$$F(n)\begin{cases}\boxed{\begin{matrix}1 & n=0\\ 1 & n=1\end{matrix}} & \text{Condiciones Iniciales} \quad \{\text{Caso base}\\ F(n-1)+F(n-2) & n>1 \quad \{\text{Caso recursivo}\end{cases}$$

n	F(n)
0	1
1	1
2	2
3	3
4	5
5	8

Caso base

n	F(n)
6	13
7	21
8	34
9	55
10	89

Cabeza

$$F(n)\begin{cases}1 & n=0 \quad \text{Caso base}\\ n\times F(n-1) & \text{Si } n>0\end{cases}$$

```
def f(n):
        if  n==0:
                return 1
        elif n==1:
                return 1
        else:
                return f(n-1)+f(n-2)
```

n	F(n)
0	1
1	$1\times 0! = 1$
2	$2\times 1! = 2$
3	$3\times 2! = 6$
4	$4\times 3! = 24$

n	F(n)
5	$5\times 4! = 120$
6	$6\times 5! = 720$
7	$7\times 6!$
8	$8\times 7!$
9	$9\times 8!$
10	$10\times 9!$

100

Dada una poblacińn inicial de bacterias de 100, sabemos que las bacterias se triplican cada hora, ¿Cuantas bacterias tendremos a las 80 horas?

$T(0)=100$ Inicial

$\boxed{T(n)=3\times T(n-1)}$

$T(1)=3\times 100$

$T(2)=3\times T(1)=3\times 3\times 100=3^2\times 100$

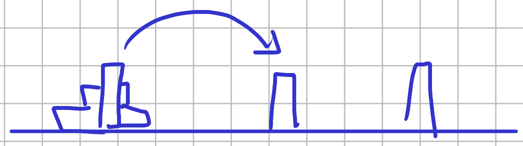
$T(3)=3\,T(2)=3^3\times 100$

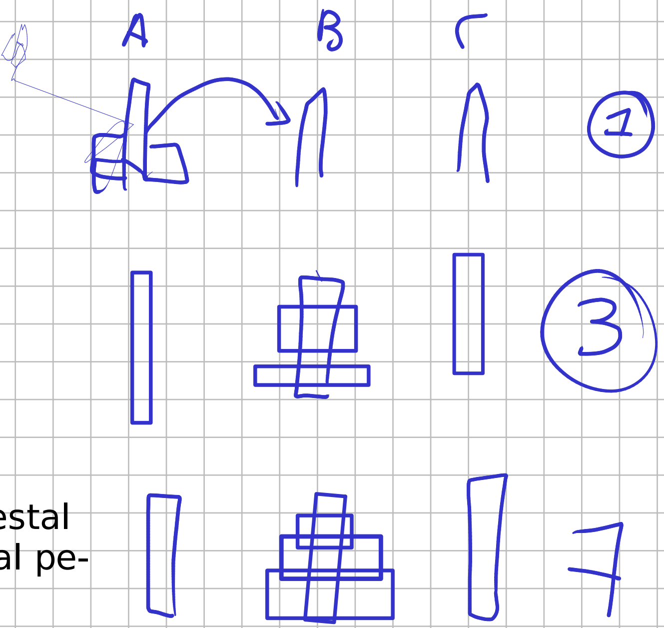
$T(4)=3\times T(3)=3^4\,100$

$T(n)=3^n\,T(0)=3^n\times 100$

$T(80)=3^{80}\times 100$

Torres de Hanoi



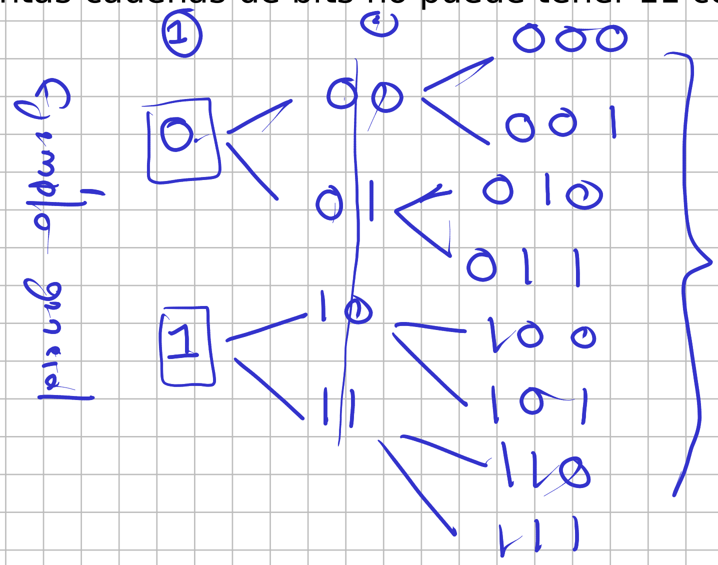


Si voy a mover 3 discos, tengo que mover 2 al pedestal auxiliar, el disco más grande lo muevo una vez al pedestal de destino y finalmente muevo los 2 al pedestal destino.

$$T(3) = 2T(2) + 1$$

$$\boxed{T(n) = 2T(n-1) + \overset{f(n)}{1}}$$

¿Cuantas cadenas de bits no puede tener 11 consecutivos? (ternario)



$$T(1) = 2$$

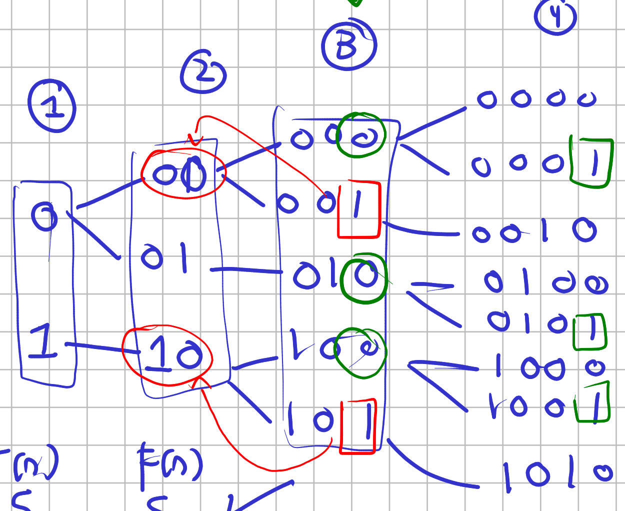
$$T(n) = \underbrace{T(n-1)}_{0} + \underbrace{T(n-1)}_{1}$$

$$T(n) = 2T(n-1) \Big\} 2^n$$

$$T(2) = \underbrace{T(1)}_{0} + \underbrace{T(1)}_{1}$$

$$T(2) = 2T(1)$$

ii)



$$T(n) = \underbrace{?}_{0} + \underbrace{?}_{1}$$

$$T(n) = \underbrace{T(n-1)}_{0} + \underbrace{T(n-2)}_{01}$$

$$T(1) = 2$$

$$T(2) = 3$$

n	$T(n)$	$F(n)$	
3	5	5	✓
4	8	8	✓

R. R Homogéneas

$$T(n) = C_1 T(n-1) + C_2 T(n-2) + C_3 T(n-3) + \dots + C_k T(n-k)$$

$C_i \neq 0$

$T(n) = r^n$

$\lambda - 1 - n + k$

$r^{n-k} \neq 0$

$$\frac{r^n}{r^{n-k}} = \frac{C_1 r^{n-1}}{r^{n-k}} + \frac{C_2 r^{n-2}}{r^{n-k}} + \dots + \frac{C_k r^{n-k}}{r^{n-k}}$$

$$\boxed{r^k = C_1 r^{k-1} + C_2 r^{k-2} + \dots + C_k}$$

Ecuación característica

$$\left. r^k - C_1 r^{k-1} - C_2 r^{k-2} - \dots - C_k = 0 \right\}$$

Raíces k raíces (distintas)

$$T(n) = A(r_1)^n + B(r_2)^n + \dots + K(r_k)^n$$

$$T(n) = 7T(n-1) - 12T(n-2)$$

$T(0) = 10$
$T(1) = 15$

1) Ec

$$\frac{r^n}{r^{n-2}} = \frac{7r^{n-1}}{r^{n-2}} - \frac{12 r^{n-2}}{r^{n-2}}$$

$$r^2 = 7r - 12 \longrightarrow r^2 - 7r + 12 = 0$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad ax^2 + bx + c = 0$$

$$r = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm \sqrt{1}}{2} \begin{cases} \frac{8}{2} = 4 \\ \frac{6}{2} = 3 \end{cases}$$

$$T(n) = A(4)^n + B(3)^n$$

$T(0) = 10$
$T(1) = 15$

$10 = A + B$

$15 = 4A + 3B$

$-30 = -3A - 3B$

$15 = 4A + 3B$

$\boxed{-15 = A}$ $\boxed{B = 25}$

$T(n) = -15 \times 4^n + 25 \times 3^n$

$T(0) = 10$

$T(1) = 15$

Sea una RR homogenea cuya EC tiene raices repetidas con multiplicidad m, la solución se plantea multiplicando un polinomo de grado m-1 a cada raiz de multiplicidad m

Ejemplo

$(\overbrace{4,4,4,4,4,4,4,4}^{8},\overbrace{3,3,3,3,3,3}^{6},\overbrace{2,2,2}^{3},1)$

$T(n) = (An^7 + Bn^6 + Cn^5 + Dn^4 + En^3 + Fn^2 + Gn + H)4^n +$
$(In^5 + Jn^4 + Kn^3 + Ln^2 + Mn + N)3^n +$
$(On^2 + Pn + Q)2^n + R1^n$

$T(n) = -4T(n-1) - 4T(n-2)$

- $T(0) = 10$
- $T(1) = -10$

$r^2 + 4r + 4 = 0$

$\frac{-4 \pm \sqrt{\cancel{16-16}}^{\,0}}{2} = \frac{-4 \pm 0}{2} \rightarrow -2$

$m = 2$

$T(n) = (An + B)(-2)^n$

$-10 = (A(1) + B)(-2)^2$

$-10 = 10 - 20$

$\boxed{10 = B}$

$\boxed{-10 = -2A - 2B}$

$-10 = -2A - 20$ $\}$ $\boxed{A = -5}$

$$\boxed{T(n) = (-5n + 10)(-2)^n}$$

$$T(n) = T(n-1) + 8T(n-2) - 12T(n-3) \qquad \begin{cases} T(0) = 4 \\ T(1) = 8 \\ T(2) = 10 \end{cases}$$

$(2, 2, -3)$

$$T(n) = T(n-1) + 8T(n-2) - 12T(n-3)$$
$$r^n = r^{n-1} + 8r^{n-2} - 12r^{n-3}$$
$$r^3 = r^2 + 8r - 12$$

$$r^3 - r^2 - 8r + 12 = 0$$

General

$$\boxed{T(n) = (A + Bn)2^n + C(-3)^n}$$

n

$$\begin{cases} 0: & 4 = A + C \\ 1: & 8 = 2A + 2B - 3C \\ 2: & 10 = 4A + 8B + 9C \end{cases}$$

[4.24 -0.6 -0.24]

Sol Total

$$\boxed{T(n) = (4.24 - 0.6n)2^n - 0.24(-3)^n}$$