

Fundamentos de análisis y diseño de algoritmos

Ecuaciones de recurrencia

```
def factorial(n):
```

```
    if n==0:
```

```
        return 1
```

```
    else:
```

```
        return n*factorial(n-1)
```

$n=0 \rightarrow f(n)=1$
 $f(0)=1$

$f(n) = n * f(n-1)$

$$f(n) = \begin{cases} 1 & n=0 \\ n * f(n-1) & n > 0 \end{cases}$$

Recurrencias

Método de iteración

Método maestro*

Método de sustitución

Recurrencias

Método de iteración

Expandir la recurrencia y expresarla como una suma de términos que dependen de n y de las condiciones iniciales

Recurrencias

$$T(n) = n + 3T(n/4), T(1) = \Theta(1) \text{ y } n \text{ par}$$

Expandir la recurrencia 2 veces

$$0) \quad T(n) = 3 \boxed{T\left(\frac{n}{4}\right)} + n$$

$$1) \quad T(n) = 3 \left(3 T\left(\frac{n}{4^2}\right) + \frac{n}{4} \right) + n$$

$$T(n) = 3^2 \boxed{T\left(\frac{n}{4^2}\right)} + 3 \frac{n}{4} + n$$

$$2) \quad T(n) = 3^2 \left(3 T\left(\frac{n}{4^3}\right) + \frac{n}{4^2} \right) + 3 \frac{n}{4} + n$$

$$T(n) = 3^3 T\left(\frac{n}{4^3}\right) + \frac{3^2 n}{4^2} + \frac{3n}{4} + n$$

Recurrencias

$$T(n) = n + 3T(n/4)$$

$$n + 3 (n/4 + 3T(n/16))$$

$$n + 3 (n/4 + 3(n/16 + 3T(n/64)))$$

$$n + 3*n/4 + 3^2*n/4^2 + 3^3T(n/4^3)$$

Recurrencias

$$T(n) = n + 3T(n/4)$$

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$$n + 3*n/4 + 3^2*n/4^2 + 3^3T(n/4^3)$$

$$T(n/4^4)$$

$$T(n/4^5)$$

¿Cuándo se detienen las iteraciones?

Recurrencias

$$1) T(n) = n + 3T(n/4)$$

$$n + 3 (n/4 + 3T(n/16))$$

$$2) n + 3 (n/4 + 3(n/16 + 3T(n/64)))$$

$$3) n + 3*n/4 + 3^2*n/4^2 + 3^3T(\underbrace{n/4^3}_{\hat{1}})$$

¿Cuándo se detienen las iteraciones?

Cuando se llega a $T(1)$

$$T\left(\frac{n}{4^i}\right) \rightarrow T(1)$$

$$\frac{n}{4^i} = 1$$

$$i = \log_4(n)$$

Recurrencias

$$T(n) = n + 3T(n/4)$$

$$n + 3 (n/4 + 3T(n/16))$$

$$n + 3 (n/4 + 3(n/16 + 3T(n/64)))$$

$$n + 3*n/4 + 3^2*n/4^2 + 3^3T(n/4^3)$$

¿Cuándo se detienen las iteraciones?

Cuando se llega a $T(1)$, esto es, cuando $(n/4^i)=1$

Recurrencias

$$T(n) = n + 3T(n/4)$$

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$$n + 3*n/4 + 3^2*n/4^2 + 3^3n/4^3 + \dots + 3^{\log_4 n}T(1)$$

¿Cuándo se detienen las iteraciones?

Cuando se llega a $T(1)$, esto es, cuando $(n/4^i)=1$

Recurrencias

$$T(n) = n + 3T(n/4)$$

$$n + 3 (n/4 + 3T(n/16))$$

$$n + 3 (n/4 + 3(n/16 + 3T(n/64)))$$

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$$n + 3*n/4 + 3^2*n/4^2 + 3^3n/4^3 + \dots + 3^{\log_4 n}T(1)$$

Después de iterar, se debe tratar de expresar como una sumatoria con forma cerrada conocida

Recurrencias

$$T(n) = n + 3T(n/4]$$

$$n + 3 (n/4] + 3T(n/16])$$

$$n + 3 (n/4] + 3(n/16] + 3T(n/64]))$$

$$n + 3*n/4] + 3^2*n/4^2] + 3^3(n/4^3]) + ... + 3^{\log_4 n} \Theta(1)$$

$$\leq n + 3n/4 + 3^2n/4^2 + 3^3n/4^3 + ... + 3^{\log_4 n} \Theta(1)$$

Recurrencias

$$T(n) = n + 3T(n/4)$$

$$n + 3 (n/4 + 3T(n/16))$$

$$n + 3 (n/4 + 3(n/16 + 3T(n/64)))$$

$$n + 3*n/4 + 3^2*n/4^2 + 3^3(n/4^3) + \dots + 3^{\log_4 n} \Theta(1)$$

$$\leq n + 3n/4 + 3^2n/4^2 + 3^3n/4^3 + \dots + 3^{\log_4 n} \Theta(1)$$

$$= \left(\sum_{i=0}^{\log_4 n} \left(\frac{3}{4} \right)^i n \right) + 3^{\log_4 n} \Theta(1)$$

$$= n \left(\frac{(3/4)^{(\log_4 n)} - 1}{(3/4) - 1} \right) + n^{\log_4 3} = n * 4 (1 - (3/4)^{(\log_4 n)}) + \Theta(n^{\log_4 3})$$

$$= O(n)$$

$$T(n) = 3T\left(\frac{n}{4}\right) + n$$

$$T(n) = 3\left(3T\left(\frac{n}{4^2}\right) + \frac{n}{4}\right) + n = 3^2 T\left(\frac{n}{4^2}\right) + \frac{3}{4}n + n$$

$$T(n) = 3^2\left(3T\left(\frac{n}{4^3}\right) + \frac{n}{4^2}\right) + \frac{3}{4}n + n$$

$$2) \quad T(n) = 3^3 T\left(\frac{n}{4^3}\right) + \frac{3^2}{4^2}n + \frac{3}{4}n + n$$

$$i) \quad T(n) = 3^i T\left(\frac{n}{4^i}\right) + \left(\frac{3}{4}\right)^{i-1}n + \left(\frac{3}{4}\right)^{i-2}n + \dots + \left(\frac{3}{4}\right)^0 n$$

$$\frac{n}{4^i} = 1$$

$$i = \log_4(n)$$

$$T(n) = 3^{\log_4(n)} T(1) + \left(\frac{3}{4}\right)^{\log_4(n)-1}n + \left(\frac{3}{4}\right)^{\log_4(n)-2}n + \dots + \left(\frac{3}{4}\right)^1 n + \left(\frac{3}{4}\right)^0 n$$

$$T(n) = n^{\log_4(3)} T(1) + \sum_{i=0}^{\log_4(n)-1} \left(\frac{3}{4}\right)^i n$$

$$\sum_{i=0}^n ar^i = \frac{ar^{n+1} - a}{r-1}$$

$$a = n \quad r = \frac{3}{4} \quad n = \log_4(n) - 1$$

$$T(n) = n^{\log_4(3)} T(1) + \frac{n \left(\frac{3}{4}\right)^{\log_4(n)} - n}{\frac{3}{4} - 1}$$

$$T(n) = n^{\log_4(3)} T(1) + \frac{n \times n^{\log_4\left(\frac{3}{4}\right)} - n}{-\frac{1}{4}}$$

$$T(1) = \Theta(1)$$

$$T(1) = C$$

Recurrencias

Resuelva por el método de iteración

$$T(n) = \underline{2T(n/2) + 1}, T(1) = \Theta(1)$$

$$1) T(n) = 2 \left(2T\left(\frac{n}{2^2}\right) + 1 \right) + 1 = 2^2 \underbrace{T\left(\frac{n}{2^2}\right)} + \underline{2 + 1}$$

$$2) T(n) = 2^2 \left(2T\left(\frac{n}{2^3}\right) + 1 \right) + 2 + 1 = 2^3 T\left(\frac{n}{2^3}\right) + 2^2 + 2^1 + 2^0$$

$$i) T(n) = 2^i T\left(\frac{n}{2^i}\right) + \underline{2^{i-1} + 2^{i-2} + \dots + 2^1 + 2^0}$$

$$T(1)$$

$$\frac{n}{2^i} = 1 \quad n = 2^i \quad i = \log_2(n)$$

$$T(n) = 2^{\log_2(n)} T(1) + 2^{\log_2(n)-1} + 2^{\log_2(n)-2} + \dots + 2^1 + 2^0$$

$$T(n) = n T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i$$

$$T(n) = Cn + \frac{2^{\log_2(n)} - 1}{2 - 1}$$

$$T(n) = Cn + n - 1 \quad O(n)$$

Recurrencias

Resuelva por el método de iteración

✕ $T(n) = 2T(n/2) + 1, T(1) = \Theta(1)$

$$T(n) = 2T(n/2) + n, T(1) = \Theta(1)$$

$$T(n) = 2T(n/2) + n, \quad T(1) = \Theta(1)$$

$$T(n) = 2 \left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2} \right) + \overset{1n}{n} = 2^2 T\left(\frac{n}{2^2}\right) + n + n$$

$$T(n) = 2^2 \left(2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} \right) + \overset{2n}{n + n}$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + \overset{3n}{n + n + n}$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + i n$$

$$T(1) = T\left(\frac{n}{2^i}\right)$$

$$\frac{n}{2^i} = 1$$

$$i = \log_2(n)$$

$$T(n) = 2^{\log_2(n)} T(1) + \log_2(n) \times n$$

$$T(n) = n \times T(1) + \log_2(n) \times n \quad O(n \log_2(n))$$

Recurrencias

Resuelva por el método de iteración

$$T(n) = 2T(n/2) + 1, T(1) = \Theta(1)$$

$$T(n) = 2T(n/2) + n, T(1) = \Theta(1)$$

$$T(n) = T(n/2) + 1, T(1) = \Theta(1)$$

Recurrencias

Resuelva por el método de iteración

$$T(n) = 2T(n/2) + 1, T(1) = \Theta(1)$$

$$T(n) = 2T(n/2) + n, T(1) = \Theta(1)$$

$$T(n) = T(n/2) + 1, T(1) = \Theta(1)$$

Demuestre que $T(n) = T(n/2) + n$, es $\Omega(n \log n)$

$$T(n) = 5T(n/4) + n,$$

$$T(n) = 5 \left(5T\left(\frac{n}{4^2}\right) + \frac{n}{4} \right) + n = 5^2 T\left(\frac{n}{4^2}\right) + \frac{5}{4}n + n$$

$$T(n) = 5^2 \left(5T\left(\frac{n}{4^3}\right) + \frac{n}{4^2} \right) + \frac{5}{4}n + n$$

$$T(n) = 5^3 T\left(\frac{n}{4^3}\right) + \frac{5^2}{4^2}n + \frac{5}{4}n + n$$

$$T(n) = 5^i T\left(\frac{n}{4^i}\right) + \left(\frac{5}{4}\right)^{i-1}n + \left(\frac{5}{4}\right)^{i-2}n + \dots + \left(\frac{5}{4}\right)^2n + \left(\frac{5}{4}\right)n$$

$$T(1)$$

$$\frac{n}{4^i} \approx 1$$

$$i = \log_4(n)$$

$$T(n) = 5^{\log_4(n)} \underline{T(1)} + \sum_{i=0}^{\log_4(n)-1} \left(\frac{s}{4}\right)^i n$$

$$T(n) = n^{\log_4(s)} C \times n + n \left(\frac{\left(\frac{s}{4}\right)^{\log_4(n)} - 1}{\frac{s}{4} - 1} \right)$$

$$T(n) = n^{\log_4(s)} \times C \times n + n \left(n^{\log_4\left(\frac{s}{4}\right)} - 1 \right)$$

$$T(2) = C \times n$$

$$\underline{T(1) = C}$$

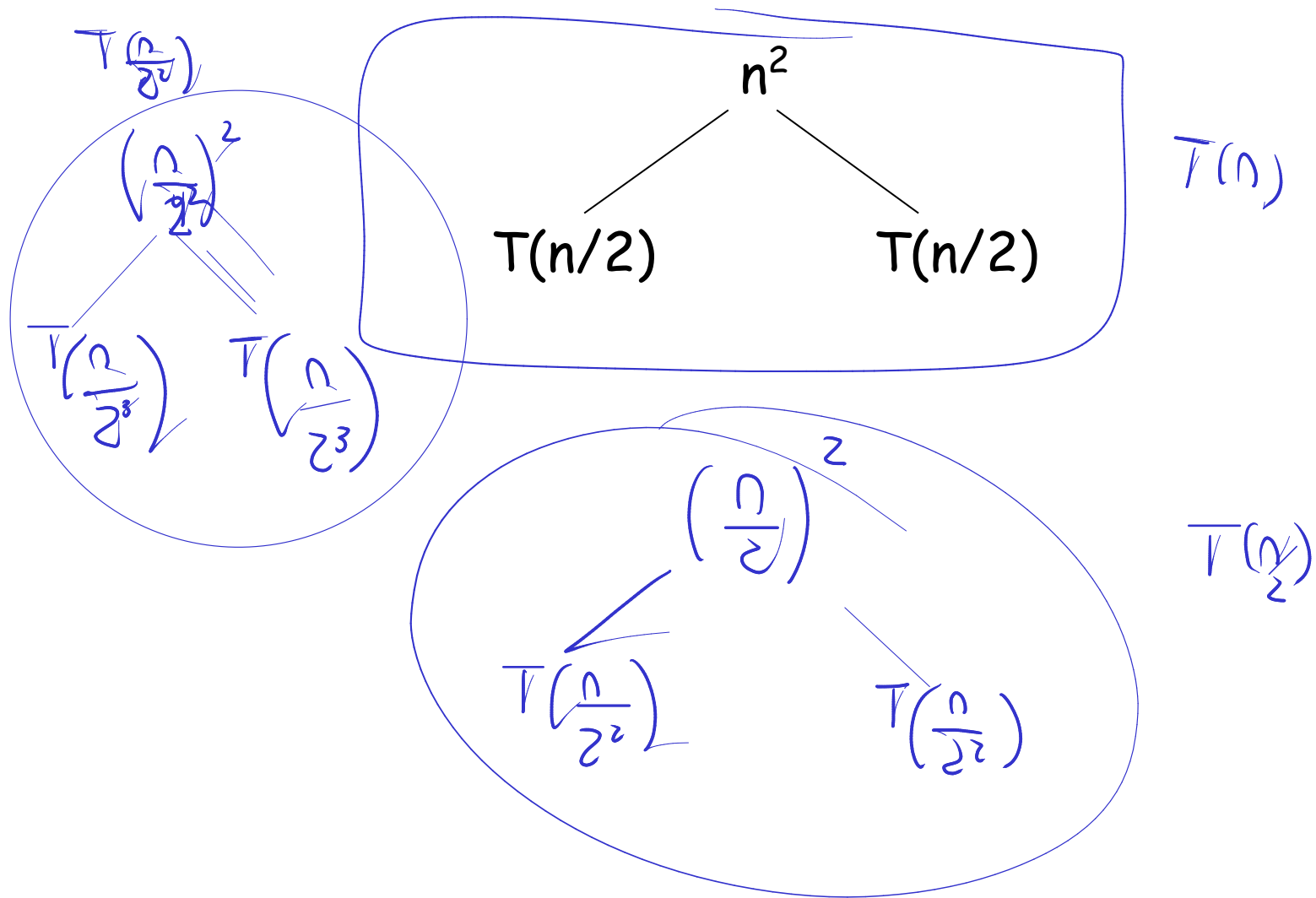
Recurrencias

Iteración con árboles de recursión

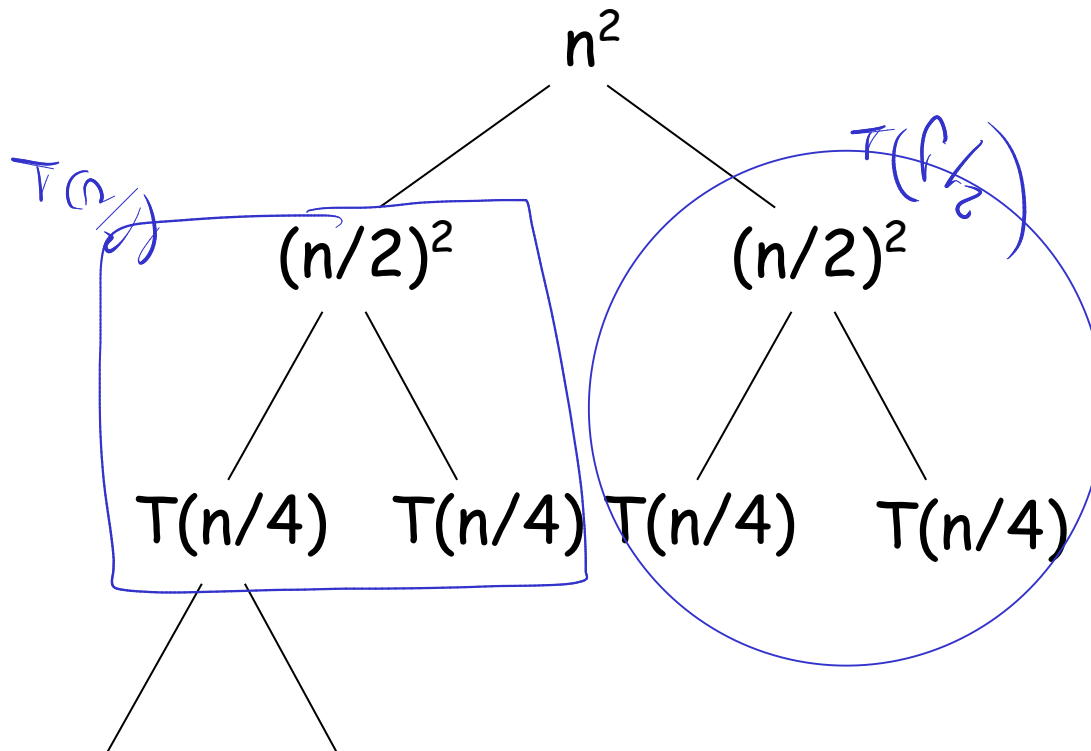
$$T(1) = 1$$

$$T(n) = 2T(n/2) + n^2$$

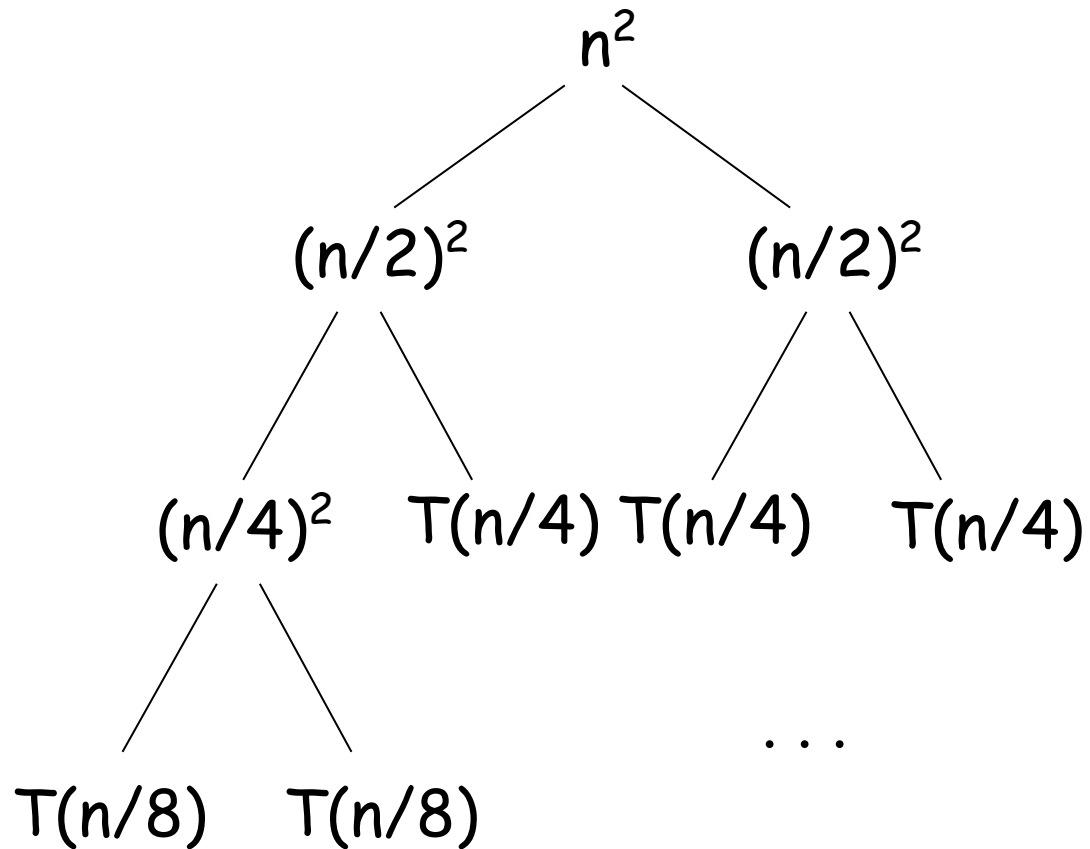
Recurrencias



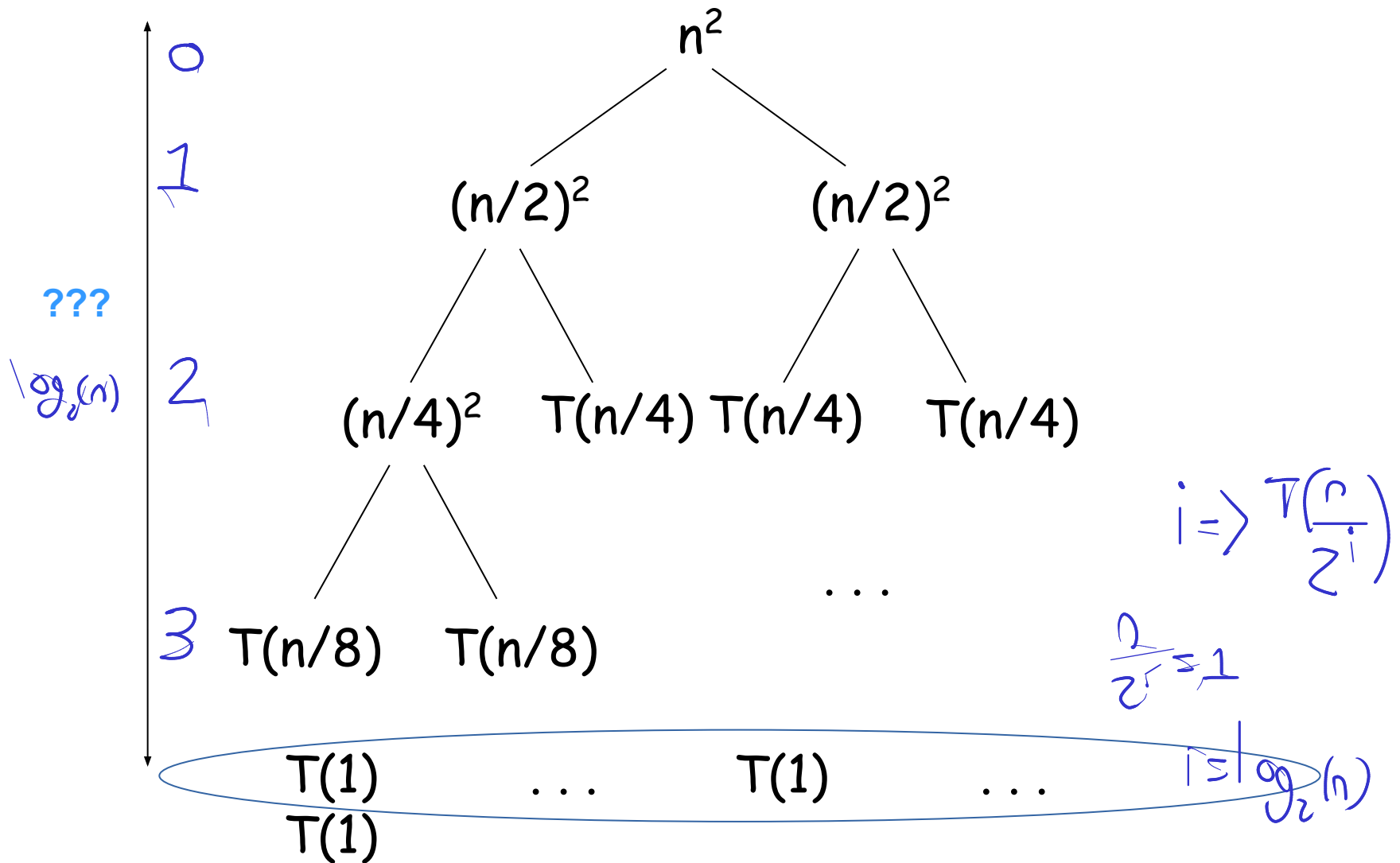
Recurrencias



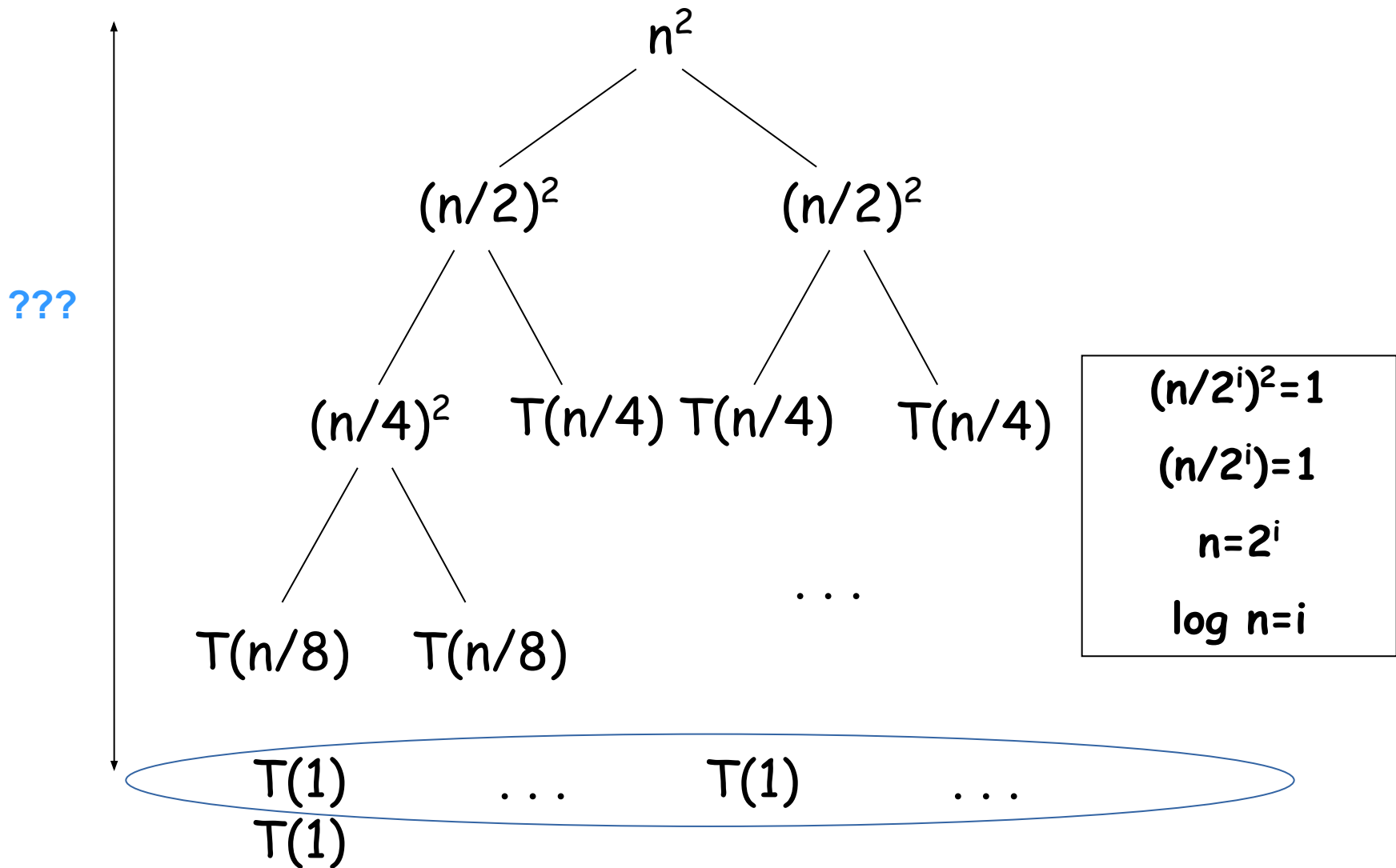
Recurrencias



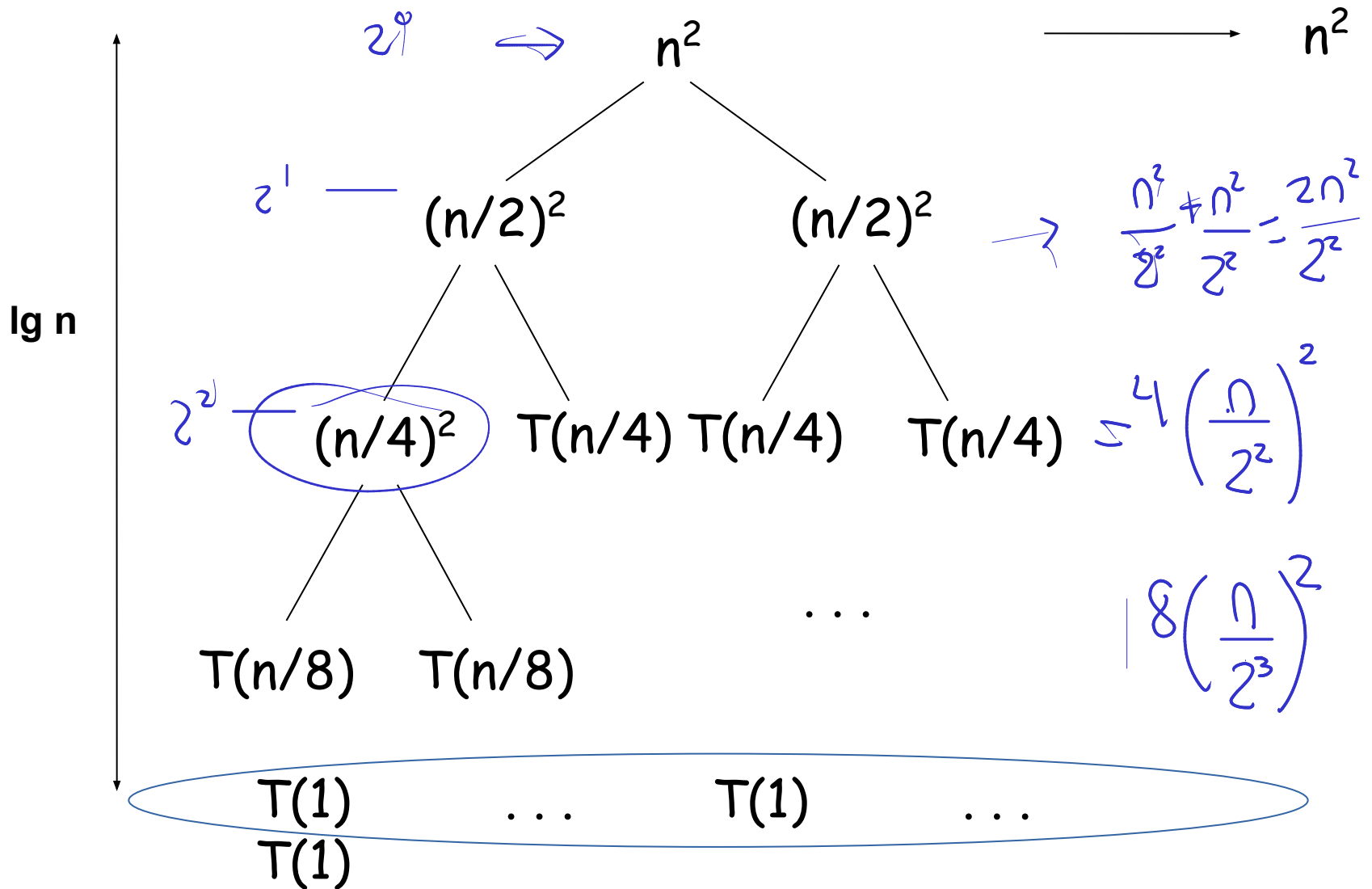
Recurrencias



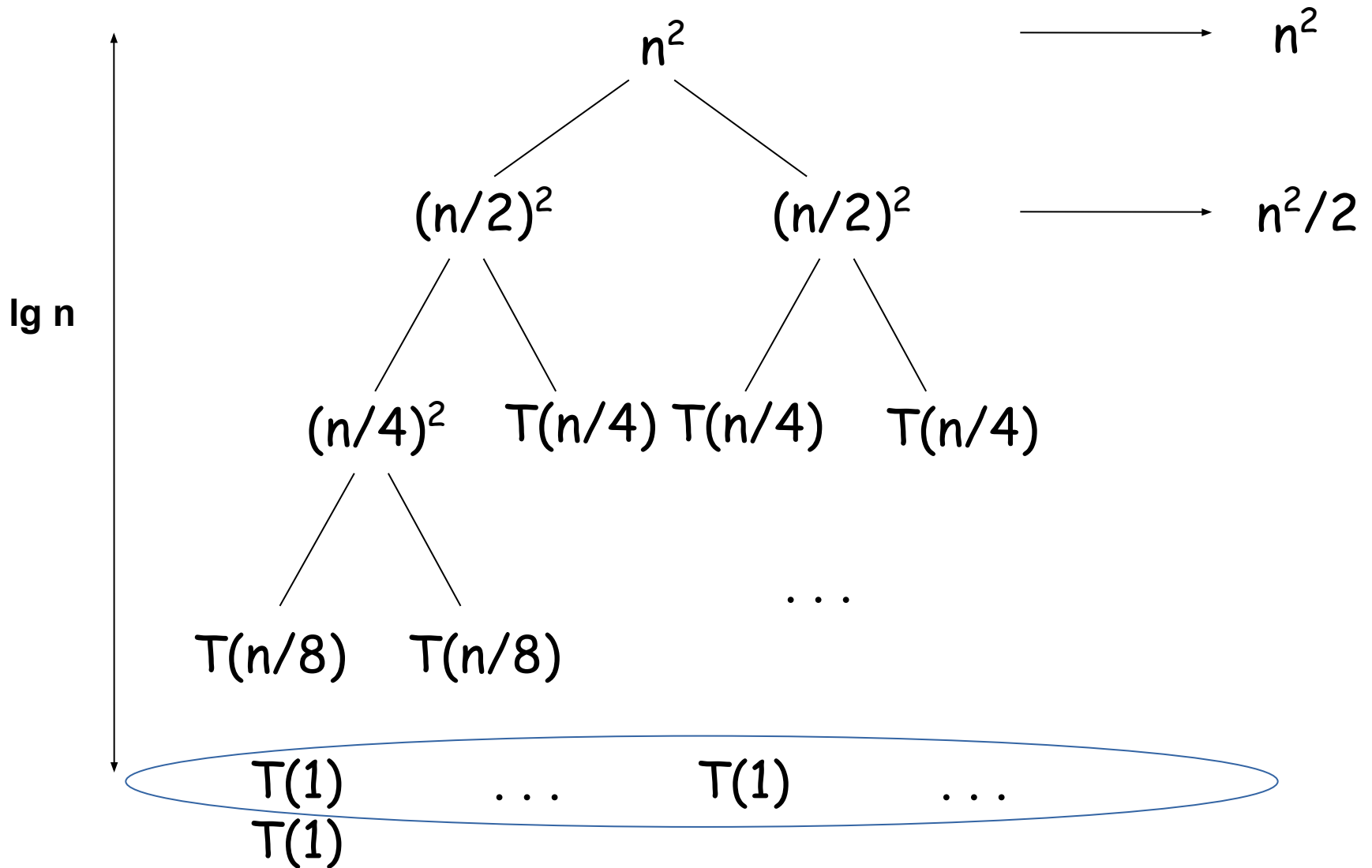
Recurrencias



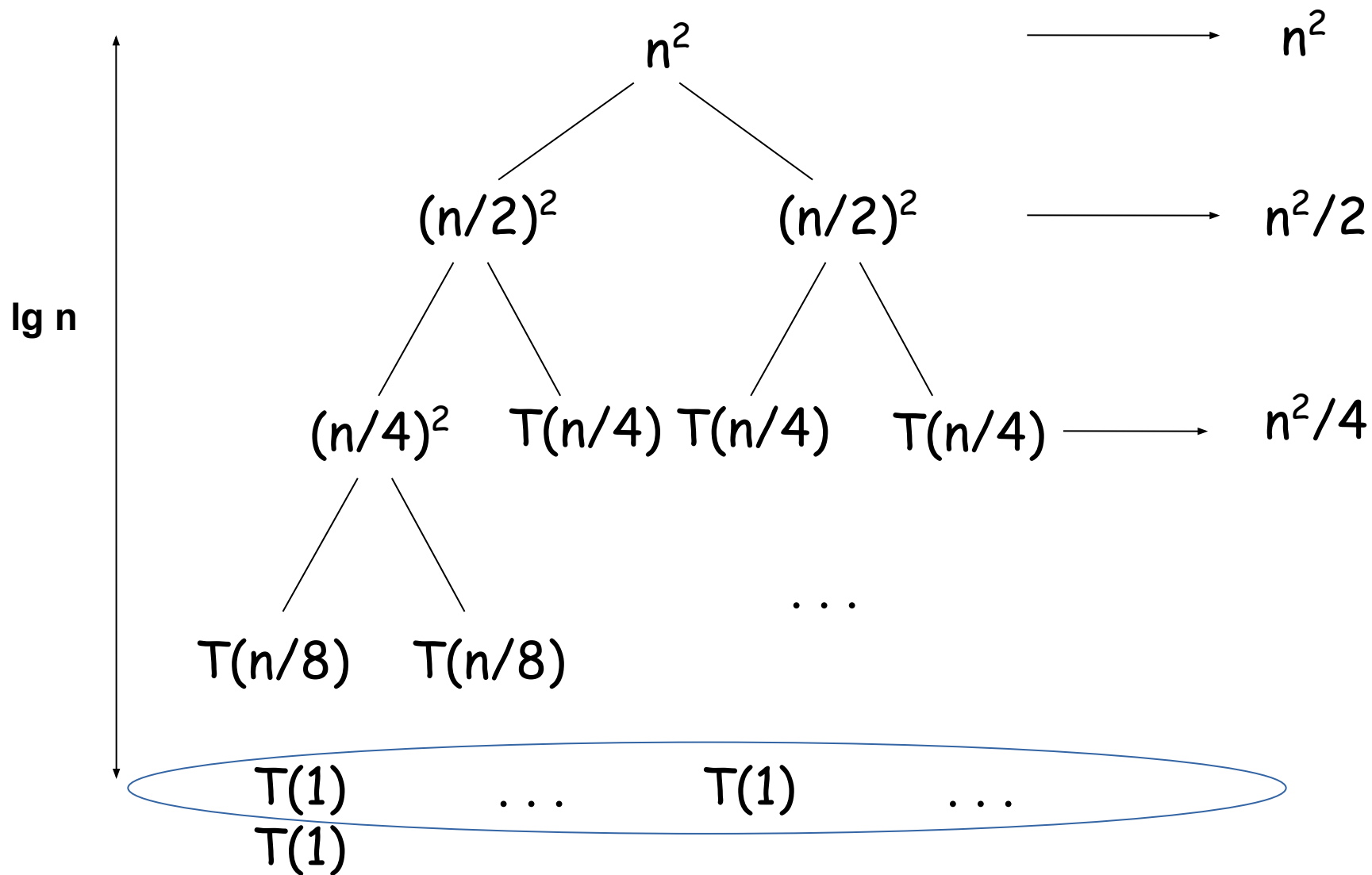
Recurrencias



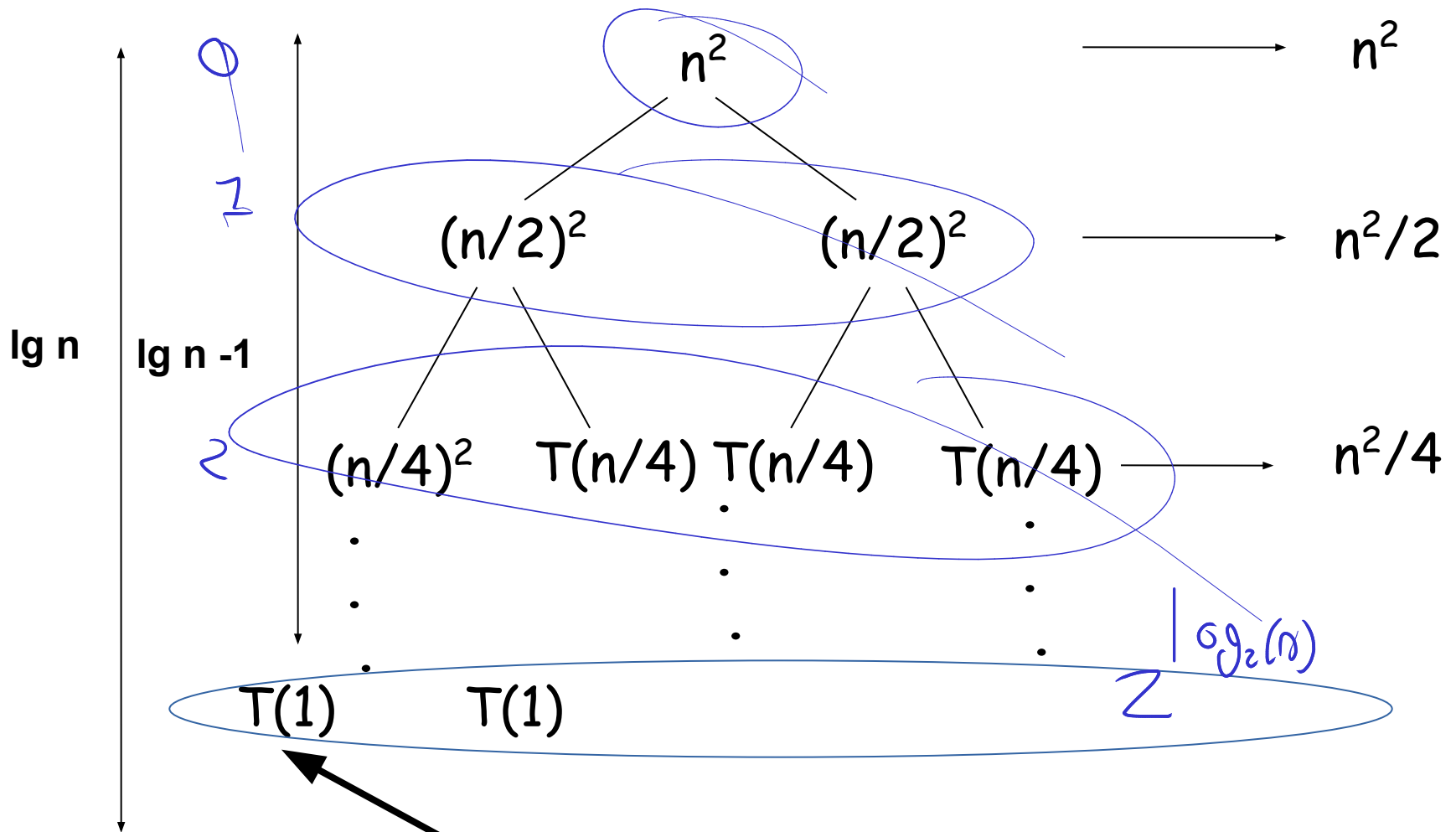
Recurrencias



Recurrencias

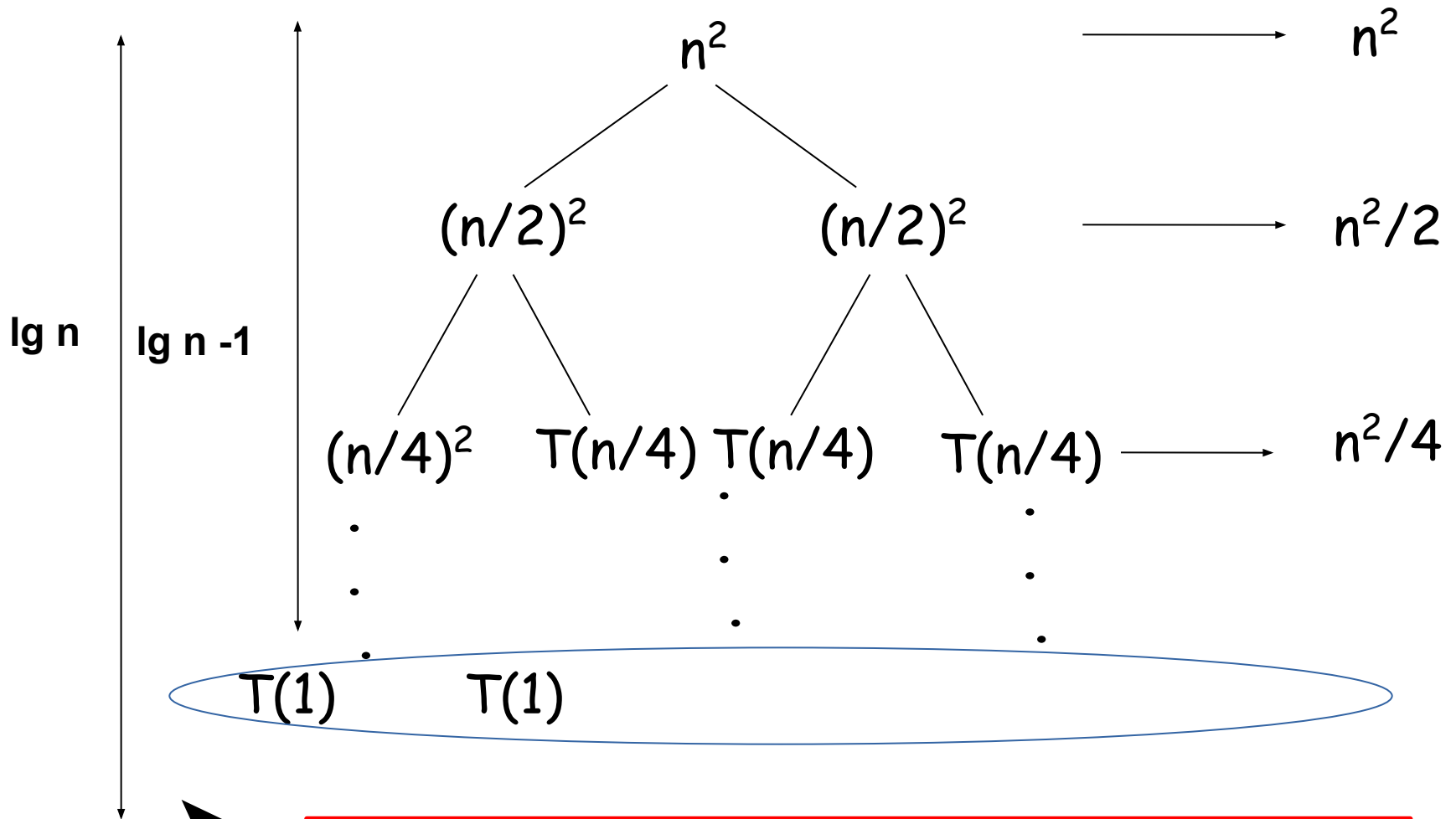


Recurrencias



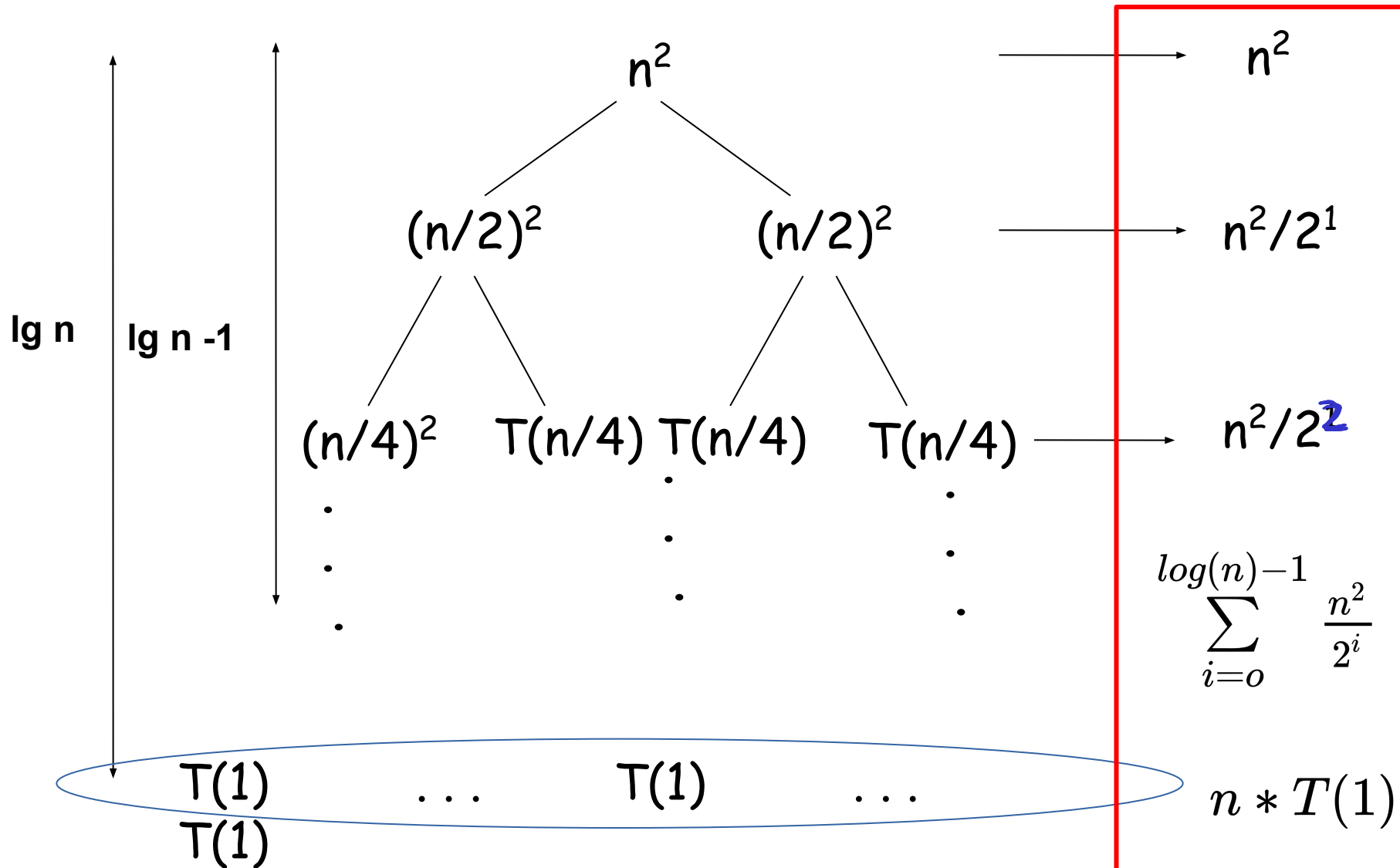
¿Cuántas hojas $T(1)$
tenemos?

Recurrencias



Si recuerda en un árbol m-ario se tienen máximo m^h . En este caso al ser árbol binario $m=2$, tenemos $2^{\lg(n)}$ hojas. Por lo tanto se

Recurrencias



Recurrencias

$$T(n) = n * T(1) + \sum_{i=0}^{\log(n)-1} \frac{n^2}{2^i}$$

$$T(n) = n * c + n^2 \frac{0.5^{\log(n)} - 1}{0.5 - 1}$$

$$T(n) = n * c + n^2 \frac{n^{\log(0.5)} - 1}{-0.5}$$

$$T(n) = n * c + n^2 \frac{n^{-1} - 1}{-0.5}$$

$$T(n) = n * c - \frac{n}{0.5} + \frac{n^2}{0.5} = O(n^2)$$

Recurrencias

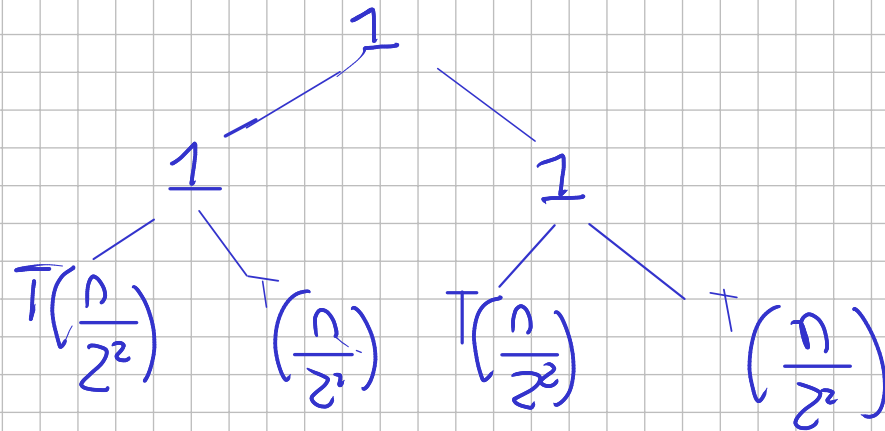
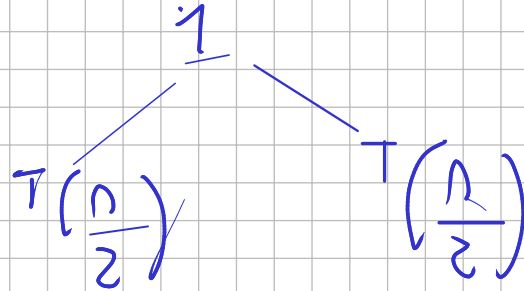
Resuelva construyendo el árbol

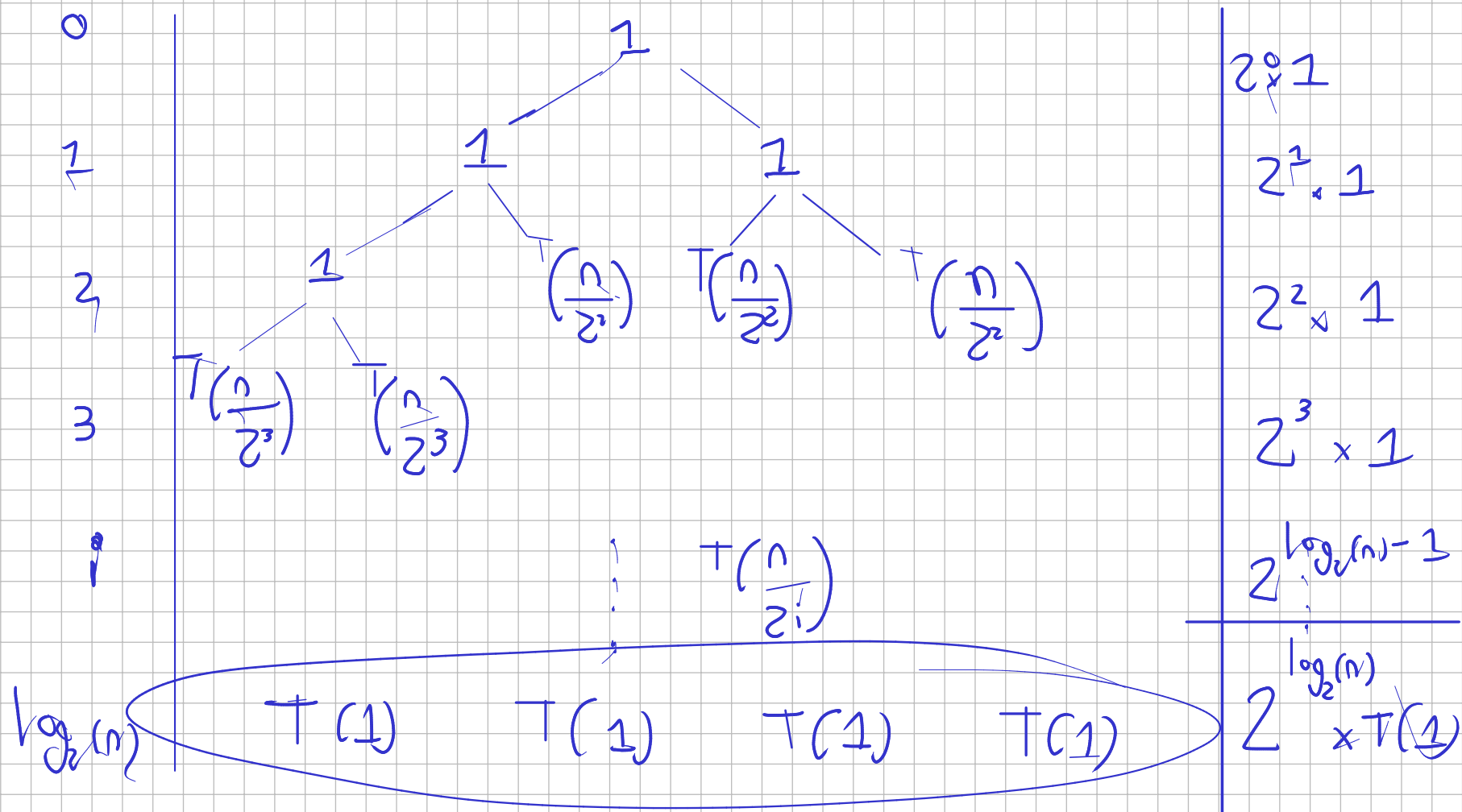
$$T(n) = 2T(n/2) + 1, T(1) = \Theta(1)$$

$$T(n) = 2T(n/2) + n, T(1) = \Theta(1)$$



$$T(n) = 2T(n/2) + 1, T(1) = \Theta(1)$$





$$\frac{n}{2^i} = 1$$

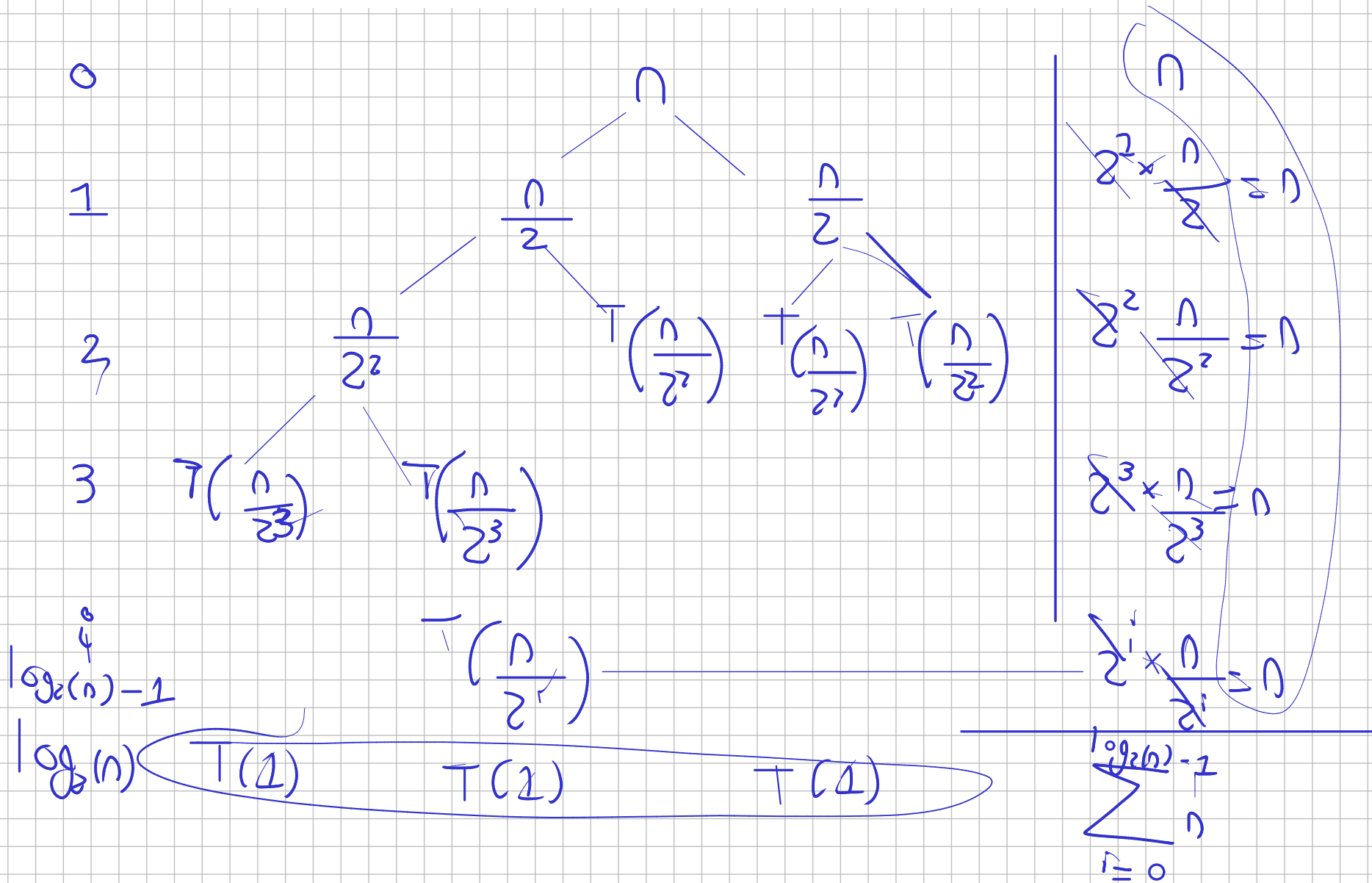
$$i = \log_2(n)$$

$$2^{\log_2(n)} T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i$$

$$T(n) = 2^{\log_2(n)} T(1) + \sum_{i=0}^{\log_2(n)-1} 2^i$$

$$T(n) = nC + \frac{2^{\log_2(n)} - 1}{2 - 1} = \boxed{nC + n - 1}$$

$$T(n) = 2T(n/2) + n, T(1) = \Theta(1)$$



$$T(n) = n T(1) + \sum_{i=0}^{\log_2(n)-1} n$$

$$T(n) \leq Cn + \log_2(n) \times n$$

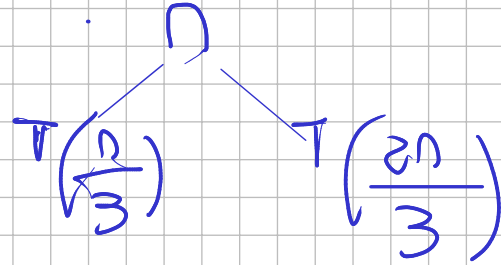
$$\sum_{i=1}^n C \leq C \times n$$

Recurrencias

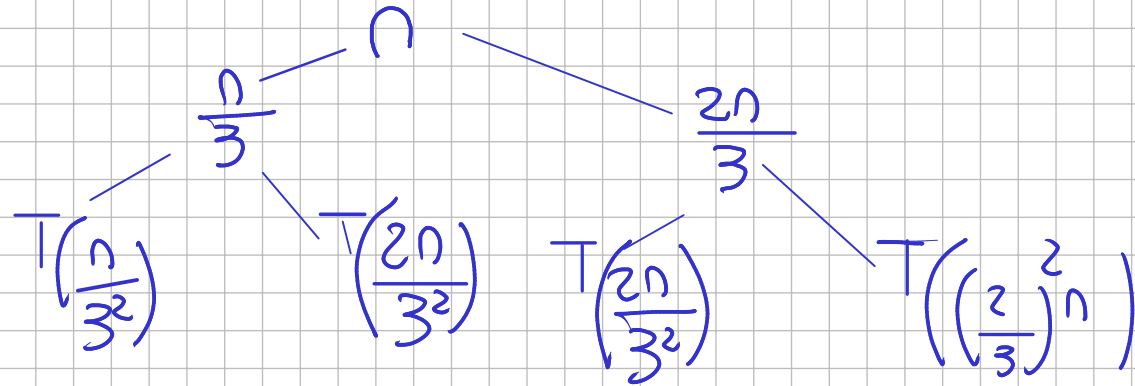
Resuelva la recurrencia $T(n) = T(n/3) + T(2n/3) + n$

Indique una cota superior y una inferior

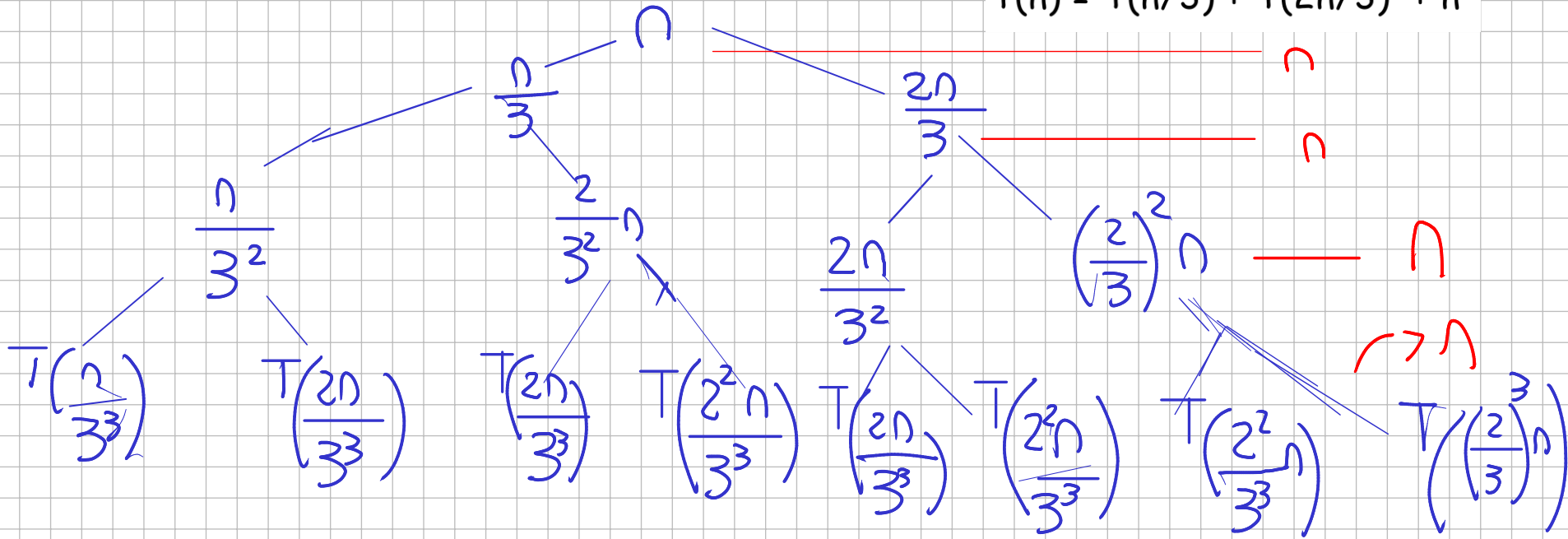
$$T(n) = T(n/3) + T(2n/3) + n$$



$$n \approx \frac{2}{3}n$$



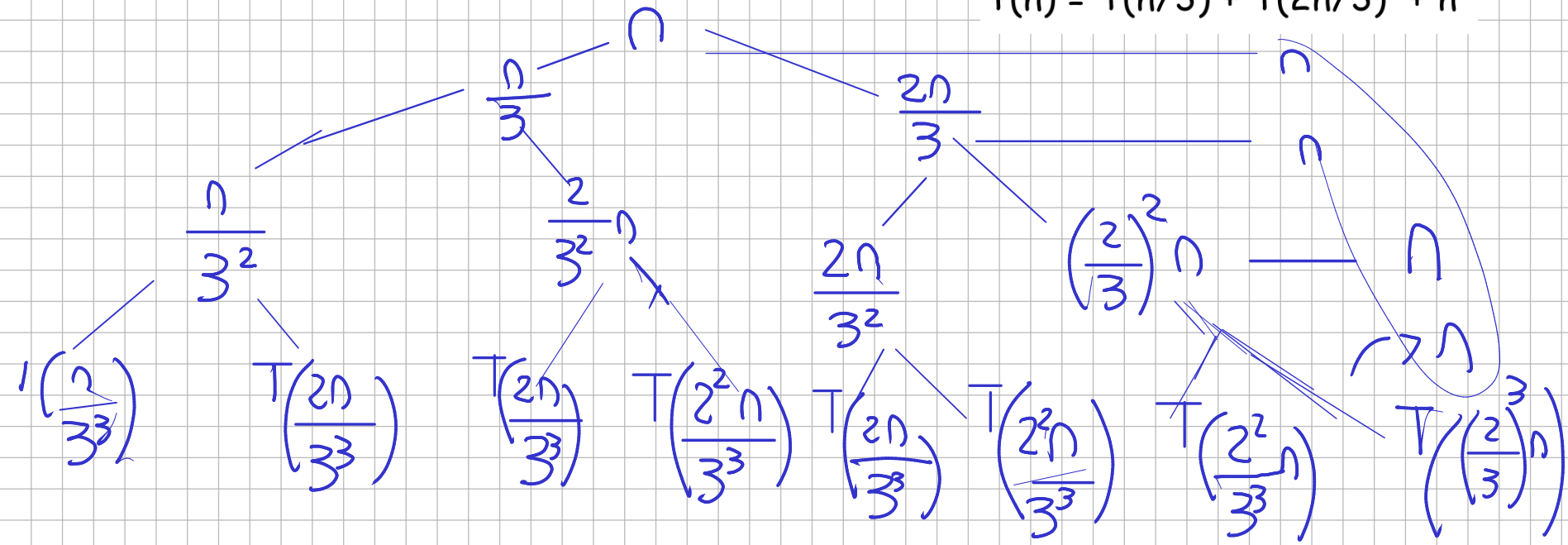
$$T(n) = T(n/3) + T(2n/3) + n$$



$$\frac{1}{3^2} + \frac{2}{3^2} + \frac{2}{3^2} + \frac{4}{3^2} = \frac{9}{3^2}$$

$$\frac{1}{3^3} + \frac{2}{3^3} + \frac{2}{3^3} + \frac{4}{3^3} + \frac{2}{3^3} + \frac{4}{3^3} + \frac{4}{3^3} + \frac{8}{3^3} = \frac{27}{3^3} = 1$$

$$T(n) = T(n/3) + T(2n/3) + n$$

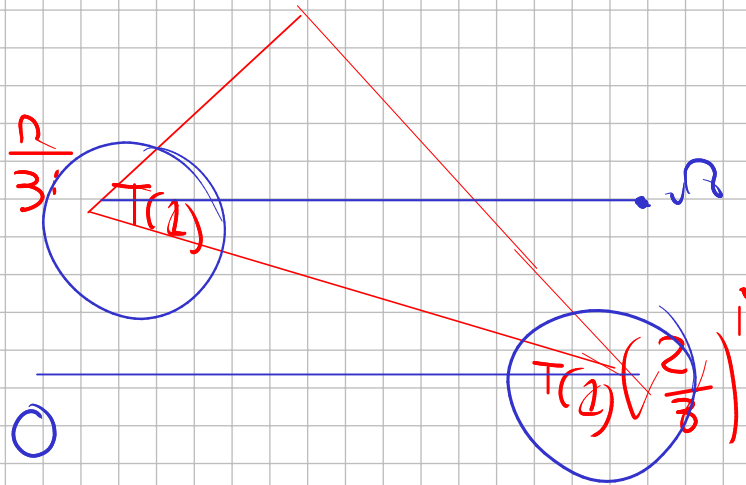


$$\frac{n}{3^i}$$

$$i_1 \leq \log_3(n)$$

$$\left(\frac{2}{3}\right)^i \leq n$$

$$i \log_{\frac{2}{3}}\left(\frac{2}{3}\right) \approx \log_{\frac{2}{3}}(n)$$



$$\left(\frac{2}{3}\right)^i n$$

$$i_2 \leq \log_{\frac{2}{3}}(n)$$

$$2^{\log_3(n)} T(1) + \sum_{i=0}^{\log_3(n)-1} n$$

$$n^{\log_3(2)} + \log_3(n) \times n$$

Cota inferior

$$\log_3(n) = \frac{\log_2(n)}{\log_3(2)}$$

$$2^{\log_{\frac{2}{3}}(n)} + \sum_{i=0}^{\log_{\frac{2}{3}}(n)-1} n$$

$$C \times \log(n) \times n$$

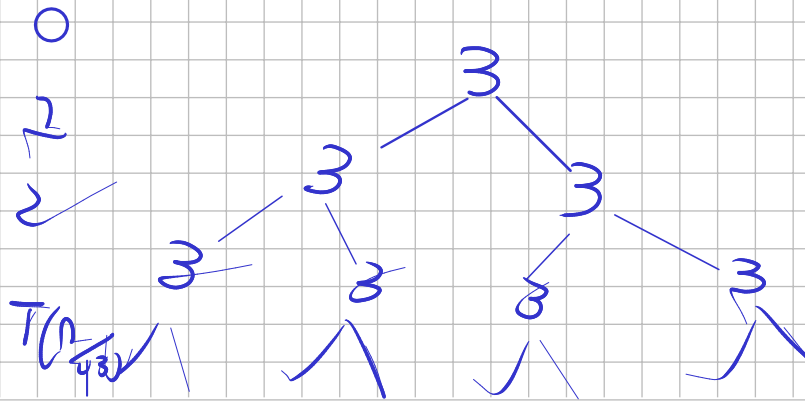
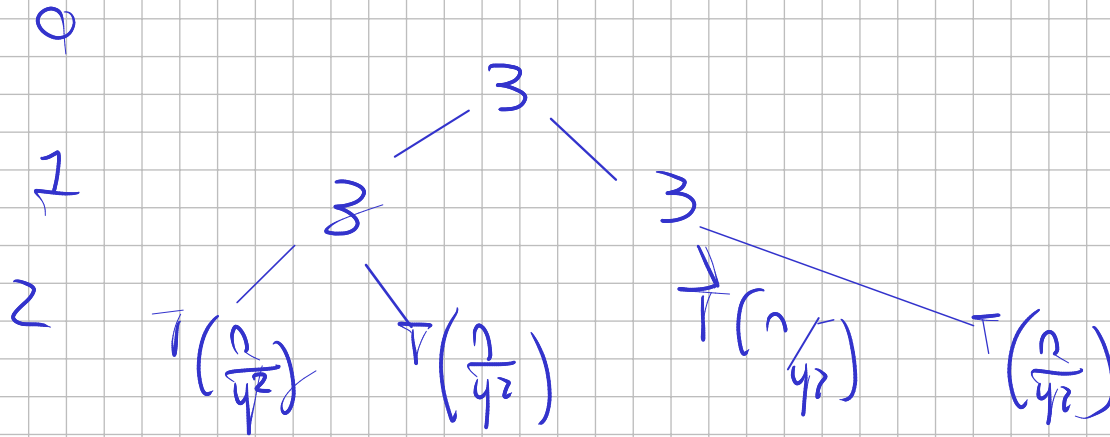
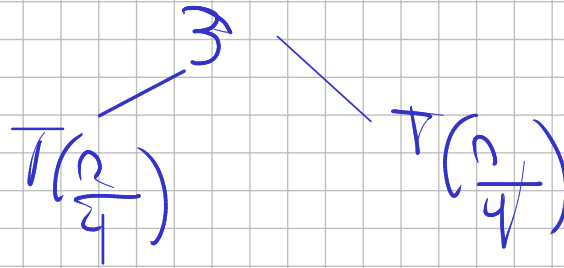
$$n^{\log_{\frac{2}{3}}(2)} + \log_{\frac{2}{3}}(n) \times n$$

Cota Superior

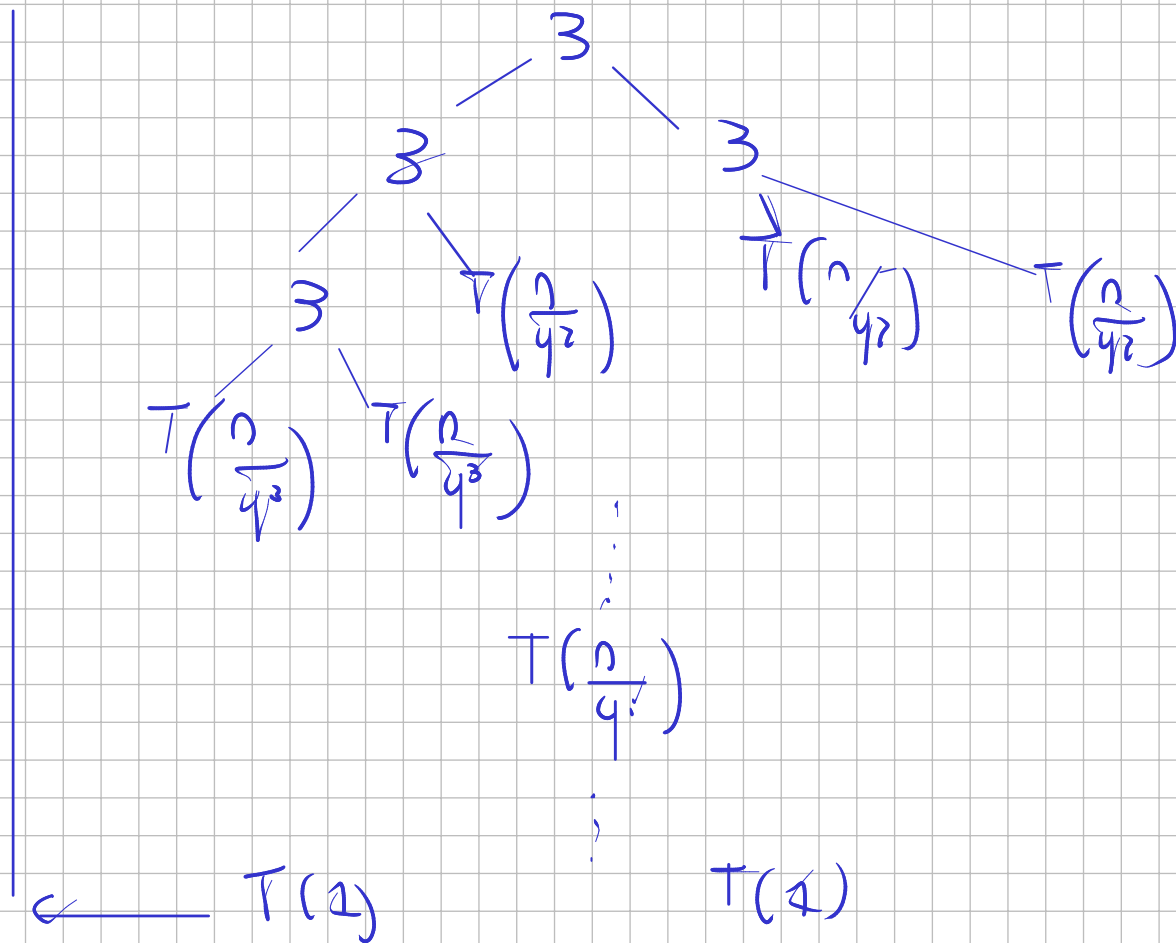
$$\log_{\frac{2}{3}}(n) = \frac{\log_2(n)}{\log_2(\frac{2}{3})}$$

$$\Theta(n \log(n))$$

$$T(n) = 2T(n/4) + 3$$



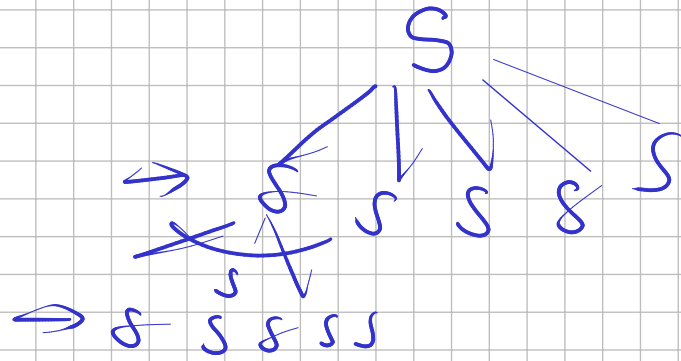
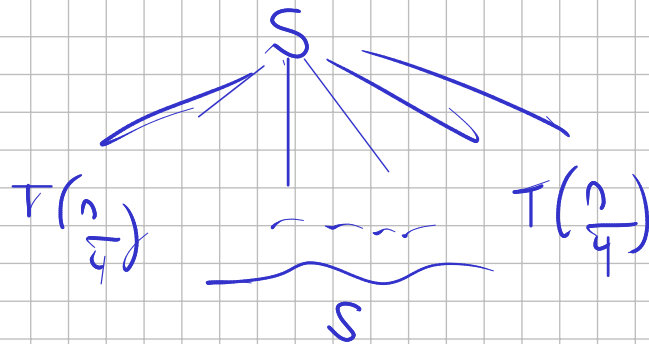
$2^0 \times 3$
$2^2 \times 3'$
$2^2 \times 3$



$$\frac{n}{4^i} \leq 1$$

$$i = \log_4(n)$$

$$1) T(n) = 5T(n/4) + 5$$

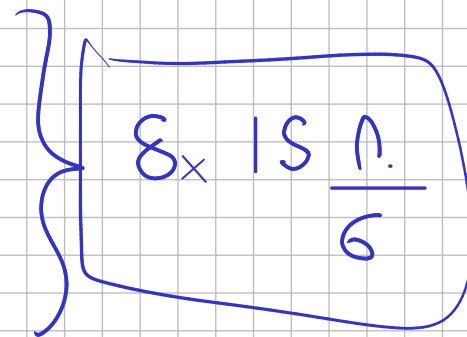
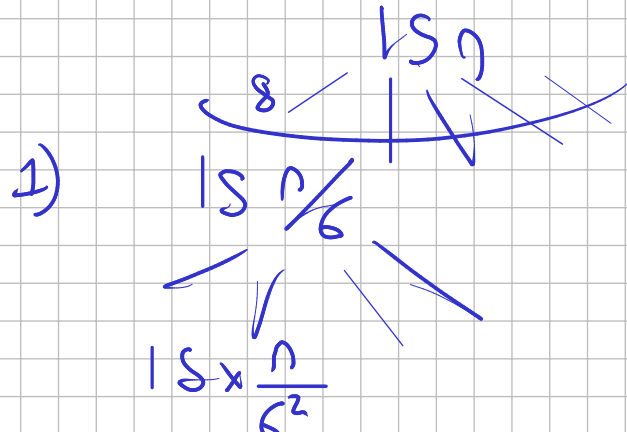


$$S^0 \times S$$

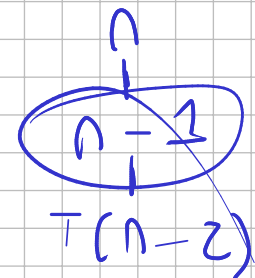
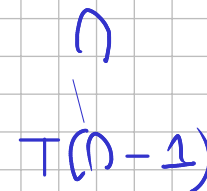
$$S^1 \times S = S^2$$

$$S^2 \times S = S^3$$

$$2) T(n) = 8T(n/6) + 15n$$



$$3) T(n) = T(n-1) + n$$



Recurrencias

Método maestro

Permite resolver recurrencias de la forma:

$$T(n) = aT(n/b) + f(n), \text{ donde } a \geq 1, b > 1$$

Recurrencias

Dado $T(n) = aT(n/b) + f(n)$, donde $a \geq 1$, $b > 1$, se puede acotar asintóticamente como sigue:

1. $T(n) = \Theta(n^{\log_b a})$

Si $f(n) = O(n^{\log_b a - \varepsilon})$ para algún $\varepsilon > 0$

2. $T(n) = \Theta(n^{\log_b a} \lg n)$

Si $f(n) = \Theta(n^{\log_b a})$ para algún $\varepsilon > 0$

3. $T(n) = \Theta(f(n))$

Si $f(n) = \Omega(n^{\log_b a + \varepsilon})$ para algún $\varepsilon > 0$ si $a * f(n/b) \leq c * f(n)$

para algún $c < 1$

Recurrencias

Dado $T(n) = 9T(n/3) + n$

$$n^{\log_3 9} = n^2 \quad \text{Vs} \quad f(n) = n$$

$$\text{Es } f(n) = O(n^{\log_b a - \varepsilon}) \quad ?$$

$$\text{Es } n = O(n^{2 - \varepsilon}) \quad ?$$

Recurrencias

Dado $T(n) = 9T(n/3) + n$

$$n^{\log_3 9} = n^2 \quad \text{vs} \quad f(n) = n$$

Es $f(n) = O(n^{\log_b a - \varepsilon})$?

Es $n = O(n^{2-\varepsilon})$?

Si $\varepsilon = 1$ se cumple que $n = O(n)$, por lo tanto, se cumple que:

$$T(n) = \Theta(n^2)$$

Recurrencias

$$T(n) = T(2n/3) + 1$$

$$n^{\log_{3/2} 1} = n^0 = 1 \quad \text{Vs} \quad f(n) = 1$$

$$\text{Es } f(n) = O(n^{\log_b a - \varepsilon}) \quad ?$$

$$\text{Es } 1 = O(n^{0 - \varepsilon}) \quad ?$$

No existe $\varepsilon > 0$

Recurrencias

$$T(n) = T(2n/3) + 1$$

$$n^{\log_{3/2} 1} = n^0 = 1 \quad \text{vs} \quad f(n) = 1$$

$$\text{Es } f(n) = \Theta(n^{\log_b a}) \quad ?$$

$$\text{Es } 1 = \Theta(1) \quad ?$$

Si, por lo tanto, se cumple que:

$$T(n) = \Theta(1 * \lg n) = \Theta(\lg n)$$

Recurrencias

$$T(n) = 3 T(n/4) + n \lg n$$

$$n^{\log_4 3} = n^{0.793} \quad \text{vs} \quad f(n) = n \lg n$$

$$\text{Es } f(n) = O(n^{\log_b a - \epsilon}) \quad ?$$

$$\text{Es } f(n) = \Theta(n^{\log_b a}) \quad ?$$

$$\text{Es } f(n) = \Omega(n^{\log_b a + \epsilon}) \quad ?$$

Si, y además, $af(n/b) \leq cf(n)$

$$3(n/4) \lg(n/4) \leq cn \lg n$$

$$3(n/4) \lg n - 3(n/4) * 2 \leq cn \lg n$$

$$(3/4)n \lg n \leq cn \lg n \rightarrow c = 3/4 \text{ y se concluye } T(n) = \Theta(n \lg n)$$

Recurrencias

$$T(n) = 2T(n/2) + n \lg n$$

Muestre que no se puede resolver por el método maestro

Recurrencias

Resuelva usando método del maestro

$$T(n) = 4T(n/2) + n$$

$$T(n) = 4T(n/2) + n^2$$

$$T(n) = 4T(n/2) + n^3$$

Recurrencias

Método de sustitución

Suponer la forma de la solución y probar por inducción matemática

Recurrencias

$$T(n)=2T(\lfloor n/2 \rfloor)+n, T(1)=1$$

Suponer que la solución es de la forma $T(n)=O(n \lg n)$

Probar que $T(n) \leq cn \lg n$.

Se supone que se cumple para $n/2$ y se prueba para n

Hipotesis inductiva: $T(n/2) \leq cn/2 \lg (n/2)$

Recurrencias

$$T(n) = 2T(\lfloor n/2 \rfloor) + n, \quad T(1) = 1$$

Probar que $T(n) \leq cn \lg n$.

Hipótesis inductiva: $T(n/2) \leq cn/2 \lg (n/2)$

Paso inductivo:

$$\begin{aligned} T(n) &\leq 2(cn/2 \lg (n/2)) + n \\ &\leq cn \lg (n/2) + n \\ &= cn \lg (n) - cn + n, \text{ para } c \geq 1, \text{ haga } c=1 \\ &\leq cn \lg n \end{aligned}$$

Recurrencias

$$T(n) = 2T(\lfloor n/2 \rfloor) + n, \quad T(1) = 1$$

Probar que $T(n) \leq cn \lg n$.

Paso base: si $c=1$, probar que $T(1)=1$ se cumple

$$T(1) \leq 1 * 1 \lg 1 ?$$

$$1 \leq 0 ?$$

No, se debe escoger otro valor para c

Recurrencias

$$T(n) = 2T(\lfloor n/2 \rfloor) + n, \quad T(1) = 1$$

Probar que $T(n) \leq cn \lg n$.

Paso base: si $c=2$, probar que $T(1)=1$ se cumple

$$T(1) \leq 2 * 1 \lg 1 ?$$

$$1 \leq 0 ?$$

No, se puede variar k .

Para esto, se calcula $T(2)$ y se toma como valor inicial

Recurrencias

Probar que $T(n) \leq cn \lg n$.

$$T(2) = 2T(0) + 2 = 4$$

Paso base: si $c=1$, probar que $T(2)=4$ se cumple

$$T(2) \leq 1 * 2 \lg 2 ?$$

$$4 \leq 2 ?$$

No, se puede variar c .

Recurrencias

Probar que $T(n) \leq cn \lg n$.

$$T(2) = 2T(0) + 2 = 4$$

Paso base: si $c=3$, probar que $T(2)=4$ se cumple

$$T(2) \leq 3 \cdot 2 \lg 2 ?$$

$$4 \leq 6 ?$$

Si, se termina la demostración

Recurrencias

$$T(n)=T(n-1)+T(n-2)+1, T(1)=O(1), T(2) = O(1)$$

Suponer que la solución es de la forma $T(n)=O(2^n)$

Probar que $T(n) \leq c2^n$.

Se supone que se cumple para $n-1$ y se $n-2$ prueba para n

Hipotesis inductiva: $T(n-1) \leq c2^{(n-1)}$ y $T(n-2) \leq c2^{(n-2)}$

Recurrencias

$$T(n)=T(n-1)+T(n-2)+1, T(1)=O(1), T(2) = O(1)$$

Ahora se debe probar que: $T(n) \leq c2^n$

$$T(1) \leq c2^1 \rightarrow 1 \leq 2*c$$

$$T(2) \leq c2^2 \rightarrow 1 \leq 4*c$$

$$T(3) \leq c2^3 \rightarrow 2 \leq 8*c$$

$$T(4) \leq c2^4 \rightarrow 3 \leq 16*c$$

$$T(5) \leq c2^5 \rightarrow 5 \leq 32*c$$

$$T(6) \leq c2^6 \rightarrow 8 \leq 64*c$$

$$T(7) \leq c2^7 \rightarrow 13 \leq 128*c$$

$$T(8) \leq c2^8 \rightarrow 21 \leq 256*c$$

Con $c = 1$, se cumple.

Referencias

Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. 2009. Introduction to Algorithms, Third Edition (3rd ed.). The MIT Press. Chapter 4

Gracias

Próximo tema:

Divide y vencerás