$F(n)$ { 1 $n=0$ — Condiciones Iniciales — { Caso base
1 $n=1$

$F(n-1)+F(n-2)$ $n>1$ — { Caso recursivo

n	F(n)
0	1
1	1
2	2
3	3
4	5
5	8

Caso base

n	F(n)
6	13
7	21
8	34
9	55
10	89

$F(n)$ { 1 $n=0$ — Caso base
$n \times F(n-1)$ Si $n>0$

Codigo

```
def f(n):
    if  n==0:
        return 1
    elif n==1:
        return 1
    else:
        return f(n-1)+f(n-2
```

n	F(n)
0	1
1	$1 \times 0! = 1$
2	$2 \times 1! = 2$
3	$3 \times 2! = 6$
4	$4 \times 3! = 24$

n	F(n)
5	$5 \times 4! = 120$
6	$6 \times 5! = 720$
7	$7 \times 6!$
8	$8 \times 7!$
9	$9 \times 8!$
10	$10 \times 9!$

100

Dada una poblaciń inicial de bacterias de 100, sabemos que las bacterias se triplican cada hora, ¿Cuantas bacterias tendremos a las 80 horas?

$T(0) = 100$ Inicial

$T(n) = 3 \times T(n-1)$

$T(1) = 3 \times 100$

$T(2) = 3 \times T(1) = 3 \times 3 \times 100 = 3^2 \times 100$

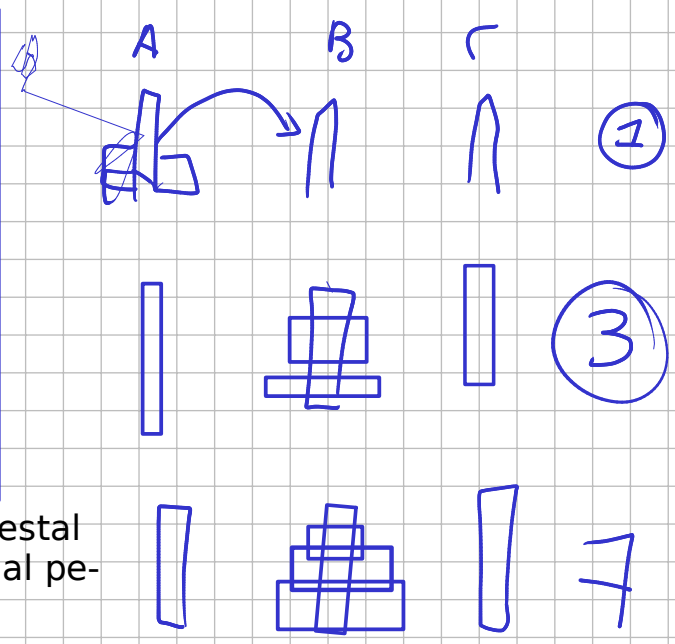
$T(3) = 3 T(2) = 3^3 \times 100$

$T(4) = 3 \times T(3) = 3^4 \times 100$

$T(n) = 3^n T(0) = 3^n \times 100$

$T(80) = 3^{80} \times 100$

Torres de Hanoi

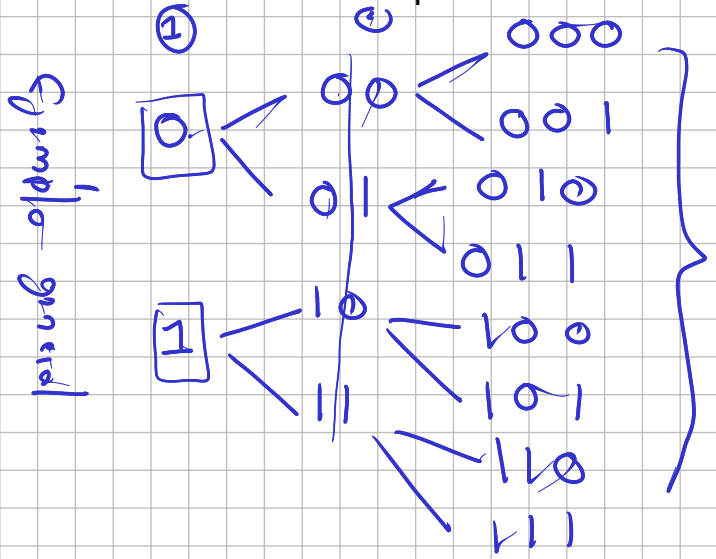


Si voy a mover 3 discos, tengo que mover 2 al pedestal auxiliar, el disco más grande lo muevo una vez al pedestal de destino y finalmente muevo los 2 al pedestal destino.

$$T(3) = 2T(2) + 1$$

$$\boxed{T(n) = 2T(n-1) + 1}$$ F(n)

¿Cuantas cadenas de bits no puede tener 11 consecutivos? (ternario)



$T(1) = 2$

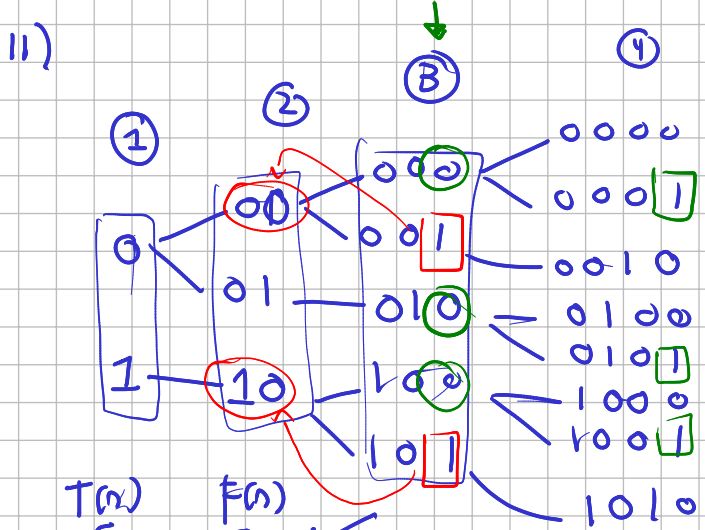
$T(n) = \underbrace{T(n-1)}_{0} + \underbrace{T(n-1)}_{1}$

$T(n) = 2T(n-1)$ } 2^n

$T(2) = \underbrace{T(1)}_{0} + \underbrace{T(1)}_{1}$

$T(2) = 2T(1)$

11)



$T(n) = \underbrace{?}_{0} + \underbrace{?}_{1}$

$T(n) = \underbrace{T(n-1)}_{0} + \underbrace{T(n-2)}_{01}$

$T(1) = 2$

$T(2) = 3$

n	T(n)	F(n)	
3	5	5	✓
4	8	8	✓

R. R Homogéneo

$$T(n) = C_1 T(n-1) + C_2 T(n-2) + C_3 T(n-3) + \dots + C_k T(n-k)$$

$C_1 \neq 0$

$$T(n) = r^n \qquad \lambda - 1 - h + k \qquad r^{n-k} \neq 0$$

$$r^n = C_1 r^{n-1} + C_2 r^{n-2} + \dots + C_k r^{n-k}$$

$$\frac{r^n}{r^{n-k}} = \frac{C_1 r^{n-1}}{r^{n-k}} + \frac{C_2 r^{n-2}}{r^{n-k}} + \dots + \frac{C_k r^{n-k}}{r^{n-k}}$$

$$r^k = C_1 r^{k-1} + C_2 r^{k-2} + \dots + C_k$$

Ecuación característica

$$r^k - C_1 r^{k-1} - C_2 r^{k-2} - \dots - C_k = 0$$

Raíces (k) raíces (distintas)

$$T(n) = A(r_1)^n + B(r_2)^n + \dots + K(r_k)^n$$

$$T(n) = 7T(n-1) - 12T(n-2) \qquad T(0) = 10 \quad T(1) = 15$$

1) Ec

$$r^n = 7r^{n-1} - 12r^{n-2}$$

$$\frac{r^n}{r^{n-2}} = \frac{7r^{n-1}}{r^{n-2}} - \frac{12r^{n-2}}{r^{n-2}}$$

$$r^2 = 7r - 12 \longrightarrow r^2 - 7r + 12 = 0$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad ax^2 + bx + c = 0$$

$$r = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm \sqrt{1}}{2} \begin{cases} \frac{8}{2} = 4 \\ \frac{6}{2} = 3 \end{cases}$$

$$T(n) = A(4)^n + B(3)^n$$

$T(0) = 10$
$T(1) = 15$

$10 = A + B$
$15 = 4A + 3B$

$-30 = -3A - 3B$
$15 = 4A + 3B$

$-15 = A$ $\quad$ $B = 25$

$T(n) = -15 \times 4^n + 25 \times 3^n$ $\quad$ $T(0) = 10$, $T(1) = 15$

Sea una RR homogenea cuya EC tiene raices repetidas con multiplicidad m, la solución se plantea multiplicando un polinomo de grado m-1 a cada raiz de multiplicidad m

Ejemplo

(4,4,4,4,4,4,4,4,3,3,3,3,3,3,2,2,2,1)

$T(n) = (An^7 + Bn^6 + Cn^5 + Dn^4 + En^3 + Fn^2 + Gn + H)4^n +$
$(In^5 + Jn^4 + Kn^3 + Ln^2 + Mn + N)3^n +$
$(On^2 + Pn + Q)2^n + R1^n$

$T(n) = -4T(n-1) - 4T(n-2)$

$r^2 + 4r + 4 = 0$

$T(n) = (An + B)(-2)^n$

$-10 = (A(1) + B)(-2)^1$

$-10 = 10 - 2Q$

$T(n) = (-5n + 10)(-2)^n$

- $T(0) = 10$
- $T(1) = -10$

$\frac{-4 \pm \sqrt{16 - 16}}{2} = \frac{-4 \pm 0}{2} \rightarrow -2$, $m = 2$

$10 = B$

$-10 = -2A - 2B$

$-10 = -2A - 20$ $\quad$ $A = -5$

$$T(n) = T(n-1) + 8T(n-2) - 12T(n-3)$$

$T(0) = 4$

$T(1) = 8$

$T(2) = 10$