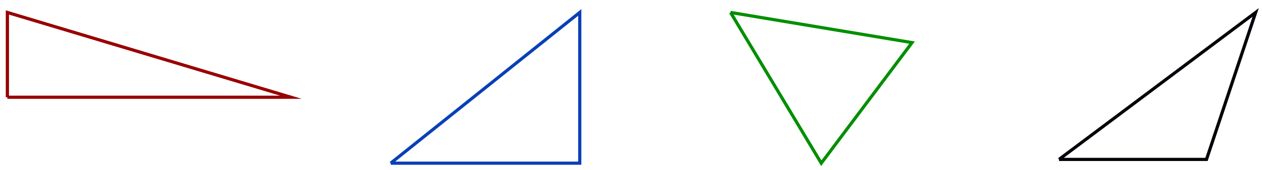
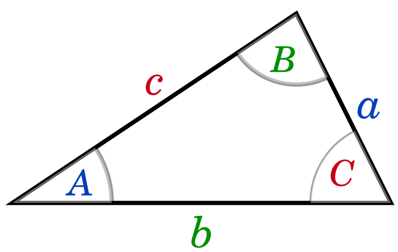
# TRIANGLES

A **triangle** is a **polygon** with 3 sides (edges) formed by three **vertices** (corners).



## Angles of a Triangle

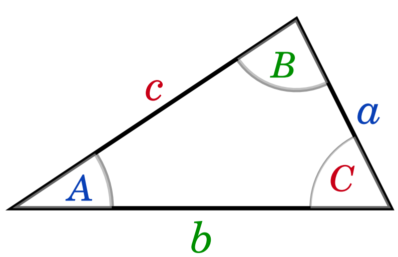
There are 3 **angles** inside a triangle. The sum of those angles is 180°.




| Sum of Angles |
| --- |
| $\angle A + \angle B + \angle C = 180°$ |

## Perimeter of a Triangle

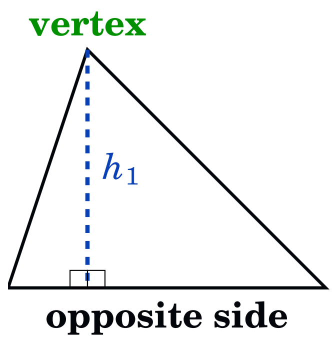
The sum of the 3 sides of a triangle is the **perimeter**.




| Perimeter of a Triangle |
| --- |
| $a + b + c$ |

# Altitudes of a Triangle

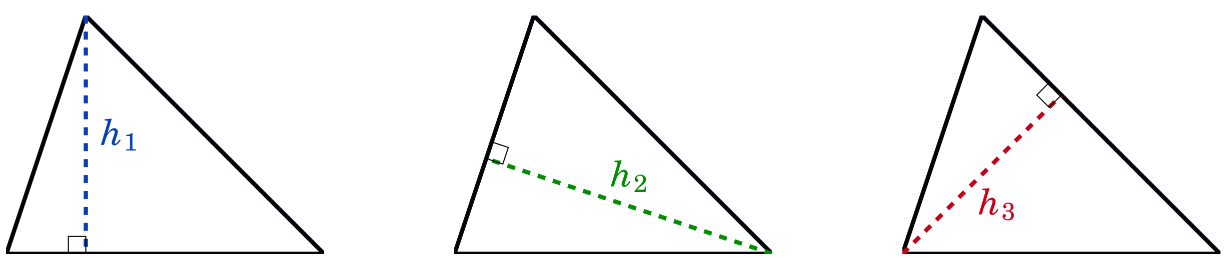
An **altitude** is a **perpendicular** line segment extending from a **vertex** to the side opposite that vertex. It is also called the **height** of a triangle.




**Perpendicular** $\perp$ means the segments form a $90^\circ$ angle (right angle).

## Altitudes of Acute Triangles

Triangles have three altitudes. If the triangle is **acute**, all three altitudes are *inside the triangle*.

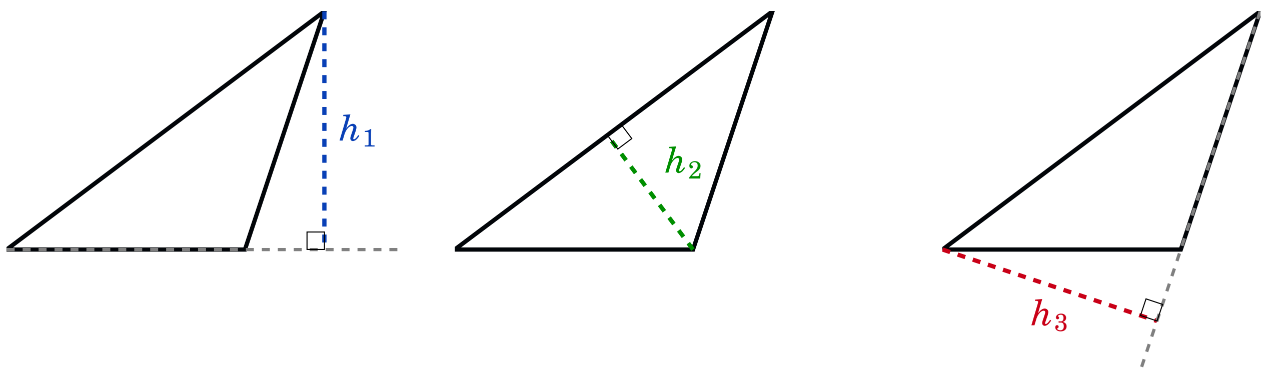



**Acute triangles** have three **acute** angles.

## Altitudes of Obtuse Triangles

If the triangle is **obtuse**, two of the altitudes are *outside the triangle*.

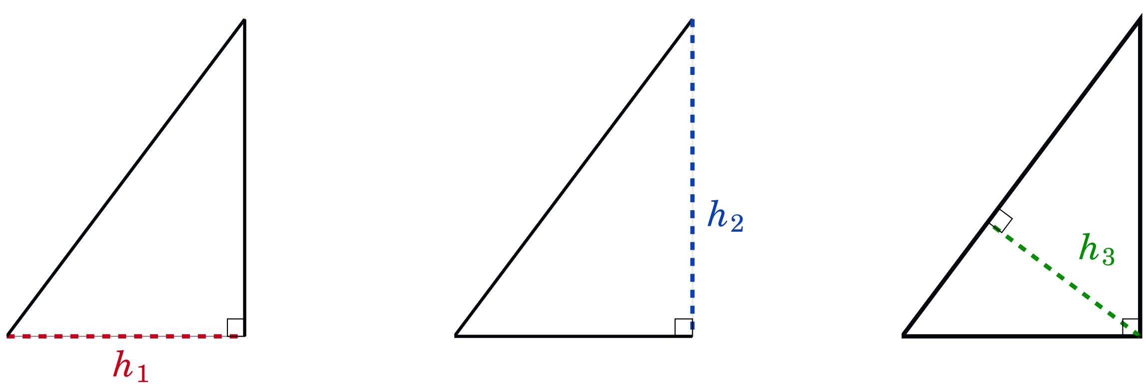
To draw the altitude, extend the line segment opposite the vertex.




- **Obtuse triangles** have one **obtuse angle** (greater than 90°).

## Altitudes of Right Triangles

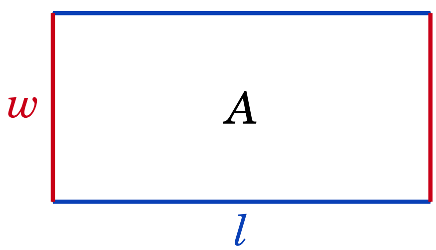
Two of the altitudes of a **right triangle** are the **legs** of the triangle.




- **Right triangles** have one **right angle** (equal to 90°).
- The **hypotenuse** of a right triangle is the side opposite the right angle.
- The **legs** are the two sides which form the right angle.

# Area of a Triangle

To find the **area** of a triangle, we will use the formula for the area of a rectangle.



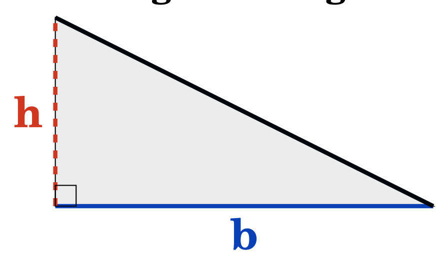

**Area of a Rectangle**

$$A = l \times w$$

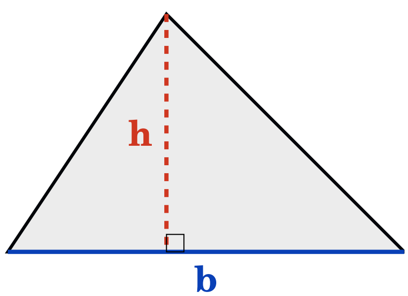
## Types of Triangles

The **height** of each triangle is an **altitude** denoted by **h**.

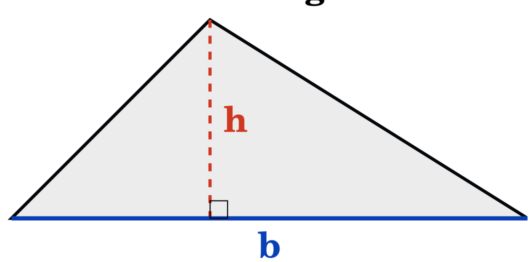
**Right Triangle**




**Acute Triangle**

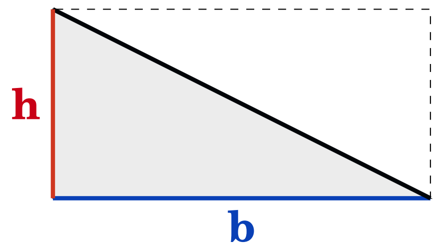



**Obtuse Triangle**



### ✿ Find the area $\mathbf{A}$ of a right triangle.

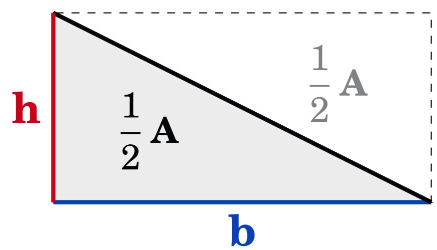
❶ Observe that the dashed lines form a rectangle whose area is $\mathbf{A} = \mathbf{b} \times \mathbf{h}$.



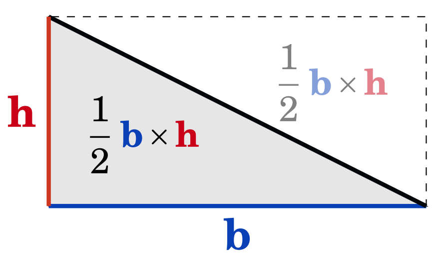

$$\mathbf{A} = \mathbf{b} \times \mathbf{h}$$

❷ The area of the triangle $\mathbf{A}$ is *one half* of the area $\mathbf{A}$ of the rectangle.

$$\mathbf{A} = \frac{1}{2}\mathbf{A}$$



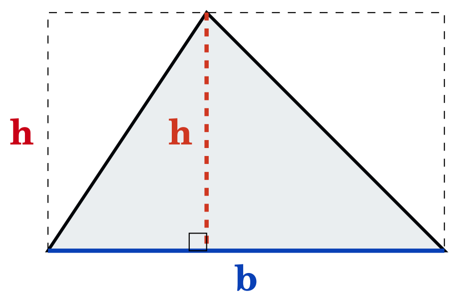

❸ Recall that $\mathbf{A} = \mathbf{b} \times \mathbf{h}$.




| **Area of a Triangle** |
|---|
| $A = \frac{1}{2} b \times h$ |

### ⚙ Find the area **A** of an acute triangle.

❶ Observe that the dashed lines form a rectangle whose area is $\mathbf{A} = \mathbf{b} \times \mathbf{h}$.

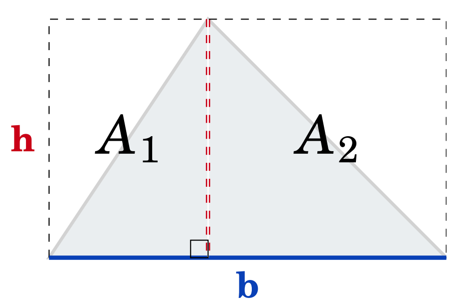



$$\mathbf{A} = \mathbf{b} \times \mathbf{h}$$

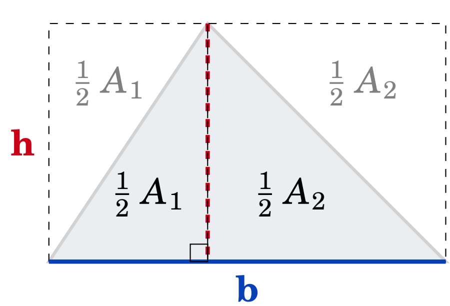
❷ The dashed lines also form two smaller rectangles with areas $A_1$ and $A_2$.

❸ The area **A** of the larger rectangle is given by $\mathbf{A} = A_1 + A_2$.

$$\mathbf{A} = A_1 + A_2$$




❹ The area of the triangle **A** is the sum of *one half* of the areas $A_1$ and $A_2$.




$$\mathbf{A} = \frac{1}{2}\,A_1 + \frac{1}{2}\,A_2$$

❺ We will now factor and substitute using $\mathbf{A} = \mathbf{b} \times \mathbf{h}$ and $\mathbf{A} = A_1 + A_2$.

$$
\begin{aligned}
\mathbf{A} &= \tfrac{1}{2} A_1 + \tfrac{1}{2} A_2 \\
\mathbf{A} &= \tfrac{1}{2} (A_1 + A_2) \\
\mathbf{A} &= \tfrac{1}{2} \mathbf{A} \\
\mathbf{A} &= \tfrac{1}{2} \mathbf{b} \times \mathbf{h}
\end{aligned}
$$

| **Area of a Triangle** |
|---|
| $A = \frac{1}{2} b \times h$ |