$\forall n\, P(n)$

$n \in$ Personas

$P(n)$ n no tiene amigos

$$\overbrace{1+3+5+\dots+}^{n} = n^2$$

$2k-1$

$$(2(1)-1)+(2(2)-1)+(2(3)-1)+\dots+(2(n)-1) = n^2$$

$$\sum_{i=1}^{n}(2i-1)$$

$P(1)$

$$\sum_{i=1}^{1} 2i-1 = 2(1)-1 = (1)^2$$

$$1=1$$

Paso base

$P(K)$

$$\sum_{i=1}^{K}(2i-1) = K^2 \quad \text{Verdad}$$

$P(K+1)$

$$\sum_{i=1}^{K+1}(2i-1) = (K+1)^2$$

$$\sum_{i=1}^{10} i = 1+2+\dots+10$$

$$\sum_{i=1}^{5} i + \sum_{i=6}^{10} i$$

$$\sum_{i=1}^{K}(2i-1) + \sum_{i=K+1}^{K+1}(2i-1) = (K+1)^2$$

$(a+b)^2$

$a^2+2ab+b^2$

$(K+1)^2$

K^2+2K+1

$$K^2 + (2(K+1)-1) = (K+1)^2$$

$$K^2+2K+2-1 = (K+1)^2$$

$$\underline{K^2+2K+1} = \underline{(K+1)^2}$$

QED!

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$P(1)$ $\qquad 1 = \frac{1(\cancel{1}+1)}{\cancel{2}} \qquad 1 = 1$ ✓

$P(K) \rightarrow P(K+1)$

$$\sum_{i=1}^{K} i = \frac{K(K+1)}{2}$$

$1+2+3+4+\dots+K+(K+1)$

$$\sum_{i=1}^{K+1} i = \frac{(K+1)(K+2)}{2}$$

$$\sum_{i=1}^{K} i + (K+1) = \frac{(K+1)(K+2)}{2}$$

$$\sum_{i=K+1}^{K+1} i$$

$$\frac{K(K+1)}{2} + (K+1) = \frac{(K+1)(K+2)}{2}$$

$\varphi \in D$

$$\frac{K(K+1)}{2} + \frac{2(K+1)}{2} = \frac{(K+1)(K+2)}{2}$$

$$\frac{(K+1)(K+2)}{2} = \frac{(K+1)(K+2)}{2}$$

$n < 2^n \qquad n \geq 1$

Paso base $\qquad 1 < 2^1 \qquad 1 < 2$

Paso inductivo

$a^b a^c = a^{b+c}$

$a + a = 2a$

$p(K) \qquad K < 2^K$

$P(K+1) \qquad K+1 < 2^{K+1}$

$K+1 < 2^1 \times 2^K$

$K+1 < 2^K + 2^K$

$\left\{ \begin{array}{l} K < 2^K \\ K+1 < 2^K + 1 \end{array} \right.$

Sucesor de un número natural

n -> n+1

n <= n+1

$K+1 \leq K+1$ ✓

$\cancel{2^K} + 1 \leq \cancel{2^K} + 2^K$

$\boxed{1 \leq 2^K}$

$1 < 2^1$

$1 < 2^2$

$1 < 2^3$

$$\sum_{j=0}^{n} ar^j = a + ar + ar^2 + \cdots + ar^n = \frac{ar^{n+1} - a}{r-1}, \text{ cuando } r \neq 1.$$

$P(0)$ $\quad ar^0 = \frac{ar^1 - a}{r-1}$ $\qquad r \neq 1$

$a = \frac{ar - a}{r-1}$ $\qquad a = a\left(\frac{r-1}{r-1}\right)$ $\qquad a = a$ ✓

$P(k) \longrightarrow P(k+1)$

$ar^0 + ar^1 + ar^2 + \dots ar^k = \frac{ar^{k+1} - a}{r-1} \Leftarrow T \quad P(k)$

$P(k+1)$

$\boxed{ar^0 + ar^1 + ar^2 + \dots ar^k} + ar^{k+1} = \frac{ar^{k+2} - a}{r-1}$

$\frac{ar^{k+1} - a}{r-1} + ar^{k+1} = \frac{ar^{k+2} - a}{r-1}$

$\frac{ar^{k+1} - a + (r-1)ar^{k+1}}{r-1} = \frac{ar^{k+2} - a}{r-1}$

$\frac{(\cancel{1} + r - \cancel{1})ar^{k+1} - a}{r-1}$ $\qquad$ QED

$\frac{r^1 a r^{k+1} - a}{r-1} = \frac{ar^{k+2} - a}{r-1}$