$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = 1+2+3+\cdots+n$$

$$P(1) \qquad \frac{1(\cancel{2})}{\cancel{2}} = 1 \qquad \sum_{i=1}^{1} i = 1$$

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2} \quad \checkmark$$

$$\frac{(k+1)(k+2)}{2}$$

$$\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + \overbrace{\sum_{i=k+1}^{k+1} i}^{} = \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$\frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$\sum_{i=1}^{n} (2i-1) = 1+3+5+7+\cdots 2n-1 = n^2$$

Paso base $\quad P(1) \qquad \sum_{i=1}^{1}(2i-1) = (1)^2 \Rightarrow 2(1)-1 = 1$

$$= 1 = 1 \quad \checkmark$$

Paso inductivo $\quad P(k) \longrightarrow \underline{P(k+1)}$

$$\sum_{i=1}^{k} 2i-1 = k^2 \quad \checkmark$$

$$\sum_{i=1}^{k+1} 2i-1 = (k+1)^2 \Rightarrow \underbrace{\sum_{i=1}^{k}(2i-1)}_{} + \sum_{i=k+1}^{k+1}(2i-1)$$

$1+3+5+7+\ldots\ldots$

$$k^2 + 2(k+1) - 1$$

$$k^2 + 2k + 2 - 1$$

$$\boxed{k^2 + 2k + 1}$$

$n < 2^n$

$2^{n+1} = 2 \times 2^n$

Paso base $\quad P(1) \quad 1 < 2^1$ ✓

$P(K)$

$\boxed{K < 2^K}$

$8 < 10$ ✓

$9 < 11$ ✓

Paso inductivo $\quad P(K) \to P(K+1)$

$n=1 \quad 1 < 2^1 \quad 1 < 2$

$n=2 \quad 2 < 2^2 \quad 2 < 4$

$K+1 < 2^{K+1}$

$K+1 < 2(2^K)$

$K+1 < 2^K + 2^K$

$1 \leq 2$

$2 \leq 4$

$\to \boxed{1 \leq 2^K} \quad K \geq 1$

$K+1 < 2^K + 1$

$P(K) \leq P(K+1)$

$K+1 \leq K+1 \Rightarrow T$

$\cancel{2^K + 1 \leq 2^K + 2^K}$

$P(K) \leq P(K+1)$

$2^0 + 2^1 + 2^2$

$$\overset{2^0}{1} + \overset{2^1}{2} + 2^2 + \cdots + 2^n = 2^{n+1} - 1$$

$$\sum_{i=0}^{n} ar^n = \frac{ar^{n+1} - a}{r-1}$$

Paso base

$P(0) \quad 1 = 2^{0+1} - 1$

$1 = 2^1 - 1$

$\boxed{1 = 1}$

Paso inductivo $\quad P(K) \to P(K+1)$

$P(K) \quad 2^0 + 2^1 + 2^2 + \cdots 2^K = 2^{K+1} - 1$

$P(K+1) \quad \underbrace{2^0 + 2^1 + 2^2 + \cdots 2^K} + 2^{K+1} = 2^{K+2} - 1$

$2^{K+1} - 1 + 2^{K+1} = 2^{K+2} - 1$

$2(2^{K+1}) - 1 = 2^{K+2} - 1$

$2^{K+1+1} - 1 = 2^{K+2} - 1$

$2^{K+2} - 1 = 2^{K+2} - 1$

QED!

$$2(-7)^0 + 2(-7)^1 + 2(-7)^2 + \dots + 2(-7)^n = \frac{1-(-7)^{n+1}}{4}$$

$P(0)$

$$2 = \frac{1-(-7)^{0+1}}{4} = \frac{1-(-7)^1}{4} = \frac{8}{4} = 2$$

$P(K) \to P(K+1)$

$$2(-7)^0 + 2(-7)^1 + 2(-7)^2 + \dots + 2(-7)^k + 2(-7)^{k+1} = \frac{1-(-7)^{k+2}}{4}$$

$$\frac{1-(-7)^{k+1}}{4} + 2(-7)^{k+1} = \frac{1-(-7)^{k+2}}{4}$$

$$\frac{1-(-7)^{k+1} + 8(-7)^{k+1}}{4}$$

$$\frac{1-\left((-7)^{k+1} - 8(-7)^{k+1}\right)}{1}$$

$$\frac{1-\left((1-8)(-7)^{k+1}\right)}{4} = \frac{1-(-7)(-7)^{k+1}}{4}$$

$$\frac{1-(-7)^{k+2}}{4}$$

QED!