

LAB 5: HEAT EXCHANGERS

ME 436 HEAT TRANSFER

*0087 Black Engineering
Iowa State University*



Section 9
Group 1

Zack Johnson
Carina Grady
Brady Suby
Megan Kowalski

November 29, 2018

1. Introduction

Heat exchangers are used in a wide variety of fields, some of them including: automotive, industrial, cooling, and heating. Cryogenic engines and beverage distillation are a couple specific examples that use heat exchangers. Having a clear understanding of how heat exchangers work allows engineering to continue to improve on these applications and create new ones.

It is important to have a good understanding of how heat exchangers work because there are many different flow arrangements to consider. The four categories include concentric tube, cross flow, shell-and-tube, and compact. There are many variations within these groups as well. For example, this lab focuses on the two different flow arrangements for concentric tubes, parallel and counter flow. Depending on the desired heat exchange outcome, some applications may favor one arrangement over another. There will be one arrangement that works best and provides the most efficient heat exchange for an application. Understanding the differences between the arrangements can help eliminate those arrangements that are not going to provide the best outcome.

The main objective of this lab is to determine the settings that provide the optimal experimental thermal performance of a heat exchanger. In this case, the desired characteristics of the heat exchanger is a high change in the cold flow temperature and a minimum change to the hot flow temperature, as this is indicative of a high heating effectiveness. The mass flow rates of the hot and cold fluids were changed systematically to get this optimal performance. This was done for a concentric tube heat exchanger operating in parallel flow and counter flow arrangements. Obtaining the specifications of the optimal performance then allows students to make theoretical predictions and draw conclusions. In order to complete the main objective, the following tasks were performed:

- Discussed main assumptions and equations used.
- Plotted the experimental temperatures for both parallel and counter flow arrangements.
- Performed an energy balance to calculate the experimental heat transfer rates.
- Estimated the theoretical outlet temperatures and heat transfer rate for both flow arrangements using the effectiveness-NTU method.
- Compared the experimental and the theoretical results to determine if there were any differences.

These points were used to form a conclusion based on the findings from this experiment.

2. Experimental Setup

To be able to explore the workings of a heat exchanger this lab uses HT31 heat exchangers. The heat exchanger was controlled by an Armfield HT30XC control. A visual representation of this apparatus is provided in Figure 1 below. The apparatus simulates concentric tube heat exchange which controlled via a computer. There are many systems within this device which include the following: a hot water vessel, recirculation pump, cold water control system, tubular heat exchanger system, and a computer interface.

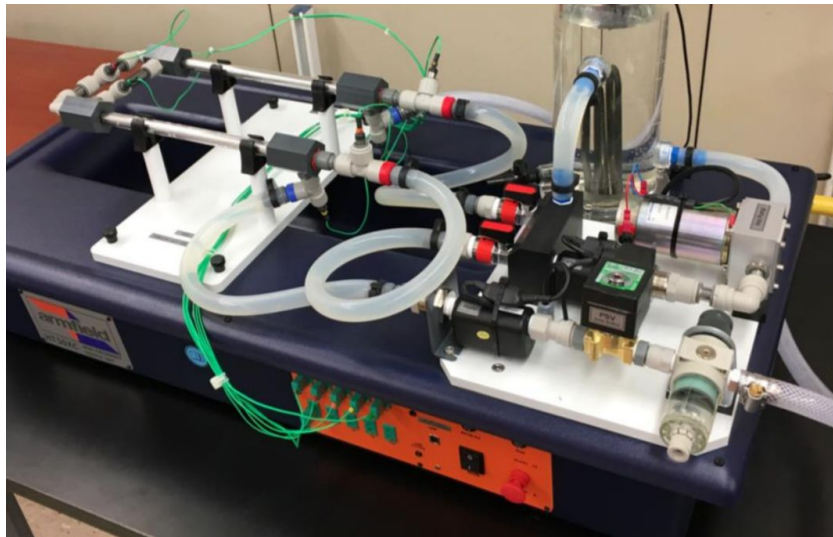


Figure 1: Experimental apparatus: HT31 heat exchanger with Armfield HT30XC control

The tubular heat exchanger system is represented in Figure 2 below. The hot fluid, which is represented as the pink section in the figure, flows through the inner tube that is made of stainless steel. The cold fluid flows through the space in between the inner and outer tubes. The outer tube is made of acrylic. Depending on the settings selected, both fluids flow the same direction to represent parallel flow, or the fluids will flow in opposite directions to represent counterflow.

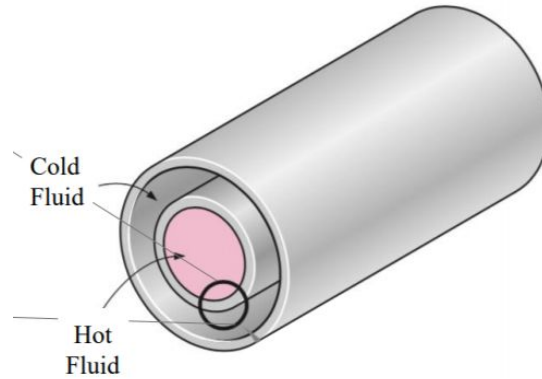


Figure 2: Schematic of tubes

The dimensional layout is given below:

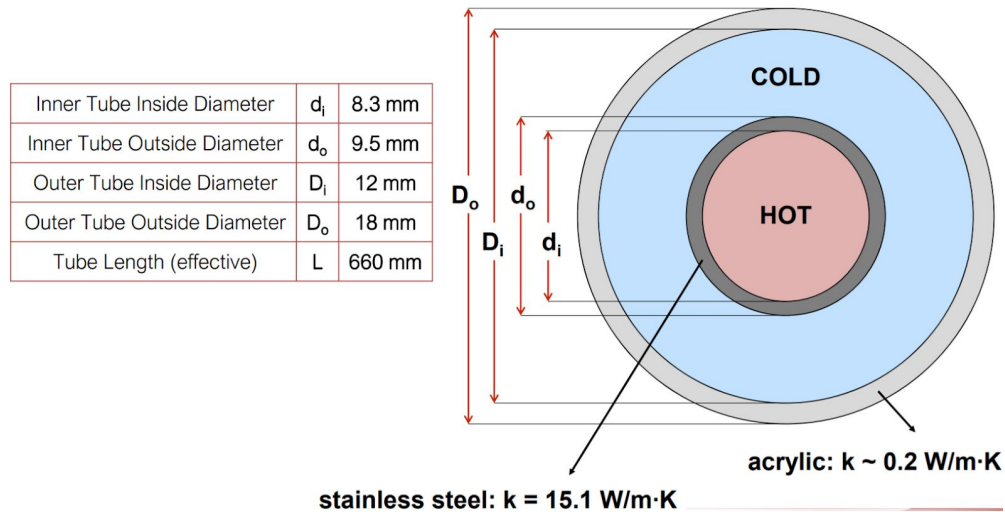


Figure 3: Dimensional Layout

Using the computer interface, the mass flow rates of the fluids can be changed. The hot fluid mass flow rate was changed by adjusting the pump speed. The cold fluid mass flow rate was changed by adjusting the valve opening. Based on the changing mass flow rates, students are then able to see how various temperatures change.

Six type-K thermocouples are placed throughout the apparatus to read temperatures at various positions. The six thermocouple placements are shown in Figure 3 below. T1, T2, and T3 measured the temperatures of the hot fluid inside the tube. T4, T5, and T6 measured the temperatures of the cold fluid inside the annulus. After changing the mass flow rates of the fluids, students are then able to determine

the settings for the best flow rate configuration by maximizing the temperature difference between T4 and T6 while minimizing the temperature difference between T1 and T3.

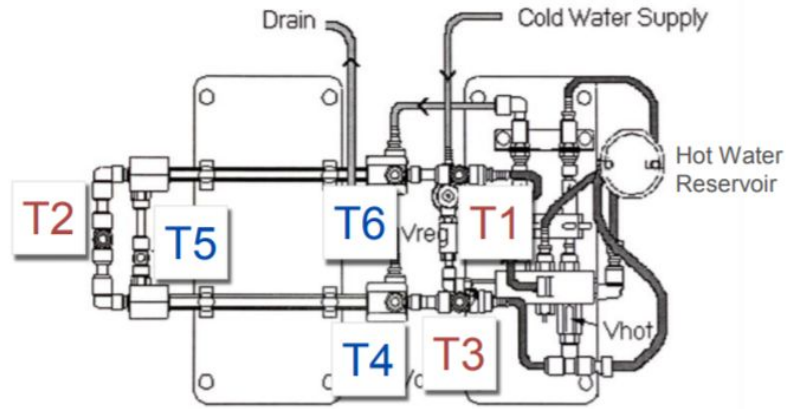


Figure 4: Schematic of apparatus with thermocouples

3. Theory and Analysis

- **Assumption 1.** Steady-State: Between changes to the system control variables, the temperature data was allowed to plateau towards a constant temperature, thus signifying that no change in temperature, and therefore energy, was occurring in the control volume.
- **Assumption 2.** Constant properties: Because water does not significantly change its specific heat with small deviations in temperature, the specific heat is taken as constant throughout a flow path during a given sample.
- **Assumption 3.** Clean surfaces: The setups have been maintained well with no build up or impurities in the tube surfaces, so no additional resistances are assumed outside of the convection heat transfers and resistance of the pure wall material.
- **Assumption 4.** No mechanical energy change: The system has a constant flow rate and cross sectional area, resulting in constant velocity. The heat exchanger is also nearly horizontal, so changes in kinetic and potential energy are negligible.
- **Assumption 5.** Fully developed flow: For simplicity, it is assumed that the flow has fully-developed boundary layers.

3.1. Control Volume Energy Balance

For a steady-state heat exchanger with no heat generation, the energy balance equations are seen below:

$$q_{out} = q_c - q_h \quad (1)$$

$$q_c = C_c(T_{c,o} - T_{c,i}) \quad (2)$$

$$q_h = C_h(T_{h,o} - T_{h,i}) \quad (3)$$

q_{atm} : Outside heat transfer(W)

q_c : Heat transfer to cold system(W)

q_h : Heat transfer from hot system(W)

C : Heat capacity rate ($\frac{W}{K}$)

T : Temperature (K)

The subscripts refers to the classification of the two flows (h=hot and c=cold) and whether the fluid is entering (i=in) or leaving the heat exchanger (o=out). The heat capacity rate is itself defined as the product:

$$C = mc_p \quad (4)$$

$$m = \rho(Q/60) \quad (5)$$

c_p : Specific heat ($\frac{J}{kg-K}$)

m : Mass flow rate ($\frac{kg}{s}$)

Q : Flow rate ($\frac{L}{min}$)

ρ : Density ($\frac{kg}{L}$)

The division by 60 is done to convert the flow rate data (recorded in the above units) into liters per second for translation into Watts. All of the property information is retrieved from water tables based on the temperature data collected at the points and averaged.

3.2. Thermal Resistance

With the temperature data and heat capacity rate providing experimental data on the heat transfer experienced by the fluid, it is compared to established models of heat exchangers based on the thermal resistance properties expected. For a clean system:

$$UA = \left(\frac{1}{(hA)_h} + \frac{\ln(\frac{d_o}{d_i})}{2\pi kL} + \frac{1}{(hA)_c} \right)^{-1} \quad (6)$$

UA : Overall heat transfer coefficient ($\frac{W}{K}$)

L : Tube Length

A_h : Inner (hot) surface area = $\pi d_i L = 1.72 * 10^{-2} m^2$

A_c : Outer (cold) surface area = $\pi d_o L = 2.01 * 10^{-2} m^2$

k : Steel tube thermal conductivity = $15.1 \frac{W}{m-K}$

h : Convection heat transfer coefficient ($\frac{W}{m^2-K}$)

Note that d_i and d_o are the smaller tube diameters given in the dimension schematic, along with the tube length. For each flow, the convection coefficient must must calculated using the Nusselt correlations for internal cylinder flow. In the case of the hot tube with $D = d_i$ and the cold annulus using $D = D_i - d_o$:

$$\bar{h} = \frac{\overline{Nu} * k_f}{D} \quad (7)$$

k_f : Water thermal conductivity

Additionally, k_f is evaluated with the temperature data. The Nusselt number itself depends on the Reynold's number of the flow, which requires conversion of flow rate data to velocity.

$$V_h = \frac{4(Q/60,000)}{\pi d_i^2}, \quad V_c = \frac{4(Q/60,000)}{\pi (D_i^2 - d_o^2)} \quad (8)$$

$$Re = \frac{V * D}{\nu} \quad (9)$$

For this lab, the Dittus-Boetler correlation was used, finally tabulating the Prandtl number as well

$$\overline{Nu} = 0.023 Re^{0.8} Pr^n, n_h = 0.3, n_c = 0.4 \quad (10)$$

3.3. NTU/Efficiency method

By calculating a theoretical maximum heat flow and calculating an efficiency based on the overall coefficient, the heat exchanger can be characterized. Using the NTU method, defined key ratios are:

$$NTU = \frac{UA}{C_{min}}, C_r = \frac{C_{min}}{C_{max}} \quad (11,12)$$

C_{min} and C_{max} are the minimum and maximum heat capacity rates from a given flow rate and temperature data set, from either the hot or cold side. These ratios are used for the following effectiveness correlations:

$$\epsilon = \frac{1 - e^{-NTU(1+C_r)}}{(1+C_r)} \quad (\text{Parallel}) \quad (13)$$

$$\epsilon = \frac{1 - e^{-NTU(1-C_r)}}{(1-C_r)e^{-NTU(1-C_r)}} \quad (\text{Counterflow}) \quad (14)$$

Assuming that the maximum heat transfer can only be as high as the minimum heat capacity rate allows, and that the actual heat flow between the heat exchanger tubes is represented by this effectiveness, the final theoretical heat transfer equation is shown below.

$$q = \epsilon C_{min}(T_{h,i} - T_{c,i}) \quad (15)$$

4. Results and Discussion

The objective of this lab is to find the parameters that results in the optimal heat exchanger performance, which results from minimizing ΔT_h and maximizing ΔT_c . Two parameters were able to be changed: mass flow rate and opening of the valve. The experimentation of changing these two parameters is seen in Appendix A and B. In short, a higher mass flow rate and lower valve opening results in optimal performance.

Note that ΔT_h is minimized more and ΔT_c is maximized more when the valve opening is <3.5. However, it was observed that an opening of less than 3.5 resulted in a very small flow rate. “Not enough flow to wash your hands” is analogous to not having enough fluid for heat transfer in a heat exchanger to be effective, which is not suitable in this experiment. As a result, any valve opening <3.5 is small enough to “not having enough flow to wash your hands” and is deemed not usable data. Thus, any results with a valve opening of less than 3.5 were used simply for comparison.

For this experiment, the tubes used in the heat exchanger were not insulated. Without insulation, heat is able to escape the apparatus via convection, resulting in heat loss from the system. The tubes that were used are considered short enough, resulting in a relatively small amount of heat loss. As shown in Table 1 in Section 4.1, only 6.1% and 8.2% of the heat was lost for the parallel and counterflow arrangements respectively. However, if insulation were to be used in this experiment there would be no heat loss resulting heat transfer only between the hot and cold fluids. This means that minimizing the heat loss would maximize the effectiveness of the heat transfer within the heat exchanger.

4.1. Experimental Results

As shown in Figures 5 and 6 below, the experimental temperatures are fairly close to the expected theoretical temperatures that were calculated using the MATLAB code. However, the cold fluid outlet temperatures for both the parallel and counter flow arrangements vary the most. The temperatures match exactly at the beginning of the tube and then start to vary away from each other. This difference in the experimental and theoretical changes is due the heat loss through convection from the tube.

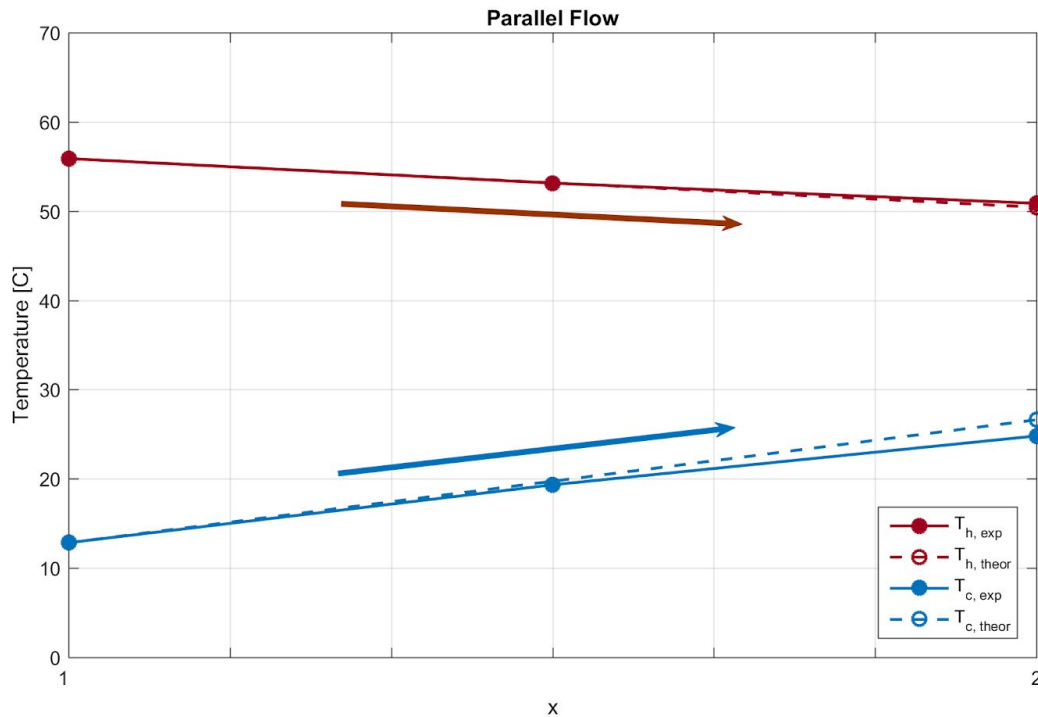


Figure 5: Experimental and theoretical temperatures of the parallel flow arrangement

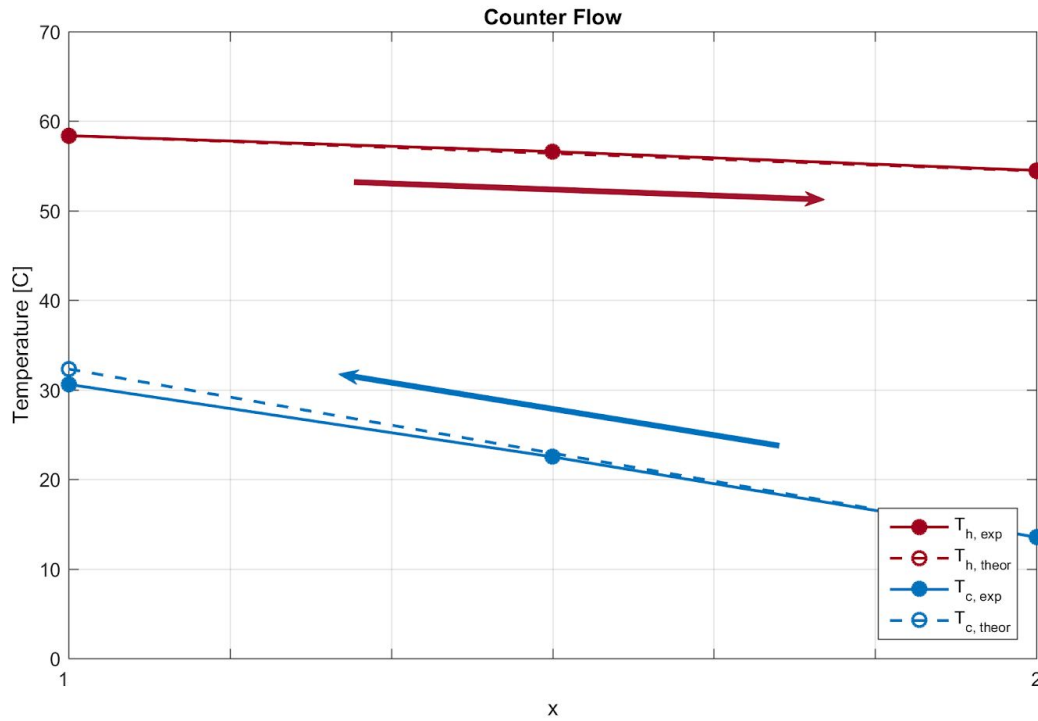


Figure 6: Experimental and theoretical temperatures of the parallel flow arrangement

Although both arrangements look very similar in the plots above, the data in Table 1 below provides insight of how different the flow arrangements performed. The data shows a slightly larger heat transfer from both flows in the parallel arrangement, with a higher ambient heat loss from counterflow. However, by the metrics of this experiment, the counterflow's lower change in hot flow temperature and higher change in the cold side indicates the higher effectiveness, and thus is the "optimal" arrangement.

Table 1: Experimental Heat Transfer Data

HX	ΔT_h [°C]	ΔT_c [°C]	C_h [W/K]	C_c [W/K]	$q_{h,exp}$ [W]	$q_{c,exp}$ [W]	q_{amb} [W]	$q_{amb,pct}$ [%]
Parallel	5.02	11.97	236.934	93.687	1189.22	1120.98	-68.24	-6.087
Counter	3.88	17.07	302.701	63.596	1174.84	1085.55	-89.29	-8.225

From the experimental data, it is shown that both the counterflow and parallel exchanger cold flows are, overall, losing heat to the 23°C environment. As expected, the counterflow setup loses slightly more to the atmosphere; this is because counterflow arrangements allow for higher temperatures that can exceed even the hot side exit temperature, thus creating a greater temperature difference between it and the atmosphere. In short, a parallel flow heat exchanger of infinite length "converge" to a value in the middle

between the hot and cold temperature so that $T_{c,o}=T_{h,o}$, while counterflow heat exchanger of infinite length can lead to values where $T_{c,o}=T_{h,i}$ and $T_{h,o}=T_{c,i}$.

In this lab set-up, the tube length is 66cm long. If the length is decreased to 50cm, then there will be more heat transfer occurring in the counter flow arrangement, and less heat transfer occurring in the parallel flow arrangement. Similarly, if the length is increased to 100cm, there will be less heat transfer occurring in the counter flow arrangement, and more heat transfer occurring in the parallel flow arrangement. The purpose of the heat exchanger is to increase heat transfer between two cold and hot streams, so the more heat transfer that occurs, the higher its effectiveness.

4.2. Theoretical Results

The table below displays the theoretical output of using the theoretical NTU analysis. In both analyses, the prediction for the hot side temperature change was 2% and 8% higher than experimental values for counterflow and parallel respectively, with the accuracy falling to 10% and 15% higher for the cold side temperature change. This is likely due to the fact that the UA model utilized ignores the ambient heat transfer loss to the atmosphere for the approximation. In reality, the heat exchanger is not perfectly insulated, and so there is a loss of heat transfer to the atmosphere, which results in a noticeable drop in the temperature gain compared to the predicted value. For heat transfer, the discrepancy again was 2% higher for the counterflow and 8% higher for the parallel. The higher error rates of the parallel flow are likely due to the more extreme temperature change of both flows that occurs in the first segments of a parallel heat exchanger. The more varied profile means that many of the fluid properties, especially viscosity used in Reynold's number calculation, do not act as constant as they are portrayed in the model compared to the less extreme temperature slope of the counterflow exchanger.

Table 2: Theoretical Heat Transfer Results

HX	h_h [W/m ² -K]	h_c [W/m ² -K]	UA [W/K]	Cr	ϵ	NTU	ΔT_h [°C]	ΔT_c [°C]	q_t [W]
Parallel	6178	3730	39.726	0.395	0.320	0.424	5.45	13.78	1290.5
Counter	7719	2838	36.271	0.210	0.419	0.570	3.94	18.78	1194.1

Given the variables already found with the theoretical analysis, the temperature at any given point in the parallel or counterflow T_h or T_c can be found. This is due to the linear fit regression line of the plotted data points. The slope of the data points is documented through ΔT and the starting temperatures of the hot and

cold fluids were held constant before conducting the experiment. Thus, given the linear correlation, the temperature at the midpoint can be located.

Referencing equations 13 and 14 for effectiveness for both flows; the parallel flow heat exchanger had a effectiveness of 0.320, while the counterflow heat exchanger had an effectiveness of 0.419. Based on these results, the counterflow heat exchanger was more effective by approximately 24%. These results were foreseeable because the counterflow arrangement had a larger change in ΔT_c and smaller change in ΔT_h , which results in increased effectiveness. The experimental and theoretical results that were conducted are very similar with minimal error. Therefore, the theoretical results are consistent with the described analysis of the experimental results.

Although heat transfer is occurring, the Reynolds number for the cold and hot stream are different, which may result in different flow regimes. With an internal flow critical Reynolds number equaling 1×10^4 , the flow regime for parallel flow and counterflow heat exchanger can be determined, as seen in Table 3.

Table 3: Theoretical Reynolds Number

HX	Re_c	Re_h	Flow Regime, c	Flow Regime, h
Parallel	1.28×10^3	1.68×10^4	Laminar	Turbulent
Counter	941	2.25×10^4	Laminar	Turbulent

As seen in Equation 10 in Section 3.2, the Dittus-Boelter Equation was used to determine the Nusselt Number. The full table of internal flow Nusselt Number approximations is seen in Appendix C, and the Dittus-Boelter Equation is one of them. To use this equation, the flow must be turbulent, fully developed, and have a Prandtl number between 0.6 and 160. Although the Reynold's number in the cold region suggests laminar flow for a cylinder, models for laminar flow require either uniform heat flux across both surfaces, or a uniform heat flux and a constant temperature. Because of the nature of parallel and counterflow heat exchangers, uniform heat flux at all points along the tube length does not occur in either of these systems and so laminar equations are unusable. Considering how an annulus will have more disturbances due to two surfaces providing shear instead of just one, it was assumed that there would be more turbulence than reported by the Reynold's number, and that the Dittus-Boelter equation would be roughly accurate for it.

5. Conclusion

This lab focused on determining parameters on what makes a parallel flow and counterflow heat exchanger the most optimal by changing the mass flow rate and opening of the valve. A heat exchanger is the most effective when ΔT_h is minimized, and ΔT_c is maximized. Gathering data consisted of test-runs of changing the parameters were done to find the optimal mass flow rate and valve opening. After determining the “best” test-run that led to the lowest ΔT_h and highest ΔT_c , the two heat exchanger results can be compared. After completing the experiment and analyzing the results of the heat transfer occurring in the heat exchanger, the following conclusions were made:

- Optimal heat exchanger design was found with a maximized mass flow rate and a minimized valve opening.
- The counterflow heat exchanger achieved better experimental results than the parallel flow heat exchanger because it had a lower ΔT_h by 23% and a higher ΔT_c by approximately 30%.
- The above statement is confirmed with theoretical results. The counterflow heat exchanger had a higher effectiveness in comparison to the parallel flow heat exchanger by 23.6%.
- The parallel flow heat exchanger had less heat losses. Ambient heat losses from the parallel flow heat exchanger was 6.1%, and ambient heat losses from the counter flow heat exchanger was 8.2%.

In conclusion, the counterflow heat exchanger was deemed more effective than the parallel flow heat exchanger for the given parameters. However, this experiment could have been improved by further analysis of different types of changing parameters. For example, if the lab was able to explore the effects of a change of tube length, there would have been additional conclusions made. Nevertheless, this lab still completed its objective of determining what makes a heat exchanger effective by comparing the mass flow rates in counterflow and parallel flow arrangements.

6. References

- [1] Dr. Pittoni, Paola and Pfiefer, Spencer. "Heat Exchangers Slides." Iowa State University Heat Transfer Lab, Fall 2018

- [2] Dr. Maxwell, Gregory. "ME436 Lab 5 Heat Exchangers - Prelab Video". Iowa State University Heat Transfer Laboratory

- [3] Incropera, F. P. and DeWitt, D. P., *Fundamentals of Heat and Mass Transfer*, **7e**, John Wiley and Sons, Inc., 2002.

7. Appendix

Raw data and MATLAB codes have been submitted in a zipped file along with this report. The raw data is in the data folder of the code.

7.1. Appendix A: Counter Flow Data Collected

Table 4: Collected mass flow rates and temperatures for the counterflow arrangement

FLOW ARRANGEMENT: COUNTER FLOW									
Run	\dot{m}_h [kg/s]	\dot{m}_c [kg/s]	T_{hi} [°C]	T_{ho} [°C]	ΔT_h [°C]	T_{ci} [°C]	T_{co} [°C]	ΔT_c [°C]	BEST?
1 st : 1.7 Valve: 3.5	.0367	.0167	58.5	52.1	6.4	13.5	26.65	13.15	
2 nd : 2.1 Valve 3.5	.0458	.0183	58.4	52.8	5.6	13.5	27.4	13.9	
3 rd : 2.6 Valve: 3.5	.0583	.015	58.4	54.0	4.4	13.7	30.2	16.5	
4 th : 3 Valve: 3.5	.0734	.0153	58.4	54.5	3.9	13.5	30.6	17.1	X
5 th : 1 Valve: 3.5	.0185	.0151	58.7	50.2	8.5	13.6	26.0	12.4	
6 th : 1.7 Valve: 4.5	.0358	.0220	58.4	51.6	6.8	13.5	25.3	11.8	
7 th : 1.7 Valve: 2	.0362	.0063	58.8	53.9	4.9	13.9	36.0	22.1	

Table 5: Changes made between each data collection

Run	Change planned:	Output expected:	Why?	Did it work?	What was the actual outcome?
1 st
2 nd	↑ Pump speed +0.4	↑ \dot{m}_{dot}	Minimize $\Delta T_h = q/(\dot{m}_{dot} * c_p)$	Yes	↓ ΔT_h -0.8°C
3 rd	↑ Pump speed +0.4	↑ \dot{m}_{dot}	Minimize $\Delta T_h = q/(\dot{m}_{dot} * c_p)$	Yes	↓ ΔT_h -1.2°C
4 th	↑ Pump speed +0.4	↑ \dot{m}_{dot}	Minimize $\Delta T_h = q/(\dot{m}_{dot} * c_p)$	Yes	↓ ΔT_h -0.5°C
5 th	↓ Pump speed -2.0	↓ \dot{m}_{dot}	Experimental check	No	↑ ΔT_h +4.6°C
6 th	↑ Valve area +1.0	↑ h_h	↓ $\Delta T < \uparrow h_h < \uparrow Nu < \uparrow v < \downarrow A$	No	↑ ΔT_h +0.4°C
7 th	↓ Valve area -2.5	↑ $R_{th,h}$	↓ $\Delta T < \uparrow R_{th,h} < \uparrow A$	Yes	↓ ΔT_h -1.9°C

7.2. Appendix B: Parallel Flow Data Collected

Table 6: Collected mass flow rates and temperatures for the parallel flow arrangement

FLOW ARRANGEMENT: PARALLEL FLOW									
Run	\dot{m}_h [kg/s]	\dot{m}_c [kg/s]	T_{hi} [°C]	T_{ho} [°C]	ΔT_h [°C]	T_{ci} [°C]	T_{co} [°C]	ΔT_c [°C]	BEST?
1 st : 1.7 Valve: 2	0.039	0.006	54.08	58.92	4.84	13.98	35.29	21.31	
2 nd : 1.7 Valve: 3.5	0.039	0.016	52.37	58.67	6.30	13.65	27.22	13.57	
3 rd : 1.7 Valve: 4.5	0.039	0.023	51.47	58.50	7.03	13.48	24.53	11.05	
4 th : 1 Valve: 3.5	0.022	0.023	48.65	58.76	10.11	13.46	22.02	8.56	
5 th : 2.1 Valve: 3.5	0.048	0.023	52.20	58.35	6.15	13.42	25.09	11.67	
6 th : 2.6 Valve: 3.5	0.059	0.023	52.98	58.20	5.22	13.38	25.84	12.46	
7 th : 3 Valve: 3.5	0.069	0.025	53.41	58.11	4.70	13.33	26.02	12.69	X

Table 7: Changes made between each data collection

Run	Change planned:	Output expected:	Why?	Did it work?	What was the actual outcome?
1 st
2 nd	↑ Valve area +1.5	↑ h_h	↓ $\Delta T < \uparrow h_h < \uparrow Nu < \uparrow v < \downarrow A$	No	↑ $\Delta T_h + 1.5^\circ\text{C}$
3 rd	↑ Valve area +1.0	↑ h_h	↓ $\Delta T < \uparrow R_{th,h} < \uparrow A$	No	↑ $\Delta T_h + 1.7^\circ\text{C}$
4 th	↓ Pump speed -0.7	↓ m_{dot}	Experimental check	No	↑ $\Delta T_h + 3.1^\circ\text{C}$
5 th	↑ Pump speed +1.1	↑ m_{dot}	Minimize $\Delta T_h = q/(m_{dot} * c_p)$	Yes	↓ $\Delta T_h - 4.0^\circ\text{C}$
6 th	↑ Pump speed +0.4	↑ m_{dot}	Minimize $\Delta T_h = q/(m_{dot} * c_p)$	Yes	↓ $\Delta T_h - 0.9^\circ\text{C}$
7 th	↑ Pump speed +0.4	↑ m_{dot}	Minimize $\Delta T_h = q/(m_{dot} * c_p)$	Yes	↓ $\Delta T_h - 0.52^\circ\text{C}$

7.3. Appendix C: Internal Flow Nusselt Number Approximations

TABLE 8.4 Summary of convection correlations for flow in a circular tube^{a,b,e}

Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q_s''
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \frac{0.0668 Gz_D}{1 + 0.04 Gz_D^{2/3}}$	(8.57)	Laminar, thermal entry (or combined entry with $Pr \geq 5$), uniform T_s , $Gz_D = (D/x) Re_D Pr$
$\overline{Nu}_D = \frac{\frac{3.66}{\tanh[2.264 Gz_D^{-1/3} + 1.7 Gz_D^{-2/3}]} + 0.0499 Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432 Pr^{1/6} Gz_D^{-1/6})}$	(8.58)	Laminar, combined entry, $Pr \geq 0.1$, uniform T_s , $Gz_D = (D/x) Re_D Pr$
$\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right]$	(8.20) ^c	Turbulent, fully developed
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21) ^c	Turbulent, fully developed, smooth walls, $3000 \leq Re_D \leq 5 \times 10^6$
$Nu_D = 0.023 Re_D^{4/5} Pr^n$	(8.60) ^d	Turbulent, fully developed, $0.6 \leq Pr \leq 160$, $Re_D \geq 10,000$, $(L/D) \geq 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0.14}$	(8.61) ^d	Turbulent, fully developed, $0.7 \leq Pr \leq 16,700$, $Re_D \geq 10,000$, $L/D \geq 10$
$Nu_D = \frac{(f/8)(Re_D - 1000) Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	(8.62) ^d	Turbulent, fully developed, $0.5 \leq Pr \leq 2000$, $3000 \leq Re_D \leq 5 \times 10^6$, $(L/D) \geq 10$
$Nu_D = 4.82 + 0.0185(Re_D Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q_s'' , $3.6 \times 10^3 \leq Re_D \leq 9.05 \times 10^5$, $3 \times 10^{-3} \leq Pr \leq 5 \times 10^{-2}$, $10^2 \leq Re_D Pr \leq 10^4$
$Nu_D = 5.0 + 0.025(Re_D Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s , $Re_D Pr \geq 100$