$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$P(0)$

$$\sum_{i=0}^{0} i^2 = 0 \qquad \frac{0(0+1)(2(0)+1)}{6} = 0$$

$P(n) \longrightarrow P(n+1)$

$$\frac{(n+1)(n+2)(2n+3)}{6}$$

QED!

$$\sum_{i=0}^{n+1} i^2 = \sum_{i=0}^{n} i^2 + \sum_{i=n+1}^{n+1} i^2 = \frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)^2}{6}$$

$$\frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{\boxed{(n+1)}(n(2n+1) + 6(n+1))}{6}$$

$$\frac{(n+1)(2n^2+n+6n+6)}{6} = \frac{(n+1)(2n^2+7n+6)}{6}$$

$$\begin{array}{r|l} 2n^2+7n+6 & 2n+3 \\ \hline -2n^2-4n & n+2 \\ \hline 3n+6 & \\ -3n-6 & \\ \hline 0 & \end{array}$$

$(n+2)(2n+3)$

$$\frac{(n+1)(n+2)(2n+3)}{6}.$$

$$\sum_{i=-10000}^{2n^2}(i^2+2) = \sum_{i=-10000}^{2n^2} i^2 + \sum_{i=-10000}^{2n^2} 2$$

$$\rightarrow \sum_{k=1}^{n} c = cn \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \rightarrow \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} \qquad \sum_{k=0}^{n} ar^k = \frac{ar^{(n+1)}-a}{r-1} \text{ Si } r \neq 1 \qquad \sum_{k=0}^{n} ar^k = (n+1)a \text{ Si } r = 1$$

$$\sum_{i=1}^{2n^2+10001}(i-10001)^2 + \sum_{i=1}^{2n^2+10001} 2$$

$$\sum_{i=1}^{2n^2+10001} i^2 - 20002i + 10001^2 + \sum_{i=1}^{2n^2+10001} 2$$

$$x = 2n^2+10001$$

$$\frac{x(x+1)(2x+1)}{6} - 20002\frac{x(x+1)}{2} + x \cdot 10001^2$$

$$+ 2x$$