Lab 11

Iterative methods for solving linear systems

Consider the system $Ax = b$, with $A = (a_{ij})_{i,j=\overline{1,n}}$, $x = (x_1, ..., x_n)'$ and $b = (b_1, ..., b_n)'$.

Input: A -matrix of coefficients; b -vector of free terms; $x^{(0)}$ -the initial approximation of the solution; ε -precision, N - maximum number of iterations; ($\omega-$parameter for relaxation method);

Output: x -vector of the solutions or *a message* - in case that the maximum number of iterations is exceeded.

Problem:

1. Solve the following system using *Jacobi, Gauss-Seidel* and *relaxation* iterative methods, for $\varepsilon = 10^{-3}$:

$$\begin{pmatrix} 3 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}.$$

Return the solution of the system and the number of iterations needed for finding the solution.

2. Solve the following system using the matriceal forms of Jacobi, Gauss-Seidel and relaxation methods, for $\varepsilon = 10^{-5}$:

$$\begin{pmatrix} 3 & 1 & 1 \\ -2 & 4 & 0 \\ -1 & 2 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 12 \\ 2 \\ -5 \end{pmatrix}.$$

(*Facultative*) **3.** Find LU decomposition of the following matrix using Doolittle method.

$$A = \begin{bmatrix} 6 & 2 & 1 & -1 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 4 & -1 \\ -1 & 0 & -1 & 3 \end{bmatrix}.$$

For $b = \begin{bmatrix} 8 \\ 7 \\ 5 \\ 1 \end{bmatrix}$, solve the system $Ax = b$.