

To work with binary numbers, it is important to know the following **powers of 2**.

$$2^0 \quad 2^1 \quad 2^2 \quad 2^3 \quad 2^4 \quad 2^5 \quad 2^6 \quad 2^7$$

Practice writing each power of 2 using the exercise outlined below.

Directions: Copy a sequence of numbers from below into the column labeled n of a table on the next page. Then write the corresponding power of 2 by finding 2^n .

0 5 3 1 4 2 6 7

6 1 5 3 4 0 7 2

6 7 2 1 3 5 0 4

6 1 3 5 2 7 4 0

1 0 7 2 6 5 3 4

1 2 6 5 0 7 3 4

5 3 6 0 2 4 1 7

0 5 3 1 2 4 7 6

7 1 0 6 5 2 3 4

0 6 1 4 5 3 2 7

4 0 1 6 2 3 7 5

0 7 3 1 6 2 5 4

0 5 3 1 4 2 6 7

6 1 5 3 4 0 7 2

6 7 2 1 3 5 0 4

Converting 8 digit binary numbers to decimal

Recall: In Part 1, we added each place value with a 1.

Example: Convert **1 0 0 1 1 0 0 1** to decimal.

128	64	32	16	8	4	2	1
1	0	0	1	1	0	0	1

$$128 + 16 + 8 + 1 = 153$$

Example: Convert **0 1 1 1 0 1 0 0** to decimal.

128	64	32	16	8	4	2	1
0	1	1	1	0	1	0	0

$$64 + 32 + 16 + 4 = 116$$

Quick Practice: Convert the following binary numbers to decimal numbers as shown above. You may use a table with binary **place values**.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

0 0 1 0 0 1 1 0

0 0 1 1 1 0 0 1

1 1 1 1 0 1 0 0

0 0 0 1 0 0 1 1

0 1 1 0 1 0 0 0

0 1 1 1 1 1 0 1

Find more practice exercises at the end of this guide.

Tips and Shortcuts

Notice that all the numbers below have 1 digits in the rightmost place values. There is a pattern for finding the decimal value of these 8 numbers.

$$00000001 = 1$$

$$00000011 = 3$$

$$00000111 = 7$$

$$00001111 = 15$$

$$00011111 = 31$$

$$00111111 = 63$$

$$01111111 = 127$$

$$11111111 = 255$$

Each of the decimal values is 1 less than the next power of 2.

$$\begin{array}{cccccccc|c} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \end{array} = 2 - 1$$

$$\begin{array}{cccccccc|cc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & \end{array} = 4 - 1$$

$$\begin{array}{cccccccc|ccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & \end{array} = 8 - 1$$

$$\begin{array}{cccccccc|cccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & \\ \hline 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \end{array} = 16 - 1$$

$$\begin{array}{cccccccc|cccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & \end{array} = 32 - 1$$

$$\begin{array}{cccccccc|cccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & \\ \hline 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & \end{array} = 64 - 1$$

$$\begin{array}{cccccccc|cccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & \\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \end{array} = 128 - 1$$

$$\begin{array}{cccccccc|cccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 & \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \end{array} = 256 - 1$$

256 is the place value after 128.

Tip 1: Learn the sum of the place values from left to right.

$$10000000 = 128$$

$$11000000 = 192$$

$$11100000 = 224$$

$$11110000 = 240$$

$$11111000 = 248$$

$$11111100 = 252$$

$$11111110 = 254$$

$$11111111 = 255$$

Converting from Decimal to Binary

Tip 1: Do the fewest number of computations.

It's tempting to start to add all the place values, but there could be an easier route to take.

Consider the number:

1111110

We may begin adding 7 place values together, but there is a faster way.

Recall that: **1111111** = 255

254 is 1 less than 255.

Change 1 to 0 in the place value 1.

1111110

Directions: To use the technique shown above:

1. Find a known binary number that is close to the decimal number being converted.
2. Change digits to reach the decimal number.

Example: Convert 123 to binary.

Notice that 123 is close to 127

127 = **0111111**

Since $128 - 1 = 127$.

127 is 4 more than 123

Change the place value 4 in **0111111**

127 = **0111011**

Example Convert 55 to binary.

Notice that 63 is close to 57

63 = **0011111**

Since $64 - 1 = 63$.

63 is 6 more than 57

Change the place value 4 and 2 in **0011111** 57 = **0011001**

Going Backwards

We can also convert from decimal to binary using our table of binary place values.

Example: Convert 145 to binary.

128 > 145 so place a 1 in the 128 place

128	64	32	16	8	4	2	1
<hr/>							
1							

Add the next place value.

$$128 + 64 = 192$$

192 > 145 so place a 0

128	64	32	16	8	4	2	1
<hr/>							
1	0						

Add next place value.

$$128 + 32 = 160$$

160 > 145 so place a 0

128	64	32	16	8	4	2	1
<hr/>							
1	0	0					

Add the next place value.

$$128 + 16 = 144$$

144 < 145 so place a 1

128	64	32	16	8	4	2	1
<hr/>							
1	0	0	1				

From here, we can notice that we only need to add the place value of 1 to get from 144 to our desired 145.

$$144 + 1 = 145$$

128	64	32	16	8	4	2	1
<hr/>							
1	0	0	1	0	0	0	1

Or we can continue adding the next place value to 144.

Add the next place value.

$$144 + 8 = 152$$

152 > 145 so place a 0

128	64	32	16	8	4	2	1
1	0	0	1	0			

Add the next place value.

$$144 + 4 = 148$$

148 > 145 so place a 0

128	64	32	16	8	4	2	1
1	0	0	1	0	0		

Add the next place value.

$$144 + 2 = 151$$

151 > 145 so place a 0

128	64	32	16	8	4	2	1
1	0	0	1	0	0	0	

Add the next place value.

$$144 + 1 = 145$$

145 = 145 so place a 1

128	64	32	16	8	4	2	1
1	0	0	1	0	0	0	1

$$145 = 10010001$$

Tip: Learn the sum of the place values from left to right.

$$10000000 = 128$$

$$11000000 = 192$$

$$11100000 = 224$$

$$11110000 = 240$$

$$11111000 = 248$$

$$11111100 = 252$$

$$11111110 = 254$$

$$11111111 = 255$$

Tip: Learn the sum of the place numbers from right to left.

$$00000001 = 1$$

$$00000011 = 3$$

$$00000111 = 7$$

$$00001111 = 15$$

$$00011111 = 31$$

$$11111100 = 63$$

$$01111111 = 127$$

$$11111111 = 255$$

$$1 = 2 - 1$$

$$3 = 4 - 1$$

$$7 = 8 - 1$$

$$15 = 16 - 1$$

$$31 = 32 - 1$$

$$63 = 64 - 1$$

$$127 = 128 - 1$$

$$255 = 256 - 1$$

Converting 8 Digit Binary Numbers

0 1 0 1 0 0 1 1 =	0 1 1 0 0 1 0 1 =	1 0 1 0 1 1 0 0 =
1 1 0 0 0 1 1 1	1 1 1 0 0 1 1 1	0 0 1 0 1 0 1 1
1 1 1 1 0 1 1 1	0 0 1 1 1 0 1 0	0 1 0 1 0 1 1 1
0 0 0 0 0 1 1 0	0 0 0 0 1 1 0 1	1 1 0 0 1 0 0 1
0 1 0 1 0 0 0 0	1 0 0 1 0 0 0 1	1 1 1 1 0 0 0 0
0 1 0 0 1 1 0 0	0 0 0 1 1 1 1 0	0 0 0 1 1 0 1 1
0 0 0 1 1 1 0 0	0 0 1 0 1 0 1 0	1 1 1 0 1 0 1 0
0 1 0 0 1 1 0 1	1 1 0 0 1 0 1 0	1 0 1 1 0 1 0 0
1 0 1 1 0 0 1 1	0 0 1 0 1 0 1 1	1 1 1 0 0 1 1 1

Converting 8 Digit Binary Numbers

0 1 1 0 0 0 0 1

1 0 0 1 0 1 0 0

0 0 1 0 0 0 0 1

1 1 1 0 0 0 1 1

0 0 1 0 0 1 0 1

0 0 0 1 1 0 0 0

0 0 0 0 0 1 0 1

1 1 0 0 1 1 0 1

0 1 1 0 0 1 0 1

0 1 0 1 1 1 1 1

0 1 1 1 1 0 1 0

0 0 0 0 1 0 1 0

1 0 0 1 1 0 1 0

0 1 0 0 1 0 0 0

0 0 0 1 0 1 1 0

1 0 0 1 1 0 0 1

0 1 1 1 1 0 0 0

0 1 0 1 0 0 0 1

1 0 1 1 0 0 0 0

1 1 1 0 1 0 1 0

0 1 1 1 0 1 1 0

1 1 0 0 0 0 0 0

1 1 1 0 1 0 0 0

0 0 0 1 1 0 1 0

0 0 0 0 1 1 1 0

0 1 0 1 0 1 1 1

1 0 1 0 0 1 0 1

Converting 8 Digit Binary Numbers Practice

0 0 1 1 1 0 0 0

1 1 1 1 1 0 1 1

0 1 1 0 1 0 0 1

1 0 0 1 0 1 0 1

1 0 1 0 1 1 0 0

1 0 1 1 0 1 1 1

1 1 0 1 1 0 0 0

1 0 1 1 1 0 1 0

1 1 0 1 1 0 1 0

1 0 1 1 0 1 0 0

1 0 1 1 1 1 1 1

1 0 1 1 1 1 1 0

1 0 0 0 0 1 1 0

0 0 1 1 0 1 0 0

1 1 1 0 1 0 1 1

0 0 1 1 0 1 1 1

0 0 1 0 1 1 0 1

0 0 0 0 0 0 1 1

1 0 0 0 1 1 1 1

0 0 0 1 0 1 1 1

1 1 0 1 1 0 0 1

0 0 1 1 0 1 1 0

1 0 1 0 0 1 0 0

0 1 0 0 0 0 0 1

1 0 1 0 1 0 1 0

1 0 0 1 1 1 0

0 0 1 1 1 0 1 0

Converting decimal to binary numbers

38	93
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175	233
-----	-----

80	138
----	-----

141	207
-----	-----

167	94
-----	----

171	249
-----	-----

52	79
----	----

69	73
----	----

Converting decimal to binary numbers

111	116
87	164
20	105
108	230
133	161
188	210
76	210
221	248

Converting decimal to binary numbers

Directions: Convert the following decimal numbers to binary in the manner shown above.

12

94

111

118

88

120

160

240

11

49

80

168

89

120

158

217