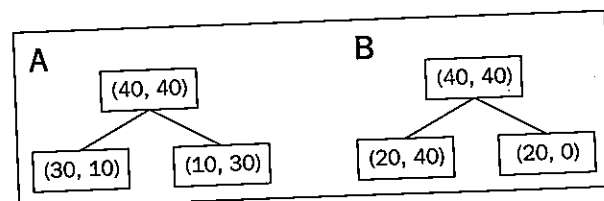


This is a useful criterion for pruning but not recommended for growing a decision tree, since it is less sensitive to changes in the class probabilities of the nodes. We can illustrate this by looking at the two possible splitting scenarios shown in the following figure:



We start with a dataset  $D_p$  at the parent node  $D_p$  that consists of 40 samples from class 1 and 40 samples from class 2 that we split into two datasets  $D_{left}$  and  $D_{right}$ , respectively. The information gain using the classification error as a splitting criterion would be the same ( $IG_E = 0.25$ ) in both scenario A and B:

$$I_E(D_p) = 1 - 0.5 = 0.5$$

$$A: I_E(D_{left}) = 1 - \frac{3}{4} = 0.25$$

$$A: I_E(D_{right}) = 1 - \frac{3}{4} = 0.25$$

$$A: IG_E = 0.5 - \frac{4}{8} \cdot 0.25 - \frac{4}{8} \cdot 0.25 = 0.25$$

$$B: I_E(D_{left}) = 1 - \frac{4}{6} = \frac{1}{3}$$

$$B: I_E(D_{right}) = 1 - 1 = 0$$

$$B: IG_E = 0.5 - \frac{6}{8} \times \frac{1}{3} - 0 = 0.25$$

However, the Gini impurity would favor the split in scenario B ( $IG_G = 0.1\bar{6}$ ) over scenario A ( $IG_G = 0.125$ ), which is indeed more pure:

$$I_G(D_p) = 1 - (0.5^2 + 0.5^2) = 0.5$$

$$A: I_G(D_{left}) = 1 - \left( \left( \frac{3}{4} \right)^2 + \left( \frac{1}{4} \right)^2 \right) = \frac{3}{8} = 0.375$$

$$A: I_G(D_{right}) = 1 - \left( \left( \frac{1}{4} \right)^2 + \left( \frac{3}{4} \right)^2 \right) = \frac{3}{8} = 0.375$$

$$A: IG_G = 0.5 - \frac{4}{8} \cdot 0.375 - \frac{4}{8} \cdot 0.375 = 0.125$$

$$B: I_G(D_{left}) = 1 - \left( \left( \frac{2}{6} \right)^2 + \left( \frac{4}{6} \right)^2 \right) = \frac{4}{9} = 0.\bar{4}$$

$$B: I_G(D_{right}) = 1 - (1^2 + 0^2) = 0$$

$$B: IG_G = 0.5 - \frac{6}{8} \cdot 0.\bar{4} - 0 = 0.1\bar{6}$$

Similarly, the entropy criterion would favor scenario B ( $IG_H = 0.31$ ) over scenario A ( $IG_H = 0.19$ ):

$$I_H(D_p) = -(0.5 \log_2(0.5) + 0.5 \log_2(0.5)) = 1$$

$$A: I_H(D_{left}) = -\left( \frac{3}{4} \log_2\left(\frac{3}{4}\right) + \frac{1}{4} \log_2\left(\frac{1}{4}\right) \right) = 0.81$$