Relaciones de recurrencia no homogenea

$$T(n) = C_1 T(n-1) + C_2 T(n-2) + \dots + T(n-k) + F(n)$$

homogenea

no homogenea

$$T(n) = \underbrace{T^h(n)}_{} + \underbrace{T^p(n)}_{}$$

1) Sol $T^h(n)$ Ecuacion general

2) Sol $T^p(n)$

n^b pol orden B $\longrightarrow$ Pol orden b

$n^2 + 8 \longrightarrow An^2 + Bn + C$

$r^n \longrightarrow Ar^n$

$n^b r^n$ $\quad$ $Pol_m^b \times r^n$

$n^2 2^n$ $\quad$ $(An^2 + Bn + C)2^n$

Tabla

Especulamos raices $T^h(n)$

$r = 2, 2$ $\quad$ $T^h(n) = A2^n + Bn2^n$

$F(n) = n + 2^n$ $\quad$ $T^p(n) = Cn + D + En^2 2^n$

$r = 1, 1, 1, 2$ $\quad$ $F(n) = n^2 + 3^n + 1$

$$T^h(n) = A1^n + Bn1^n + Cn^2 1^n + D2^n$$

$$\rightarrow T^h(n) = A + Bn + Cn^2 + D2^n$$

$$T^p(n) = (En^2 + Fn + G)n^3 + H3^n$$

$$T(n) = 3T(n-1) - 2T(n-2) - 5n + 3 + n2^n \qquad \begin{cases} T(0) = 10 \\ T(1) = 22 \end{cases}$$

1) Sol $T^h(n)$

$r = 1, 2$

E.O $\quad r^2 - 3r + 2 = 0$

$$\boxed{T^h(n) = A + B2^n}$$

2) Sol $T^p(n)$

pol ord 1

$F(n) = \boxed{-5n+3} + n2^n$

$2^{n-2} = 2^n \times 2^{-2}$

$\frac{2^n}{2^2} = \frac{2^n}{4}$

$$T^p(n) = (Cn + D)n + n(En + F)2^n$$

3) $T(n) = 3T(n-1) - 2T(n-2) - 5n + 3 + n2^n$

$\to Cn^2 + Dn + En^2 2^n + Fn2^n = 3C(n-1)^2 + 3D(n-1) + \frac{3E(n-1)^2 2^n}{2} + \frac{3F(n-1)2^n}{2}$

$- 2C(n-2)^2 - 2D(n-2) - \frac{2E(n-2)^2 2^n}{4}$

$- \frac{2F(n-2)2^n}{4}$

$- 5n + 3 + n2^n$

$(n-1)^2$

$n^2 - 2n + 1$

$(n-2)^2$

$n^2 - 4n + 4$

$\to n^2 2^n \quad E = \frac{6E}{4} - \frac{2E}{4} \qquad E = \frac{4}{4}E \qquad E = E$

$\to n2^n \quad \cancel{F} = -3E + \frac{3}{2}F + 2E - \frac{1}{2}F + 1$

$2F = -6E + 3F + 4E - F + 2$

$0 = -2E + 2 \qquad \boxed{E = 1}$

$2^n \quad 0 = \frac{3}{2}E - \frac{3}{2}F - 2E + F$

$\boxed{F = -1}$

$0 = 3E - 3F - 4E + 2F \qquad 0 = 3 - F - 4$

$1 = -F$

$$\to Cn^2 + Dn + En^2 2^n + Fn 2^n = 3C(n-1)^2 + 3D(n-1) + 3E\frac{(n-1)^2 2^n}{2} + 3F\frac{(n-1)2^n}{2}$$

$$-2C(n-2)^2 - 2D(n-2) - 2E\frac{(n-2)^2 2^n}{4} - 2F\frac{(n-2)2^n}{4} - 5n + 3 + n2^n$$

$(n-1)^2$

$n^2 - 2n + 1$

$(n-2)^2$

$n^2 - 4n + 4$

n^2 | $C = 3C - 2C \qquad C = C \quad :)$

n | $D = -6C + 3D + 8C - 2D - 5$

$0 = 2C - 5 \qquad \boxed{C = \frac{5}{2}}$

cte | $0 = 3C - 3D - 8C + 4D + 3$

$0 = -5C + D + 3 \qquad 0 = -\frac{25}{2} + D + 3$

$\frac{25}{2} - \frac{6}{2} = D \qquad \boxed{\frac{19}{2} = D}$

$E = 1 \qquad F = -1 \qquad C = \frac{5}{2}$

$T^h(n) = A + B2^n$

$D = \frac{19}{2}$

Sol $T^p(n)$ $\qquad F(n) = -5n + 3 + n2^n$

$\to T^p(n) = (Cn + D)n + n(En + F)2^n$

$$T(n) = A + B2^n + \frac{5}{2}n^2 + \frac{19}{2}n + n^2 2^n - n2^n$$

$T(0)=10$
$T(1)=22$

$10 = A+B$

$22 = A+2B+\frac{5}{2}+\frac{19}{2}+2-2$

$\frac{44}{2}-\frac{24}{2} = A+2B \qquad 10 = A+2B$

$$\begin{array}{r} 10 = A+B \\ 10 = A+2B \\ \hline 0 = B \end{array} \qquad A=10$$

$$\boxed{T(n)=10+\frac{5}{2}n^2+\frac{19}{2}n+n^2 2^n-n2^n}$$

Cambio variable

$T(n)=AT(n/B)+F(n)$

$T(n)=A_0T\left(\frac{n}{B}\right)+A_1T\left(\frac{n}{B^2}\right)+A_2T\left(\frac{n}{B^3}\right)\cdots$

$T(n)=2T(n/2)+2\log_2(n) \qquad T(1)=10$
$n\in 2^x$

$n=2^k$

$T(2^k)=2T\left(\frac{2^k}{2}\right)+2\log_2(2^k) \qquad \log_2(2^k)= k\log_2(2)=k$

$T(2^k)=2T(2^{k-1})+2k$

$T_k=T(2^k)$

$$\boxed{T_k=2T_{k-1}+2k}$$

$T_k=T_k^{(h)}+T_k^{(p)}$

$r-2=0 \qquad r=2$

$T_k^{(h)}=A2^k$

$F(n) = 2K$

$$B K + C = 2(B(K-1)+C) + 2K$$

$$T_K^{(P)} = BK + C$$

$$\cancel{BK} + C = \cancel{2BK} - 2B + 2C + 2K$$

K	$B = 2B + 2$
ct_e	$C = -2B + 2C$

$\boxed{B = -2}$

$-C = 4$ $\quad \boxed{C = -4}$

$\boxed{T_K = A 2^K - 2K - 4}$ $\quad \boxed{T(1) = 10}$

$n = 2^K$ $\quad K = \log_2(n)$

$a^{\log_b c} = c^{\log_b a}$

$$T(n) = A 2^{\log_2(n)} - 2\log_2(n) - 4$$

$$T(n) = A n^{\log_2(2)} - 2\log_2(n) - 4$$

$$T(n) = An - 2\log_2(n) - 4 \qquad O(n)$$

$10 = A - 4$ $\quad 14 = A$

$$\boxed{T(n) = 14n - 2\log_2(n) - 4}$$

$$T(n) = 4T(n/2) - 4T(n/4) + 2n + n\ln(n) \qquad T(1) = 10 \quad T(2) = 24$$

$$n = 2^k$$

$$T(2^k) = 4T(2^{k-1}) - 4T(2^{k-2}) + 2\times 2^k + 2^k \times \ln(2^k)$$

$$T(2^k) = 4T(2^{k-1}) - 4T(2^{k-2}) + 2\times 2^k + \ln(2)\,k\,2^k$$

$$T_k = 4T_{k-1} - 4T_{k-2} + 2\times 2^k + \ln(2)\,k\,2^k$$

$$r^2 - 4r + 4 = 0 \qquad r = 2, 2$$

$$T_k^{h} = A2^k + Bk2^k$$

$$F(k) = 2\,2^k + \ln(2)\,k\,2^k$$

$$F(k) = (\underbrace{2 + \ln(2)\,k}_{\text{pol orden } 1})\,2^k$$

$$T_k^{(p)} = (Ck + D)\,2^k \times k^2$$

$$Ck^3 2^k + Dk^2 2^k = \frac{4C(k-1)^3 2^k}{2} + \frac{4D(k-1)^2 2^k}{2} - \frac{4C(k-2)^3 2^k}{4} - \frac{4D(k-2)^2 2^k}{4} + 2\times 2^k + \ln(2)\,k\,2^k$$

$$Ck^3 2^k + Dk^2 2^k = 2C(k^3 - 3k^2 + 3k - 1)2^k + 2D(k^2 - 2k + 1)2^k - C(k^3 - 6k^2 + 12k - 8)2^k - D(k^2 - 4k + 4)2^k + 2\times 2^k + \ln(2)\,k\,2^k$$

$k^3 2^k$ | $C = 2C - C$ $\qquad C = C$:)

$k^2 2^k$ | $D = -6C + 2D + 6C - D$ $\qquad D = D$:D

$k 2^k$ | $0 = 6C - 4D - 12C + 4D + \ln(2)$

$6C = \ln(2)$ $\qquad \boxed{C = \frac{\ln(2)}{6}}$

2^k | $0 = -2C + 2D + 8C - 4D + 2$

$0 = 6C - 2D + 2$

$$0 = \ln(2) - 2D + 2 \qquad 2D = \ln(2) + 2$$

$$\boxed{D = \frac{\ln(2)+2}{2}}$$

$$T_k^h = A2^k + Bk2^k$$

$$T_k^{(p)} = (Ck+D)2^k \times k^2$$

$$T_k = A2^k + Bk2^k + \left(\frac{\ln(2)}{6}k + \left(\frac{\ln(2)+2}{2}\right)\right)2^k k^2$$

$$n = 2^k \qquad k = \log_2(n)$$

$$T(n) = An + B\log_2(n) \times n + \left(\frac{\ln(2)}{6} \times \log_2(n) + \left(\frac{\ln(2)+2}{2}\right)\right) n \times (\log_2(n))^2$$

$$T(1) = 10 \qquad T(2) = 24$$

$$10 = A$$

$$24 = 10 \times 2 + B \times 2 + \left(\frac{\ln(2)}{6} + \frac{\ln(2)+2}{2}\right) 2 \times 1^2$$

$$4 = 2B + \frac{\ln(2)}{3} + \ln(2) + 2$$

$$2 = 2B + \ln(2)\frac{4}{3}$$

$$\boxed{B = \frac{2 - \ln(2)\frac{4}{3}}{2}} \quad :)$$

$$T(n) = 10n + B\log_2(n) \times n + \left(\frac{\ln(2)}{6} \times \log_2(n) + \left(\frac{\ln(2)+2}{2}\right)\right) n \times (\log_2(n))^2$$