Assume data.csv is the observed data in form of input-output pair. Please use the family of exponential function, $y = \alpha e^{\beta x}$ with $\beta < 0$, for the regression analysis and find the optimal coefficients, α and β. You may use JavaScript, python, or other language to solve this problem. In addition, compare the difference of estimation error for linear regression and exponential regression.

```
let x=[0,1,2,3,4,5,6,7,8,9,10]
let y=[9735,4597,2176,1024,483,229,108,52,24,11,6]
```

Linear Regression Results: $y=\alpha+\beta x$

Intercept (alpha): 5120.636
Slope (beta): -688.764
Correlation: -0.757

Put alpha / beta into the equationo we get **y=5120.636+(-688.764x)**

i	yi	(y-yi)	sum(y-yi)
0	5120.636	4614.364	0.024
1	4431.872	165.128	
2	3743.108	-1567.11	
3	3054.344	-2030.34	
4	2365.58	-1882.58	
5	1676.816	-1447.82	
6	988.052	-880.052	
7	299.288	-247.288	
8	-389.476	413.476	
9	-1078.24	1089.24	
10	-1767	1773.004	

Alpha represents the estimated value of Y when X equals 0, which is 5120.636.
Beta, being -688.764, indicates a negative correlation between Y and X.
The correlation coefficient ranges from -1 to 1, where values closer to 1 denote a stronger positive linear relationship, and values closer to -1 denote a stronger negative linear relationship. In this linear regression result, the correlation coefficient of -0.757 indicates a moderate negative correlation between X and Y. This means that as X increases, Y tends to decrease, but the strength of this relationship is moderate and not very strong.

Exponential Regression Results: $y = ae^{\beta x}$

Coefficient (alpha): 9620.329

Coefficient (beta): -0.746

Put alpha / beta into the equation we get **y=** $9620.329e^{-0.746x}$

i	yi	y-yi	**sum(y-yi)**
0	9620.329	114.671	**151.3588**
1	4562.535	34.46466	
2	2163.827	12.17289	
3	1026.216	-2.21622	
4	486.6931	-3.6931	
5	230.819	-1.81897	
6	109.4682	-1.46816	
7	51.91635	0.083651	
8	24.62184	-0.62184	
9	11.67715	-0.67715	
10	5.538002	0.461998	

The coefficient alpha represents the estimated value of Y when X equals 0, which is 9620.329.
The coefficient beta, -0.746, signifies an exponential decay trend of Y with increasing X. A larger absolute value of beta indicates a faster decay rate.

Linear regression is primarily used to measure the linear relationship between a dependent variable (Y) and one or more independent variables (X). The goal of linear regression is to find the best-fit line that minimizes the error between the line and observed data points. Linear regression can be significantly affected by outliers, which can greatly influence the regression line's results.

In **linear regression**, it is assumed that there is a linear relationship between X and Y, expressed as $y=\alpha+\beta x$, where α is the intercept and β is the slope.

In contrast, exponential regression assumes that the relationship between the dependent variable Y and the independent variable X is in exponential form, suitable for situations where the dependent variable exhibits nonlinear growth or decay patterns. Exponential regression is often used to describe patterns where growth rates or decay rates change with the independent variable.

The **exponential regression** equation is $y = ae^{\beta x}$, where α and β are regression parameters, and $\beta < 0$.

Researchers can choose the appropriate regression model based on the characteristics of their data and the objectives of their analysis to achieve the best predictive and analytical results.

linear regression

0 9735 linearexponential regression.js:12
1 4597 linearexponential regression.js:12
2 2176 linearexponential regression.js:12
3 1024 linearexponential regression.js:12
4 483 linearexponential regression.js:12
5 229 linearexponential regression.js:12
6 108 linearexponential regression.js:12
7 52 linearexponential regression.js:12
8 24 linearexponential regression.js:12
9 11 linearexponential regression.js:12
10 6 linearexponential regression.js:12
alpha 5120.636363636364 beta -688.7636363636364
linearexponential regression.js:14
correlation is -0.757019814984881

exponential regression.

a 9620.329408784626 b -0.7461780351670065 linearexponential regression.js:30
a 9620.329408784626 b -0.7461780351670065 linearexponential regression.js:21

0 9620.329408784626 linearexponential regression.js:22
1 4561.723310041821 linearexponential regression.js:22
2 2163.056863559917 linearexponential regression.js:22
3 1025.6683005508223linearexponential regression.js:22
4 486.34665157320825 linearexponential regression.js:22
5 230.61360614288708 linearexponential regression.js:22
6 109.35129329295124 linearexponential regression.js:22
7 51.85169056084271 linearexponential regression.js:22
8 24.586794843062844 linearexponential regression.js:22
9 11.658452677555992 linearexponential regression.js:22
10 5.528151176368649 linearexponential regression.js:22