$$T(n) = 4T(n/4) + 2n + 4$$

$$\boxed{T(1) = 1}$$

$$T(n) = 8T(n/4) + 2n^2$$

1) Expansion

2) Arbol

3) Maestro

Dado $T(n) = aT(n/b) + f(n)$, donde $a \geq 1$, $b > 1$, se puede acotar asintóticamente como sigue:

1. $T(n) = \Theta(n^{\log_b a})$

Si $f(n) = O(n^{\log_b a - \varepsilon})$ para algún $\varepsilon > 0$

2. $T(n) = \Theta(n^{\log_b a} \lg n)$

Si $f(n) = \Theta(n^{\log_b a})$ para algún $\varepsilon > 0$

3. $T(n) = \Theta(f(n))$

Si $f(n) = \Omega(n^{\log_b a + \varepsilon})$ para algún $\varepsilon > 0$ si $a*f(n/b) \leq c*f(n)$

para algun $c<1$

$T(n) = 4T(n/4) + 2n + 4 \qquad T(1) = 1$

1) $T(n) = 2n + 4 + 4T(n/4)$

$= 2n + 4 + 4\left(\frac{2n}{4} + 4 + 4T(n/4^2)\right)$

(1) $= 2n + 4 + 2n + 4^2 + 4^2 T\left(\frac{n}{4^2}\right)$

$2n + 4 + 2n + 4^2 + 4^2\left(\frac{2n}{4^2} + 4 + 4T(n/4^3)\right)$

$2n + 4 + 2n + 4^2 + 2n + 4^3 + \boxed{4^3} T\left(\frac{n}{4^{\boxed{3}}}\right)$

$(\boxed{3}) 2n + \boxed{\overset{1}{4} + \overset{2}{4^2} + \overset{3}{4^3}} + 4^3 T\left(\frac{n}{4^3}\right)$

k

$\boxed{(k) 2n + 4 + 4^2 + \ldots 4^k + 4^k T\left(\frac{n}{4^k}\right)}$

¿Hasta cuando expando? $T(1)$

$1 = \frac{n}{4^k} \qquad k = \log_4(n)$

$\log_4(n) \times 2n + \sum_{i=1}^{\log_4(n)} 4^i + 4^{\log_4(n)} T(1)$

$2n \log_4(n) + \frac{4^{\log_4(n)+1} - 1}{3} - 4^0 + nT(1)$

ajuste

$T(n) = 2n \log_4(n) + \frac{4}{3}n - \frac{4}{3} + n$

$\sum_{i=0}^{n} ar^i = \frac{ar^{n+1} - a}{r - 1} \qquad r \neq 1$

$\sum_{i=1}^{n} a = n \times a$

$a^{\log_b n} = n^{\log_b a}$

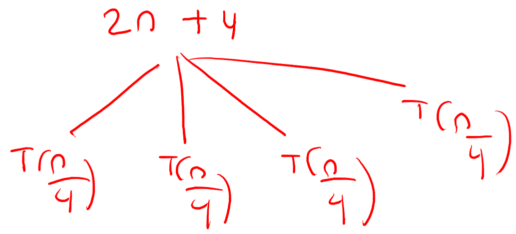
$4^{\log_4(n)+1} = 4 \times 4^{\log_4(n)}$

$= 4 \times n^{\log_4(4)}$

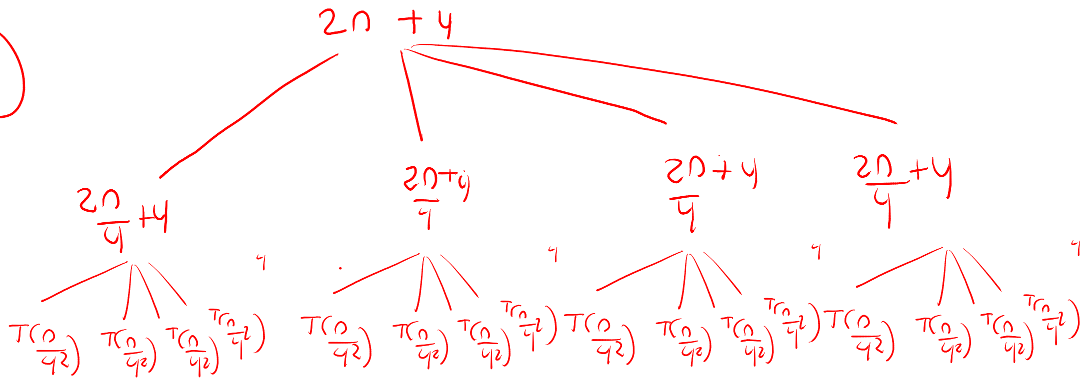
$= 4n$

$\log_a b = \frac{\log_c b}{\log_c a}$

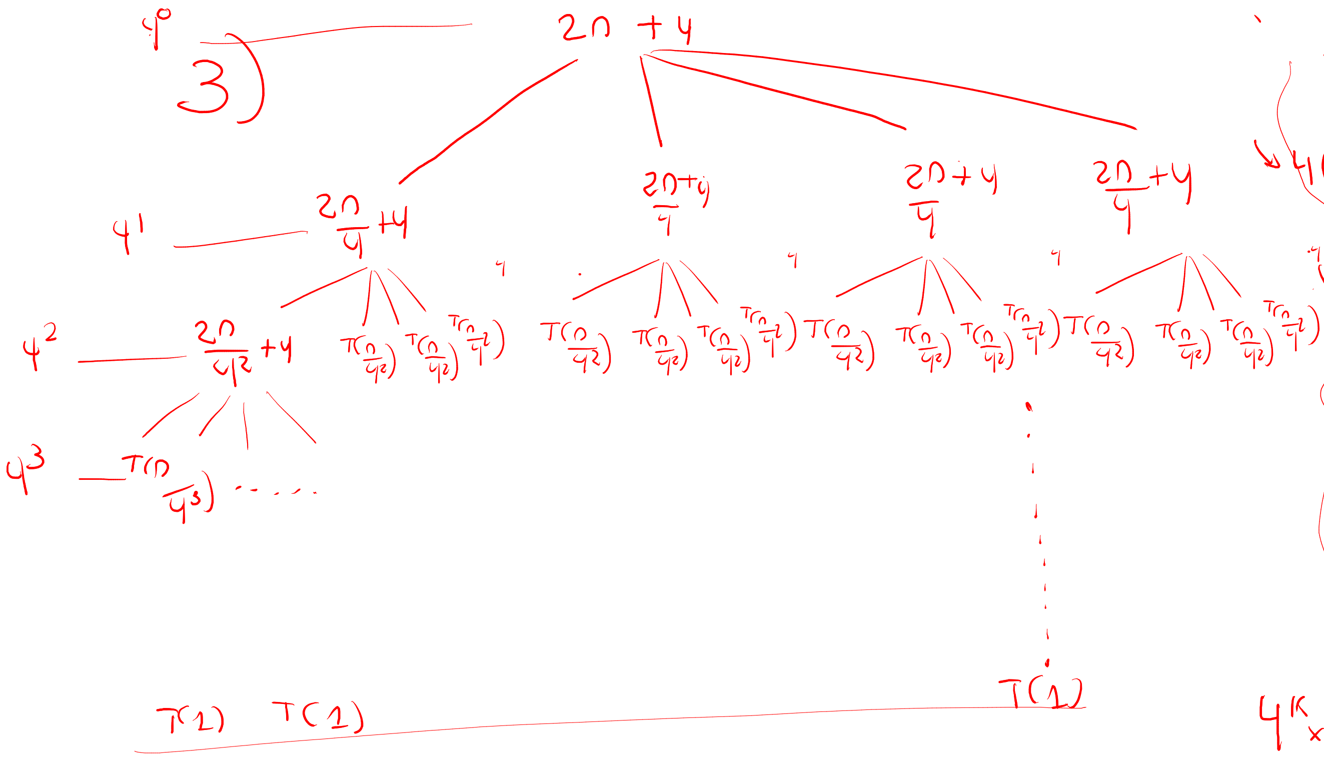
1)



2)



3)



$2n + 4$

$4\left(\frac{2n}{4} + 4\right)$

$4^2\left(\frac{2n}{4^2} + 4\right)$

$4^3\left(\frac{2n}{4^3} + 4\right)$

$\vdots$

$4^k\left(\frac{2n}{4^k} + 4\right)$

$4^k \times T(1)$

$4^{\log_4(n)}\, T(1)$

$nT(1)$

$\frac{n}{4^k} = 1 \qquad k = \log_4(n)$

$2n + 4 + 2n + 4^2 + 2n + 4^3 + \dots + 2n + 4^k + nT(1)$

$\log_4(n) \times 2n + \left(\sum_{i=1}^{\log_4(n)} 4^i\right) + nT(1)$

$T(n) = 4T(n/4) + 2n + 4$

$a = 4 \quad b = 4 \quad f(n) = 2n+4$

$\log_b a = \log_4 4 = 1 \qquad n^1$

1) $2n+4$ es $O(n^{1-\overset{0.1}{\epsilon}})$ X

2) $2n+4$ es $\Theta(n^1) \leftarrow O(n)$ y $\Omega(n)$

$\boxed{\Theta(n \log(n))}$

Dado $T(n) = aT(n/b) + f(n)$, donde $a \geq 1$, $b>1$, se puede acotar asintóticamente como sigue:

1. $T(n)=\Theta(n^{\log_b a})$

   Si $f(n)=O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon>0$

2. $T(n)=\Theta(n^{\log_b a} \lg n)$

   Si $f(n)=\Theta(n^{\log_b a})$ para algún $\varepsilon>0$

3. $T(n)=\Theta(f(n))$

   Si $f(n)=\Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon>0$ si $a*f(n/b) \leq c*f(n)$

   para algun $c<1$

$$T(n) = 8T(n/4) + 2n^2$$

$$T(n) = 2n^2 + 8T(n/4)$$

$$T(n) = 2n^2 + 8\left(2\frac{n^2}{4^2} + 8T\left(\frac{n}{4^2}\right)\right)$$

$$T(n) = 2n^2 + 8 \times \frac{2n^2}{4^2} + 8^2 T\left(\frac{n}{4^2}\right)$$

$$T(n) = 2n^2 + \frac{8 \times 2n^2}{4^2} + 8^2\left(2\left(\frac{n}{4^2}\right)^2 + 8T\left(\frac{n}{4^3}\right)\right)$$

$$\frac{8^2 \times 2}{(4^2)^2} = \frac{8^2 \times 2}{4^2 \times 4^2} \to \frac{64 \times 2}{16 \times 16}$$

$$T(n) = 2n^2 + \frac{8 \times 2n^2}{4^2} + 8^2 \times 2 \times \left(\frac{n}{4^2}\right)^2 + 8^3 T\left(\frac{n}{4^3}\right)$$

$$T(n) = 2n^2 + n^2 + \frac{1}{2}n^2 + 8^3 T\left(\frac{n}{4^3}\right)$$

$$T(n) = 2n^2 + n^2 + \left(\frac{1}{2}\right)n^2 + 8^3\left(2\left(\frac{n}{4^3}\right)^2 + 8T\left(\frac{n}{4^4}\right)\right)$$

$$\frac{1024}{4096} = \frac{1}{4}$$

$$T(n) = 2n^2 + n^2 + \frac{1}{2}n^2 + \frac{1}{2^2}n^2 + 8^4 T\left(\frac{n}{4^4}\right)$$

$$\frac{1}{2^x} = \left(\frac{1}{2}\right)^x$$

$$T(n) = 2n^2 + \left(\frac{1}{2}\right)^0 n^2 + \left(\frac{1}{2}\right)^1 n^2 + \dots + \left(\frac{1}{2}\right)^{k-2} n^2 + 8^k T\left(\frac{n}{4^k}\right)$$

$$n^{\log_4(8)}$$

$$\frac{n}{4^k} = 1 \qquad k = \log_4(n)$$

$$T(n) = 2n^2 + \left(\sum_{i=0}^{\log_4(n)-2} \left(\frac{1}{2}\right)^i n^2\right) + 8^{\log_4(n)} T(1)$$

dominante

$$\frac{1}{2} \times \frac{1}{2}^{\log_4(n)}$$

$$\frac{1}{2} n^{\log_4(1) - \log_4(2)}$$

$$T(n) = 2n^2 + n^2\left(\frac{\left(\frac{1}{2}\right)^{\log_4(n)-1} - 1}{\frac{1}{2} - 1}\right) + n^{\log_4(8)}(1)$$

$$n^2 \times n^{-\frac{1}{2}} = n^{\frac{3}{2}}$$

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para algun $c<1$

$$T(n) = 8T(n/4) + 2n^2$$

$$n^{\log_9(8)} = n^{\log_8(4)}$$

1) $2n^2$ es $O(n^{\log_8(4)-\epsilon})$ X

2) $2n^2$ es $\Theta(n^{\log_8(4)})$

3) $2n^2$ es $\Omega(n^{0.66+\epsilon})$

$$8 \times 2\left(\frac{n}{4}\right)^2 \leq c\,2n^2$$

$$\frac{16n^2}{16} < c \times 2n^2 \qquad 1 < c \times 2$$

$$\frac{1}{2} < c \qquad c > \frac{1}{2}$$

$$c = 0.75$$

$$\boxed{T(n) = \Theta(n^2)}$$