Week 3 notes

Lecture

SAMPLING WITH REPLACEMENT, WITHOUT ORDERING Picking k objects from n distinct objects with replacement, recording result without regard to order

- How many different ways can you choose k objects from n, with replacement and ignoring order? The answer is the number of different sequences of k for X's and n-1 for '/'s

  To work this out, think back to Example 2.21 - the no. of distinct permutations of k white balls, n-k black balls = $\big(*k ^n) = \big(*{n-l} ^n)$

  The total number of symbols is $\big(*k ^{n+k-1}) = \big(*{n-1} ^{n+k-1}$

Condition Probability

The Conditional Probability of Event A given that event B occurs (or Conditional on B occurring) is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sometimes $P(A|B) = P(A)$ ie. $P(Raining \ in \ Denmark | Wearing \ Socks \ in Melbourne)$, In this case one event does not affect the other as in they are independent events.

In General howver extra information changes the conditional prob of an event. ie:

$$P(it \ rains \ in \ hawthorn|it \ is \ cloudy)$$

in this case $P(A|B) > P(A)$ (fair assumption).In General,

$$P(A|B) \neq P(A)$$

for $P(A|B) = \frac{P(A\capB)}{P(B)}$ effectively, the sample sapce has been reduced to the outcomes in B

** Note ** Remember to distinguish between independent and mutually exclusive events. Independent events do not affect each but can both occur at the same time whereas mutually exclusive events cannot occur at the same time.

If A and B are Independent then:

$$ \begin{aligned} P(A|B) &= P(A) \ P(B|A) &= P(A)P(B) \ P(A \cap B) &= P(A)P(B) \end{aligned} $$

Example 2.24

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Urn - 4 Balls $\rightarrow$ #1 and #2 are black, #3 and #4 are white.
    Select one ball.
        * A = black ball selected$
        * B = even-numbered ""$
        * C = no.on selected ball > 2
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$$ \begin{aligned} P(A) &= \frac{2}{4} = \frac{1}{2} \ P(B) &= \frac{2}{4} = \frac{1}{2} \ P(C) &= \frac{2}{4} = \frac{1}{2} \end{aligned} $$

Probability of A given B

$$ \begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} = \frac{P(even numbered black ball)}{\frac{1}{2}} \ &= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} \end{aligned} $$

Since $P(A|B) = P(A)$ this means that A and B are independent

Probability of A given C

$$ \begin{aligned} P(A|C) &= \frac{A \cap C}{P(C)} \ &= \frac{0}{1/2} \ &= 0 \end{aligned} $$

Since $(P(A|C) \neq P(A))$ they are not independent

For two events A,B to be both independent and mutually exclusive,

$$ \begin{aligned} P(A \cap B) &= P(A)P(B) = 0 \ & \rightarrow P(A) / or / P(B) / or / both = 0 \end{aligned} $$

This means that for all practical purposes events can't be Independent and Mutually Exclusive.

If A,B are independent, are A and $\bar{B}$? $\rightarrow$ Yes

Example 2.25

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Urn with 2 Black Balls, 3 White Balls. Two balls selected at random w/o
replacement, sequence of colours is noted. What is P(Both Balls Black)?
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$$ \begin{aligned} \frac{\big(_2 ^2) \big(_0 ^3)}{\big(_2 ^ 5)} &= \frac{\frac{2!}{2!0!}\cdot \frac{3!}{0!3!}}{\frac{5!}{2!3!}} \ &= \frac{1}{\frac{5!}{2!3!}} \ &= \frac{1}{10} P(B_1 \cap B_2) &= P(B_1) \times P(B_2 | B_1) \ &= \frac{2}{5} \times \frac{1}{4} \ &= \frac{1}{10} \end{aligned} $$