Lenguajes y gramaticas

- Aplicaciones para el desarrollo de lenguajes de programación.
- Sintaxis: Forma de escribir en un lenguaje de programación
- Verificación de programas (Formal) <-- Plantas nucleares matematicamente nos aseguramos que no existan errores, para esto se utilizan los automatas.
- Expresiones regulares: Identificar cadenas en textos muy largos, caracteres, correos

Alfabeto Σ conjunto de símbolos

{a,b,c,d,...,z} Alfabeto inglés
{0,1} Binario
{+,-,*,/} Operaciones

Cerradura de un alfabeto o lenguaje universal Σ^*

$\Sigma = \{0,1\}$ $\Sigma^* = \{\epsilon, 0, 1, 10, 11, 100, \ldots\}$

Lenguaje: Es un conjunto de palabras que podemos formar a partir de un alfabeto

$$\Sigma^* \supseteq L$$

Ejemplos de lenguajes: {0,1}

L1 = Lenguajes de todas las cadenas de tamaño par $L1 = \{01, 10, 11, 00, 1000,$
L2 = Lenguaje de todas las cadenas que inician en 0 y terminan en 1

$L2 = \{01, 001, 011, 00001, \ldots\}$

Operaciones sobre los lenguajes

$L1 \cup L2$

$L1 \cdot L2$

$L1 = \{a, aa, aab\}$

$L2 = \{a, bb, aabb\}$

$L1 \cup L2 = L2 \cup L1$

$\{a, aa, aab, bb, aabb\}$

$L1 \cdot L2$

$\{aa, abb, aaabb, aaa, aabb, aaaabb, aaba, aabbb, aabaabb\}$

$L1 . L2 \neq L2 . L1$

$$L_1^n = \begin{cases} \epsilon & n=0 \\ L \cdot L^{n-1} & \text{en otro caso} \end{cases}$$

$n \geq 0$

$L_1 = \{a, b, bb\}$

$L_1^0 = \{\epsilon\}$

$L_1^1 = L_1 \times L_1^0$

$L_1^1 = \{a, b, bb\}\{\epsilon\}$

$L_1^2 = L_1^1 \cdot L_1$

$\{a, b, bb\}\{a, b, bb\}\{\epsilon\}$

$= \{aa, ab, abb, ba, bb, bbb, bba, bbbb\}$

$L_1^4 = \{abbbb$

$babbb$

$L_1^* = \{L_1^0 \cup L_1^1 \cup L_1^2 \cup L_1^3 \cup L_1^4 \cup \dots L_1^n\}$

$L_1 = \{aa, ba\}$

$L_1^* = \{\epsilon, aa, ba, aaba, baaa, baba, \dots\}$

$L_1^+ = \{aa, ba, aaba, \dots\}$

$L_1^+ = L_1 \times L_1^*$

$L_1^+ = L_1\{\epsilon \cup L_1 \cup L_1^2 \cup L_1^3 \cup L_1^4 \cup \dots\}$

$= \{L_1 \cup L_1^2 \cup L_1^3 \cup \dots\}$

Lenguajes Regulares

$\Sigma = \{a, b, c, \dots\}$

$\{a\}$ es un lenguaje regular

$\{a\}\{b\}$ es un lenguaje regular

$\{\epsilon\}$ es un lenguaje regular

$\{a\} \cup \{b\}$ es un lenguaje regular

$\emptyset$ es un lenguaje regular

$\{a\}^* \{b\}^+$

$L = \{aa, bb\}$ $\quad \Sigma = \{a, b\}$

$\{a\}\{a\} \cup \{b\}\{b\}$

$L = \{a^2a, b^2b^2\}$

$L = \{a\}\{a\}\{a\} \cup \{b\}\{b\}\{b\}\{b\}$

$L = a^n b^m \quad n, m \geq 0$

$a^n = \{\epsilon, a, aa, aaa, \dots\}$

$b^m = \{\epsilon, \cancel{b}, bb, bbb, \dots\}$

$\{\epsilon, b, bb, \underbrace{\boxed{bbb}}_{a^0b^3}, ab, \underbrace{abb}_{a^1b^2}, abb\}$

$a^n b^n$

$a^n = \{\epsilon, a, aa, aaa, \dots\}$

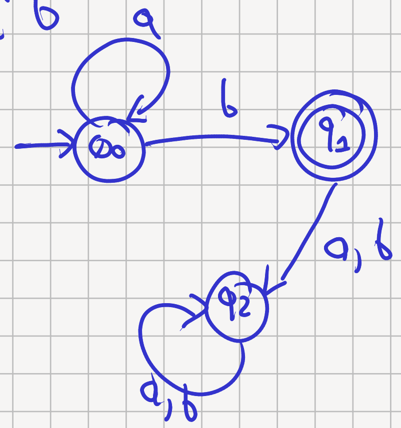
$b^n = \{\epsilon, b, bb, bbb, \dots\}$

$a^n b^n = \{\epsilon, b, bb, \underbrace{\boxed{bbb}}_{a^0 b^3}, a, ab, abb, \dots\}$

Automatas finitos D AFD

$\Sigma, Q, q_0, T, F(q_i, \Sigma) \to q_j$

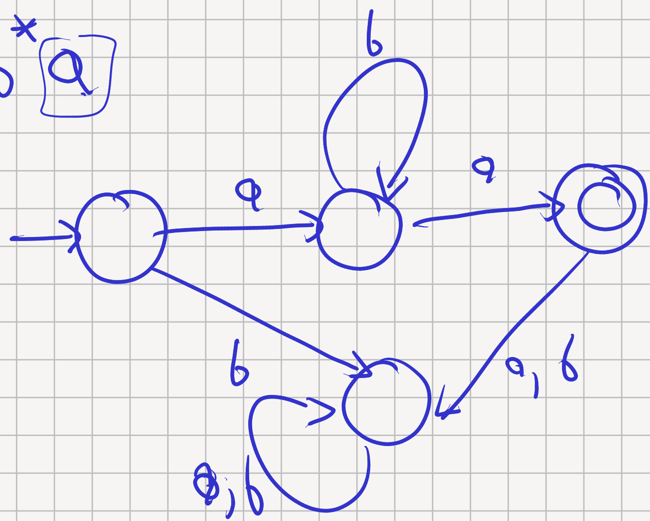
$a^* b$



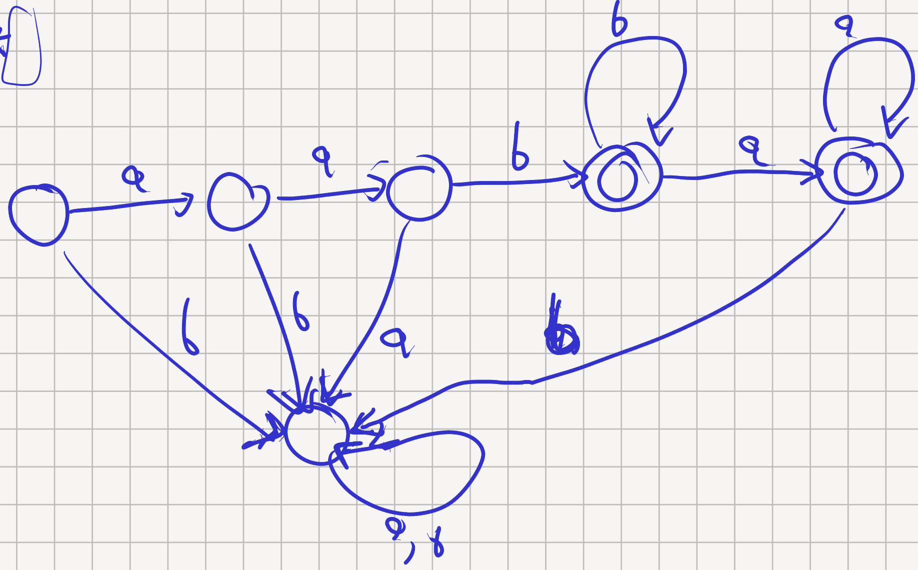
Q	a	b
q_0	q_0	q_1
q_1	q_2	q_2
q_2	q_2	q_2

$ab^*\boxed{a}$

$\boxed{aba}a$



$aa\boxed{b^+}\boxed{a^*}$

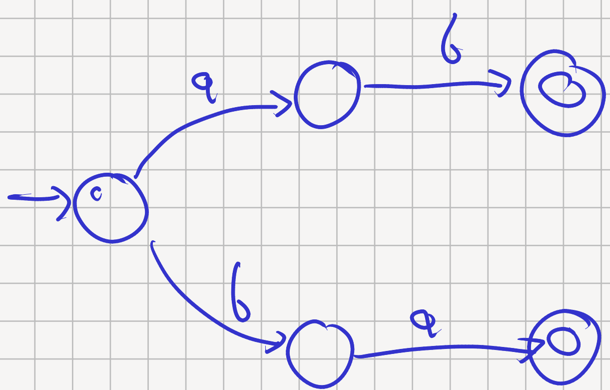


AFN

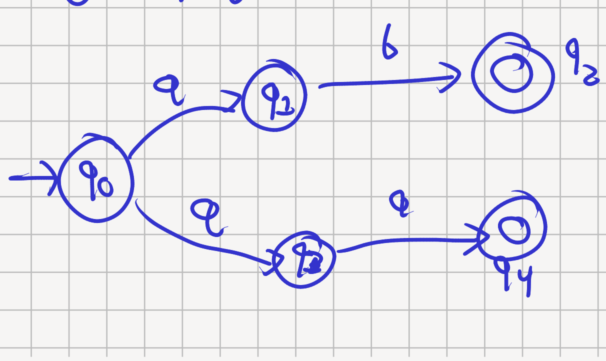
Automata finito no determinista

$$\Sigma, Q, q_0, T, \Delta(q_i, \Sigma_i) \to \underbrace{2^{Q}}_{\text{Conjunto potencia}}$$

$ab \cup ba$

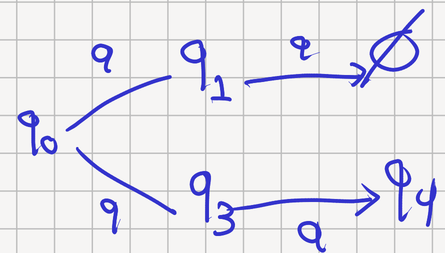


$aa \cup ab$



Δ	a	b
q_0	$\{q_1, q_3\}$	$\emptyset$
q_1	$\emptyset$	q_2
q_2	$\emptyset$	$\emptyset$
q_3	q_4	$\emptyset$
q_4	$\emptyset$	$\emptyset$

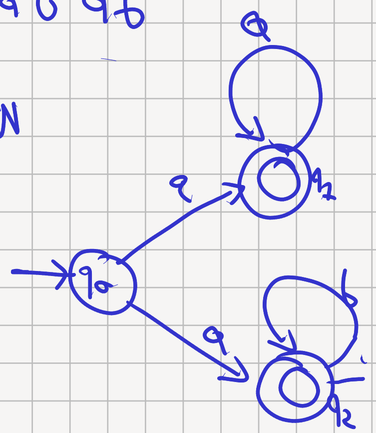
aa



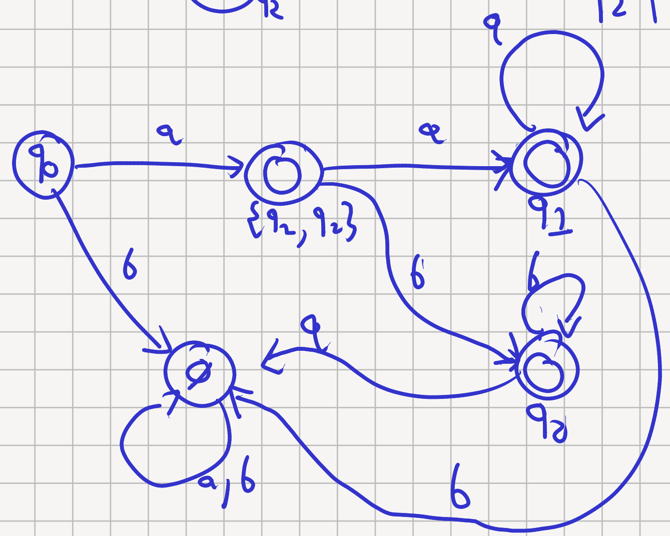
AFN $\longrightarrow$ AFD

$\underbrace{aa^*}_{a^+} \cup ab^*$

AFN



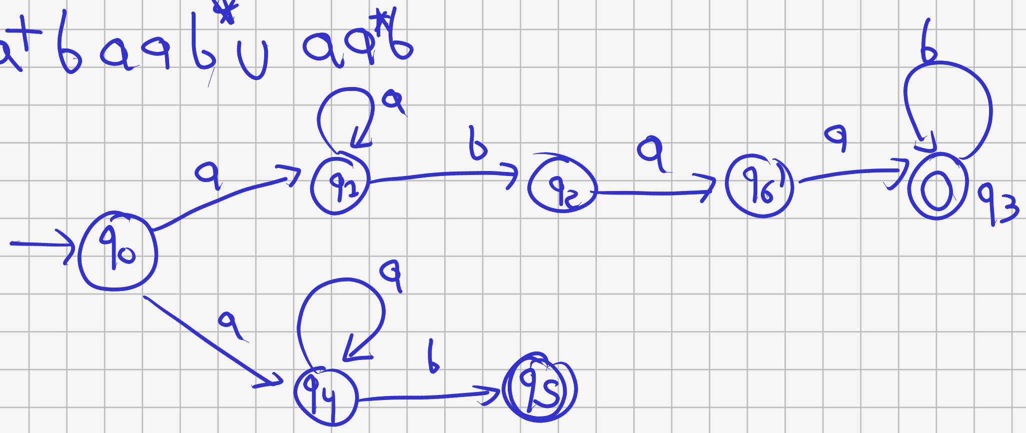
	a	b
q_0	$\{q_1, q_2\}$	$\emptyset$
$\{q_1, q_2\}$	q_1	q_2
q_1	q_1	$\emptyset$
q_2	$\emptyset$	q_2



a ✓
aaa ✓
ab ✓
abb ✓
abbb ✓

abba ✗
abbbb ab ✗

$a^+baab^* \cup aa^*b$



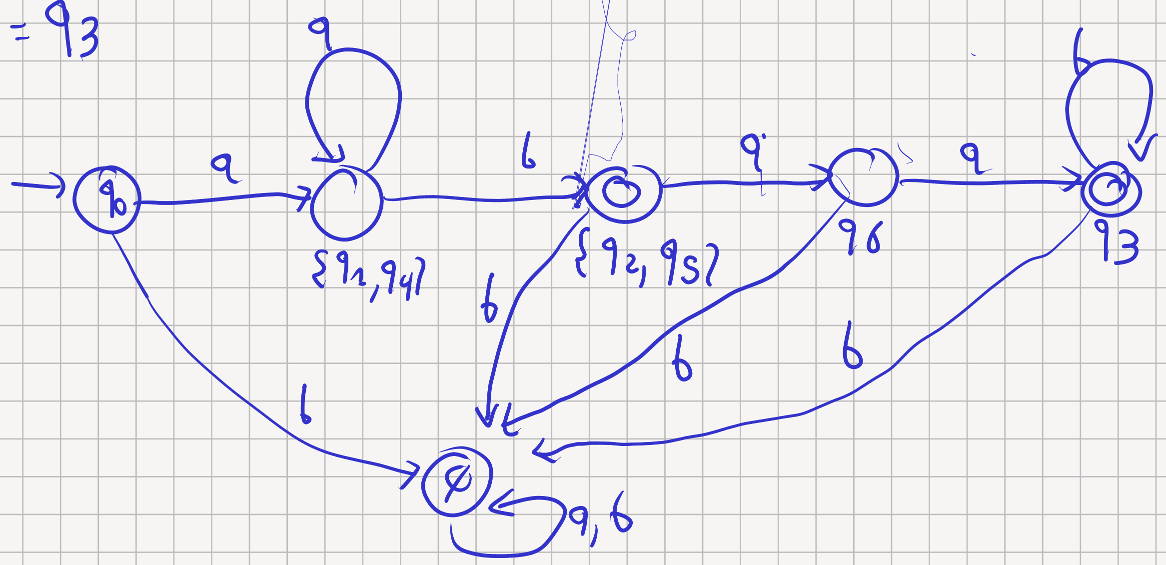
$(q_0, a) \Rightarrow \{q_1, q_4\}$ $\quad (q_0, b) \Rightarrow \emptyset$

$(\{q_1, q_4\}, a) \Rightarrow \{q_1, q_4\}$ $\quad (\{q_1, q_4\}, b) \Rightarrow \{q_2, q_5\}$

$(\{q_2, q_5\}, a) \Rightarrow \{q_6\}$ $\quad (\{q_6\}, a) \Rightarrow q_3$ $\quad (\{q_2, q_5\}, b) \Rightarrow \emptyset$

$(\{q_0\}, b) = \emptyset$ $\quad (\{q_3\}, a) = \emptyset$

$(\{q_3\}, b) = q_3$

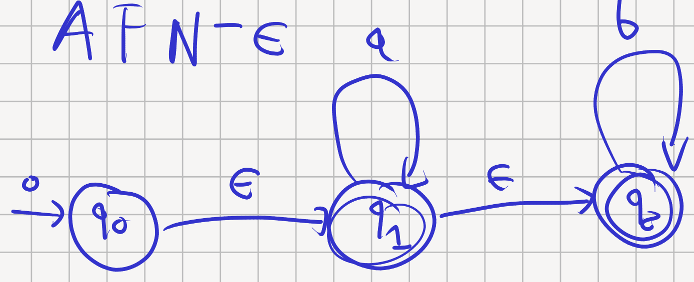


$(a \cup b)^* = \{\epsilon, a, b, aa, ab, ba, bb, \ldots\}$

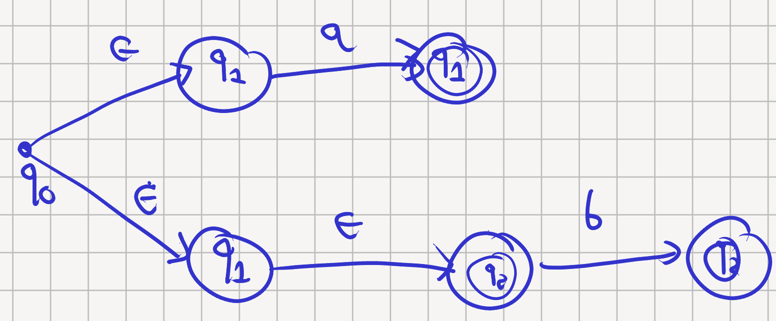
$\{a, b\}^*$

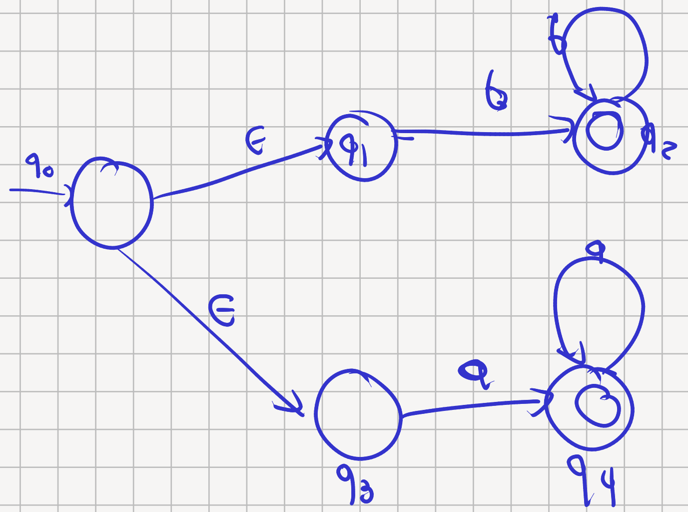
$(a \cup b) = \{a, b\}$

AFN-ϵ



$a^* \cup b^*$





$AFN_{\epsilon} \longrightarrow AFN$

$\epsilon\text{-}c(q_0) = \{q_0, q_1, q_3\}$

$\epsilon\text{-}c(q_1) = \{q_2\}$

$\epsilon\text{-}c(q_2) = \{q_2\}$

$\epsilon\text{-}c(q_3) = \{q_3\}$

$\epsilon\text{-}c(q_4) = \{q_4\}$

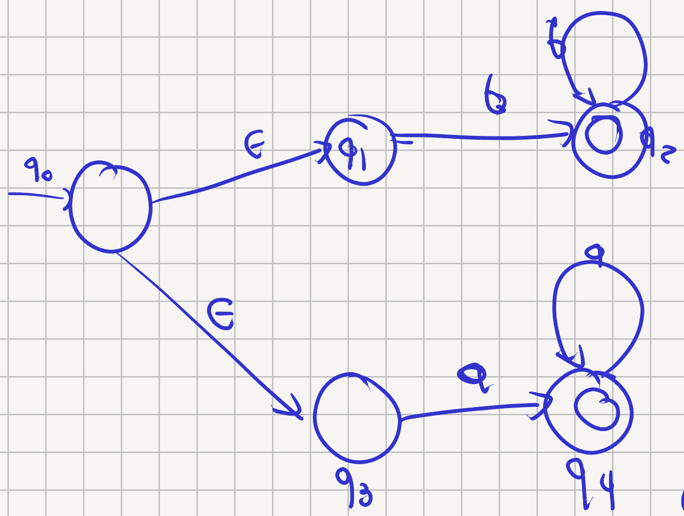
$\epsilon - c(q)$

$d(q, x)$

$d(q_0, a) = \emptyset$ $\quad d(q_2, a) = \emptyset$

$d(q_0, b) = \emptyset$ $\quad d(q_1, b) = q_2$

$d(q_2, a) = \emptyset$ $\quad d(q_2, b) = q_2$



$\{q_0, q_1, q_3\}$

$d(\epsilon\text{-}c(q_0), a) = \{q_4\}$

$d(\epsilon\text{-}c(q_1), b) = \{q_2\}$

$\epsilon\text{-}c(d(\epsilon\text{-}c(q_0), a)) = \{q_4\}$

ϵa $\quad AFN\text{-}\epsilon \longrightarrow AFN \longrightarrow AFD$

$\epsilon\text{-}c(q_0) = \{q_0, q_1, q_3\}$

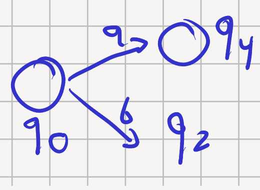
$d(e\text{-}c(q_0), a) = \{q_4\}$

$d(e\text{-}c(q_0), b) = \{q_2\}$

$\epsilon\text{-}c(d(e\text{-}c(q_0), a)) = \{q_4\} \leftarrow$

$a\epsilon^*$

$\epsilon\text{-}c(d(\epsilon\text{-}c(q_0), b)) = \{q_2\} \leftarrow$



$\epsilon - c(q_1) = \{q_1\}$

$d(\epsilon - c(q_1), a) = \emptyset$

$c - \epsilon(d(\epsilon - c(q_1), a)) = \emptyset$

$d(\epsilon - c(q_1), b) = q_2$

$c - \epsilon(d(\epsilon - c(q_1), b)) = q_2$

$\epsilon - c(q_2) = \{q_2\}$

$d(\epsilon - c(q_2), a) = \emptyset$

$c - \epsilon(d(\epsilon - c(q_2), a)) = \emptyset$

$d(\epsilon - c(q_2), b) = q_2$

$c - \epsilon(d(\epsilon - c(q_2), b)) = q_2$

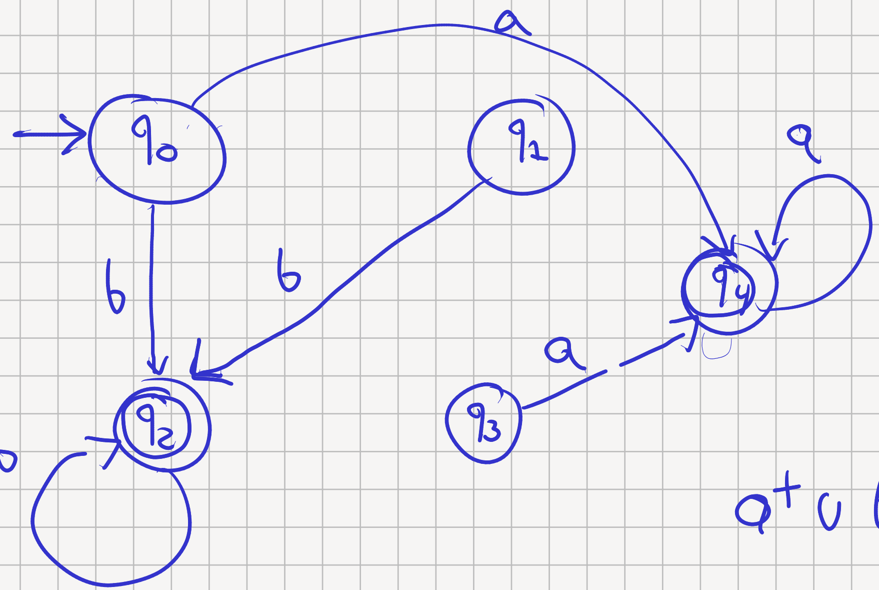
$\epsilon - c(q_3)$

$c - \epsilon(d(\epsilon - c(q_3)), a) = q_4$

$c - \epsilon(d(\epsilon\, c(q_3)), b) = \emptyset$

$c - \epsilon(d(\epsilon - c(q_4)), a) = q_4$

$c - \epsilon(d(\epsilon - c(q_4)), b) = \emptyset$

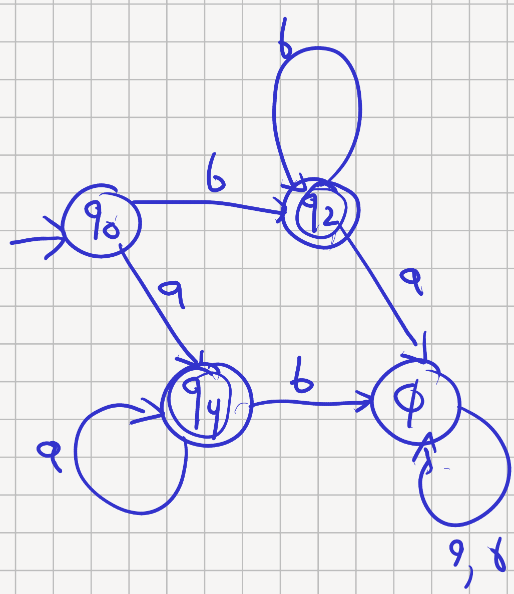


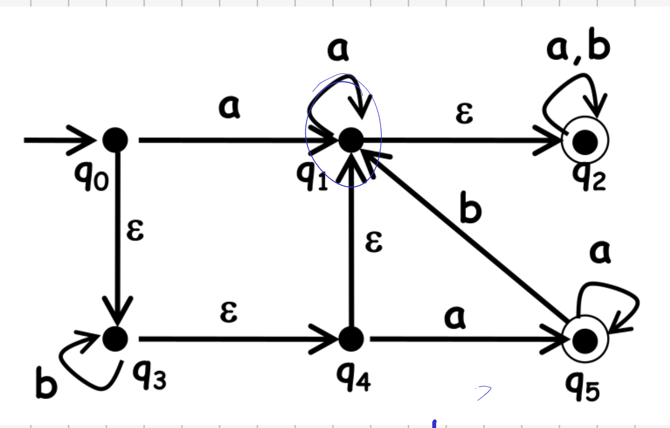
$a^+ \cup b^+$

$(q_0, a) \to q_4 \qquad (q_0, b) = q_2$

$(q_4, a) \to q_4 \qquad (q_4, b) = \emptyset$

$(q_2, a) \to \emptyset \qquad (q_2, b) = q_2$





• $\epsilon - c(q_0) = \{q_0, q_3, q_4, q_1, q_2\}$

a $\quad d(\epsilon - c(q_0), a) = \{q_1, q_5, q_2\}$

$\epsilon - c(d(\epsilon - c(q_0), a)) = \{q_1, q_5, q_2\}$

$d(\epsilon - c(q_0), b)) = \{q_3, q_2\}$

b. $\epsilon - c(\quad - - \quad , b) = \{q_3, q_4, q_1, q_2\}$

• $\epsilon - c(q_1) = \{q_1, q_2\}$

$\epsilon - c(d(\epsilon - c(q_1), a)) = \{q_1, q_2\}$

$\epsilon - c(d(\epsilon - c(q_1), b)) = \{q_2\}$

• $\epsilon - c(q_2) = \{q_2\}$

$\epsilon - c(d(\epsilon - c(q_2), a)) = \{q_2\}$

$\epsilon - c(d(\epsilon - c(q_2), b)) \{q_2\}$

$\epsilon - c(q_3) = \{q_3, q_4, q_1, q_2\}$

$\epsilon - c(d(\epsilon - c(q_3), a)) = \epsilon - c(\{q_1, q_2, q_5\}) = \{q_1, q_2, q_5\}$

$\epsilon - c(d(\epsilon - c(q_3), b)) = \epsilon - c(\{q_3, q_2\} = \{q_3, q_4, q_1, q_2\}$

$\epsilon - c(q_4) = \{q_4, q_1, q_2\}$

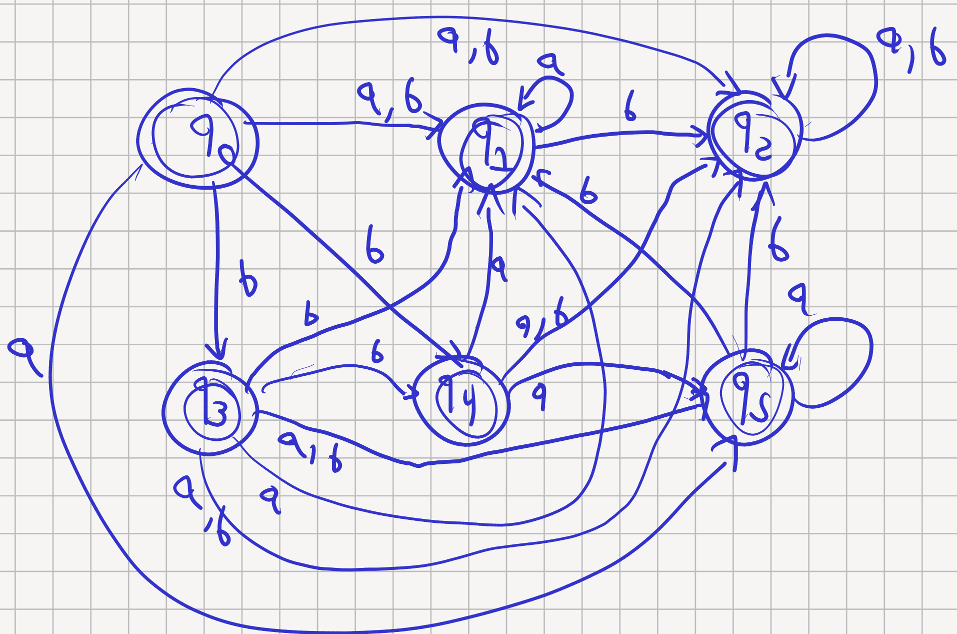
$\epsilon - c(d(\epsilon - c(q_4), a) = \epsilon c\{q_5, q_1, q_2\} = \{q_5, q_1, q_2\}$

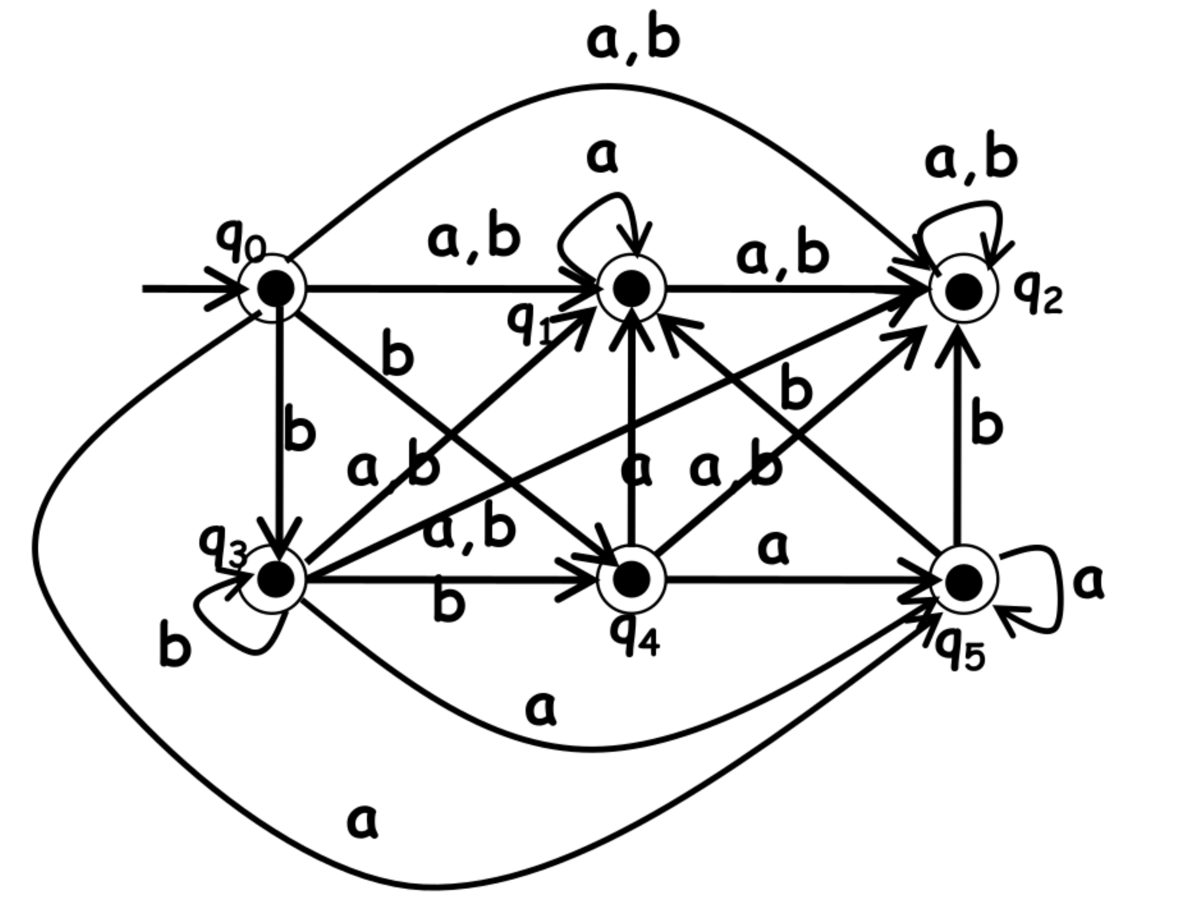
$\epsilon - c(d(\epsilon - c(q_4), b) = \epsilon c\{q_2\} = \{q_2\}$

$\epsilon - c(q_5) = \{q_5\}$

$\epsilon - c(d(\epsilon - c(q_5), a)) = \epsilon - c(q_5) = \{q_5\}$

$\epsilon - c(d(\epsilon - c(q_5), b)) = \epsilon - c(q_1) = \{q_1, q_2\}$





a,b
a
a,b
q0
a,b
a,b
q1
q2
b
b
b
b
a,b
a
a,b
q3
a,b
a
a
b
q4
b
q5
a
a