$$1+2+3+4+5+6+\cdots+100$$

$$\sum_{i=1}^{100} i$$

Valor $2+4+6+8+10+\cdots+1000$

$$\begin{array}{ccccccc} 2 & 4 & 6 & 8 & 10 & \cdots & 1000 \\ i: 1 & 2 & 3 & 4 & 5 & & 500 \end{array}$$

$$\sum_{i=1}^{500} 2i$$

$$\sum_{k=1}^{n} c = cn \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} \qquad \sum_{k=0}^{n} ar^k = \frac{ar^{(n+1)}-a}{r-1} \text{ Si } r \neq 1 \qquad \sum_{k=0}^{n} ar^k = (n+1)a \text{ Si } r = 1$$

$$\sum_{k=1}^{100} 200 = 20000$$

$$\sum_{i=-3}^{n} i^2 = \sum_{i=1}^{n+4} (i-4)^2$$

$$\sum_{i=1}^{n+4} i^2 - 8i + 16 = \sum_{i=1}^{n+4} i^2 - 8\sum_{i=1}^{n+4} i + \sum_{i=1}^{n+4} 16$$

$$\frac{(n+4)(n+5)(2n+9)}{6} - \frac{8(n+4)(n+5)}{2} + 16(n+4)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Paso base

$P(1)$

$$\sum_{i=1}^{1} i = 1 \qquad \frac{1(\cancel{2})}{\cancel{2}} \qquad 1 = 1 \checkmark$$

$P(n) \longrightarrow P(n+1)$

$$\frac{(n+1)(n+2)}{2}$$

$P(n)$

$$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + \sum_{i=n+1}^{n+1} i = \frac{n(n+1)}{2} + (n+1)$$

$$\frac{n(n+1)}{2} + \frac{2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$$

1. Demuestre por inducción matemática que:

$$\sum_{i=-3}^{n} i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} + 14$$

Paso base $P(-3)$

$$\sum_{i=-3}^{-3} i^2 = (-3)^2 = 9 \qquad \frac{(-3)^3}{3} + \frac{(-3)^2}{2} + \frac{-3}{6} + 14$$

$$-9 + \frac{9}{2} - \frac{1}{2} + 14$$

$$-9 + \frac{8}{2} + 14 = -5 + 14 = 9$$

Paso inductivo

$$\frac{\overbrace{(n+1)^3}^{n^3+3n^2+3n+1}}{3} + \frac{(n+1)^2}{2} + \frac{n+1}{6} + \boxed{14}$$

$$\sum_{i=-3}^{n+1} i^2 = \underbrace{\sum_{i=-3}^{n} i^2}_{P(n)} + (n+1)^2$$

$$\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} + \boxed{14} + (n+1)^2$$

$$\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} + n^2 + 2n + \underbrace{1}_{\frac{1}{6}+\frac{5}{6}} + 14$$

$$\frac{n^3}{3} + \frac{n^2}{2} + n^2 + \underbrace{2n}_{\frac{2n}{2}+n} + \frac{5}{6} + \frac{n}{6} + \frac{1}{6} + 14$$

$$\frac{n^3}{3} + n^2 + \boxed{\frac{n^2}{2} + \frac{2n}{2} + \frac{1}{2}} + n + \frac{3}{6} + \frac{n+1}{6} + 14$$

$$\frac{n^3}{3} + \frac{3n^2}{3} + \frac{3n}{3} + \frac{2}{6}\frac{1}{3} + \frac{(n+1)^2}{2} + \frac{n+1}{6} + 14$$

$$\frac{(n+1)^3}{3} + \frac{(n+1)^2}{2} + \frac{n+1}{6} + 14 \qquad \phi AD!$$

2. Demuestre por inducción matemática que:

$$\sum_{i=0}^{n}(8i^2 + \frac{3}{6}) = \frac{(n+1)(8n(2n+1)+3)}{6}$$

$P(0)$

$\frac{3}{6}$, $\frac{(1)(3)}{6}$ ✓

$$\frac{(n+2)(8(n+1)(2n+3)+3)}{6}$$

$$\frac{(n+2)((8n+8)(2n+3))+3}{6} = \frac{(n+2)(16n^2+24n+16n+24+3)}{6}$$

$$\frac{(n+2)(16n^2+40n+27)}{6} = \frac{16n^3+40n^2+27n+32n^2+80n+54}{6}$$

$$\frac{16n^3+72n^2+107n+54}{6}$$

$$\sum_{i=0}^{n+1}\left(8i^2+\frac{3}{6}\right) = \boxed{\sum_{i=0}^{n}\left(8i^2+\frac{3}{6}\right)} + 8(n+1)^2 + \frac{3}{6}$$

$$= \frac{(n+1)(8n(2n+1)+3)}{6} + \frac{48(n+1)^2}{6} + \frac{3}{6} = \frac{(n+1)(16n^2+8n+3)+48(n+1)^2+3}{6}$$

$$\frac{16n^3+8n^2+3n+16n^2+8n+3+48n^2+96n+48+3}{6}$$

24+48
72+80

$$\frac{16n^3+72n^2+107n+54}{6}$$

1, 2, 3 $= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

A

1, 2, ..., n $= \boxed{\{\emptyset, \{1\}, \ldots A\}} = 2^n$

1, 2, n, n+1

0, 1, 2, 3

$P(A_2) \subset P(A_3)$