$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

\# probls — Tamaño — Costo conquis

$\underbrace{T(1) = \Theta(1)}_{\text{Caso base}}$

$T(n) = 2T\left(\frac{n}{2}\right) + n$

3 5 7 8 2 4 8

2 3 4 5 7 8

$T(n) = T\left(\frac{n}{2}\right) + 1 \simeq O(\log(n))$

1 0

|

1 1

|

$T\left(\frac{n}{2^2}\right)$ 2

1 0

|

$T\left(\frac{n}{2}\right)$ 1

$T\left(\frac{n}{2}\right) = T\left(\frac{n}{2^2}\right) + 1$

$T\left(\frac{n}{2^2}\right) = T\left(\frac{n}{2^3}\right) + 1$

1 0

|

1 1

|

1 2

|

$T\left(\frac{n}{2^3}\right)$ 3

h

1 0

|

1 1

|

1 2

|

1 3

⋮

$T\left(\frac{n}{2^i}\right)$ i

$T(1) = \Theta(1) = c$

$T\left(\frac{n}{2^i}\right) = T(1)$

$\frac{n}{2^i} = 1 \qquad 2^i = n$

$\log_2(n) = \log_2(2^i)$

$\log_2(n) = i$

1	1	0
1	1	1
1	1	2
	⋮	
1	1	$\log_2(n) - 1$
	$T(1)$	$\log_2(n)$

$$\sum_{i=0}^{\log_2(n)-1} 1 = 1 + \sum_{i=1}^{\log_2(n)-1} 1 + T(1)$$

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

$$1 + (\log_2(n) - 1)(1) + T(1)$$

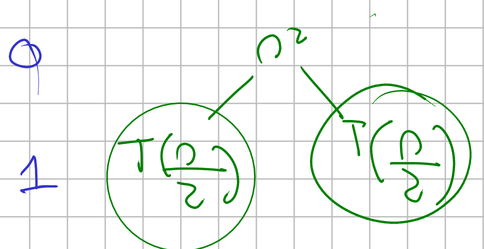
$$\cancel{1} + \log_2(n) \cancel{- 1} + T(1)$$

$$\boxed{\log_2(n) + c} \to O(\log_2(n))$$

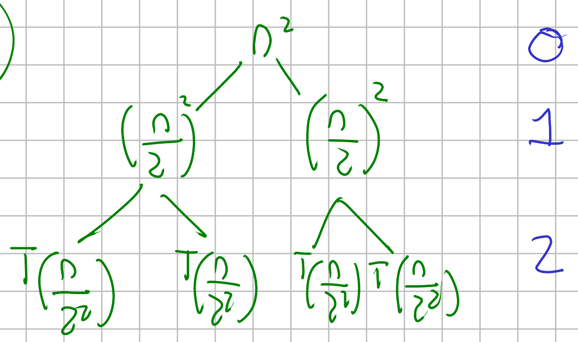
Ω

Θ

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2 \qquad T(1) = \Theta(1)$$



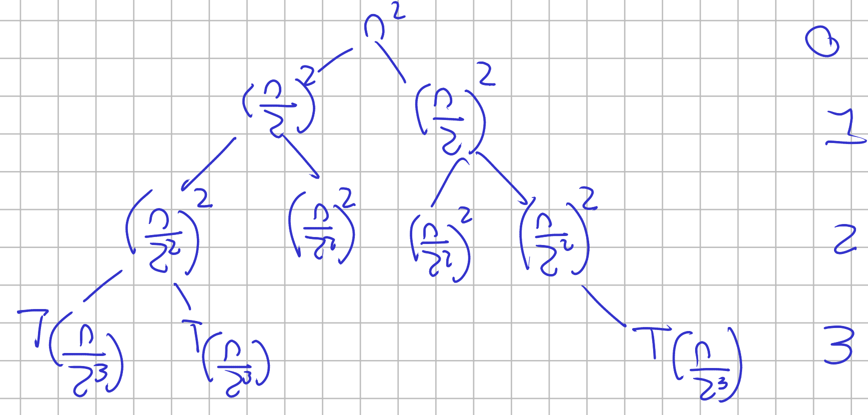
$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2$$

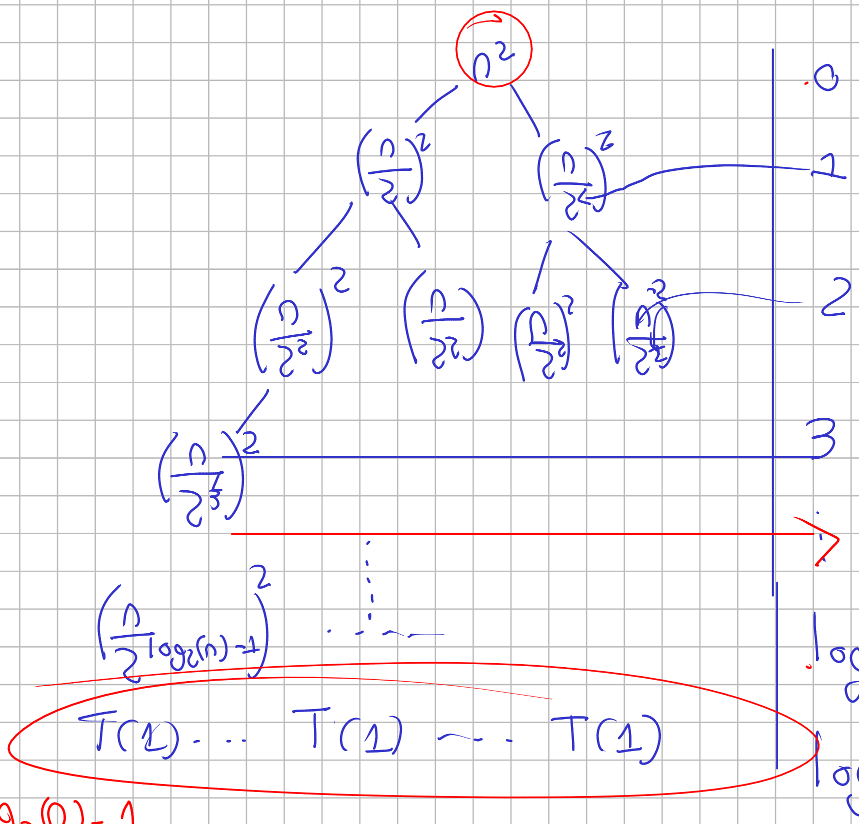


$T(1)$

$1 = \frac{n}{2^i}$

$i = \log_2(n)$





$$n^2$$
$$2\left(\frac{n}{2}\right)^2$$
$$2^2\left(\frac{n}{2^2}\right)^2$$
$$2^3\left(\frac{n}{2^3}\right)^2$$
$$2^i\left(\frac{n}{2^i}\right)^2$$
$$2^{\log_2(n)-1}\left(\frac{n}{2^{\log_2(n)-1}}\right)^2$$
$$2^{\log_2(n)} = n$$
$$n^{\log_2(2)} = n$$

$$\sum_{i=0}^{\log_2(n)-1} 2^i\left(\frac{n}{2^i}\right)^2 + T(1)\times n$$

$$\left(\frac{n}{2^i}\right)^2 = \frac{n^2}{(2^i)^2} = \frac{n^2}{(2^2)^i}$$

$$\left(\frac{1}{2}\right)^i n^2 = \left(\frac{2}{4}\right)^i n^2 = \frac{2^i n^2}{4^i} = \frac{n^2}{4^i}$$

$$\sum_{i=0}^{\log_2(n)-1}\left(\frac{1}{2}\right)^i n^2 + T(1)n$$

$$\sum_{i=0}^{n} ar^i = \frac{ar^{n+1}-a}{r-1} \qquad a = n^2,\ r = \frac{1}{2}$$

$$F^{-1}(F(x)) = x$$
$$F^{-1}(x) = \frac{1}{x}$$
$$F(x) = x$$
$$F(F(x)) = x$$

$$\frac{n^2\left(\frac{1}{2}\right)^{\log_2(n)} - n^2}{\frac{1}{2} - 1} = \frac{n^2\times n^{\log_2(2^{-1})} - n^2}{-\frac{1}{2}} + T(1)n$$

$$= -2(n - n^2) + T(1)n$$

$$\boxed{2n^2 - 2n + T(1)n} \rightarrow \Theta(n^2)$$

$$\boxed{T(1) = C}$$

$$n^2 \times n^{\log_2\left(\frac{1}{2}\right)}$$

$$n^2 \times n^{\cancel{\log(2^{-1})}\ -1}$$

$$n^2 \times n^{-1} = n^{2-1} = n$$

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n) = \Theta(n^{\log_b a})$

Si $f(n) = O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon > 0$

2. $T(n) = \Theta(n^{\log_b a} \lg n)$

Si $f(n) = \Theta(n^{\log_b a})$ para algún $\varepsilon > 0$

3. $T(n) = \Theta(f(n))$

Si $f(n) = \Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon > 0$ y si af(n/b)≤cf(n) para algun c<1

$F(n)$ a b

$T(n) = 2T(\frac{n}{2}) + n$

n es $O(n^{\log_2 2 - \epsilon})$

n es $O(n^{1-\epsilon})$

n es $\Theta(n^{\log_2 2})$

n es $\Theta(n)$

$\boxed{T(n) \in (n \log(n))}$

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n) = \Theta(n^{\log_b a})$

Si $f(n) = O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon > 0$

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Si $f(n) = \Theta(n^{\log_b a})$ para algún $\varepsilon > 0$

3. $T(n) = \Theta(f(n))$

Si $f(n) = \Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon > 0$ y si af(n/b)≤cf(n) para algun c<1

$a = 1$ $b = 2$

$T(n) = T(\frac{n}{2}) + 1$

1 es $O(n^{\log_2 1 - \epsilon})$

1 es $O(n^{0-\epsilon})$

1 es $\Theta(n^0)$

1 es $\Theta(1)$

$T(n) = n^0 \log(n)$

$T(n) = \log(n)$

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n) = \Theta(n^{\log_b a})$

Si $f(n) = O(n^{\log_b a-\varepsilon})$ para algún $\varepsilon > 0$

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Si $f(n) = \Theta(n^{\log_b a})$ para algún $\varepsilon > 0$

3. $T(n) = \Theta(f(n))$

Si $f(n) = \Omega(n^{\log_b a+\varepsilon})$ para algún $\varepsilon > 0$ y si af(n/b)≤cf(n) para algun c<1

$\log_2 2 = 1$

$a = 2$ $b = 2$ $F(n) = n^2$

$T(n) = 2T(\frac{n}{2}) + n^2$

n^2 es $O(n^{1-\epsilon})$

n^2 es $\Theta(n)$

n^2 es $\Omega(n^{1+\epsilon})$

$2\left(\frac{n}{2}\right)^2 \leq c \times n^2$

$\frac{1}{2} n^2 \leq c \times n^2$

$c \leq \frac{1}{2}$

$\boxed{\Theta(n^2)}$

Dado T(n) = aT(n/b) + f(n), donde a≥1, b>1, se puede acotar asintóticamente como sigue:

1. $T(n) = \Theta(n^{\log_b a})$

Si $f(n) = \underline{O(n^{\log_b a - \varepsilon})}$ para algún $\varepsilon > 0$

$T(n) = 2T\left(\frac{n}{2}\right) + n^3$

$\log_b a = \log_2 2 = 1$

2. $T(n) = \Theta(n^{\log_b a} \lg n)$

Si $f(n) = \Theta(n^{\log_b a})$ para algún $\varepsilon > 0$

$f(n) \sim O(n^{1-\varepsilon})$ X

$f(n) \sim \Theta(n)$ X

3. $T(n) = \Theta(f(n))$

Si $f(n) = \Omega(n^{\log_b a + \varepsilon})$ para algún $\varepsilon > 0$ y si af(n/b)≤cf(n) para algun c<1

$f(n) = \Omega(n^{1+\varepsilon})$

$n^3 = \Omega(n^{2+\varepsilon})$

$2\left(\frac{n}{2}\right)^3 \leq c\, n^3$ $\quad \frac{1}{4} \leq c$ ✓

$\Theta(n^3)$

$\boxed{T(n) = 3T\left(\frac{n}{3}\right) + n}$