Lab 12

Numerical methods for solving nonlinear equations

1. Solve the equation

$$x = cosx.$$

using Newton's method for: $x_0 = \frac{\pi}{4}$, $\varepsilon = 10^{-4}$ and maximum number of iterations $N = 100$.

2. For finding the position of a satellite for $t = 9$ minutes, we have to solve Kepler's equation

$$f(E) = E - 0.8 \sin E - \frac{2\pi}{10} = 0.$$

Type the results obtained applying Newton's method 6 times, starting with $E = 1$. (Notice the quadratic precision.)

3. Use the secant method with $x_0 = 1$ and $x_1 = 2$ to solve $x^3 - x^2 - 1 = 0$, with $\varepsilon = 10^{-4}$ and maximum number of iterations $N = 100$.

4. Let $f : [1, 2] \to \mathbb{R}$, $f(x) = (x - 2)^2 - \ln x$. Solve the equation $f(x) = 0$, using bisection and false position methods, for $\varepsilon = 10^{-4}$ and maximum number of iterations $N = 100$. (Use $abs(f(c) < \varepsilon$ as a stopping criterion.)

(*Facultative*) **5.** Check the performances of Newton's method in two versions:

standard: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

root of multiplicity m: $x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)}$

to approximate the multiple zero $\alpha = 1$ of the function $f(x) = (x^2 - 1)^p \log x$ (for $p \geq 1$ and $x > 0$). The desired root has multiplicity $m = p + 1$. Consider the value $p = 2$ and $x_0 = 0.8$, $\varepsilon = 10^{-10}$. Type the number of iterations required to converge for each case.