Matemáticas Discretas

Oscar Bedoya

oscar.bedoya@correounivalle.edu.co

- * Inducción matemática
- * Ejemplos

Inducción matemática

 Muchos teoremas establecen que P(n) es verdad para todos los enteros positivos n, donde P(n) es una expresión matemática

Inducción matemática

Una prueba por inducción matemática consiste de dos pasos

- Paso base. Se muestra que la proposición P(1) se cumple
- Paso inductivo. Se supone que P(n) es cierto y se intenta demostrar que P(n+1) también. $P(n) \rightarrow P(n+1)$

La suma de los n primeros enteros 1+2+3+...+n es $n\cdot(n+1)/2$

La suma de los n primeros enteros 1+2+3+...+n es $n\cdot(n+1)/2$

• Paso base. P(1)

$$1 = 1.2/2 = 1$$

La suma de los n primeros enteros 1+2+3+...+n es n·(n+1)/2

• Paso base. P(1)

$$1 = 1.2/2 = 1$$

$$1+2+3+...+n = n \cdot (n+1)/2$$





La suma de los n primeros enteros 1+2+3+...+n es n·(n+1)/2

Paso base. P(1)

$$1 = 1.2/2 = 1$$

$$1+2+3+...+n = n \cdot (n+1)/2$$

$$\frac{(n+1)(n+2)}{2}$$

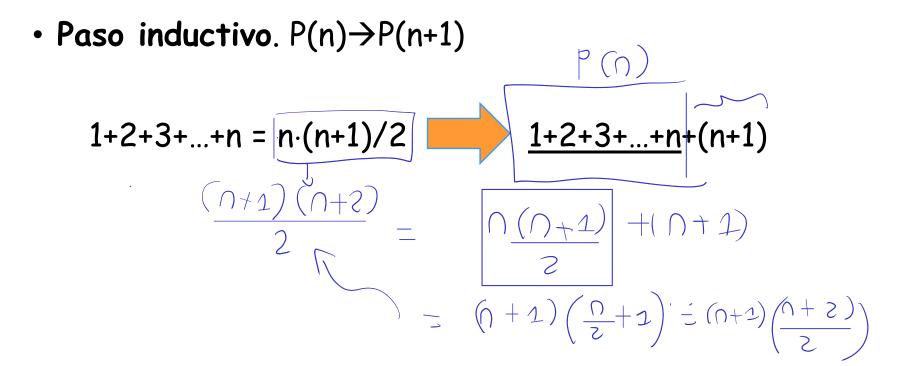
$$\frac{(n+1)(n+2)}{2}$$

$$\frac{(n+4)(n+4)}{2} \cdot (n+4)(n+4)$$

La suma de los n primeros enteros 1+2+3+...+n es $n\cdot(n+1)/2$

Paso base. P(1)

$$1 = 1.2/2 = 1$$



La suma de los n primeros enteros 1+2+3+...+n es n·(n+1)/2

Paso base. P(1)

$$1 = 1.2/2 = 1$$

$$1+2+3+...+n = n \cdot (n+1)/2$$

$$= n \cdot (n+1)/2 + (n+1)$$

$$= (n+1) \cdot (n+2)/2$$

$$= P(n+1)$$

Demuestre que
$$2^{0}+2^{1}+2^{2}+...+2^{n}=2^{n+1}-1$$

$$V = 0$$

$$\sum_{i=0}^{n} 2^{i} = 2^{n+1} - 1$$

$$\sum_{i=0}^{n} z^{i} = z^{0} = 1$$

$$\sum_{i=0}^{n} z^{i} = z^{0} = 1$$

$$\sum_{i=0}^{n+1} z^{i} = 2^{n+2} - 1$$

Demuestre que $2^{0}+2^{1}+2^{2}+...+2^{n}=2^{n+1}-1$

• Paso base. P(0)

$$2^{\circ} = 1 \text{ y } 2^{\circ +1} - 1 = 1$$

Demuestre que $2^{0}+2^{1}+2^{2}+...+2^{n}=2^{n+1}-1$

Paso base. P(0)

$$2^{\circ} = 1 \text{ y } 2^{\circ +1} - 1 = 1$$

$$2^{0}+2^{1}+2^{2}+...+2^{n}=2^{n+1}-1$$



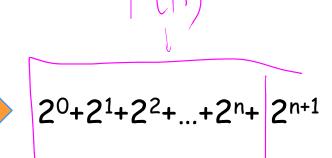


Demuestre que
$$2^{0}+2^{1}+2^{2}+...+2^{n}=2^{n+1}-1$$

Paso base. P(0)

$$2^{\circ} = 1 \text{ y } 2^{\circ +1} - 1 = 1$$

$$2^{0}+2^{1}+2^{2}+...+2^{n}=2^{n+1}-1$$



Demuestre que $2^{0}+2^{1}+2^{2}+...+2^{n}=2^{n+1}-1$

Paso base. P(0)

$$2^{\circ} = 1 \text{ y } 2^{\circ +1} - 1 = 1$$

$$2^{0}+2^{1}+2^{2}+...+2^{n}=2^{n+1}-1$$

$$= (2^{n+1}-1) + 2^{n+1}$$

$$= 2 \cdot 2^{n+1}-1$$

$$= 2^{(n+1)+1}-1 = P(n+1)$$

Demuestre que la suma de los primeros n impares es n^2 , es decir, $1+3+5+...+(2n-1)=n^2$

$$P(A) = \sum_{i=1}^{1} (2n-1) = 1$$

$$P(A) = \sum_{i=1}^{1} (2n-1) = 1$$

$$P(A) = \sum_{i=1}^{1} (2n-1) = 0^{2}$$

$$P(A) = \sum_{i=1}^{1} (2n-1) = 0^{2}$$

$$P(A) = \sum_{i=1}^{1} (2n-1) = 1$$

$$P(A) = \sum_{i=1}^{1} ($$

Demuestre que la suma de los primeros n impares es n^2 , es decir, $1+3+5+...+(2n-1)=n^2$

Paso base. P(1)

$$1 = 1^2$$

Demuestre que la suma de los primeros n impares es n^2 , es decir, $1+3+5+...+(2n-1)=n^2$

Paso base. P(1)

$$1 = 1^2$$

$$1+3+5+...+(2n-1)=n^2$$

Demuestre que la suma de los primeros n impares es n^2 , es decir, $1+3+5+...+(2n-1)=n^2$

Paso base. P(1)

$$1 = 1^2$$

$$1+3+5+...+(2n-1)=n^2$$
 $1+3+...+(2n-1)+(2n+1)$

Demuestre que la suma de los primeros n impares es n^2 , es decir, $1+3+5+...+(2n-1)=n^2$

Paso base. P(1)

$$1 = 1^2$$

$$1+3+5+ ... + (2n-1) = n^{2}$$

$$= n^{2} + 2n + 1$$

$$= (n + 1)^{2}$$

$$= P(n+1)$$

Demuestre que $1^2+2^2+3^2+...+n^2=n(n+1)(2n+1)/6$

$$P(1) = \frac{1}{1 - 1}$$

$$P(1) = \frac{1}{1 - 1}$$

$$P(2) = \frac{1}{1 - 1}$$

$$P(3) = \frac{1}{1 - 1}$$

$$P(4) = \frac{1}{1 - 1}$$

$$P(5) = \frac{1}{1 - 1}$$

$$P(6) = \frac{1}{1 - 1}$$

$$P(7) = \frac{1}{1 - 1}$$

$$\frac{n(n+1)(2n+1)}{6} + \frac{6}{6}(n+1)^{2} = \frac{n(n+1)(2n+1)+6(n+1)}{6}$$

$$\frac{(n+1)(n+1)+6(n+1)}{6} + \frac{6}{6}(n+1)^{2} = \frac{(n+1)(2n^{2}+n+6n+6)}{6}$$

$$\frac{(n+1)(2n+4)(2n+4)}{6} = \frac{(n+1)(2(2n^{2})+7(2n)+12)}{6}$$

$$\frac{(n+1)((2n+4)(2n+3))}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

Demuestre que $1^2+2^2+3^2+...+n^2=n(n+1)(2n+1)/6$

• Paso base. P(1)

$$1^2 = (1.2.3)/6$$

Demuestre que
$$1^2+2^2+3^2+...+n^2=n(n+1)(2n+1)/6$$

Paso base. P(1)

$$1^2 = (1.2.3)/6$$

$$1^2+2^2+3^2+...+n^2=n(n+1)(2n+1)/6$$
 $1^2+2^2+3^2+...+n^2+(n+1)^2$

Demuestre que
$$1^2+2^2+3^2+...+n^2=n(n+1)(2n+1)/6$$

Paso base. P(1)

$$1^2 = (1.2.3)/6$$

$$1^{2}+2^{2}+3^{2}+...+n^{2}=n(n+1)(2n+1)/6$$
 $1^{2}+2^{2}+3^{2}+...+n^{2}+(n+1)^{2}$

Demuestre que
$$1^2+2^2+3^2+...+n^2=n(n+1)(2n+1)/6$$

• Paso base. P(1)

$$1^2 = (1.2.3)/6$$

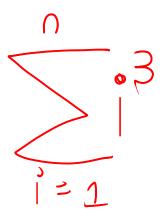
$$1^{2}+2^{2}+3^{2}+...+n^{2}=n(n+1)(2n+1)/6 \longrightarrow \frac{1^{2}+2^{2}+3^{2}+...+n^{2}}{n(n+1)(2n+1)/6}+(n+1)^{2}$$

$$= (n+1)(2n^{2}+7n+6)/6$$

$$= (n+1)(2n+3)(n+2)/6$$

$$= (n+1)[(n+1)+1][2(n+1)+1]=P(n+1)$$

Demuestre que $1^3+2^3+...+n^3=[n(n+1)/2]^2$



$$\sum_{i=1}^{3} = 1^{3} \left(\frac{1}{2} + \frac{1}{2} \right)^{2} - 1$$

$$\sum_{i=1}^{3} \left(\frac{n(n+1)}{2}\right)^{2} \left(\frac{n+1}{n+2}\right)^{2}$$

$$\sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (n+1)^3 = \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$$

$$\left(\begin{array}{c} 2 \\ 1 \end{array}\right) \left(\begin{array}{c} 1 \\ 1 \end{array}\right) \left(\begin{array}{c} 1 \\ 1 \end{array}\right) + \left(\begin{array}{c} 1 \\ 1 \end{array}\right)^{3}$$

$$(n+1)^{2} \left(\frac{n^{2}}{4} \right) + (n+1)$$

$$(n+1)^{2} \left(\frac{n^{2} + 4n + 4}{4} \right) = \frac{(n+1)^{2}}{4} \left(\frac{(n+2)(n+2)}{4} \right)$$

$$(n+1)^{2} \left(\frac{n^{2} + 4n + 4}{4} \right) = \frac{(n+1)^{2}}{4} \left(\frac{(n+2)(n+2)}{4} \right)$$

Demuestre que $1^3+2^3+...+n^3=[n(n+1)/2]^2$

Paso base. P(1)

$$1^3 = [(1.2)/2]^2$$

Demuestre que
$$1^3+2^3+...+n^3=[n(n+1)/2]^2$$

• Paso base. P(1)

$$1^3 = [(1.2)/2]^2$$

$$1^{3}+2^{3}+...+n^{3}=[n(n+1)/2]^{2}$$
 $1^{3}+2^{3}+...+n^{3}+(n+1)^{3}$

Demuestre que
$$1^3+2^3+...+n^3=[n(n+1)/2]^2$$

• Paso base. P(1)

$$1^3 = [(1.2)/2]^2$$

$$1^{3}+2^{3}+...+n^{3}=[n(n+1)/2]^{2}$$
 $1^{3}+2^{3}+...+n^{3}+(n+1)^{3}$

Demuestre que
$$1^3+2^3+...+n^3=[n(n+1)/2]^2$$

• Paso base. P(1)

$$1^3 = [(1.2)/2]^2$$

$$1^{3}+2^{3}+...+n^{3}=[n(n+1)/2]^{2}$$

$$= [n(n+1)/2]^{2}+(n+1)^{3}$$

$$= [n(n+1)/2]^{2}+(n+1)^{3}$$

Demuestre que
$$1^3+2^3+...+n^3=[n(n+1)/2]^2$$

Paso base. P(1)

$$1^3 = [(1.2)/2]^2$$

$$1^{3}+2^{3}+...+n^{3}=[n(n+1)/2]^{2}$$

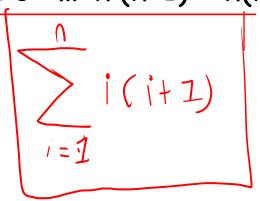
$$= [n(n+1)/2]^{2}+(n+1)^{3}$$

$$= n^{2}(n+1)^{2}/4+(n+1)^{3}$$

$$= (n+1)^{2}[n^{2}/4+(n+1)]$$

$$= (n+1)^{2}(n+2)^{2}/4 = [(n+1)(n+2)/2]^{2}$$

Demuestre que $(1.2) + 2.3 + ... + n \cdot (n+1) = n(n+1)(n+2)/3$



$$\sum_{i=1}^{n} \frac{1}{i} + \sum_{i=1}^{n} \frac{1}{i}$$

$$0=1 \sum_{i=1}^{1} i(i+1) = 1(2) = 2 \qquad 1(2)(3) = 2 \qquad i)$$

$$\frac{1(z)(3)-2}{3}$$

$$\frac{1}{1} = \frac{1}{3} = \frac{1$$

(n+1)(n+2)(n+3)

Demuestre que
$$1.2 + 2.3 + ... + n \cdot (n+1) = n(n+1)(n+2)/3$$

• Paso base. P(1)

$$1.2=(1.2.3)/3=2$$

Demuestre que
$$1.2 + 2.3 + ... + n \cdot (n+1) = n(n+1)(n+2)/3$$

• Paso base. P(1)

$$1.2=(1.2.3)/3=2$$

$$1.2+2.3+...+n\cdot(n+1)=n(n+1)(n+2)/3$$
 $1.2+2.3+...+n\cdot(n+1)+(n+1)\cdot(n+2)$

Demuestre que
$$1.2 + 2.3 + ... + n \cdot (n+1) = n(n+1)(n+2)/3$$

• Paso base. P(1)

$$1.2=(1.2.3)/3=2$$

$$1.2+2.3+...+n\cdot(n+1)=n(n+1)(n+2)/3$$
 $1.2+2.3+...+n\cdot(n+1)+(n+1)\cdot(n+2)$

Demuestre que
$$1.2 + 2.3 + ... + n \cdot (n+1) = n(n+1)(n+2)/3$$

Paso base. P(1)

$$1.2=(1.2.3)/3=2$$

$$1 \cdot 2 + 2 \cdot 3 + ... + n \cdot (n+1) = n(n+1)(n+2)/3$$

$$= n(n+1)(n+2)/3 + (n+1) \cdot (n+2)$$

$$= (n+1)(n+2)[n/3 + 1]$$

$$= (n+1)(n+2)(n+3)/3$$

$$= P(n+1)$$

Demuestre que
$$(1\cdot1!)$$
 + 2·2! +...+n·n! = $(n+1)!-1$

$$(n+2)!-1$$

$$(n+2)!-1$$

$$(n+2)!-1$$

$$(n+2)!-1 = 2-1=1$$

$$(n+1)!(n+1)!-1+(n+1)(n+1)!$$

$$(n+1)!(1+n+1)-1$$

$$(n+2)!-1$$

$$(n+2)!-1$$

Demuestre que 1.1! + 2.2! + ... + n.n! = (n+1)! - 1

• Paso base. P(1)

Demuestre que
$$1.1! + 2.2! + ... + n.n! = (n+1)! - 1$$

• Paso base. P(1)

$$1 \cdot 1! = (1+1)! - 1 = 1$$

$$1.1! + 2.2! + ... + n.n! = (n+1)! - 1$$

$$1.1! + 2.2! + ... + n.n! + (n+1).(n+1)!$$

Demuestre que
$$1.1! + 2.2! + ... + n.n! = (n+1)! - 1$$

• Paso base. P(1)

$$1 \cdot 1! = (1+1)! - 1 = 1$$

$$1.1! + 2.2! + ... + n.n! = (n+1)! - 1$$

$$1.1! + 2.2! + ... + n.n! + (n+1).(n+1)!$$

Demuestre que
$$1.1! + 2.2! + ... + n.n! = (n+1)! - 1$$

Paso base. P(1)

$$1 \cdot 1! = (1+1)! - 1 = 1$$

$$1 \cdot 1! + 2 \cdot 2! + ... + n \cdot n! = (n+1)! - 1$$

$$= (n+1)! - 1 + (n+1) \cdot (n+1)!$$

$$= (n+1)! [1 + (n+1)] - 1$$

$$= (n+1)! (n+2) - 1$$

$$= (n+2)! - 1 = P(n+1)$$

Demuestre que la suma de los primeros n pares es $n \cdot (n+1)$, es decir, $2+4+6+...+2n = n \cdot (n+1)$