Machine learning : HW 07

王上澤 111598067

1. Find $\Delta w_1 = \eta \frac{\partial J}{\partial w_1}$ of the following CNN using backprop. The activation function from $q1$ to $h1$ and $q2$ to $h2$ is ReLU, the outputs $y1$ and $y2$ are softmax ouput, and the cost function is $J = -\log y1$. Let $w1$ to $w6$ be 1.0, $x1$ = 0.5, $x2$ = 1.0, $x3$ = -0.5, and η = 0.1

   1. Find the y_1

   $$y_1 = \frac{e^{z_1}}{e^{z_1} + e^{z_2}}$$

   Find the z_1 and z_2

   $$z_1 = w_3 q_1 + w_4 q_2 \ z_2 = w_5 q_1 + w_6 q_2$$

   then find the q_1 and q_2

   $$q_1 = max(0, w_1 x_1 + w_2 x_2) \ q_2 = max(0, w_1 x_2 + w_2 x_3)$$

   2. So the y_1 is 0.5, then the cost function is

   $$J = -\log y_1$$

   and we assume the base of cost function to be e so

   $$\frac{\partial J}{\partial y_1} = -\frac{1}{y_1}$$

   than the first backprop is

   $$\frac{\partial J}{\partial y_1} = -\frac{1}{y_1} = -2 \ \frac{\partial J}{\partial y_2} = -\frac{1}{y_2} = -2$$

   3. The derivative of the softmax activation function has the following form:

   $$\frac{\partial}{\partial z_i} y_l = \{ \ y_l(1 - y_l), if \ i = l \ -y_l y_i, if \ i \ \neq l$$

   so next prop is

   $$\frac{\partial y_1}{\partial z_1} = y_1(1 - y_1) = -6 \ \frac{\partial y_2}{\partial z_2} = y_2(1 - y_2) = -6$$

   4. Due to $-6(1) + -6(1) = -12 < 0$ so the derivative of q_1 is 0, so the w_1 won't update.

2. The following shows the LeNet-5 architecture. If we follow the modern design of the convolutional layers (taught in the lecture), compute the number of connections and trainable weights of the network. To compute the results, you need the following parameters for the convolutional layers: kernel size = 5×5, stride = 1, the first convolutional layer is padded, and the second convolutional layer is not padded. To simplify the computations, ignore the bias weights and let the output units be 10.

   1. First layer is a convolution layer
      - input size = 28×28
      - kernel size = 5×5
      - feature number = 6
      - output size = 28×28 (due to the padding)
      - There are 6 feature map, so the total number of trainable weight is $6 \times (5 \times 5)$ for one filter per feature map , and the total number of connection $6 \times (5 \times 5) \times 28 \times 28$
   2. Second layer is sampling layer
      - sampling size = 2×2 (we assume that)
      - input size = $6 \times 28 \times 28$

- output size = $6 \times 14 \times 14$
- numbers of trainable weight = 6 (for every feature map)
- numbers of total connection = $6 \times (2 \times 2) \times 14 \times 14$

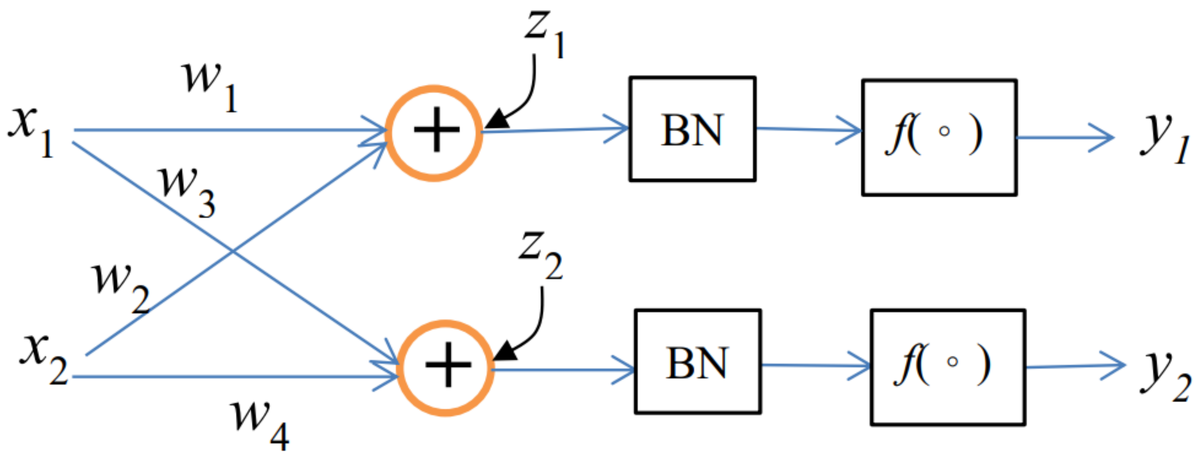
3. Third layer is convolution layer
   - input size = $6 \times 14 \times 14$
   - kernel size = 5×5
   - feature number = 16
   - output size = $16 \times (14 - 5 + 1) \times (14 - 5 + 1)$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	X				X	X	X			X	X	X	X		X	X
1	X	X				X	X	X			X	X	X	X		X
2	X	X	X				X	X	X			X		X	X	X
3		X	X	X			X	X	X	X			X		X	X
4			X	X	X			X	X	X	X		X	X		X
5				X	X	X			X	X	X	X		X	X	X

   - numbers of trainable weight = $6 \times\ (3 \times 5 \times 5 + 1)\ + 6 \times\ (4 \times 5 \times 5 + 1)\ + 3 \times\ (4 \times 5 \times 5 + 1)\ + 1 \times\ (6 \times 5 \times 5 + 1)$
   - numbers of total connection = $10 \times 10 \times [6 \times\ (3 \times 5 \times 5 + 1)\ + 6 \times\ (4 \times 5 \times 5 + 1)\ + 3 \times\ (4 \times 5 \times 5 + 1)\ + 1 \times\ (6 \times 5 \times 5 + 1)\ ]$
4. Fourth layer is sampling layer
   - input size = $16 \times 10 \times 10$
   - sampling size = 2×2
   - sample number = 16
   - output size = $16 \times 5 \times 5$
   - numbers of trainable weight = 16 (for every feature map)
   - numbers of total connection = $16 \times (2 \times 2) \times 5 \times 5$
5. Fifth layer is convolution layer
   - input size = $16 \times 5 \times 5$
   - kernel size = 5×5
   - feature number = 120
   - output size = $120 \times (5 - 5 + 1) \times (5 - 5 + 1)$
   - numbers of trainable weight = $120 \times (5 \times 5 \times 16)$
   - numbers of total connection = $120 \times (5 \times 5 \times 16)$
6. Sixth layer is full connect layer
   - input size = 120
   - output size = 84
   - numbers of trainable weight = 120×84
   - numbers of total connection = 120×84
7. Seventh layer is full connect layer
   - input size = 84
   - output size = 10
   - numbers of trainable weight = 84×10
   - numbers of total connection = 84×10

3. For the network shown below, derive the updating rule for β and γ in the batch normalization (pp. 16). Let the desired outputs be d_1 and d_2, and the mini-batch contains N samples. You may use notations of $x_1(i)$, $y_1(i)$, etc. to indicate parameters

associated with the i-th sample. To simplify the discussion, let the activation function be sigmoid and the cost function is MSE.



1. Forward propagation
    1. Input to the batch normalization layer (pre-normalization):

    $$z_j^{(i)} = \sum_k^N w_{jk}^{(i)} x_k^{(i)}$$

    2. Batch normalization transformation:

    $$\hat{z}_j^{(i)} = \gamma_j(\frac{z_j^{(i)} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}) + \beta_j$$

    where $\gamma_j$is the scaling parameter (learnable), β_j is the shifting parameter (learnable), μ_j is the mean of z_j over the mini-batch, σ_j is the standard deviation of z_j over the mini-batch, ϵ is a small constant for numerical stability.
    3. Activation function (sigmoid in this case):

    $$h_j^{(i)} = \sigma(\hat{z}_j^{(i)}) = \frac{1}{1 + e^{-\hat{z}_j^{(i)}}}$$

    4. Output of the batch normalization layer:

    $$y_j^{(i)} = w_j h_j^{(i)}$$

2. Back propagation
    1. Derivative of the cost function with respect to the output of the batch normalization layer:

    $$\frac{\partial L}{\hat{z}_1} = \frac{\partial L}{\partial \hat{y}_1} \cdot \frac{\partial y_1}{\partial \hat{z}_1}$$

    2. Derivative of the cost function with respect to the scaling parameter, γ:

    $$\frac{\partial L}{\partial \gamma_1} = \sum_{i=1}^N \frac{\partial L}{\partial \hat{z}_1} \cdot \frac{\partial \hat{z}_1}{\partial \gamma_1}$$

    since $\hat{z}_1 = \gamma_1(\frac{z_1 - \mu_1}{\sqrt{\sigma_1^2 + \epsilon}}) + \beta_1$,the partial derivative is:

    $$\frac{\partial \hat{z}_1}{\partial \gamma_1} = \frac{z_1 - \mu_1}{\sqrt{\sigma_1^2 + \epsilon}}$$

    Therefore,

    $$\frac{\partial L}{\partial \gamma_1} = \sum_{i=1}^N \frac{\partial L}{\partial \hat{z}_1} \cdot \frac{z_1 - \mu_1}{\sqrt{\sigma_1^2 + \epsilon}}$$

3. Derivative of the cost function with respect to the shifting parameter, β :

$$\frac{\partial L}{\partial \beta_1} = \sum_{i=1}^{N} \frac{\partial L}{\partial \hat{z}_1} \cdot \frac{\partial \hat{z}_1}{\partial \beta_1}$$

Since $\hat{z}_1 = \gamma_1(\frac{z_1 - \mu_1}{\sqrt{\sigma_1^2 + \epsilon}}) + \beta_1$, the partial derivative is:

$$\frac{\partial \hat{z}_1}{\partial \beta_1} = 1$$

Therefore,

$$\frac{\partial L}{\partial \beta_1} = \sum_{i=1}^{N} \frac{\partial L}{\partial \hat{z}_1}$$

3. Now, you can extend this derivation to other nodes in the network and calculate the derivatives with respect to β and γ for those nodes as well. Remember to use the chain rule and sum the contributions from all samples in the mini-batch when calculating the derivatives.

4. Build a 3-layer neural network by busing Keras to classify the Iris dataset. Vary the hidden units from 10 to 100 in the increment of 10 to observe the change of accuracy along with the number of hidden units. As usual, repeat the experiments 10 times to obtain the average accuracy. Use 10 epochs to train the network.

   - Code:

```
import numpy as np
from sklearn.datasets import load_iris
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import LabelBinarizer
from tensorflow import keras

# Load the Iris dataset
iris = load_iris()
X = iris.data
y = iris.target

# One-hot encode the target variable
encoder = LabelBinarizer()
y = encoder.fit_transform(y)

# Set the number of hidden units to vary
hidden_units = range(10, 110, 10)

# Initialize a list to store the accuracies
accuracies = []

# Repeat the experiment 10 times
for _ in range(10):
    # Split the data into training and testing sets
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)

    # Iterate over different hidden units
    for units in hidden_units:
        # Build the neural network model
        model = keras.Sequential([
            keras.layers.Dense(units, input_shape=(4,), activation='relu'),
            keras.layers.Dense(units, activation='relu'),
            keras.layers.Dense(3, activation='softmax')
        ])

        # Compile the model
        model.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
```

```
        # Train the model
        model.fit(X_train, y_train, epochs=10, verbose=0)

        # Evaluate the model on the test set
        _, accuracy = model.evaluate(X_test, y_test, verbose=0)
        accuracies.append(accuracy)

# Calculate the average accuracy
average_accuracy = np.mean(accuracies)

# Print the average accuracy
print("Average accuracy:", average_accuracy*100, "%")
```

- output:

```
Average accuracy: 77.63333275541663 %
```

5. Build the modified LeNet-5, shown on Problem 2, by using Keras to classify the MNIST dataset. Use maxpolling in the pooling layer, the ReLU in the middle layers, and the softmax in the output layer. The MNIST dataset is available in keras via tf.keras.datasets.mnist.load_data(). Use 20 epochs to train the CNN and report the average accuracy after 10 trials.

    - code:

```
import numpy as np
import tensorflow as tf
from tensorflow import keras
from tensorflow.keras import layers
from sklearn.model_selection import train_test_split
from tqdm import tqdm

# Set random seed for reproducibility
np.random.seed(42)
tf.random.set_seed(42)

# Load MNIST dataset
(X_train, y_train), (X_test, y_test) = keras.datasets.mnist.load_data()

# Reshape and normalize the input data
X_train = X_train.reshape(-1, 28, 28, 1).astype("float32") / 255.0
X_test = X_test.reshape(-1, 28, 28, 1).astype("float32") / 255.0

# Split training data into training and validation sets
X_train, X_val, y_train, y_val = train_test_split(X_train, y_train, test_size=0.1,
random_state=42)

# Define the modified LeNet-5 model
model = keras.Sequential([
    layers.Conv2D(6, kernel_size=5, strides=1, activation="relu", input_shape=(28, 28, 1),
padding="same"),
    layers.MaxPooling2D(pool_size=2, strides=2),
    layers.Conv2D(16, kernel_size=5, strides=1, activation="relu"),
    layers.MaxPooling2D(pool_size=2, strides=2),
    layers.Flatten(),
    layers.Dense(120, activation="relu"),
    layers.Dense(84, activation="relu"),
    layers.Dense(10, activation="softmax")
])

# Compile the model
model.compile(loss="sparse_categorical_crossentropy", optimizer="adam", metrics=["accuracy"])
```

```python
# Train the model for 20 epochs and report average accuracy after 10 trials
num_trials = 10
num_epochs = 20
accuracy_scores = []

for _ in tqdm(range(num_trials)):
    model.fit(X_train, y_train, epochs=num_epochs, validation_data=(X_val, y_val), verbose=0)
    _, accuracy = model.evaluate(X_test, y_test, verbose=0)
    accuracy_scores.append(accuracy)

average_accuracy = sum(accuracy_scores) / num_trials
print("Average accuracy after 10 trials:", average_accuracy)
```

- output:

```
Downloading data from https://storage.googleapis.com/tensorflow/tf-keras-datasets/mnist.npz
11490434/11490434 [==============================] - 0s 0us/step
100%|██████████| 10/10 [22:37<00:00, 135.78s/it]
Average accuracy after 10 trials: 0.9915199935436249
```