

1. (20%) With the following training dataset, use the Naïve Bayes classification algorithm to predict the class label of $(A=0, B=1, C=0)$. Please set the Laplacian smoothing parameter to 1 to prevent the "0-probability problem". Please give all the calculation steps, not just give a random guess for the answer. Please note that the attributes of the dataset include "A", "B", "C" and "Class". Furthermore, "Class" is the class label (the response variable)

$C_1 = \text{Class} = Y \Rightarrow P(C_1) = \frac{5}{10}$

$C_2 = \text{Class} = N = 5 \Rightarrow P(C_2) = \frac{5}{10}$

$P(C_1|X) = \frac{P(C_1) \cdot P(X|C_1)}{P(X)}$

$P(X|C_1) = \frac{2+1}{5+2} \times \frac{1+1}{5+2} \times \frac{1+1}{5+2}$

$= \frac{3}{7} \times \frac{2}{7} \times \frac{2}{7} \times \frac{5}{10} = 0.017$

$P(X|C_2) = \frac{2+1}{5+2} \times \frac{2+1}{5+2} \times \frac{5+1}{5+2}$

$= \frac{3}{7} \times \frac{3}{7} \times \frac{6}{7} \times \frac{5}{10} = 0.079$

$\Rightarrow \text{Ans} = N$

| Index | A | B | C | Class |
|-------|---|---|---|-------|
| 1 | 0 | 0 | 0 | Y |
| 2 | 1 | 0 | 1 | N |
| 3 | 0 | 1 | 1 | N |
| 4 | 0 | 1 | 1 | N |
| 5 | 0 | 0 | 1 | Y |
| 6 | 1 | 0 | 1 | Y |
| 7 | 1 | 0 | 1 | N |
| 8 | 1 | 0 | 1 | N |
| 9 | 1 | 1 | 1 | Y |
| 10 | 1 | 0 | 1 | Y |



Err \rightarrow Gini

2. (20%) With the following dataset, answer the following questions.

Error = 4

Gini = $2 \times \frac{2}{5} \times \frac{3}{5} = 0.98$

Gini = $2 \times \frac{2}{4} \times \frac{2}{4} = 0.5$

$C_1 = \text{TC} = + = 4, \frac{4}{9}$

$C_2 = \text{TC} = - = 5, \frac{5}{9}$

Err

a1

Data Set

(3, 1)

(1, 4)

Error = 2

$A = \frac{7-2}{9} = 0.778$

| Instance | a1 | a2 | a3 | Target Class |
|----------|----|----|-----|--------------|
| 1 | T | T | 1.0 | + |
| 2 | T | T | 6.0 | + |
| 3 | T | F | 5.0 | - |
| 4 | F | F | 4.0 | + |
| 5 | F | T | 7.0 | - |
| 6 | F | T | 3.0 | - |
| 7 | F | F | 8.0 | - |
| 8 | T | F | 7.0 | + |
| 9 | F | T | 5.0 | - |

a. Gini(a1) = $2 \times \frac{3}{9} \times \frac{6}{9} = 0.778$

Gini(a2) = $2 \times \frac{1}{5} \times \frac{4}{5} = 0.32$

- (a) Using Gini function as attribute selection measure; select the better splitting attribute among attributes a_1 and a_2 for the root. (5%)
- (b) For a_3 , which is a continuous attribute, compute the Gini value for every possible split. What is the best splitting condition for this attribute? (10%)
- (c) According to the results of questions a and b, draw a two-level decision tree and calculate its classification accuracy based on the training dataset. (5%)

An example for splitting a continuous attribute is shown in the following:

$$Gini = 1 - \sum p_i^2 = \sum_{i \neq j} p_i * p_j = 2p_1 * p_2 \text{ (for two-class case)}$$

$$= 1 - \left(\frac{2}{5} \right)^2 = 1 - \frac{4}{25} = \frac{21}{25} = 0.84$$

$$2 * \frac{2}{5} * \frac{3}{5} = 0.48$$

$$2 * \frac{1}{5} * \frac{4}{5} = 0.32$$

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| Class | Annual Income | | | | | | | | | | | |
|-----------------|---------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----|
| | No | No | No | Yes | Yes | Yes | No | No | No | No | No | No |
| | 60 | 70 | 75 | 85 | 90 | 95 | 100 | 120 | 125 | 172 | 220 | |
| Sorted Values | 55 | 65 | 72 | 80 | 87 | 92 | 97 | 110 | 122 | 172 | 230 | |
| Split Positions | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= |
| Yes | 0 | 3 | 0 | 3 | 0 | 3 | 1 | 2 | 3 | 0 | 3 | 0 |
| No | 0 | 7 | 1 | 6 | 2 | 5 | 3 | 4 | 3 | 5 | 2 | 6 |
| Gini | 0.420 | 0.400 | 0.375 | 0.343 | 0.417 | 0.400 | 0.300 | 0.343 | 0.375 | 0.400 | 0.420 | |

Figure 4.16. Splitting continuous attributes-

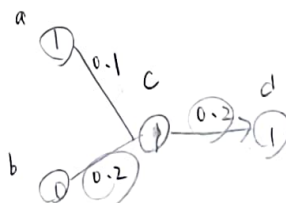
$$0.23 + 0.10 = 0.33$$

3. (20%) Given a simple MLP with two inputs a and b , one hidden node c and one output node d , please find the updated weight of W_{cd} and W_{bc} after training the network once with the following training example. Assume that the initial values of the weights of w_{ac} , w_{bc} , w_{cd} are 0.1, 0.2, 0.2, respectively, and the learning rate is 0.9.

$$w_{cd} = w_{cd} + \eta * \text{Error}_d * \text{Out}_c$$

The training example:

| a | b | d |
|---|---|---|
| 1 | 0 | 1 |



$$w_{bc} = w_{bc} + \eta * \text{Error}_c * \text{Out}_b$$

$$\text{Error}_c = 0.911 * (1 - 0.911) * (1 - 0.911) = 0.059$$

(Notes: 1. Let not consider the biases (i.e., no biases).

2. w_{ac} denotes the weight of the link from node a to node c . Similarly, w_{bc} and w_{cd} denote the weight of the link from node b to node c and the weight of the link from node c to node d , respectively.)

$$O_c = \frac{1}{1 + e^{-0.9}} = 0.411$$

$$\text{Error}_d = 0.536 * (1 - 0.536) * (1 - 0.536) = 0.116$$

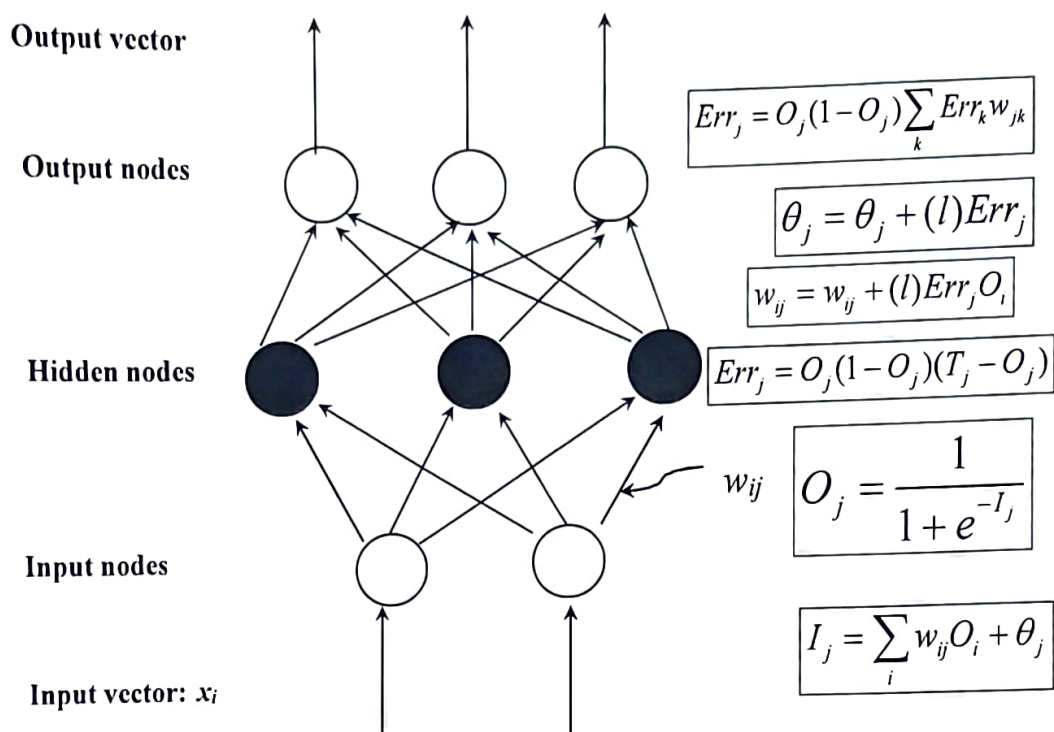
$$w_{cd} = 0.2 + 0.9 * 0.536 * 0.116 = 0.2564$$

$$O_d = \frac{1}{1 + e^{-0.142}} = 0.536$$

$$O_b = \frac{1}{1 + e^{-0.142}} = 0.536$$

$$w_{bc} = 0.2 + 0.9 * 0.059 * 0.536 = 0.2288$$

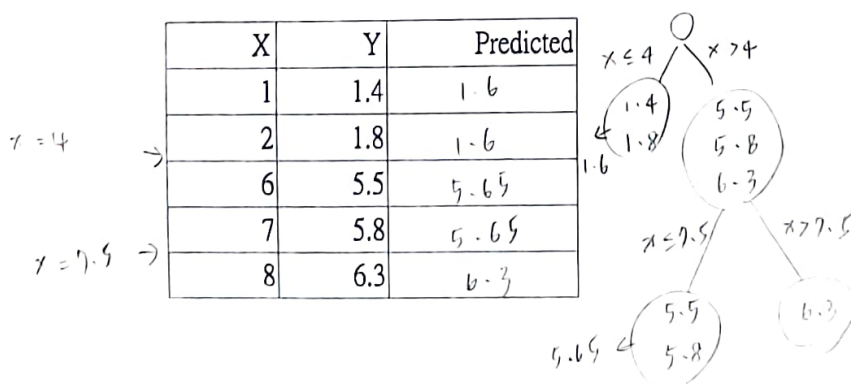
Appendix :



4. (20%) The following dataset contains two columns, X and Y. Please build a regression tree with X as the input variable and Y as the output variable (10%). Then, fill in the predicted value for each X using the regression tree. (10%)

Note: Set the minimum number of samples in a node to three to allow splitting the node in building the regression tree. In other words, if the number of samples in a node is less than or equal to two (≤ 2), no splitting of this node is allowed.

| X | Y | Predicted |
|---|-----|-----------|
| 1 | 1.4 | 1.6 |
| 2 | 1.8 | 1.6 |
| 6 | 5.5 | 5.65 |
| 7 | 5.8 | 5.65 |
| 8 | 6.3 | 6.3 |



5. (20%, Gradient Boosting, with learning rate of 0.5) With the same dataset for problem 4, use gradient boosting to construct a model for predicting Y values. To reduce the required computation, use only one one-level regression tree. That is, use $F_0 = 4.16$ as the first prediction for all Y's; construct a one-level regression tree h_1 to predict the residuals. Then, compute the predicted values for all Y's

using the **learning rate of 0.5**

| x | Y | \hat{y}_e |
|---|-----|-------------|
| 1 | 1.4 | 1.6 |
| 2 | 1.8 | 1.6 |

$$Y - \hat{y} = -2.76$$

$$-2.36$$

$$x=4 \rightarrow 1.31$$

