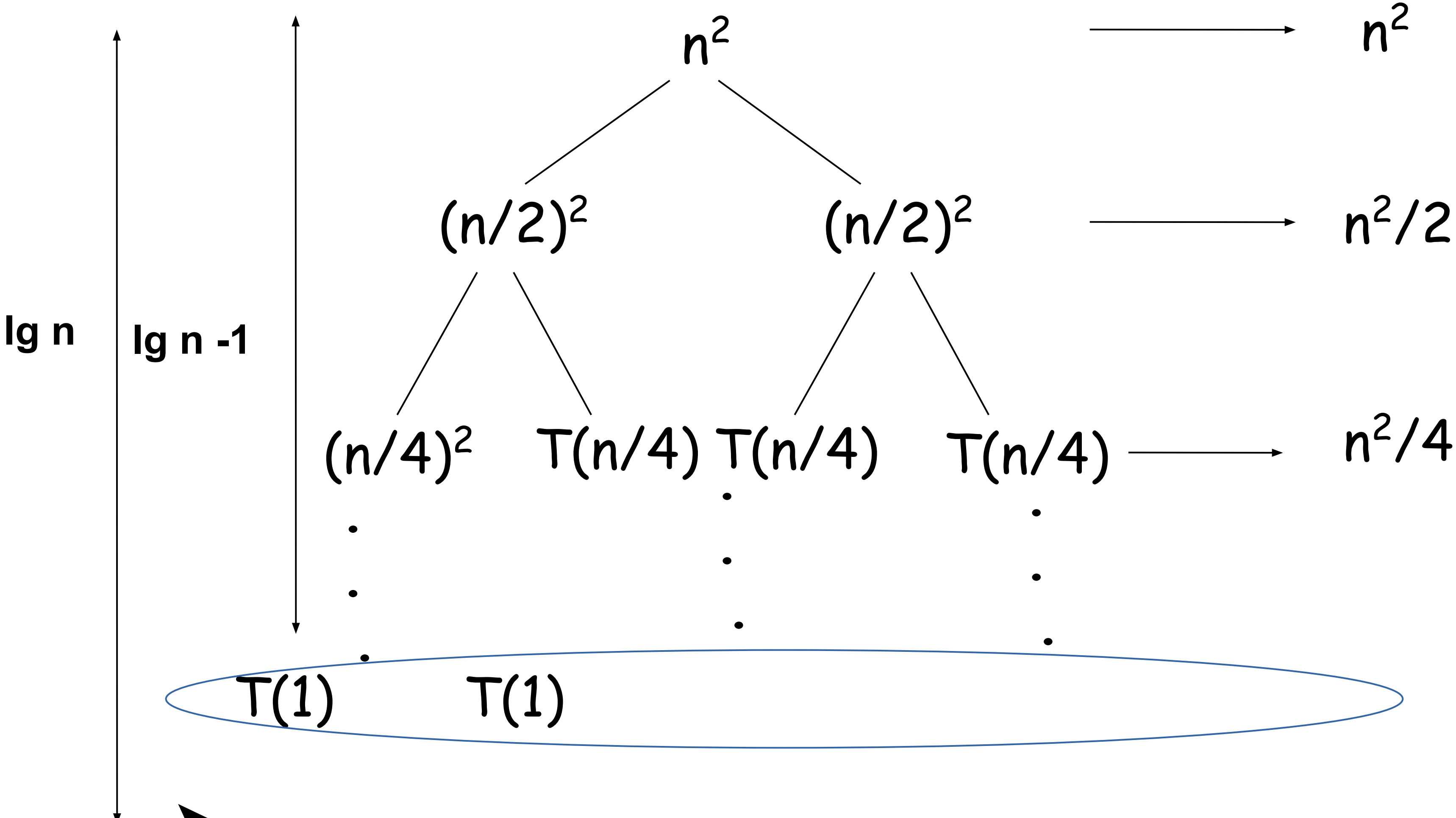


Recurrencias



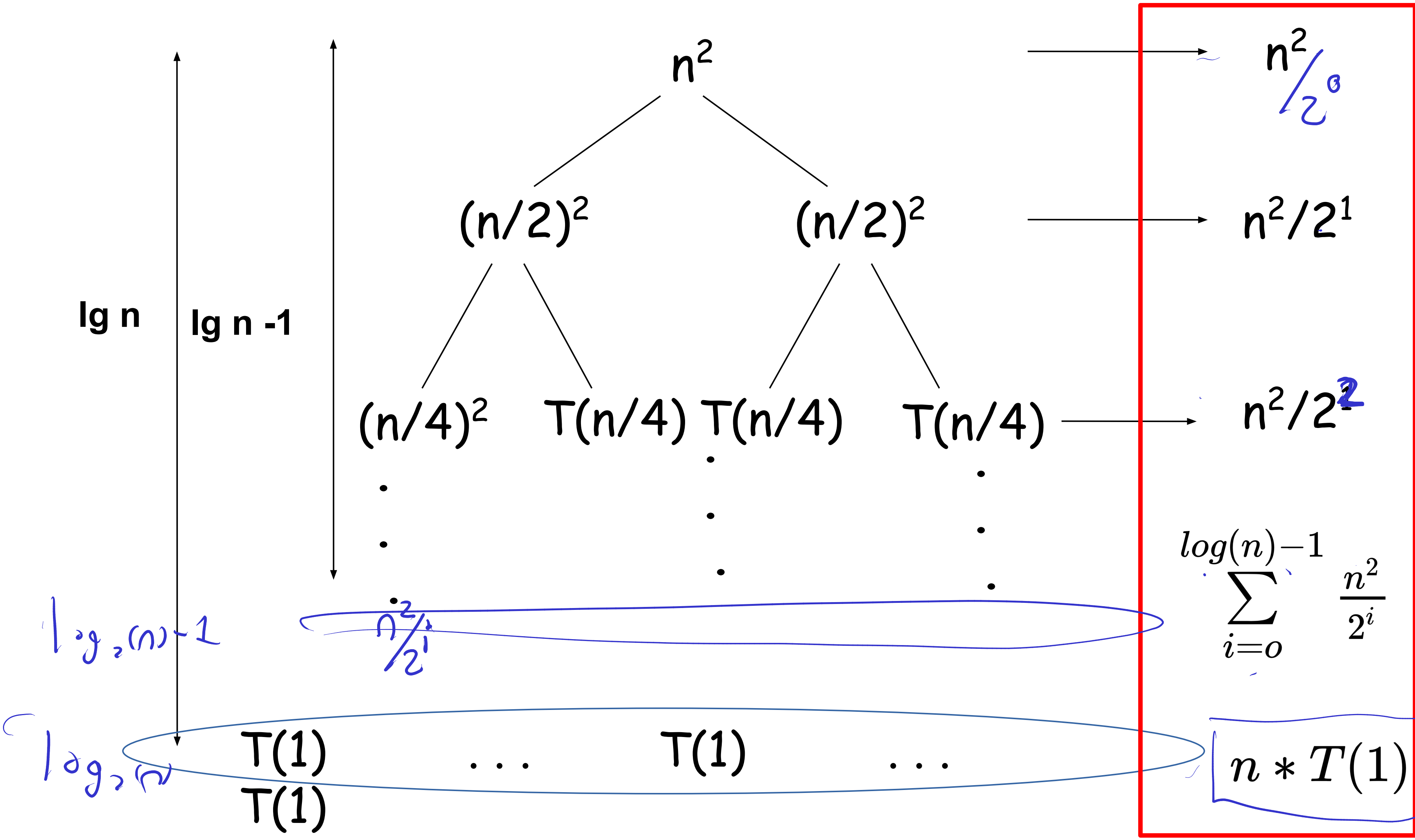
¿Cuantas hojas T(1) tenemos?

Recurrencias



Si recuerda en un árbol m-ario se tienen máximo m^h. En este caso al ser arbol binario m=2, tenemos $2^{\log(n)}$ hojas. Por lo tanto se

Recurrencias



Recurrencias

$\Theta(1)$

$$T(n) = n * T(1) + \sum_{i=o}^{log(n)-1} \frac{n^2}{2^i}$$

$$T(n) = n * c + n^2 \, \frac{0.5^{log(n)} - 1}{0.5 - 1}$$

$$T(n) = n * c + n^2 \, \frac{n^{log(0.5)} - 1}{-0.5}$$

$$T(n) = n * c + n^2 \, \frac{n^{-1} - 1}{-0.5}$$

$$T(n) = n * c - \frac{n}{0.5} + \frac{n^2}{0.5} = O(n^2)$$

Recurrencias

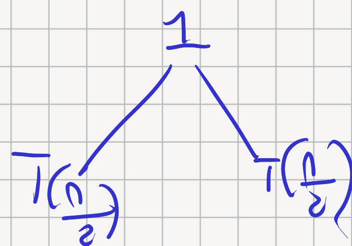
Resuelva construyendo el árbol

$T(n) = 2T(n/2) + 1$, $T(1)= \Theta(1)$

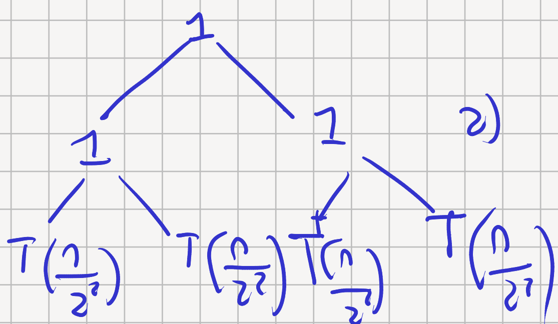
$\boxed{T(n) = 2T(n/2) + n,\ T(1)= \Theta(1)}$

$2T\left(\frac{n}{2}\right)+1$

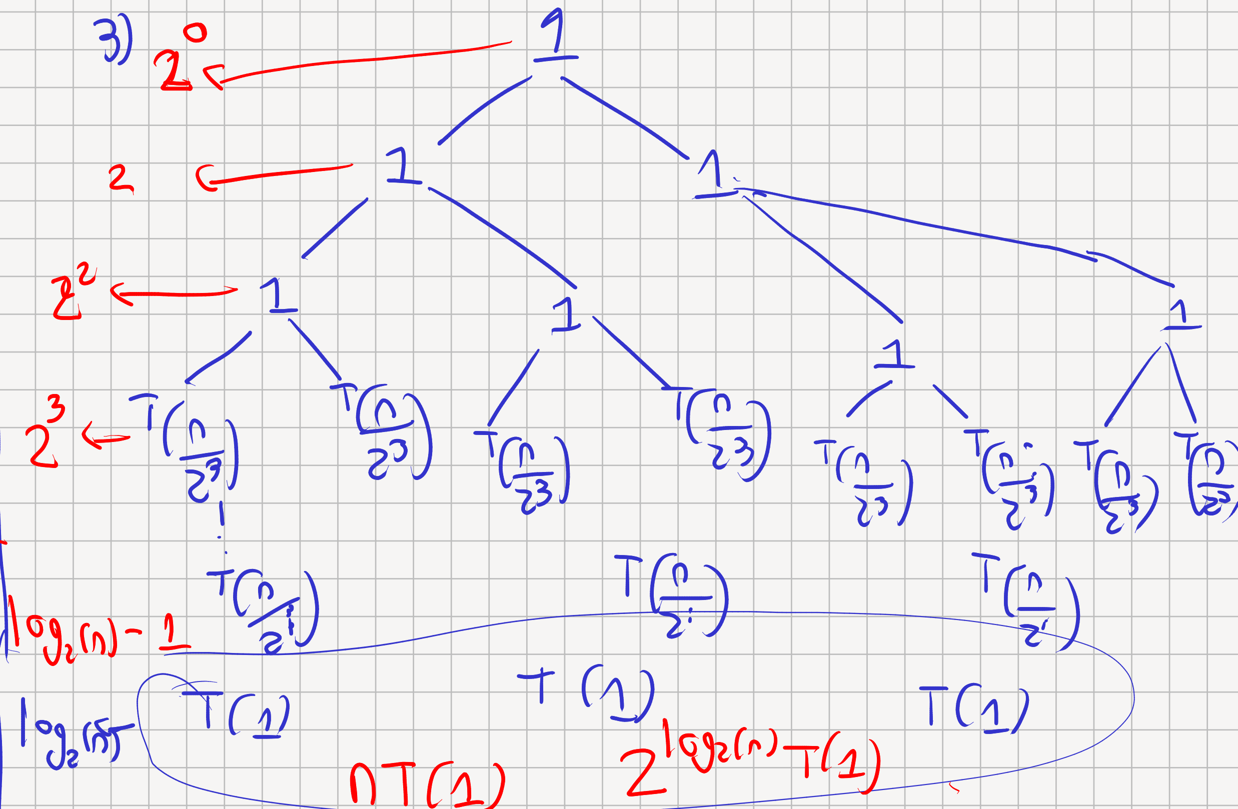
1)



2)



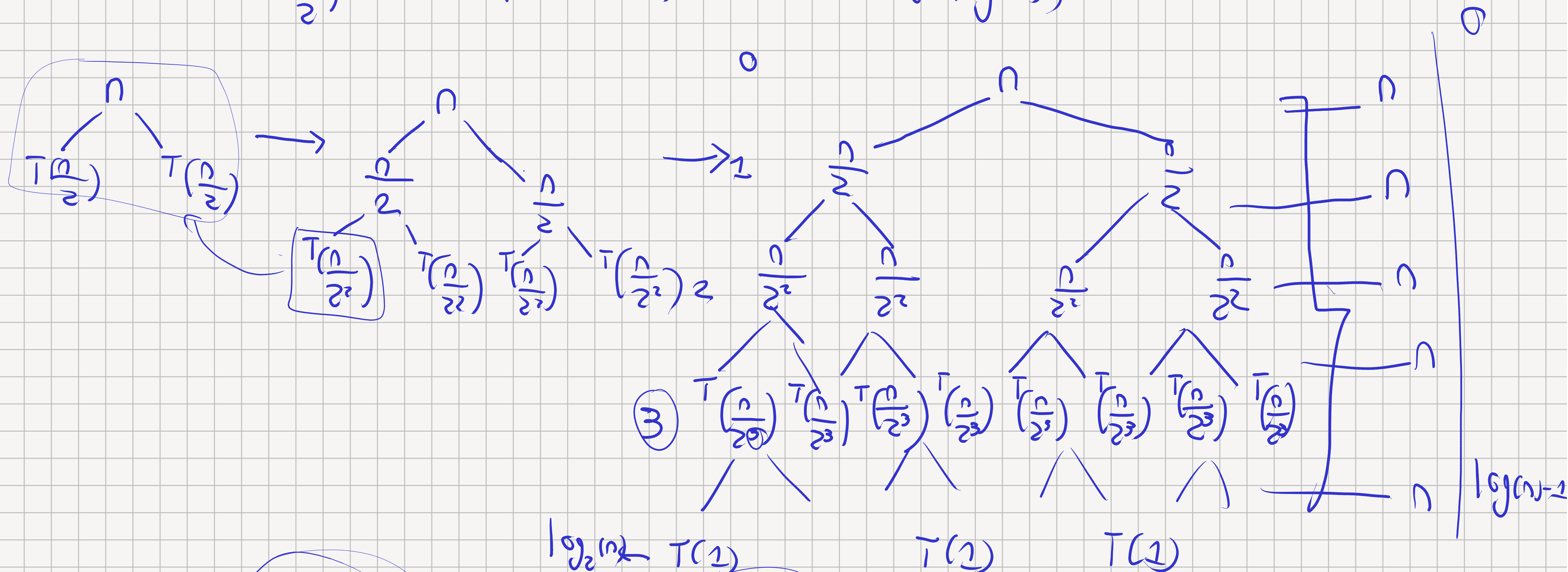
3)



$$T(n) = \underbrace{nT(1)}_{\text{Hojas}} + \sum_{i=0}^{\log_2(n)-1} 2^i = n \times C + n - 1$$

$$O(n)$$

$T(n) = 2T\left(\frac{n}{2}\right) + n \qquad T(1) = \Theta(1) \qquad T(n) = O(n\log(n))$



$$2^{\log_2(n)} \times T(1) + \sum_{i=0}^{\log_2(n)-1} n = cn + \underset{i=0}{n} + \sum_{i=1}^{\log_2(n)-1} n = cn + n + n \times (\log_2(n) - 1)$$

$$cn + n\log_2(n) = O(n\log_2(n))$$

Recurrencias

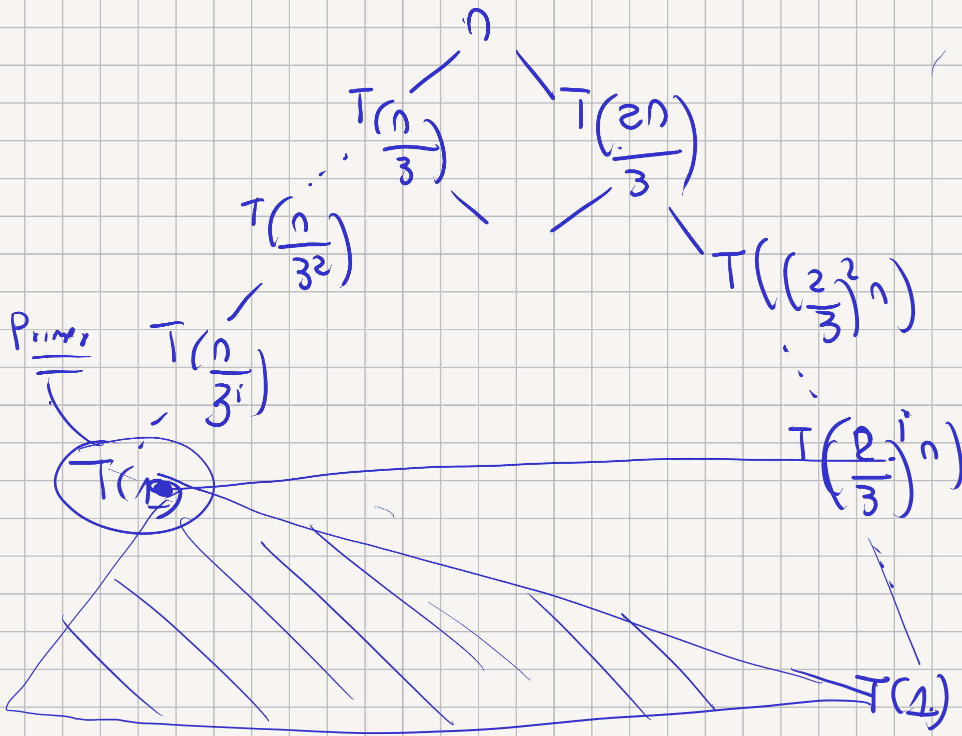
Resuelva la recurrencia T(n) = T(n/3) + T(2n/3) + n

Indique una cota superior y una inferior

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n$$

$$T(n) = T\left(\frac{n}{3^2}\right) + T\left(\frac{2n}{3^2}\right) + \frac{n}{3} + T\left(\frac{2n}{3^2}\right) + T\left(\frac{2^2}{3^2}n\right) + \frac{2}{3}n + 2$$

$$T(n) = T\left(\left\lfloor \frac{n}{3} \right\rfloor\right) + T\left(\left\lfloor \frac{2n}{3} \right\rfloor\right) + n$$

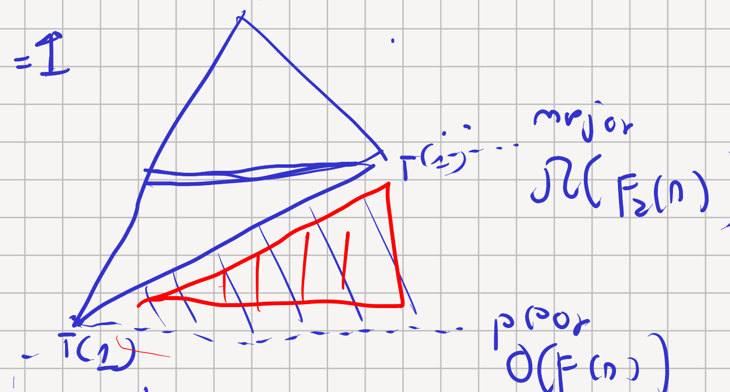


$$\frac{n}{3^i} = \quad i = \log_3(n)$$

$$\left(\frac{2}{3}\right)^i n = \quad i = \log_{\frac{3}{2}}(n)$$

$$\left(\frac{2}{3}\right)^i n = 1$$

$$T(1)$$



$$\log_{\frac{2}{3}}(n) = \frac{\log_3(n)}{\log_3\left(\frac{2}{3}\right)}$$

$$\log_{\frac{2}{3}}(n) = \frac{\log_3(n)}{\log_3(2) - \log_3(3)} \doteq -0.36$$

$$\log_{\frac{2}{3}}(n) = -2.70 \times \log_3(n)$$

$$\Omega(T(n)) = n + 2T\left(\frac{n}{3}\right)$$

$$O(T(n)) = n + 2T\left(\frac{2}{3}n\right)$$

$$\Omega(T(n)) = n + 2T\left(\frac{n}{3}\right)$$

$$n \quad \frac{n}{3} \quad T\left(\frac{n}{3}\right) \quad \frac{n}{3^2} \quad T\left(\frac{n}{3^2}\right) \quad T\left(\frac{n}{3^3}\right) \quad T(1)$$

$$2^0 n$$
$$2^1 \times \frac{n}{3}$$
$$2^2 \times \frac{n}{3^2}$$
$$2^3 \frac{n}{3^3}$$

$$\log_3(n)$$

$$\vdots$$

$$2^{\log_3(n)} \times T(1)$$

$$n^{\log_3(2)} \times T(1)$$

$$\sum_{i=0}^{\log_3(n)-1} \left(\frac{2}{3}\right)^i \times n$$

$$n^x$$

$$n\left(\frac{\cancel{\left(\frac{2}{3}\right)^{\log_3(n)}} - 1}{2/3 - 1}\right)$$

$$cn$$

$$O(n)$$

$$O(T(n)) = n + 2T\left(\frac{2}{3}n\right)$$

$$n \quad \frac{2}{3}n \quad \left(\frac{2}{3}n\right) \quad \left(\frac{2}{3}\right)^2 n \quad T\left(\left(\frac{2}{3}\right)^3 n\right) \quad T\left(\left(\frac{2}{3}\right)^i n\right) \quad 2^i\left(\frac{2}{3}\right)^i$$

$$2^0 n$$
$$2^1\left(\frac{2}{3}n\right)$$
$$2^2\left(\frac{2}{3}\right)^2 n$$
$$2^3\left(\frac{2}{3}\right)^3 n$$
$$2^i\left(\frac{2}{3}\right)^i n$$

$$\log_{3/2}(n)$$

$$\frac{2^i 2^i}{3^i} = \frac{2^{i+i}}{3^i} = \frac{2^{2i}}{3^i} = \frac{4^i}{3^i}$$

$$2^{\log_{\frac{2}{3}}(n)} \times T(1)$$

$$T(n) = \cancel{n^{\log_{\frac{2}{3}}(2)}} \times c + \sum_{i=0}^{\log_{\frac{2}{3}}(n)-1} \left(\frac{4}{3}\right)^i \times n$$

negative

$$T(n) = n^{\frac{\log_2(2)}{\log_2\left(\frac{2}{3}\right)}} + n\left(\frac{\left(\frac{4}{3}\right)^{\log_{\frac{2}{3}}(n)} - 1}{4/3 - 1}\right)$$

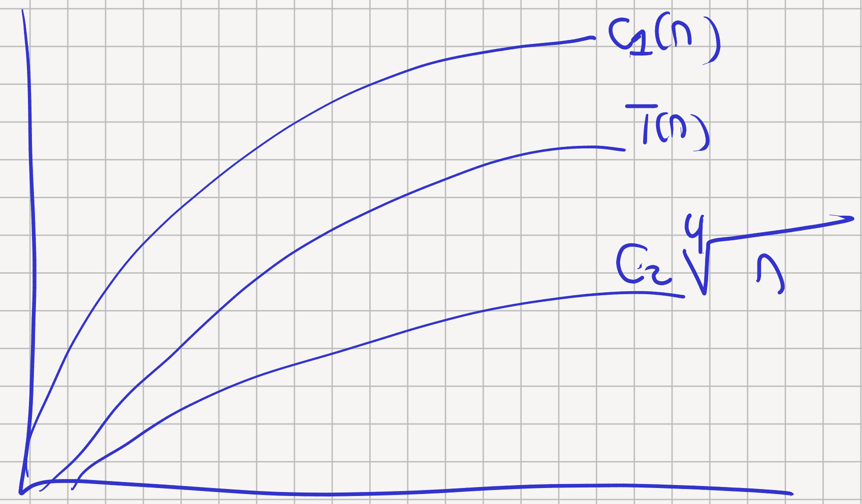
$$> 0$$

$$n \, n^{\log_{\frac{2}{3}}\left(\frac{4}{3}\right)}$$

$$n \times n^{\frac{\log_2\left(\frac{4}{3}\right)}{\log_2\left(\frac{2}{3}\right)}}$$

$$n^{\frac{\log_2(2)}{\log_2\left(\frac{2}{3}\right)}} \quad \Omega(n^{0.25})$$

$T(n) \rightarrow O(n)$ y $\Omega(n^{0.25} = \sqrt[4]{n})$



Recurrencias

Método maestro

Permite resolver recurrencias de la forma:

$T(n) = aT(n/b) + f(n)$, donde $a \geq 1$, $b > 1$

Recurrencias

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n) = \Theta(n^{\log_b a})$

Si $f(n) = O(n^{\log_b a - \varepsilon})$ para algún $\varepsilon > 0$

2. $T(n) = \Theta(n^{\log_b a} \lg n)$

Si $f(n) = \Theta(n^{\log_b a})$ para algún $\varepsilon > 0$

3. $T(n) = \Theta(f(n))$

Si $f(n) = \Omega(n^{\log_b a + \varepsilon})$ para algún $\varepsilon > 0$ si a*f(n/b) ≤c*f(n)

para algun c<1

Recurrencias

Dado T(n) = 9T(n/3) + n

$a = 9$

$b = 3$

$f(n) = n$

$$n^{\log_3 9} = n^2 \quad \textbf{Vs} \quad f(n) = n$$

$n^{\log_b a}$

$n^{\log_3 9} = n^2$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $n = O(n^{2-\varepsilon})$?

$\Theta(n^{\log_b a})$

$\Theta(n^2)$

n es $O(n^{2-\epsilon})$

n es $O(n)$ ✓

Recurrencias

Dado T(n) = 9T(n/3) + n

$$n^{\log_3 9} = n^2 \quad \textbf{Vs} \quad f(n) = n$$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $n = O(n^{2-\varepsilon})$?

Si $\varepsilon = 1$ se cumple que $n = O(n)$, por lo tanto, se cumple que:

$$T(n) = \Theta(n^2)$$

$aT(\frac{n}{b})+F(n)$ Recurrencias

$\frac{2}{3} = \frac{1}{b}$

$T(n) = T(2n/3) + 1$

$$n^{\log_{3/2} 1} = n^0 = 1 \quad \textbf{Vs} \quad f(n) = 1$$

$a = 1$

$b = \frac{3}{2}$

$n^{\log_b a}$

$n^{\log_{\frac{3}{2}} 1} = n^0$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $1 = O(n^{0-\varepsilon})$?

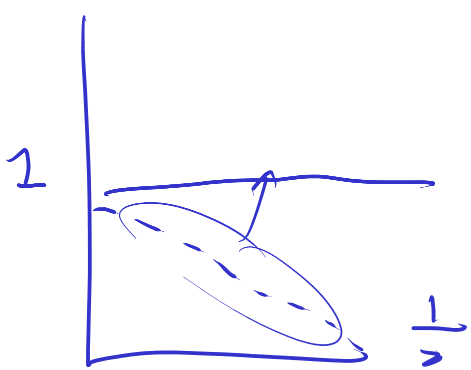
No existe $\varepsilon > 0$

1) $F(n)$ es $O(n^{\log_b a - \epsilon})$

1 es $O(n^{0-\epsilon})$

1 es $O(n^{-1})$

1 es $O(\frac{1}{n})$



Recurrencias

T(n) = T(2n/3) + 1

$$n^{\log_{3/2} 1} = n^0 = 1 \quad \text{vs} \quad f(n) = 1$$

Es $f(n) = \Theta(n^{\log_b a})$?

1 es $\Theta(n^{\log_b a})$

Es $1 = \Theta(1)$?

1 es $\Theta(n^0)$

1 es $\Theta(1)$

Si, por lo tanto, se cumple que:

$$T(n) = \Theta(1 * \lg n) = \Theta(\lg n)$$

$$T(n) = \Theta(n^{\log_b a} \log(n))$$

n a

Recurrencias

$T(n) = 3\,T(n/4) + n\lg n$ $\quad a=3 \quad b=4$

$$n^{\log_4 3} = n^{0.793} \quad \text{vs} \quad f(n) = n\lg n \qquad n^{\log_b a} = n^{\log_3 4}$$

1) Es $f(n) = O(n^{\log_b a - \varepsilon})$? $\quad n\lg(n) \in O(n^{0.793-\epsilon})$ $\quad O(\sqrt{n})$ X

2) Es $f(n) = \Theta(n^{\log_b a})$? $\quad n\log(n)$ $\quad \Theta(n^{0.793})$ $\quad O$ X $\quad \Omega$ ✓

3) Es $f(n) = \Omega(n^{\log_b a + \varepsilon})$? $\quad n\log(n) \in \Omega(n^{0.793+\epsilon})$ $\quad \in \Omega(n)$ SI

Si, y además, $af(n/b) \le cf(n)$

$3(n/4)\lg(n/4) \le cn\lg n$

$3(n/4)\lg n - 3(n/4)*2 \le cn\lg n$

$(3/4)n\lg n \le cn\lg n \rightarrow c=3/4$ y se concluye que $T(n) = \Theta(n\lg n)$

$c<1$

Recurrencias

$T(n) = 2T(n/2) + n\lg n$

Muestre que no se puede resolver por el método maestro

Recurrencias

Resuelva usando método del maestro

$T(n) = 4T(n/2) + n$

$T(n) = 4T(n/2) + n^2$

$T(n) = 4T(n/2) + n^3$

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n)=\Theta(n^{\log_b a})$

Si $f(n)=O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon>0$

2. $T(n)=\Theta(n^{\log_b a}\lg n)$

Si $f(n)=\Theta(n^{\log_b a})$ para algún $\varepsilon>0$

3. $T(n)=\Theta(f(n))$

Si $f(n)=\Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon>0$ si a*f(n/b) ≤c*f(n)

$c<1$

Recurrencias

Método de sustitución

Suponer la forma de la solución y probar por inducción matemática

Recurrencias

$T(n)=2T(\lfloor n/2 \rfloor)+n$, $T(1)=1$

Suponer que la solución es de la forma $T(n)=O(n\lg n)$

Probar que $T(n) \leq cn\lg n$.

Se supone que se cumple para $n/2$ y se prueba para n

Hipotesis inductiva: $T(n/2) \leq cn/2 \lg (n/2)$

Recurrencias

$T(n)=2T(\lfloor n/2 \rfloor)+n$, $T(1)=1$

Probar que $T(n) \leq cn\lg n$.

Hipótesis inductiva: $T(n/2) \leq cn/2 \lg (n/2)$

Paso inductivo:

$$
\begin{aligned}
T(n) &\leq 2(cn/2 \lg (n/2)) + n \\
&\leq cn \lg (n/2) + n \\
&= cn \lg (n) - cn + n, \text{ para } c \geq 1, \text{ haga } c=1 \\
&\leq cn \lg n
\end{aligned}
$$

Recurrencias

$T(n)=2T(\lfloor n/2 \rfloor)+n$, $T(1)=1$

Probar que $T(n) \le cn\lg n$.

Paso base: si $c=1$, probar que $T(1)=1$ se cumple

$T(1) \le 1*1\lg 1$?

$1 \le 0$?

No, se debe escoger otro valor para c

Recurrencias

$T(n)=2T(\lfloor n/2 \rfloor)+n$, $T(1)=1$

Probar que $T(n) \leq cn\lg n$.

Paso base: si $c=2$, probar que $T(1)=1$ se cumple

$T(1) \leq 2*1\lg 1$?

$1 \leq 0$?

No, se puede variar k.

Para esto, se calcula $T(2)$ y se toma como valor inicial

Recurrencias

Probar que $T(n) \leq cn\lg n$.

$T(2)=2T(0)+2=4$

Paso base: si $c=1$, probar que $T(2)=4$ se cumple

$T(2) \leq 1*2\lg 2$?

$4 \leq 2$?

No, se puede variar c.

Recurrencias

Probar que $T(n) \leq cn\lg n$.

$T(2)=2T(0)+2=4$

Paso base: si $c=3$, probar que $T(2)=4$ se cumple

$T(2) \leq 3*2\lg 2$?

$4 \leq 6$?

Si, se termina la demostración

Recurrencias

$T(n)=T(n-1)+T(n-2)+1$, $T(1)=O(1)$, $T(2) = O(1)$

Suponer que la solución es de la forma $T(n)=O(2^n)$

Probar que $T(n)\leq c2^n$.

Se supone que se cumple para $n-1$ y se $n-2$ prueba para n

Hipotesis inductiva: $T(n-1) \leq c2^{(n-1)}$ y $T(n-2) \leq c2^{(n-2)}$

Recurrencias

$T(n)=T(n-1)+T(n-2)+1,\ T(1)=O(1),\ T(2) = O(1)$

Ahora se debe probar que: $T(n) \leq c2^n$

$$T(1) \leq c2^1 \rightarrow 1 \leq 2*c$$

$$T(2) \leq c2^2 \rightarrow 1 \leq 4*c$$

$$T(3) \leq c2^3 \rightarrow 2 \leq 8*c$$

$$T(4) \leq c2^4 \rightarrow 3 \leq 16*c$$

$$T(5) \leq c2^5 \rightarrow 5 \leq 32*c$$

$$T(6) \leq c2^6 \rightarrow 8 \leq 64*c$$

$$T(7) \leq c2^7 \rightarrow 13 \leq 128*c$$

$$T(8) \leq c2^8 \rightarrow 21 \leq 256*c$$

Con $c = 1$, se cumple.

Referencias

Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. 2009. Introduction to Algorithms, Third Edition (3rd ed.). The MIT Press. Chapter 4

Gracias

Próximo tema:

Divide y vencerás