Fundamentos de análisis y diseño de algoritmos

Ecuaciones de recurrencia

Recurrencias

Método de iteración

Método maestro*

Método de sustitución

Recurrencias

Método de iteración

Expandir la recurrencia y expresarla como una suma de términos que dependen de n y de las condiciones iniciales

Recurrencias

$T(n) = n + 3T(n/4)$, $T(1) = \Theta(1)$ y n par

Expandir la recurrencia 2 veces

Recurrencias

$T(n) = n + 3\boxed{T(n/4)}$

$n + 3\ (\ n/4 + 3T(n/16))$

$n + 3\ (\ n/4 + 3(n/16\ + 3T(n/64)\ ))$

$n + 3*n/4 + 3^2*n/4^2\ + 3^3T(n/4^3)$

$T(1)$

$\boxed{T\left(\frac{n}{4}\right) = \frac{n}{4} + 3T\left(\frac{n}{4^2}\right)}$

$T\left(\frac{n}{4^2}\right) = \frac{n}{4^2} + 3T\left(\frac{n}{4^3}\right)$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\ (\ n/4 + 3T(n/16))$

$n + 3\ (\ n/4 + 3(n/16\ \ + 3T(n/64)\ ))$

$n + 3*n/4 + 3^2*n/4^2\ \ + 3^3T(n/4^3)$

¿Cuándo se detienen las iteraciones?

Recurrencias

$T(n) = n + 3T(n/4)$

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$n + 3\ (\ n/4 + 3(n/16\ \ + 3T(n/64)\ ))$

$n + 3*n/4 + 3^2*n/4^2\ \ + 3^3T(n/4^3)$

¿Cuándo se detienen las iteraciones?

Cuando se llega a $T(1)$

Recurrencias

$3^i T\left(\frac{n}{4^i}\right)$

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16 + 3T(n/64)\,))$

$\hookrightarrow n + 3{*}n/4 + 3^2{*}n/4^2 + 3^3T(n/4^3)$

$n + 3\frac{n}{4} + n\left(\frac{3}{4}\right)^2 + n\left(\frac{3}{4}\right)^3 + n\left(\frac{3}{4}\right)^4 + \dots + n\left(\frac{3}{4}\right)^{i-1} + 3^i T\left(\frac{n}{4^i}\right)$

¿Cuándo se detienen las iteraciones?

Cuando se llega a T(1), esto es, cuando $(n/4^i)=1$

$4^i = n \qquad i = \log_4(n)$

$\left(\frac{3}{4}\right)^0 n + \left(\frac{3}{4}\right)n + \left(\frac{3}{4}\right)^2 n + \left(\frac{3}{4}\right)^3 n + \dots + \left(\frac{3}{4}\right)^{\log_4(n)-1} n + 3^{\log_4(n)} T(1)$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\ (\ n/4 + 3T(n/16))$

$n + 3\ (\ n/4 + 3(n/16\ + 3T(n/64)\ ))$

$n + 3*n/4 + 3^2*n/4^2\ + 3^3T(n/4^3)$

$n + 3*n/4 + 3^2*n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}T(1)$

¿Cuándo se detienen las iteraciones?

Cuando se llega a $T(1)$, esto es, cuando $(n/4^i)=1$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ \ + 3T(n/64)\,))$

$n + 3^*n/4 + 3^{2*}n/4^2\ \ + 3^3T(n/4^3)$

$n + 3^*n/4 + 3^{2*}n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}T(1)$

Después de iterar, se debe tratar de expresar como una sumatoria con forma cerrada conocida

Recurrencias

$T(n) = n + 3T(n/4\rfloor)$

$n + 3\ (n/4\rfloor + 3T(n/16\rfloor))$

$n + 3\ (n/4\rfloor + 3(n/16\rfloor\ + 3T(n/64\rfloor)\ ))$

$n + 3{*}n/4\rfloor + 3^2{*}n/4^2\rfloor\ + 3^3(n/4^3\rfloor) + \ldots + 3^{\log 4n}\Theta(1)$

$\leq\ n + 3n/4 + 3^2n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}\Theta(1)$

Recurrencias

$T(n) = n + 3T(n/4)$

$n + 3\,(\,n/4 + 3T(n/16))$

$n + 3\,(\,n/4 + 3(n/16\ + 3T(n/64)\,))$

$n + 3^*n/4 + 3^{2*}n/4^2\ + 3^3(n/4^3) + \ldots + 3^{\log 4n}\Theta(1)$

$\leq\ n + 3n/4 + 3^2n/4^2 + 3^3n/4^3 + \ldots + 3^{\log 4n}\Theta(1)$

$$= \left(\sum_{i=0}^{\log_4 n} \left(\frac{3}{4}\right)^i n\right) + 3^{\log_4 n}\Theta(1) \qquad n^{\log_4 \left(\frac{3}{4}\right)}$$

$$= n\left(\frac{(3/4)^{(\log_4 n)} - 1}{(3/4) - 1}\right) + n^{\log_4 3} = n * 4(1 - (3/4)^{(\log_4 n)}) + \Theta(n^{\log_4 3})$$

$= O(n)$

Recurrencias

Resuelva por el método de iteración

$$T(n) = 2\boxed{T(n/2)} + 1,\ T(1) = \Theta(1)$$

$$T(n) = 2\left(\boxed{2T\left(\frac{n}{2^2}\right)} + 1\right) + 1$$

$$T(n) = 2\left(2\left(\boxed{2T\left(\frac{n}{2^3}\right)} + 1\right) + 1\right) + 1$$

$$T(n) = 2\left(2\left(2\left(2T\left(\frac{n}{2^4}\right) + 1\right) + 1\right) + 1\right) + 1$$

$$T(n) = 2^4\, T\left(\frac{n}{2^4}\right) + 2^3 + 2^2 + 2 + 2^0$$

$$T(n) = 2^i\, T\left(\frac{n}{2^i}\right) + \underline{2^{i-1} + 2^{i-2} + \dots + 2^2 + 2^1 + 2^0}$$

$$T(1) \qquad 1 = \frac{n}{2^i} \qquad i = \log_2(n) \qquad\qquad a^{\log_b(c)} = c^{\log_b(a)}$$

$$2^{\log_2(n)}\, T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i$$

$$n^{\log_2(2)} \times C + \frac{2^{\log_2(n)-\cancel{1}+\cancel{1}} - 1}{2-1}$$

$$Cn + n - 1 = O(n)$$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1$, $T(1)= \Theta(1)$

$T(n) = 2T(n/2) + n$, $T(1)= \Theta(1)$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

$$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + 1\right) + 1$$

$$T(n) = 2\left(2\left(2T\left(\frac{n}{2^3}\right) + 1\right) + 1\right) + 1$$

$$T(n) = 2\left(2\left(2\left(2T\left(\frac{n}{2^4}\right) + 1\right) + 1\right) + 1\right) + 1$$

$$T(n) = 2^4 T\left(\frac{n}{2^4}\right) + 2^3 + 2^2 + 2^1 + 2^0$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + 2^{i-1} + 2^{i-2} + \dots + 2^1 + 2^0$$

$$T(1) \qquad \frac{n}{2^i} = 1 \qquad i = \log_2(n)$$

$$T(n) = 2^{\log_2(n)} T(1) + 2^{\log_2(n)-1} + 2^{\log_2(n)-2} + \dots + 2^1 + 2^0$$

$$\boxed{\sum_{k=0}^{n} ar^i = \frac{ar^{n+1} - a}{r-1}}$$

$$T(n) = n\,T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i$$

$$T(n) = n\,\Theta(1) + \frac{2^{\log_2(n)-1+1} - 1}{2-1}$$

$$T(n) = n\,\Theta(1) + n - 1 \longrightarrow \Theta(1)$$

$$T(n) = 2\boxed{T\left(\frac{n}{2}\right)} + n$$

$$2\left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right) + n$$

$$2^2T\left(\frac{n}{2^2}\right) + n + n$$

$$2^2\left(2\left(2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right) + \frac{n}{2}\right) + n$$

$$2^3T\left(\frac{n}{2^3}\right) + \underline{n + n + n}$$

$$2^3T\left(\frac{n}{2^3}\right) + 3n$$

$$2^i T\left(\frac{n}{2^i}\right) + in \qquad \frac{n}{2^i} = 1 \qquad i = \log_2(n)$$

$$2^{\log_2(n)} T(1) + \log_2(n)\, n$$

$$n\,\Theta(1) + n\log_2(n) = O(n\log(n))$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2 \qquad \Theta(1) = T(1)$$

1) $$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right) + n^2$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2\left(\frac{n}{2}\right)^2 + n^2$$

$$T(n) = 2^2\left(2T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right) + 2\left(\frac{n}{2}\right)^2 + n^2$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 2^2\left(\frac{n}{2^2}\right)^2 + 2\left(\frac{n}{2}\right)^2 + n^2$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + 2^{i-1}\left(\frac{n}{2^{i-1}}\right)^2 + 2^{i-2}\left(\frac{n}{2^{i-2}}\right)^2 + \dots + 2\left(\frac{n}{2}\right)^2 + 2^0\left(\frac{n}{2^0}\right)^2$$

$$i = \log_2(n)$$

$$T(n) = n\,T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i \left(\frac{n}{2^i}\right)^2$$

$$\frac{\cancel{2^i}\, n^2}{(2^i)^{\cancel{2}}}$$

$$T(n) = n\,\Theta(1) + \sum_{i=0}^{\log_2(n)-1} n^2 \left(\frac{1}{2^i}\right)$$

$$T(n) = n\,\Theta(1) + n^2 \sum_{i=0}^{\log_2(n)-1} \left(\frac{1}{2}\right)^i$$

r

$$T(n) = n\,\Theta(1) + n^2\left(\frac{\left(\frac{1}{2}\right)^{\log_2(n)} - 1}{\frac{1}{2} - 1}\right)$$

$$T(n) = n\,\Theta(1) + n^2\left(\frac{n^{\log_2(0.5)} - 1}{-\frac{1}{2}}\right)$$

$$T(n) = n\,\Theta(1) + n^2\left(-2\left(n^{-1} - 1\right)\right)$$

$$T(n) = n\,\Theta(1) - 2n + 2n^2$$

$$\Theta(n^2)$$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1, T(1)= \Theta(1)$

$T(n) = 2T(n/2) + n, T(1)= \Theta(1)$

$T(n) = T(n/2) + 1, T(1)= \Theta(1)$

$$T(n) = 4T\left(\frac{n}{3}\right) + n + 1 \qquad T(1) = 1$$

$$T(n) = 4\left(4T\left(\frac{n}{3^2}\right) + \frac{n}{3} + 1\right) + n + 1$$

$$T(n) = 4^2 T\left(\frac{n}{3^2}\right) + \left(\frac{4}{3}\right) n + 4 + n + 1$$

$$T(n) = 4^2\left(4T\left(\frac{n}{3^3}\right) + \frac{n}{3^2} + 1\right) + \frac{4}{3} n + 4 + n + 1$$

$$T(n) = 4^3 T\left(\frac{n}{3^3}\right) + \frac{4^2}{3^2} n + 4^2 + \frac{4}{3} n + 4 + \left(\frac{4}{3}\right)^0 n + 4^0$$

$$T(n) = 4^i T\left(\frac{n}{3^i}\right) + \left(\frac{4}{3}\right)^{i-1} n + 4^{i-1} + \left(\frac{4}{3}\right)^{i-2} n + 4^{i-2} \dots + \left(\frac{4}{3}\right)^0 n + 4^0$$

$$T(1) \qquad 1 = \frac{n}{3^i} \qquad i = \log_3(n)$$

$$T(n) = 4^{\log_3(n)} \times \underbrace{T(1)}_{1} + \sum_{i=0}^{\log_3(n)-1} \left(\left(\frac{4}{3}\right)^i n + 4^i \right)$$

$$T(n) = 1 n^{\log_3(4)} + n \left(\frac{\frac{4}{3}^{\log_3(n)} - 1}{4/3 - \cancel{1}\,3/3} \right) + \left(\frac{4^{\log_3(n)} - 1}{4 - 1} \right)$$

$$T(n) = n^{\log_3(4)} + 3n \times n^{\log_3\left(\frac{4}{3}\right)} - 3n + \frac{n^{\log_3(4)} - 1}{3}$$

$$\Theta\left(n^{\log_3(4)}\right)$$

$$\log_3\left(\frac{4}{3}\right) + 1$$

$$\frac{\log_3(4) - \log_3(3) + 1}{1}$$

Recurrencias

Resuelva por el método de iteración

$T(n) = 2T(n/2) + 1$, $T(1)= \Theta(1)$

$T(n) = 2T(n/2) + n$, $T(1)= \Theta(1)$

$T(n) = T(n/2) + 1$, $T(1)= \Theta(1)$

Demuestre que $T(n) = T(n/2\rfloor) + n$, es $\Omega(n\log n)$

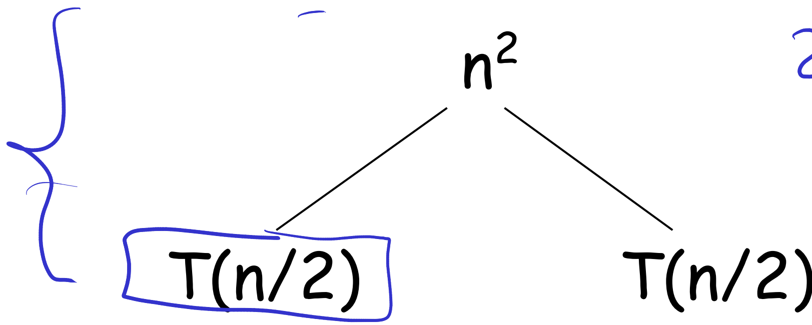
Recurrencias

Iteración con árboles de recursión

$T(1) = 1$

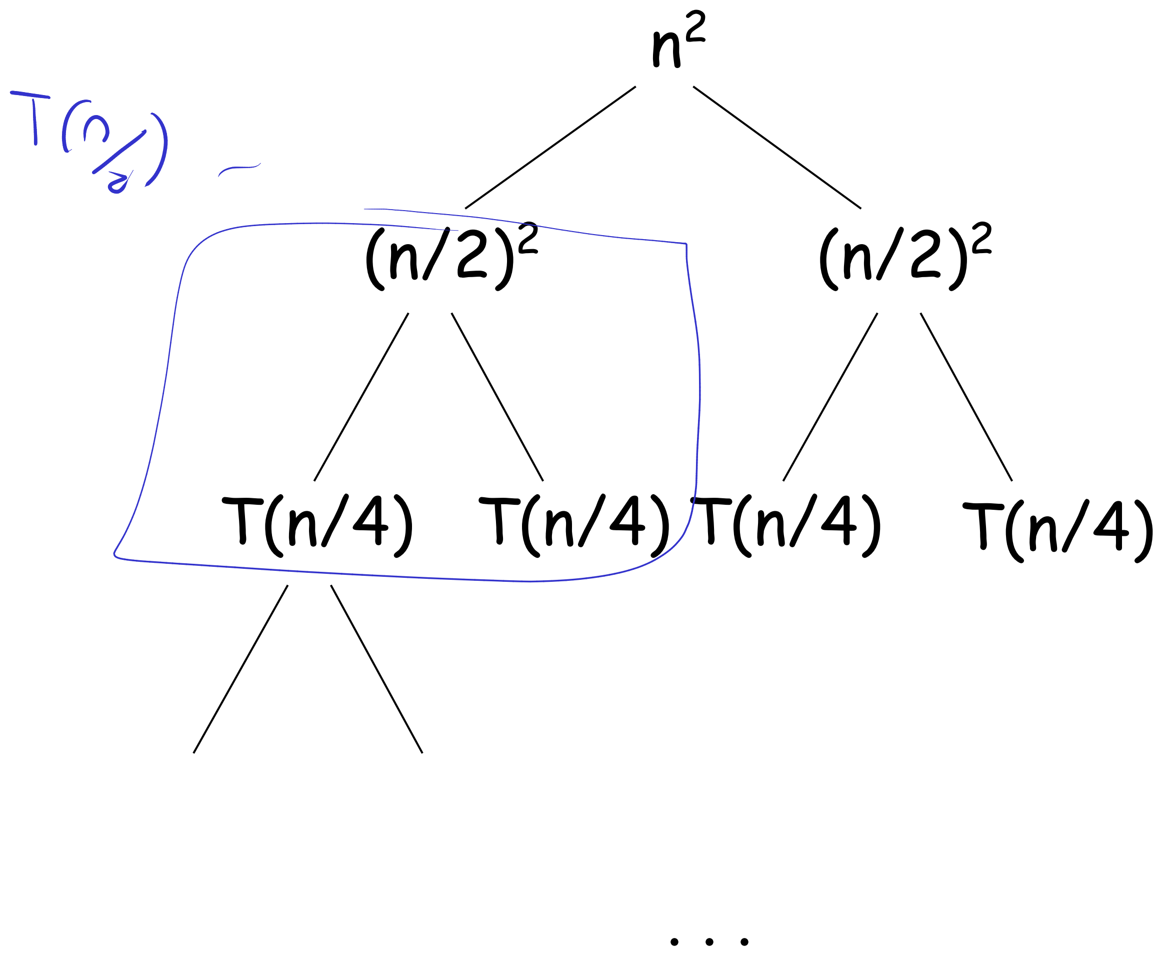
$T(n) = 2T(n/2) + n^2$

Recurrencias



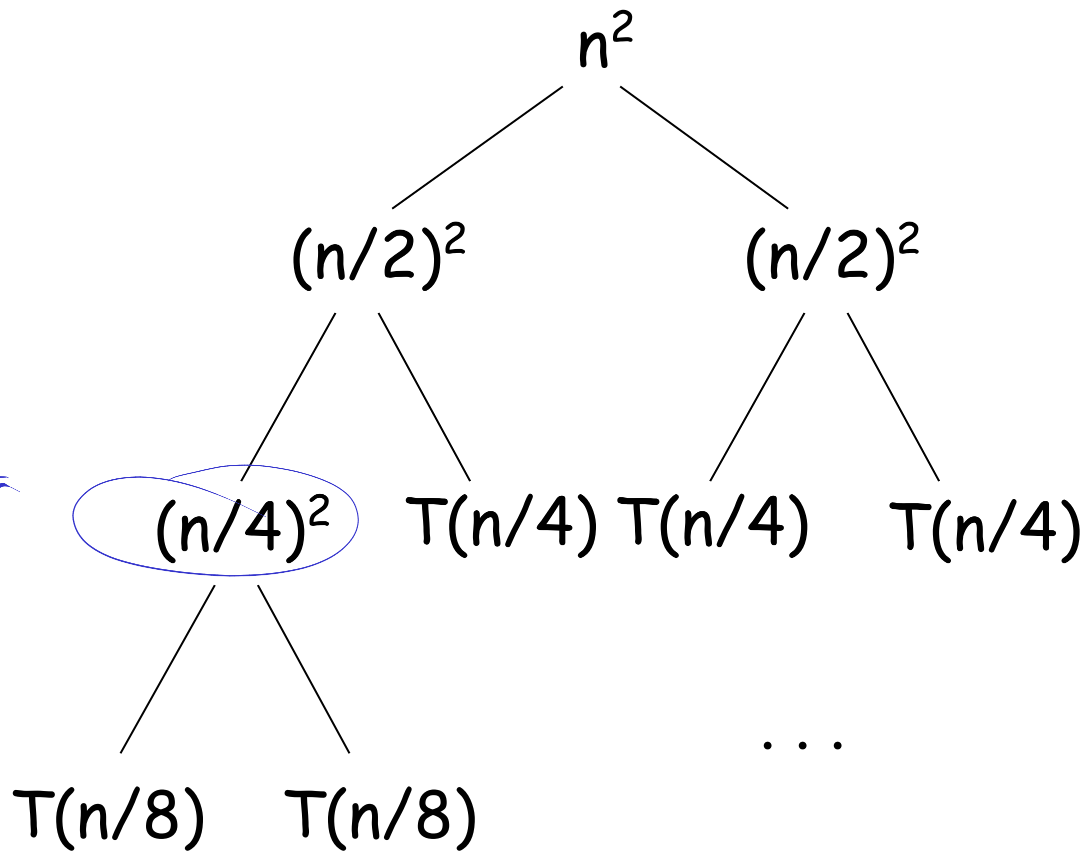
$2T(n/2) + n^2$

Recurrencias



Recurrencias

$4\left(\frac{n}{4}\right)^2$



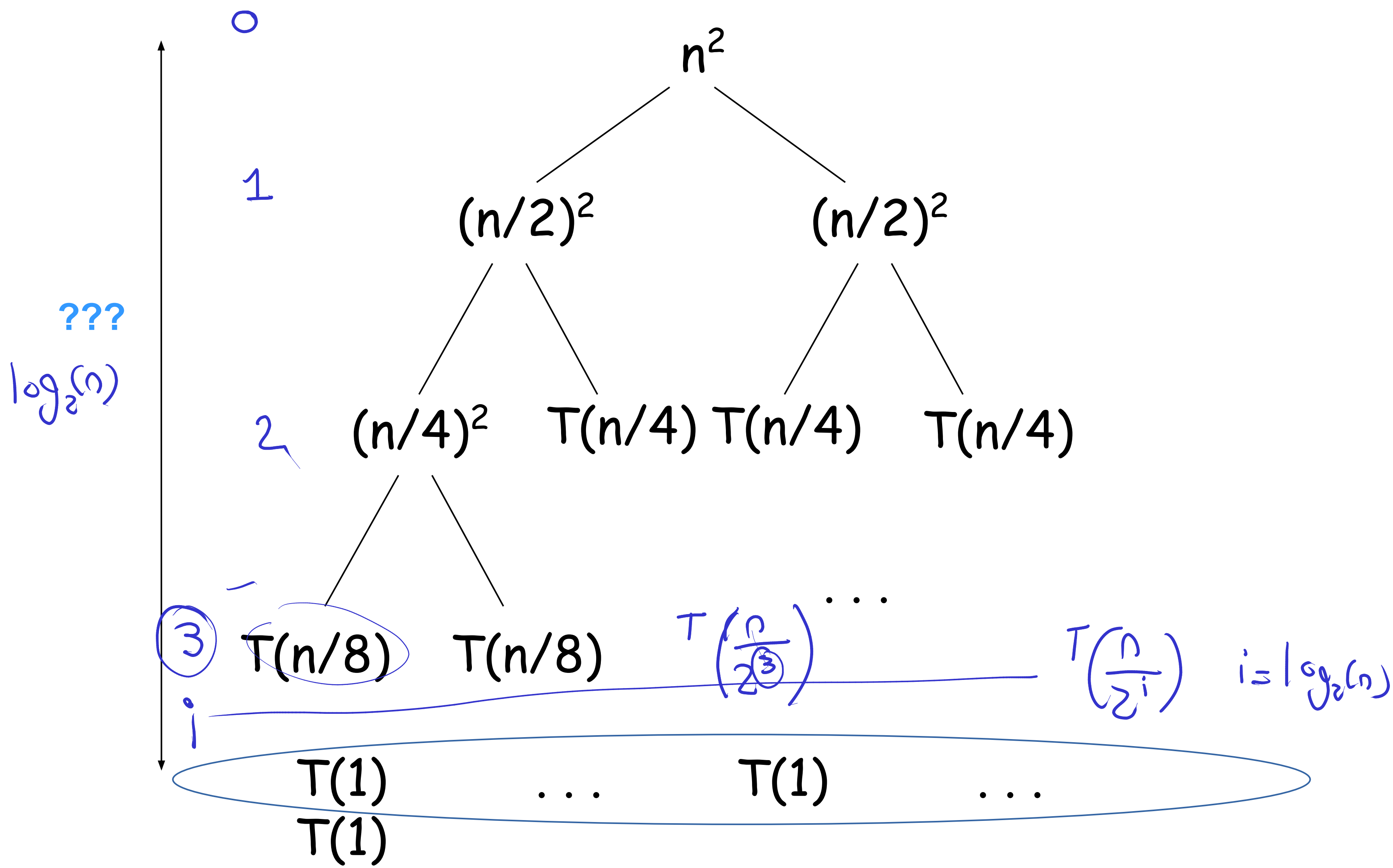
$n^2 = \frac{n^2}{2^0}$

$\frac{n^2}{2} = \frac{n^2}{2^1}$

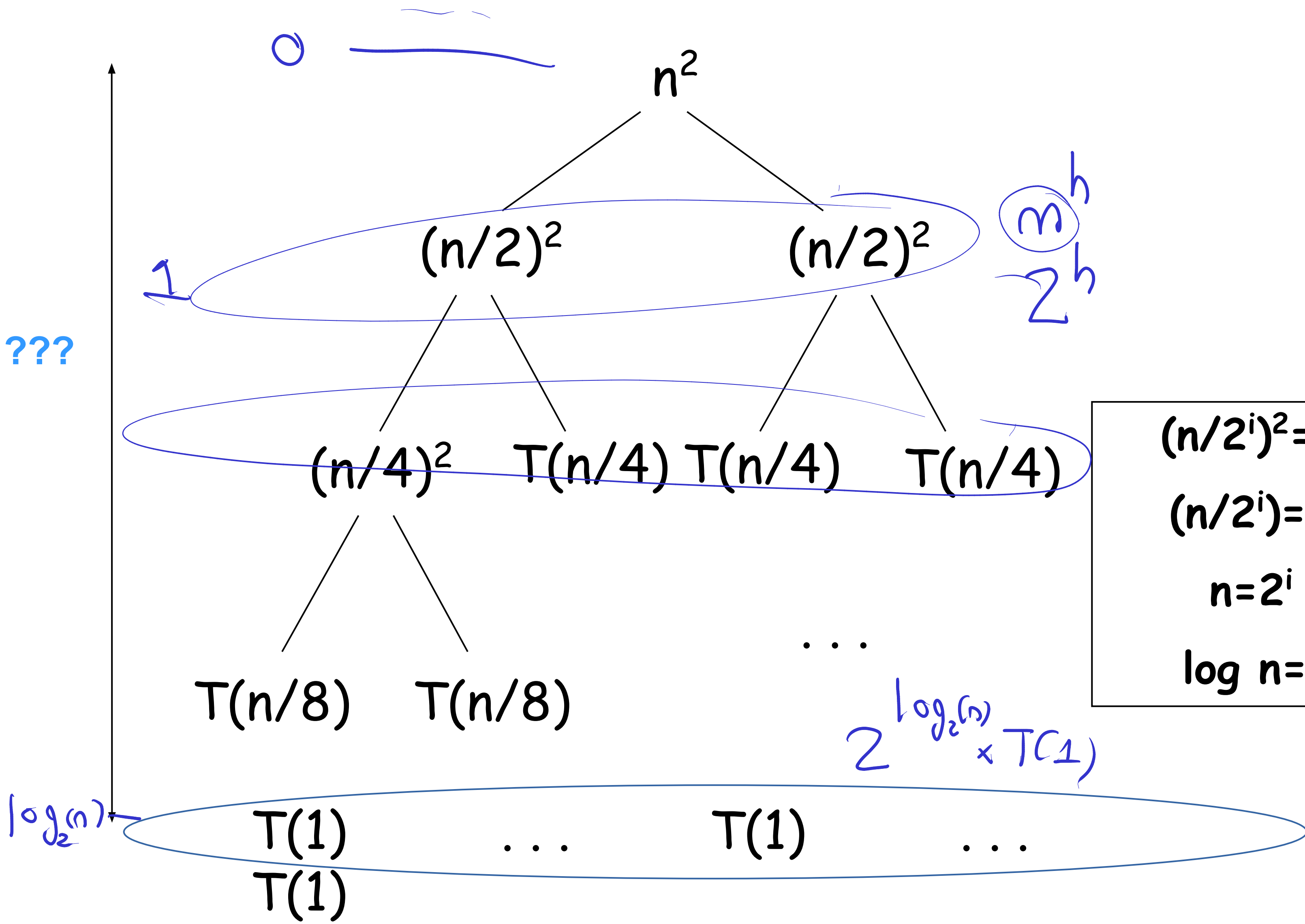
$\frac{n^2}{4} = \frac{n^2}{2^2}$

$\frac{n^2}{8} \quad \frac{n^2}{2^3}$

Recurrencias



Recurrencias



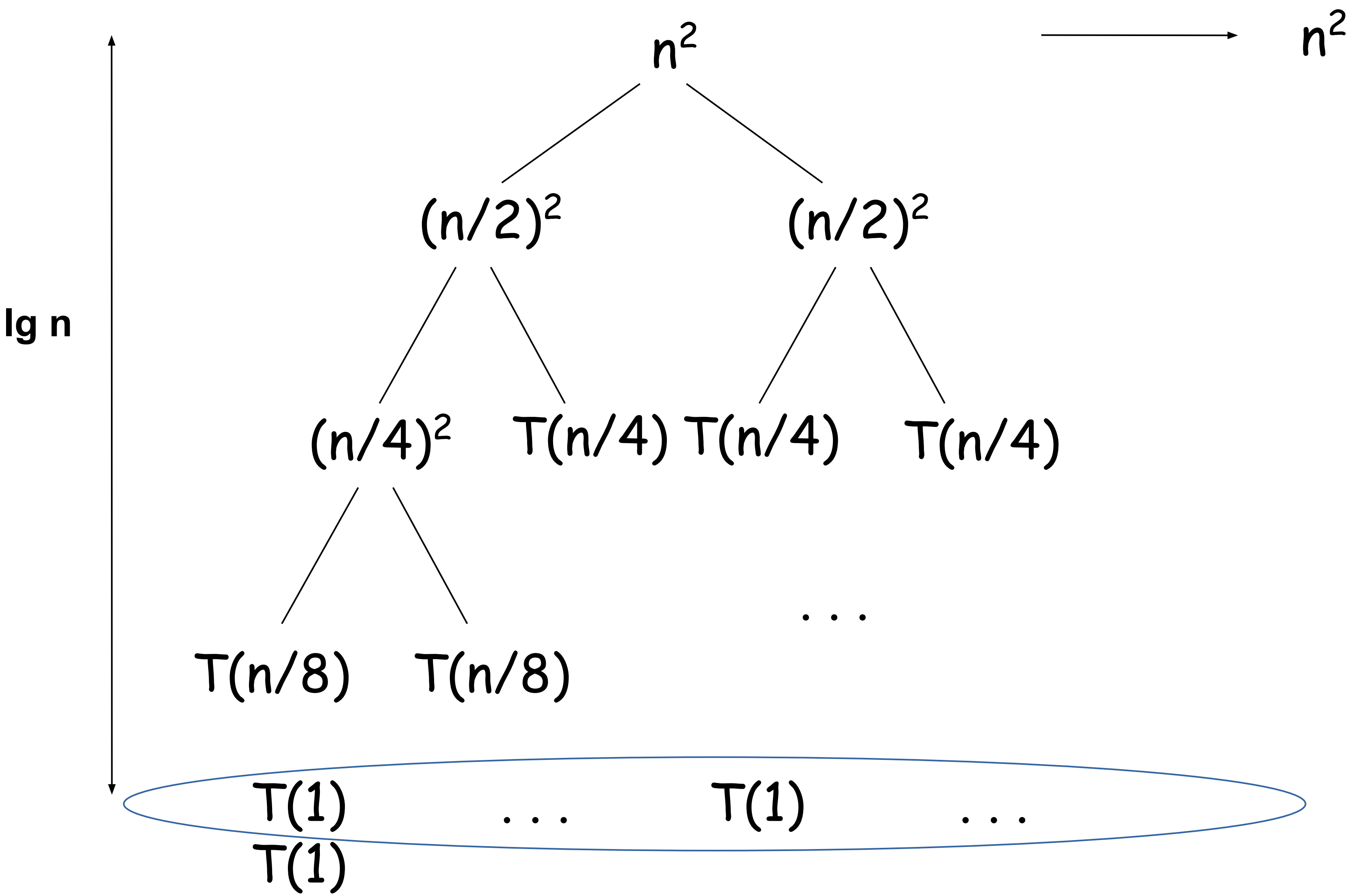
$(n/2^i)^2=1$

$(n/2^i)=1$

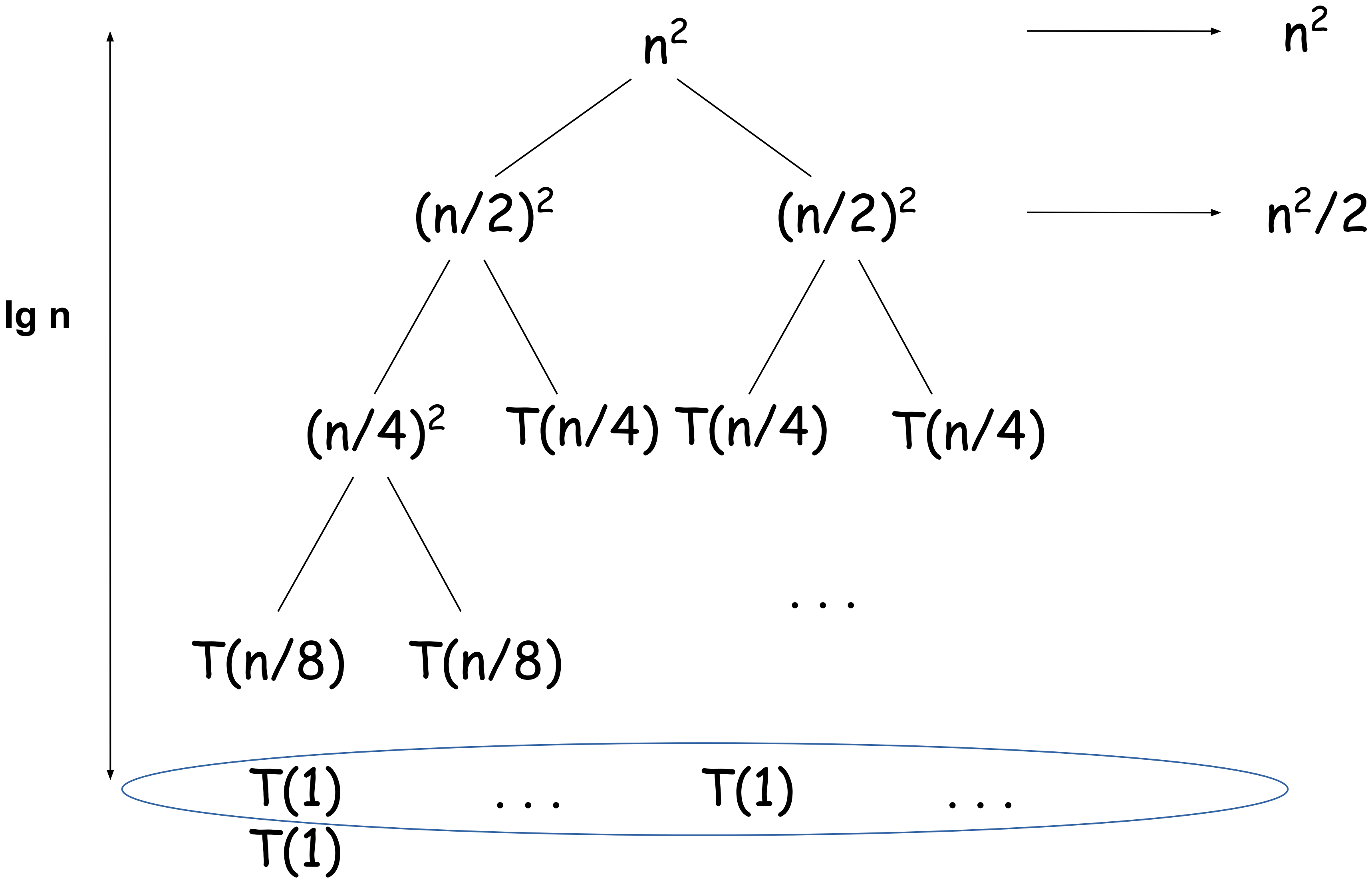
$n=2^i$

$\log n=i$

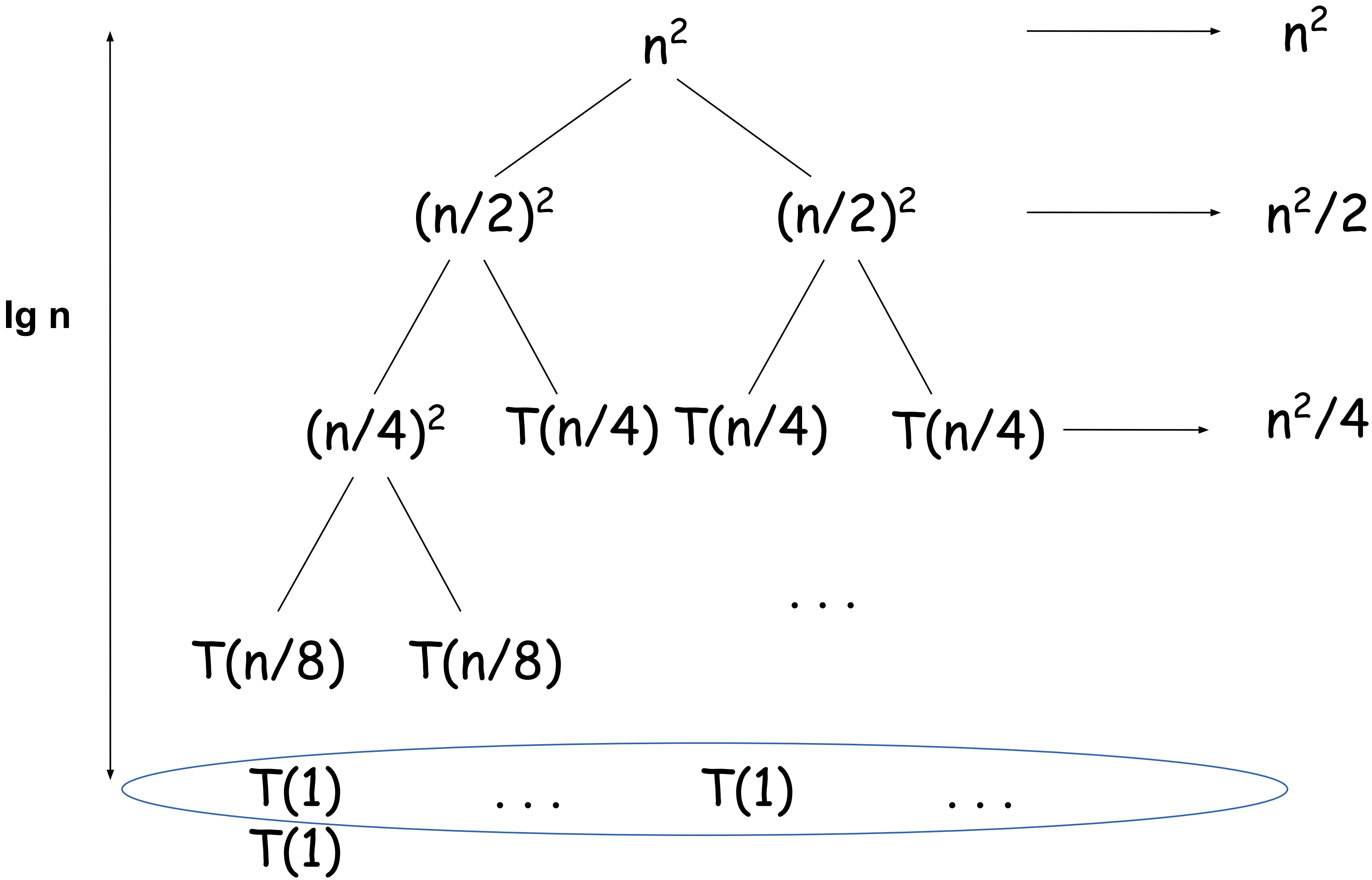
Recurrencias



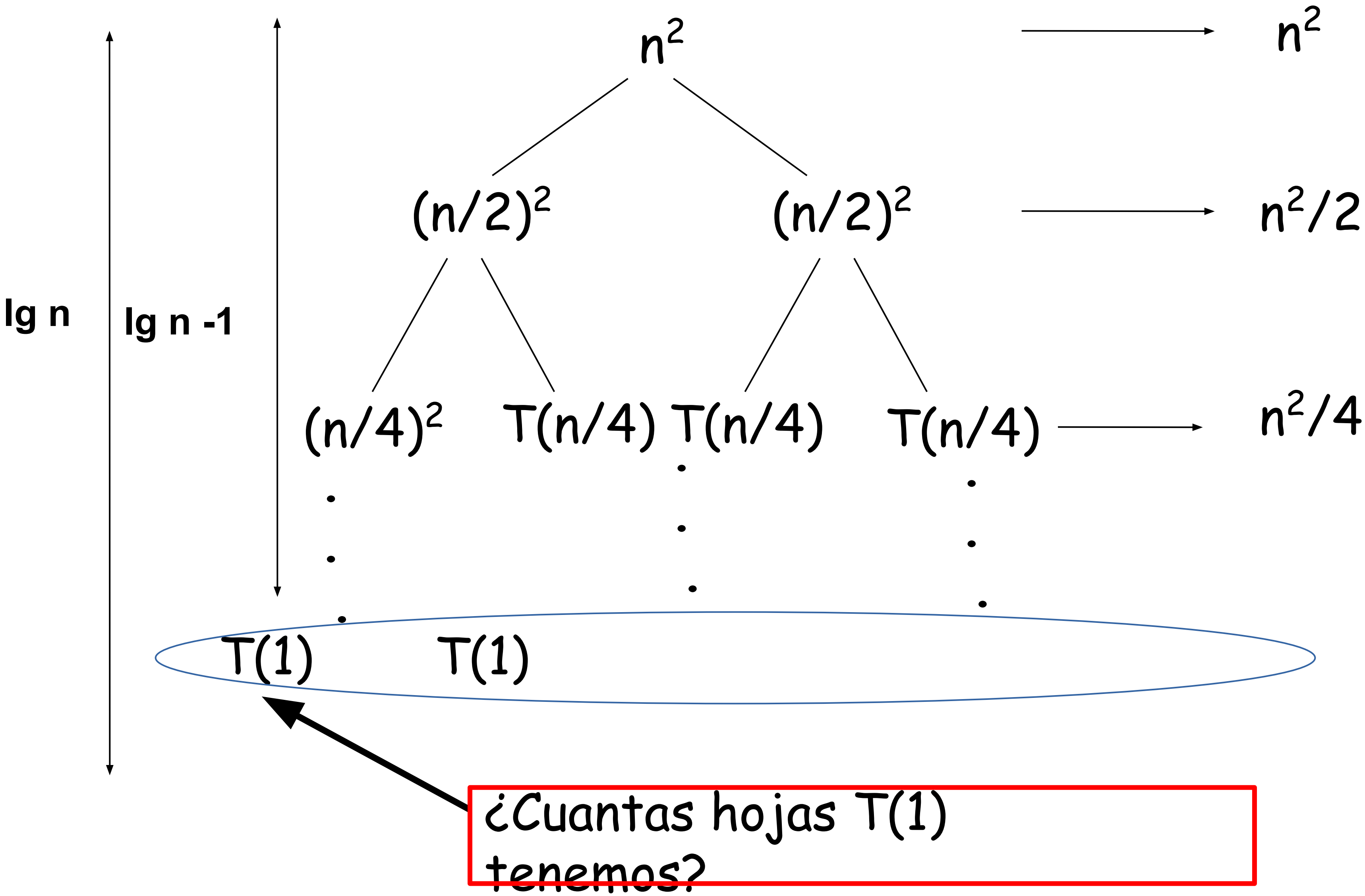
Recurrencias



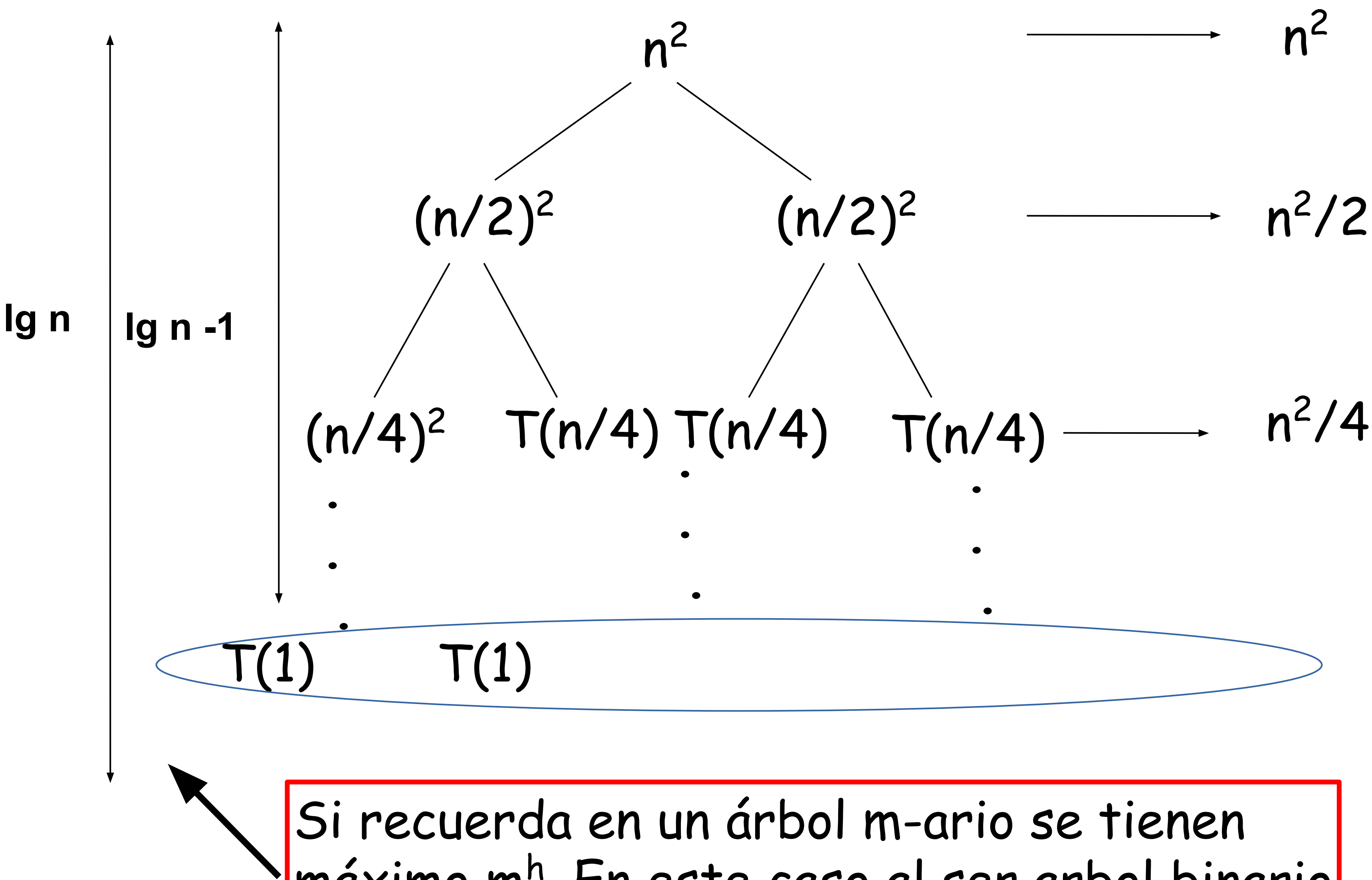
Recurrencias



Recurrencias

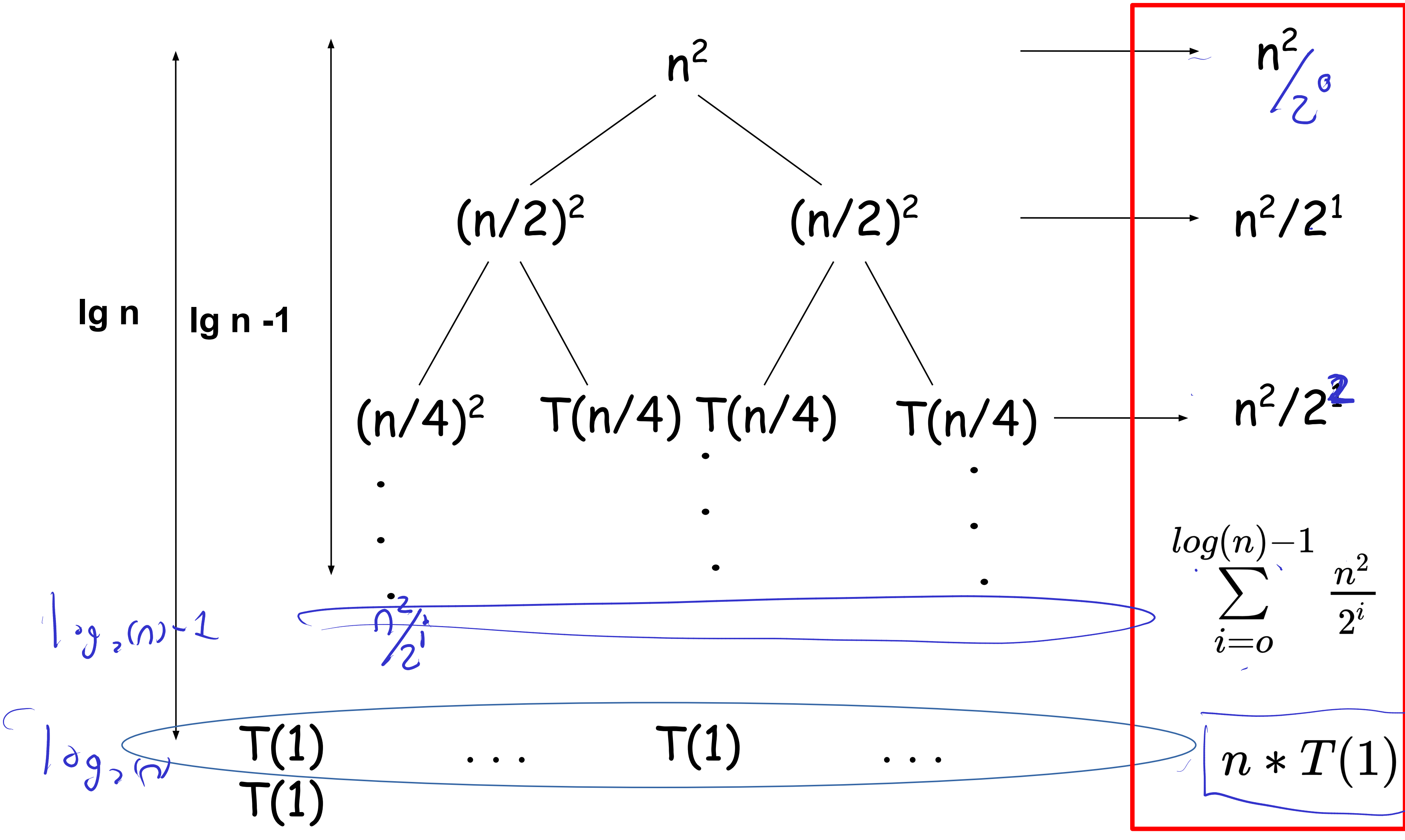


Recurrencias



Si recuerda en un árbol m-ario se tienen máximo m^h. En este caso al ser arbol binario m=2, tenemos $2^{\log(n)}$ hojas. Por lo tanto se

Recurrencias



Recurrencias

$\Theta(1)$

$$T(n) = n * T(1) + \sum_{i=o}^{log(n)-1} \frac{n^2}{2^i}$$

$$T(n) = n * c + n^2 \frac{0.5^{log(n)} - 1}{0.5 - 1}$$

$$T(n) = n * c + n^2 \frac{n^{log(0.5)} - 1}{-0.5}$$

$$T(n) = n * c + n^2 \frac{n^{-1} - 1}{-0.5}$$

$$T(n) = n * c - \frac{n}{0.5} + \frac{n^2}{0.5} = O(n^2)$$

Recurrencias

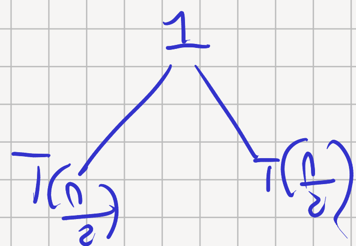
Resuelva construyendo el árbol

$T(n) = 2T(n/2) + 1, T(1) = \Theta(1)$

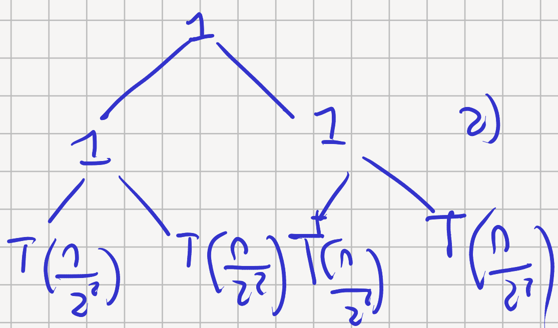
$\boxed{T(n) = 2T(n/2) + n, T(1) = \Theta(1)}$

$$2T\left(\frac{n}{2}\right)+1$$

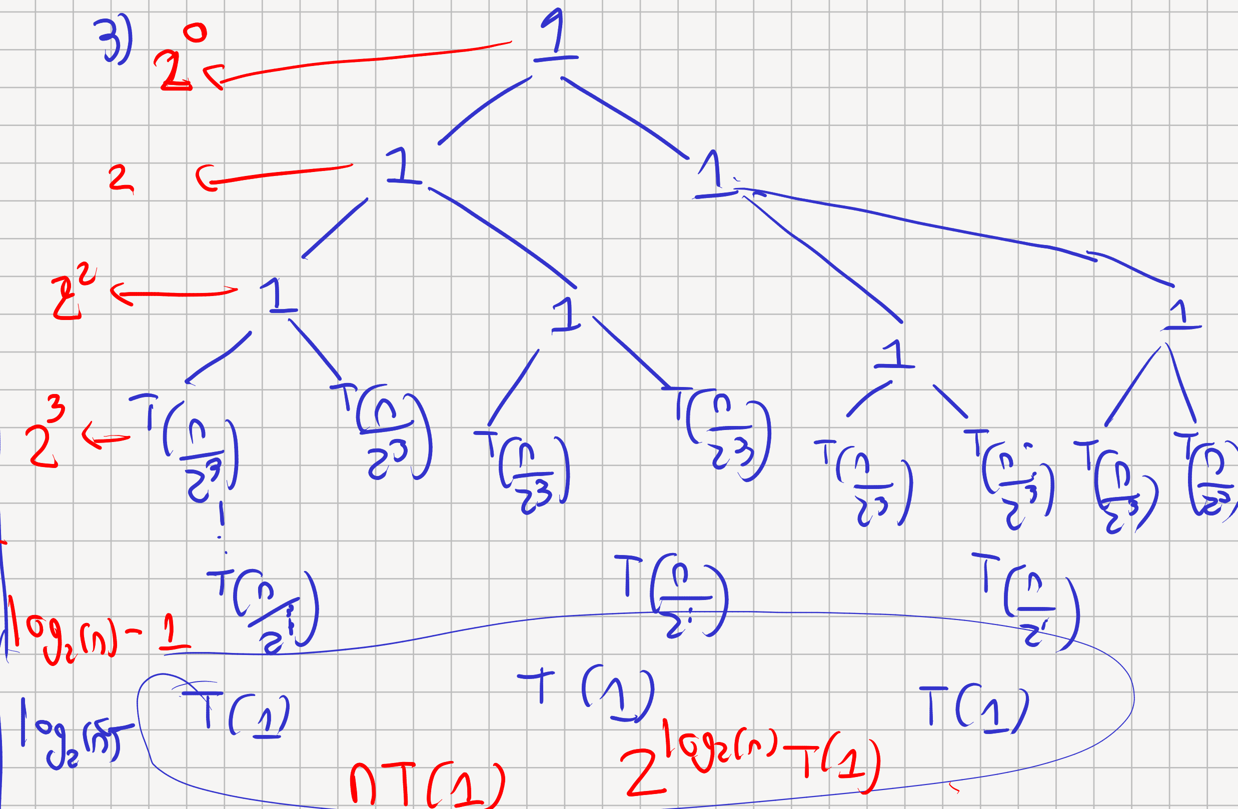
1)



2)



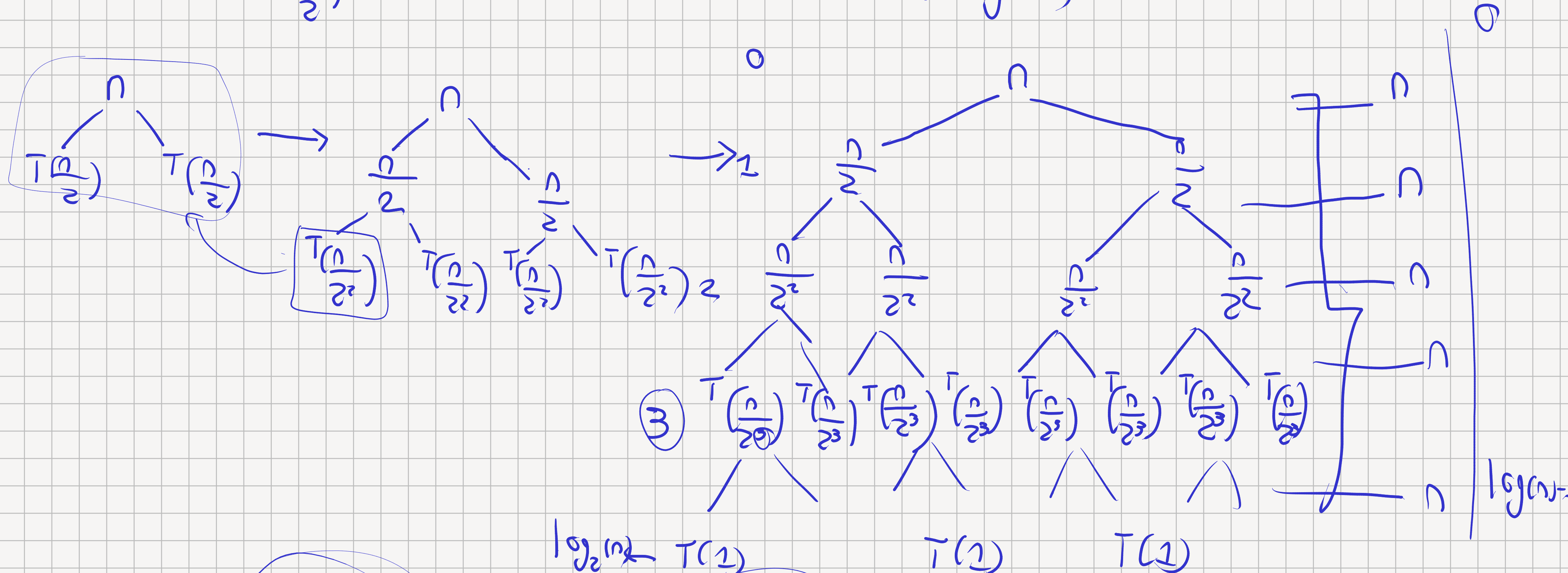
3)



$$T(n) = \underbrace{n\,T(1)}_{\text{Hojas}} + \sum_{i=0}^{\log_2(n)-1} 2^i = n \times C + n - 1$$

$$O(n)$$

$T(n) = 2T\left(\frac{n}{2}\right) + n \qquad T(1) = \Theta(1) \qquad T(n) = O(n\log(n))$



$$2^{\log_2(n)} \times T(1) + \sum_{i=0}^{\log_2(n)-1} n = cn + \underset{i=0}{n} + \sum_{i=1}^{\log_2(n)-1} n = cn + n + n \times (\log_2(n) - 1)$$

$$cn + n\log_2(n) = O(n\log_2(n))$$

Recurrencias

Resuelva la recurrencia T(n) = T(n/3) + T(2n/3) + n

Indique una cota superior y una inferior

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n$$

$$T(n) = T\left(\frac{n}{3^2}\right) + T\left(\frac{2n}{3^2}\right) + \frac{n}{3} + T\left(\frac{2n}{3^2}\right) + T\left(\frac{2^2}{3^2}n\right) + \frac{2}{3}n + 2$$

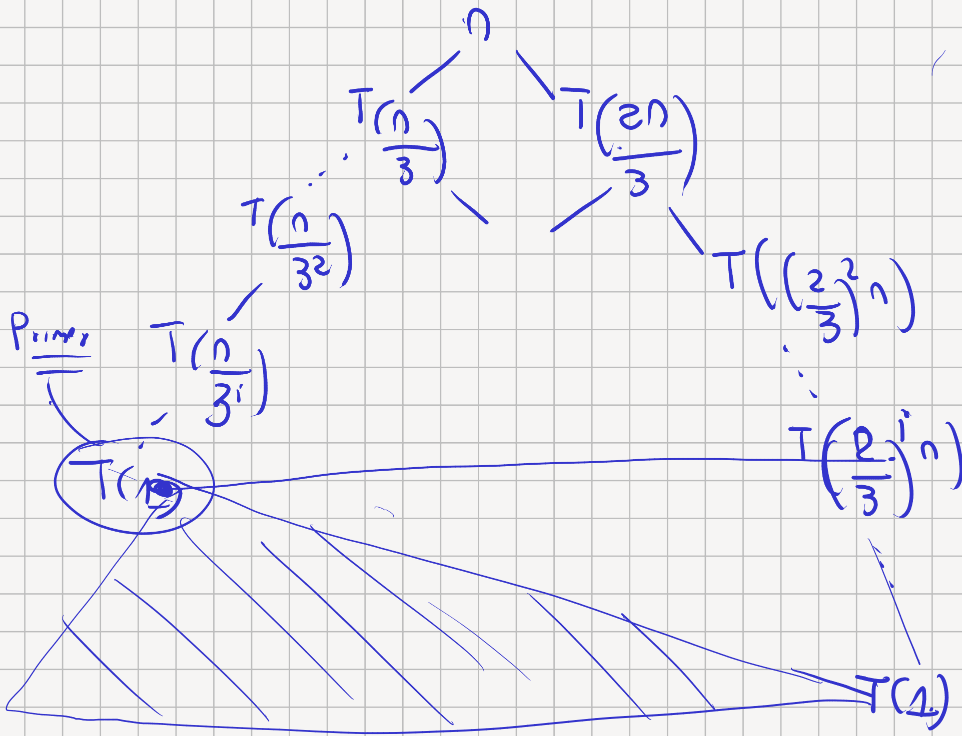
$$T(n) = T\left(\left\lfloor \frac{n}{3} \right\rfloor\right) + T\left(\left\lfloor \frac{2n}{3} \right\rfloor\right) + n$$

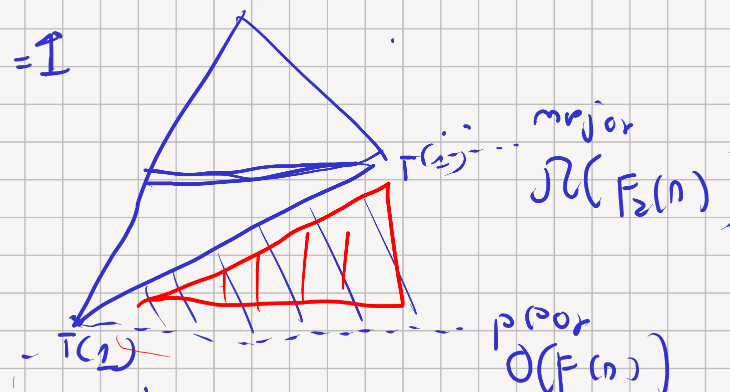
$$\frac{n}{3^i} = \quad i = \log_3(n)$$

$$\left(\frac{2}{3}\right)^i n = \quad i = \log_{\frac{3}{2}}(n)$$

$$\left(\frac{2}{3}\right)^i n = 1$$

$T(1)$





$$\log_{\frac{2}{3}}(n) = \frac{\log_3(n)}{\log_3\left(\frac{2}{3}\right)}$$

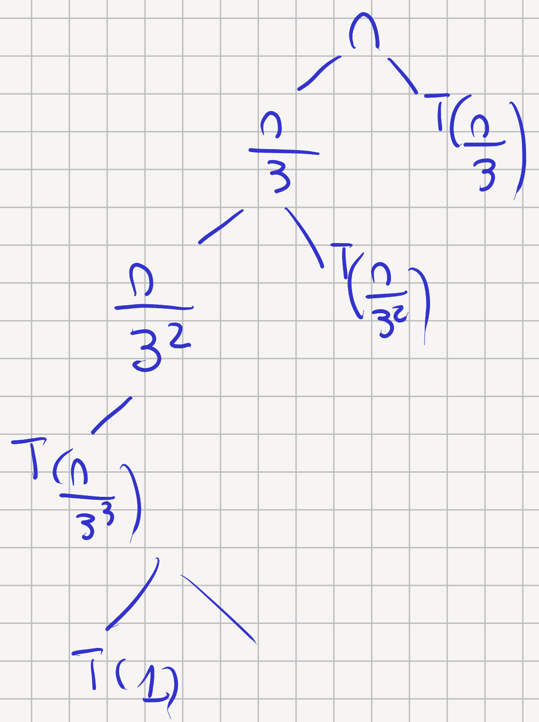
$$\log_{\frac{2}{3}}(n) = \frac{\log_3(n)}{\log_3(2) - \log_3(3)} \doteq -0.36$$

$$\log_{\frac{2}{3}}(n) = -2.70 \times \log_3(n)$$

$$\Omega(T(n)) = n + 2T\left(\frac{n}{3}\right)$$

$$O(T(n)) = n + 2T\left(\frac{2}{3}n\right)$$

$$\Omega(T(n)) = n + 2T\left(\frac{n}{3}\right)$$



$2^0 n$

$2^1 \times \frac{n}{3}$

$2^2 \times \frac{n}{3^2}$

$2^3 \frac{n}{3^3}$

$\log_3(n)$

$\vdots$

$$2^{\log_3(n)} \times T(1)$$

$$n^{\log_3(2)} \times T(1)$$

$$\sum_{i=0}^{\log_3(n)-1} \left(\frac{2}{3}\right)^i \times n$$

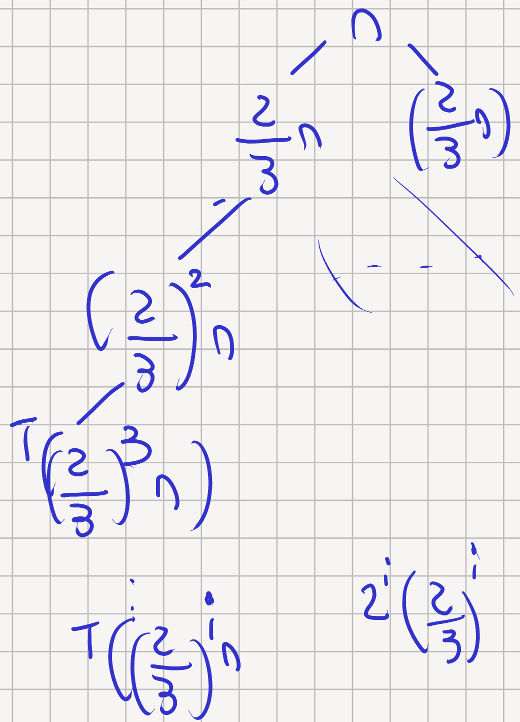
n^x

$$n\left(\frac{\left(\frac{2}{3}\right)^{\log_3(n)} - 1}{2/3 - 1}\right)$$

$$O(n)$$

cn

$$O(T(n)) = n + 2T\left(\frac{2}{3}n\right)$$



$2^0 n$

$2^1\left(\frac{2}{3}n\right)$

$2^2\left(\frac{2}{3}\right)^2 n$

$2^3\left(\frac{2}{3}\right)^3 n$

$2^i\left(\frac{2}{3}\right)^i n$

$\log_{3/2}(n)$

$2^i\left(\frac{2}{3}\right)^i$

$$\frac{2^i 2^i}{3^i} = \frac{2^{i+i}}{3^i} = \frac{2^{2i}}{3^i} = \frac{4^i}{3^i}$$

$$2^{\log_{\frac{2}{3}}(n)} \times T(1)$$

$$T(n) = n^{\log_{\frac{2}{3}}(2)} \times c + \sum_{i=0}^{\log_{\frac{2}{3}}(n)-1} \left(\frac{4}{3}\right)^i \times n$$

negativo

$$T(n) = n^{\frac{\log_2(2)}{\log_2\left(\frac{2}{3}\right)}} + n\left(\frac{\left(\frac{4}{3}\right)^{\log_{\frac{2}{3}}(n)} - 1}{4/3 - 1}\right)$$

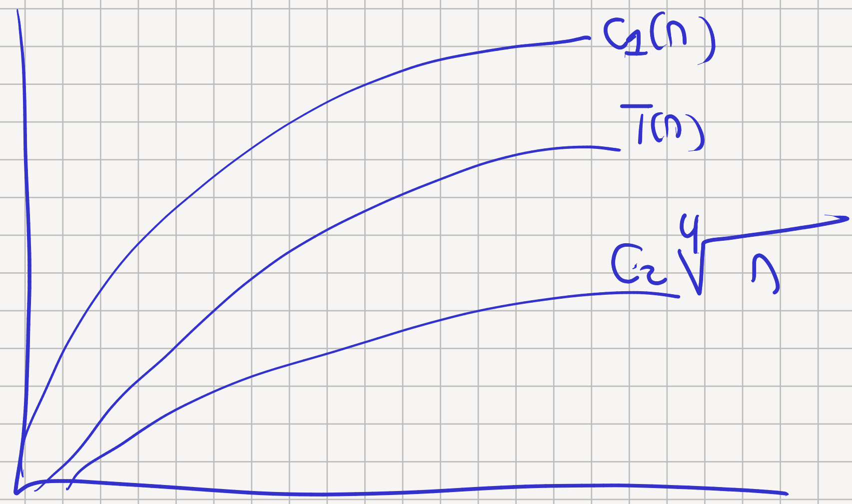
> 0

$$n\, n^{\log_{\frac{2}{3}}\left(\frac{4}{3}\right)}$$

$$n \times n^{\frac{\log_2\left(\frac{4}{3}\right)}{\log_2\left(\frac{2}{3}\right)}}$$

$$n^{\frac{\log_2(2)}{\log_2\left(\frac{2}{3}\right)}} \quad \Omega(n^{0.25})$$

$T(n) \rightarrow O(n)$ y $\Omega(n^{0.25} = \sqrt[4]{n})$



Recurrencias

Método maestro

Permite resolver recurrencias de la forma:

$T(n) = aT(n/b) + f(n)$, donde $a \geq 1$, $b > 1$

Recurrencias

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n)=\Theta(n^{\log_b a})$

Si $f(n)=O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon>0$

2. $T(n)=\Theta(n^{\log_b a}\lg n)$

Si $f(n)=\Theta(n^{\log_b a})$ para algún $\varepsilon>0$

3. $T(n)=\Theta(f(n))$

Si $f(n)=\Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon>0$ si a*f(n/b) ≤c*f(n)

para algun c<1

Recurrencias

Dado T(n) = 9T(n/3) + n

$a = 9$

$b = 3$

$f(n) = n$

$$n^{\log_3 9} = n^2 \quad \textbf{Vs} \quad f(n) = n$$

$n^{\log_b a}$

$n^{\log_3 9} = n^2$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $n = O(n^{2-\varepsilon})$?

$\Theta(n^{\log_b a})$

$\Theta(n^2)$

n es $O(n^{2-\epsilon})$

n es $O(n)$ ✓

Recurrencias

Dado T(n) = 9T(n/3) + n

$$n^{\log_3 9} = n^2 \quad \textbf{Vs} \quad f(n) = n$$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $n = O(n^{2-\varepsilon})$?

Si $\varepsilon = 1$ se cumple que $n = O(n)$, por lo tanto, se cumple que:

$$T(n) = \Theta(n^2)$$

$aT(\frac{n}{b})+F(n)$ Recurrencias

$\frac{2}{3} = \frac{1}{b}$

$T(n) = T(2n/3) + 1$

$a = 1$

$b = \frac{3}{2}$

$n^{\log_b a}$

$n^{\log_{\frac{3}{2}} 1} = n^0$

$$n^{\log_{3/2} 1} = n^0 = 1 \quad \textbf{Vs} \quad f(n) = 1$$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $1 = O(n^{0-\varepsilon})$?

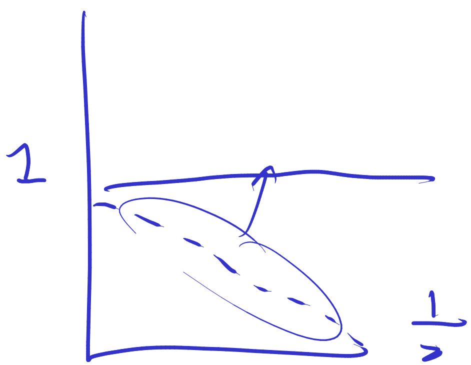
No existe $\varepsilon > 0$

1) $f(n)$ es $O(n^{\log_b a - \epsilon})$

1 es $O(n^{0-\epsilon})$

1 es $O(n^{-1})$

1 es $O(\frac{1}{n})$



Recurrencias

T(n) = T(2n/3) + 1

$$n^{\log_{3/2} 1} = n^0 = 1 \quad \text{vs} \quad f(n) = 1$$

Es $f(n) = \Theta(n^{\log_b a})$?

Es $1 = \Theta(1)$?

Si, por lo tanto, se cumple que:

1 es $\Theta(n^{\log_b a})$

1 es $\Theta(n^0)$

1 es $\Theta(1)$

$$T(n) = \Theta(1 * \lg n) = \Theta(\lg n)$$

$T(n) = \Theta(n^{\log_b a} \log(n))$

n^a

Recurrencias

$T(n) = 3\,T(n/4) + n\lg n$ $a=3$ $b=4$

$$n^{\log_4 3} = n^{0.793} \quad \text{vs} \quad f(n) = n\lg n$$

$n^{\log_b a} = n^{\log_3 4}$

1) Es $f(n) = O(n^{\log_b a - \varepsilon})$?

$n\lg(n) \in O(n^{0.793-\epsilon})$

$O(\sqrt{n})$ ✗

2) Es $f(n) = \Theta(n^{\log_b a})$?

$\to n\log(n)$ $\Theta(n^{0.793})$

O ✗ Ω ✓

3) Es $f(n) = \Omega(n^{\log_b a + \varepsilon})$?

$\to n\log(n) \in \Omega(n^{0.793+\epsilon})$

$\in \Omega(n)$ SI

Si, y además, $af(n/b) \le cf(n)$

$3(n/4)\lg(n/4) \le cn\lg n$

$3(n/4)\lg n - 3(n/4)*2 \le cn\lg n$

$(3/4)n\lg n \le cn\lg n \to c=3/4$ y se concluye que $T(n) = \Theta(n\lg n)$

$c<1$

Recurrencias

$T(n) = 2T(n/2) + n\lg n$

Muestre que no se puede resolver por el método maestro

Recurrencias

Resuelva usando método del maestro

$T(n) = 4T(n/2) + n$

$T(n) = 4T(n/2) + n^2$

$T(n) = 4T(n/2) + n^3$

Dado $T(n) = aT(n/b) + f(n)$, donde $a \geq 1$, $b > 1$, se puede acotar asintóticamente como sigue:

1. $T(n) = \Theta(n^{\log_b a})$

Si $f(n) = O(n^{\log_b a - \varepsilon})$ para algún $\varepsilon > 0$

2. $T(n) = \Theta(n^{\log_b a} \lg n)$

Si $f(n) = \Theta(n^{\log_b a})$ para algún $\varepsilon > 0$

3. $T(n) = \Theta(f(n))$

Si $f(n) = \Omega(n^{\log_b a + \varepsilon})$ para algún $\varepsilon > 0$ si $a * f(n/b) \leq c * f(n)$

$c < 1$

$T(n) = 4T\left(\frac{n}{2}\right) + n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

1) n es $O(n^{2-\epsilon})$

n es $O(n)$

$\Theta(n^2)$

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n)=\Theta(n^{\log_b a})$

Si $f(n)=O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon>0$

2. $T(n)=\Theta(n^{\log_b a}\lg n)$

Si $f(n)=\Theta(n^{\log_b a})$ para algún $\varepsilon>0$

3. $T(n)=\Theta(f(n))$

Si $f(n)=\Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon>0$ si a*f(n/b) ≤c*f(n)

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

1) $n^2 \in O(n^{2-\epsilon})$ ✗

2) $n^2 \in \Theta(n^2)$ ✓

$$\Theta(n^2 \lg(n))$$

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n)=\Theta(n^{\log_b a})$

Si $f(n)=O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon>0$

2. $T(n)=\Theta(n^{\log_b a}\lg n)$

Si $f(n)=\Theta(n^{\log_b a})$ para algún $\varepsilon>0$

3. $T(n)=\Theta(f(n))$

Si $f(n)=\Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon>0$ si a*f(n/b) ≤c*f(n)

3) $T(n) = 4T\left(\frac{n}{2}\right) + n^3$

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n)=\Theta(n^{\log_b a})$

Si $f(n)=O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon>0$

2. $T(n)=\Theta(n^{\log_b a}\lg n)$

Si $f(n)=\Theta(n^{\log_b a})$ para algún $\varepsilon>0$

3. $T(n)=\Theta(f(n))$

Si $f(n)=\Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon>0$ si a*f(n/b) ≤c*f(n)

$c<1$

1) $n^3 \in O(n^{2-\epsilon})$ X

2) $n^3 \in \Theta(n^2)$ X

3) $n^3 \in \Omega(n^{2+\epsilon})$

$\Omega(n^3)$

$4f(n/2) \leq c * f(n)$

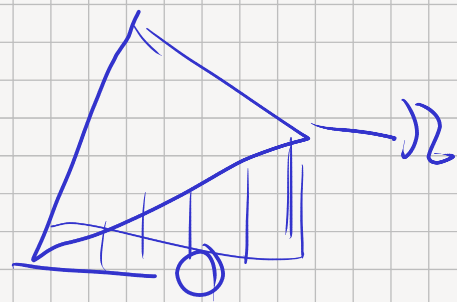
$4\left(\frac{n}{2}\right)^3 \leq cn^3 \longrightarrow \frac{4\cancel{n^3}}{8} \leq c\cancel{n^3}$ $\quad \frac{1}{2} \leq c$

$c \geq \frac{1}{2}$

$\Theta(n^3)$

$$T\left(\frac{n}{3}\right)$$

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n$$



$$T(n) = 2T\left(\frac{n}{3}\right) + n$$

$$n^{\log_b a} = n^{\log_3 2}$$

$$n^{0.63}$$

1) n is $O(n^{0.63-\epsilon})$ ✗

2) n is $\Theta(n^{0.63})$ ✗

3) n is $\Omega(n^{0.63+\epsilon})$

$\Omega(n)$ ✓

$a\, f(n/b) \leq c \times f(n)$

$2\left(\frac{n}{3}\right) \leq c \times n$ $\quad \Theta(n)$

$c \geq \frac{2}{3}$

$$T(n) = 2T\left(\frac{2}{3}n\right) + n$$

$$n^{\log_{\frac{3}{2}} 2} \qquad n^{\frac{\log_2 2}{\log_2 \frac{3}{2}}} \quad \underline{1}$$

1) n is $O(n^{1.70-\epsilon})$

$O(n)$

$$T(n) = \Theta(n^{1.70})$$

$$\Theta(n\, n^{0.7})$$

$$\Theta(n\sqrt[\sqrt{2}]{n})$$

$0.7 = \frac{1}{x}$

$x = \frac{1}{0.7}$

Recurrencias

Método de sustitución

Suponer la forma de la solución y probar por inducción matemática

Recurrencias

$T(n)=2T(\lfloor n/2 \rfloor)+n,\ T(1)=1$

Suponer que la solución es de la forma $T(n)=O(n\lg n)$

Probar que $T(n) \le cn\lg n$.

Se supone que se cumple para $n/2$ y se prueba para n

Hipotesis inductiva: $T(n/2) \le c(n/2)\lg(n/2)$

Recurrencias

$T(n)=2T(\lfloor n/2 \rfloor)+n$, $T(1)=1$

Probar que $T(n) \leq cn\lg n$.

Hipótesis inductiva: $T(n/2) \leq cn/2 \lg (n/2)$

Paso inductivo:

$T(n) \leq 2(c(n/2)\lg (n/2)) + n$

$\leq cn \lg (n/2) + n$

$= cn \lg (n) - cn + n$, para $c \geq 1$, haga $c=1$

$\leq cn \lg n$

$n \log(n) - \cancel{n} + \cancel{n}$

Recurrencias

$T(n)=2T(\lfloor n/2 \rfloor)+n$, $T(1)=1$

Probar que $T(n) \leq cn\lg n$.

Paso base: si c=1, probar que T(1)=1 se cumple

$T(1) \leq 1 * 1 \lg 1$?

$1 \leq 0$?

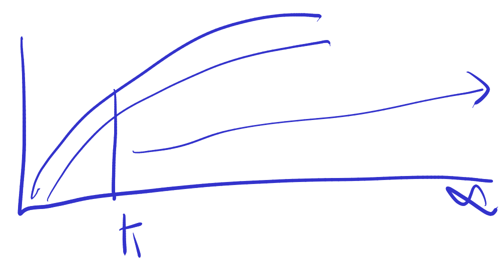
No, se debe escoger otro valor para c

Recurrencias

$T(n)=2T(\lfloor n/2 \rfloor)+n$, $T(1)=1$

Probar que $T(n) \leq cn\lg n$.

Paso base: si $c=2$, probar que $T(1)=1$ se cumple



$T(1) \leq 2{*}1\lg 1$?

$1 \leq 0$?

No, se puede variar k.

Para esto, se calcula T(2) y se toma como valor inicial

Recurrencias

Probar que $T(n) \leq cn\lg n$.

$T(2)=2T(0)+2=4$

Paso base: si $c=1$, probar que $T(2)=4$ se cumple

$T(2) \leq 1*2\lg 2$?

$4 \leq 2$?

No, se puede variar c.

Recurrencias

Probar que $T(n) \leq cn\lg n$.

$T(2)=2T(0)+2=4$

Paso base: si $c=3$, probar que $T(2)=4$ se cumple

$T(2) \leq 3*2\lg 2$?

$4 \leq 6$?

Si, se termina la demostración

$T(n) = 2\boxed{T\left(\frac{n}{2}\right)} + 1 \qquad T(1) = 1$

$n^{\log_b a} = n^{\log_2 2} = n^1$

1) 1 es $O(n^{1-\epsilon})$

1 es $O(\sqrt{n})$

$\Theta(n)$

$\boxed{T(n) \le c \times n} \qquad n \qquad \boxed{T(n) \ge c \times n}$

$\Theta(n)$

$T(n) \le \cancel{2} c \frac{n}{\cancel{2}} + 1$

$T(n) \le c\,n + 1 \qquad c = 4$

$T(n) \le 4n + 1$

n	
1	$1 \le 5$ ✓
2	$3 \le 13$ ✓
4	$7 \le 17$ ✓

$T(n) \ge c \times n \qquad \Omega(n)$

$c = 1$

$T(n) \ge \frac{\cancel{2} c n}{\cancel{2}} + 1$

$T(n) \ge c\,n + 1 = n + 1$

$T(n)$ es $\Omega(n)$

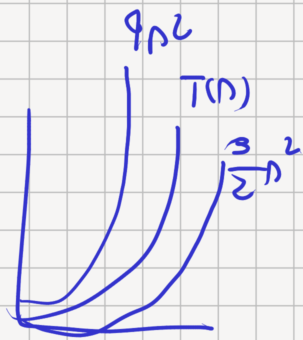
$k = 2$

n	
1	$1 \ge 2$ ✗
2	$3 \ge 3$ ✓
4	$7 \ge 5$ ✓

$T(n) = 2T\left(\frac{n}{2}\right) + n^2 \qquad T(1) = 1$

$n^{\log_b a} = n^{\log_2 2} = n$

$T(n)$ es $\Theta(n^2)$



$T(n)$ es $O(n^2)$

$T(n) \leq cn^2$

$T(n) \leq 2\left(c\left(\frac{n}{2}\right)^2\right) + n^2 \qquad c = 16$

$T(n) \leq 2\frac{cn^2}{4} + n^2 = 8n^2 + n^2 = 9n^2$

n	$T(n)$	$9n^2$	
1	1	9	✓
2	6	36	✓
4	28	144	✓
8	120	576	✓

1) n^2 es $O(n^{-\epsilon})$ X

2) n^2 es $\Theta(n)$ X

3) n^2 es $\Omega(n^{2+\epsilon})$

$\Omega(n^2)$

$a f\left(\frac{n}{b}\right) \leq c \times n^2 \qquad c < 1$

$2\left(\frac{\cancel{n}}{2}\right)^2 \leq c \times \cancel{n}^2 = \frac{1}{2} \leq c$ ✓

$\Theta(f(n)) = \Theta(n^2)$

$T(n)$ es $\Omega(n^2)$

$T(n) \geq 2\frac{cn^2}{4} + n^2 = \frac{1}{2}n^2 + n^2 = \frac{3}{2}n^2$

n	$T(n)$	$\frac{3}{2}n^2$	
1	1	$\frac{3}{2}$	X
2	6	6	✓
4	28	24	✓
8	120	96	✓

$n \geq 2$

Recurrencias

$T(n)=T(n-1)+T(n-2)+1,\ T(1)=O(1),\ T(2) = O(1)$

Suponer que la solución es de la forma $T(n)=O(2^n)$

Probar que $T(n)\leq c2^n$.

Se supone que se cumple para $n-1$ y se $n-2$ prueba para n

Hipotesis inductiva: $T(n-1) \leq c2^{(n-1)}$ y $T(n-2) \leq c2^{(n-2)}$

Recurrencias

$T(n)=T(n-1)+T(n-2)+1$, $T(1)=O(1)$, $T(2) = O(1)$

Ahora se debe probar que: $T(n) \leq c2^n$

$$T(1) \leq c2^1 \rightarrow 1 \leq 2*c$$

$$T(2) \leq c2^2 \rightarrow 1 \leq 4*c$$

$$T(3) \leq c2^3 \rightarrow 2 \leq 8*c$$

$$T(4) \leq c2^4 \rightarrow 3 \leq 16*c$$

$$T(5) \leq c2^5 \rightarrow 5 \leq 32*c$$

$$T(6) \leq c2^6 \rightarrow 8 \leq 64*c$$

$$T(7) \leq c2^7 \rightarrow 13 \leq 128*c$$

$$T(8) \leq c2^8 \rightarrow 21 \leq 256*c$$

Con $c = 1$, se cumple.

Referencias

Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. 2009. Introduction to Algorithms, Third Edition (3rd ed.). The MIT Press. Chapter 4

Gracias

Próximo tema:

Divide y vencerás