Sistema congruencia lineal

$$\begin{cases} X \equiv a_1 \bmod m_1 \\ X \equiv a_2 \bmod m_2 \\ \vdots \\ X \equiv a_n \bmod m_n \end{cases}$$

$$\begin{cases} X \equiv 2 \bmod 3 \\ X \equiv 3 \bmod 5 \\ X \equiv 2 \bmod 7 \end{cases}$$

1) $m = m_1 \times m_2 \times \dots \times m_n$

2) $M_1 = m/m_1 \quad M_2 = m/m_2 \quad \dots \quad M_n = m/m_n$

3) $x = a_1 M_1 y_1 + a_2 M_2 y_2 + \dots + a_n M_n y_n$

4) $M_k y_k \equiv 1 \bmod m_k$ Inverso

$M_k a_k (y_k) \equiv a_k \bmod m_k$

Inver $M_k \bmod m_k = y_k$

a_1: $X \equiv 2 \bmod 3$ (m_1)

a_2: $X \equiv 3 \bmod 5$ (m_2)

a_3: $X \equiv 2 \bmod 7$ (m_3)

$m = 3 \times 5 \times 7 = 105$

1) $M_1 = \frac{105}{3} = 35 \qquad M_2 = \frac{105}{5} = 21$

$M_3 = \frac{105}{7} = 15$

$y_1 = -1$

$35 \bmod 3$

$M_1 \bmod m_1$

a b

1) $\text{mcd}(35, 3) = 1$ ✓

$35 \bmod 3 = 2 \qquad 2 = 35 - (11)3$

$3 \bmod 2 = 1 \qquad 1 = 3 - 2$

$2 \bmod 1 = 0$ ✗

$1 = as + bt$

$1 = 3 - 2$

$1 = 3 - (35 - 11(3))$

$1 = (12)3 - 35$

$1 = 35(-1) + (12)3$

$s \qquad t$

$y_2 = 1$

$M_2 \bmod m_2$

$21 \bmod 5$

1) $\mathrm{mcd}(21, 5) = 1$

$21 \bmod 5 = 1 \quad 1 = 21 - 5(4)$

$1 = 21(1) + 5(-4)$

$s \quad t$

$y_3 = 1$

$M_3 \bmod m_3$

$15 \bmod 7$

$\mathrm{mcd}(15, 7) = 1$

$15 \bmod 7 = 1 \quad 1 = 15 - 7(2)$

$1 = 15(1) + 7(-2)$

$X = a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3$

$X = (2)(35)(-1) + (3)(21)(1) + (2)(15)(1)$

$X = -70 + 63 + 30 = 23$

$\rightarrow 23 \equiv 2 \bmod 3$ ✓

$\rightarrow 23 \equiv 3 \bmod 5$

$\rightarrow 23 \equiv 2 \bmod 7$