$$1+2+3+4+5+6+\cdots+100$$

$$\sum_{i=1}^{100} i$$

Valor $2+4+6+8+10+\cdots+1000$

i	1	2	3	4	5	500

$$\sum_{i=1}^{500} 2i$$

$$\sum_{k=1}^{n} c = cn \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} \qquad \sum_{k=0}^{n} ar^k = \frac{ar^{(n+1)} - a}{r-1} \text{ Si } r \neq 1 \qquad \sum_{k=0}^{n} ar^k = (n+1)a \text{ Si } r = 1$$

$$\sum_{k=1}^{100} 200 = 20000$$

$$\sum_{i=-3}^{n} i^2 = \sum_{i=1}^{n+4} (i-4)^2$$

$$\sum_{i=1}^{n+4} i^2 - 8i + 16 = \sum_{i=1}^{n+4} i^2 - 8\sum_{i=1}^{n+4} i + \sum_{i=1}^{n+4} 16$$

$$\frac{(n+4)(n+5)(2n+9)}{6} - \frac{8(n+4)(n+5)}{2} + 16(n+4)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Passo base

$P(1)$

$$\sum_{i=1}^{1} i = 1 \qquad \frac{1(2)}{2} \qquad 1 = 1 \checkmark$$

$P(n) \longrightarrow P(n+1)$

$$\frac{(n+1)(n+2)}{2}$$

$P(n)$

$$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + \sum_{i=n+1}^{n+1} i = \frac{n(n+1)}{2} + (n+1)$$

$$\frac{n(n+1)}{2} + \frac{2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$$

1. Demuestre por inducción matemática que:

$$\sum_{i=-3}^{n} i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} + 14$$

Paso base $P(-3)$

$$\sum_{i=-3}^{-3} i^2 = (-3)^2 = 9 \qquad \frac{(-3)^3}{3} + \frac{(-3)^2}{2} + \frac{-3}{6} + 14$$

$$-9 + \frac{9}{2} - \frac{1}{2} + 14$$

$$-9 + \frac{8}{2} + 14 = -5 + 14 = 9$$

Paso inductivo

$$\frac{(n+1)^3}{3} + \frac{(n+1)^2}{2} + \frac{n+1}{6} + 14$$

n^3+3n^2+3n+1

$P(n)$

$$\sum_{i=-3}^{n+1} i^2 = \sum_{i=-3}^{n} i^2 + (n+1)^2$$

$$\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} + 14 + (n+1)^2$$

$$\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} + n^2 + 2n + 1 + 14$$

$\frac{1}{6} + \frac{5}{6}$

$$\frac{n^3}{3} + \frac{n^2}{2} + n^2 + 2n + \frac{5}{6} + \frac{2n}{6} + \frac{1}{6} + 14$$

$\frac{2n}{2} + n$

$\frac{2}{6} + \frac{3}{6}$

$$\frac{n^3}{3} + n^2 + \frac{n^2}{2} + \frac{2n}{2} + \frac{1}{2} + n + \frac{3}{6} + \frac{n+1}{6} + 14$$

$$\frac{n^3}{3} + \frac{3n^2}{3} + \frac{3n}{3} + \frac{1}{3} + \frac{(n+1)^2}{2} + \frac{n+1}{6} + 14$$

$$\frac{(n+1)^3}{3} + \frac{(n+1)^2}{2} + \frac{n+1}{6} + 14 \quad \text{QED}$$