

by a translation subroutine. A new subroutine that effects certain scanning and related operations and which can be called in FORTRAN programs has been coded recently [15]. This SAPST subroutine adopts the syntax machine approach of Glennie [16] and can be used to deal with expanded meta-vocabularies. Scanning time, particularly of verbal material, depends on comparisons of quotations in the definition table with portions of the input string. Simultaneous comparisons of input material with several quotations would speed some scans considerably—a computer with several accumulators would be suited to this type of work.

Publication of this material has been delayed for some years in the hope that the implications of scanning and syntax definition would become clearer. There is no doubt now that syntax description and analysis are important, and they may provide new bridges between computing and biology [6] and between science and scholarship.

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## On Ambiguity in Phrase Structure Languages

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Let a *phrase structure language* be defined informally as a language defined by a set of definitions in the Backus notation [1] (more formal definitions may be found in [2, 3, 4]). The set of definitions is itself called a *phrase structure grammar*. A phrase structure grammar is *ambiguous* if there exists a sentence in the defined language which may be assigned more than one structure consistent with the grammar. Such an ambiguity in ALGOL 60, for example, is

if  $\beta_1$  then  $\phi$  if  $\beta_2$  then  $\Sigma_1$  else  $\Sigma_2$

where  $\beta$  represents any Boolean expression,  $\phi$  a **for** clause, and  $\Sigma$  an unconditional statement.

Attempts have been made to devise an algorithm to determine for an arbitrarily given phrase structure grammar whether or not it is ambiguous. Such attempts may yield useful heuristic procedures, but, as will be shown below, a general algorithm does not exist.

Consider two sets of strings  $\{x_i\}$  and  $\{y_i\}$  over an alphabet, where  $i$  ranges from 1 to  $k$ . Let  $n_i$  signify the digit string which represents the integer  $i$  in decimal notation. Define a language by the following grammar:<sup>1</sup>

$$\begin{aligned} S_1 &\sim x_1 ; n_1 \cup x_2 ; n_2 \cup \cdots \cup x_k ; n_k \cup x_1 S_1, n_1 \cup x_2 S_1, n_2 \\ &\quad \cup \cdots \cup x_k S_1, n_k \\ S_2 &\sim y_1 ; n_1 \cup y_2 ; n_2 \cup \cdots \cup y_k ; n_k \cup y_1 S_2, n_1 \cup y_2 S_2, n_2 \\ &\quad \cup \cdots \cup y_k S_2, n_k \end{aligned}$$

sentence  $\sim S_1 \cup S_2$

Then  $S_1$  contains those strings of the form

$$x_{i_1} x_{i_2} x_{i_3} \cdots x_{i_j} ; n_{i_j}, n_{i_j-1}, \cdots, n_{i_2}, n_{i_1}$$

with  $j \geq 1$ , and  $S_2$  contains those strings of the form

$$y_{i_1} y_{i_2} y_{i_3} \cdots y_{i_j} ; n_{i_j}, n_{i_j-1}, \cdots, n_{i_2}, n_{i_1}$$

with  $j \geq 1$ . Each string in  $S_1$  (or  $S_2$ ) has only one syntactic structure consistent with the definition. Now it is readily seen that the language is ambiguous if and only if there is a string belonging to both  $S_1$  and  $S_2$ , i.e., if there is a non-empty sequence of integers  $i_1, i_2, \cdots, i_j$  such that  $x_{i_1} x_{i_2} \cdots x_{i_j}$  is the same string as  $y_{i_1} y_{i_2} \cdots y_{i_j}$ . For an arbitrary choice of the paired sets  $\{x_i\}$  and  $\{y_i\}$ , however, it has been proven that no decision procedure (algorithm) exists to determine in each case whether such a string exists [3]. Thus there can be no algorithm to determine in each case the ambiguity of a phrase structure grammar.

Certain grammars may be proven unambiguous. Whatever axiom system and rules of operation may be allowed for such proofs, the proofs themselves may be enumerated by an appropriate algorithm. The distinct ways of generating strings in a phrase structure language may also be enumerated. Consider an algorithm which alternates between proving languages unambiguous and generating the strings of the given language. If the given language

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<sup>1</sup> The symbol  $\cup$  is used for union of two classes of strings, rather than the symbol  $|$  used in ALGOL 60 for reasons peculiar to that language.

derived from history, items related to physical and laboratory examinations are also included. On the pages of the Information Code, the code numbers appear alongside the data to which they are assigned. Thus, the same pages can be used to record and also to transfer the data to punched cards and tape, making the data available for analyses including various statistical and significance measurements. The patient's Past Illness examination and Family History codes contain spaces for entering Standard Nomenclature Code numbers of all diseases. If the specific disease is not known, the general area of illness can be recorded and coded (e.g., "The father has arteriosclerosis" would be coded as 460-942). Other sections contain blank code spaces for insertion of additional data which may be of interest to individual investigators.

The fact that the Information Code is quite large would preclude its use in recording medical data routinely. However, it could be used to record data in special studies, either from hospital records or from patient interviews. In addition, individual sections of the code could be removed and used separately to record data in areas of special interest without using the entire code. Additions and modifications of the input data can also be made without changing the code numbers. In its present form, the Information Code would not enable patients to record information directly on the code sheets, and it would best be used by physicians.

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CARD #48				Bromsulphalein (% ret.)			
P. BLOOD CHEMISTRIES				Spec. I	<input type="checkbox"/>	<input type="checkbox"/>	Col. 35-36
Amylase (units)				" II	<input type="checkbox"/>	<input type="checkbox"/>	Col. 37-38
Spec. I				" III	<input type="checkbox"/>	<input type="checkbox"/>	Col. 39-40
" II				" IV	<input type="checkbox"/>	<input type="checkbox"/>	Col. 41-42
" III				CARD #50			
" IV				Phosphatase, Alkaline (units)			
Bilirubin Total (mg %) (3 dig. 1 dec.)				(3 dig. 1 dec.)			
Spec. I				Spec. I	<input type="checkbox"/>	<input type="checkbox"/>	Col. 71-73
" II				" II	<input type="checkbox"/>	<input type="checkbox"/>	Col. 74-76
" III				" III	<input type="checkbox"/>	<input type="checkbox"/>	Col. 77-79
" IV				Blank	<input type="checkbox"/>	<input type="checkbox"/>	Col. 80
CARD #49				CARD #51			
History no.-Card no.				History no.-Card no.			
Bilirubin, direct (mg %)				Phosphatase, Alkaline (units)			
(3 dig. 1 dec.)				Spec. IV	<input type="checkbox"/>	<input type="checkbox"/>	Col. 11-13
Spec. I				Protein, total (gms %) (3 dig. 1 dec.)			
" II				Spec. I	<input type="checkbox"/>	<input type="checkbox"/>	Col. 14-16
" III				" II	<input type="checkbox"/>	<input type="checkbox"/>	Col. 17-19
" IV				" III	<input type="checkbox"/>	<input type="checkbox"/>	Col. 20-22
Bilirubin, indirect (mg %)				" IV	<input type="checkbox"/>	<input type="checkbox"/>	Col. 23-25
(3 dig. 1 dec.)				Albumin (gms %) (3 dig. 1 dec.)			
Spec. I				Spec. I	<input type="checkbox"/>	<input type="checkbox"/>	Col. 26-28
" II				" II	<input type="checkbox"/>	<input type="checkbox"/>	Col. 29-31
" III				" III	<input type="checkbox"/>	<input type="checkbox"/>	Col. 32-34
" IV				" IV	<input type="checkbox"/>	<input type="checkbox"/>	Col. 35-37

FIG. 7

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#### ROBERT W. FLOYD—continued

may be proven unambiguous, the algorithm will eventually do so. If it is ambiguous, the algorithm will eventually generate the same string twice. Since no algorithm can decide the ambiguity of every phrase structure grammar, there must be grammars which are unambiguous, but for which no proof exists of freedom from ambiguity.

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