Introduction

The numbering system we use on a daily basis is *base 10* or the **decimal** system. It turns out that while this numbering system is a natural choice for humans, when computers are concerned, a *base 2* or **binary** numbering system is more suitable.

Decimal Numbers

Let's revisit how we were taught to read and write **decimal numbers**. This will help us understand the structure of **binary numbers**.

The quantity a number represents depends on two components:

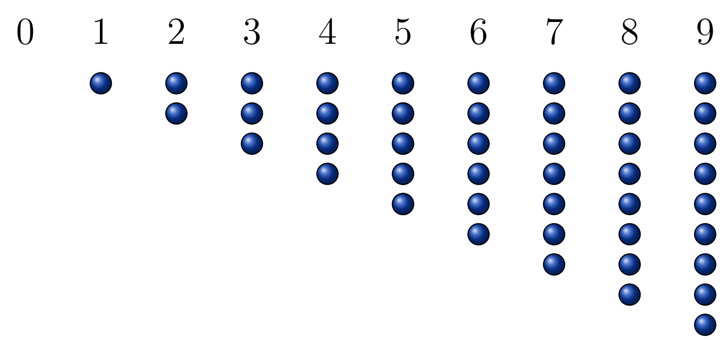
Digit value	**Place Value**

Digits

Digits are a numerical symbol used to indicate the quantity of something. We use *Arabic numerals*, which are actually Indian in origin, but were transmitted to the West via Arab merchants.

There are 10 digits in a **decimal** system: 0 1 2 3 4 5 6 7 8 9

They represent the following quantities:



Understanding Place Value

Using digits alongside a place value makes writing larger numbers more compact. We will see how unwieldy numbers becomes if there are no place values in use.

Tally system

Suppose you want to count the number of days since a full moon. After each day, you make a mark | to denote one day.

After several weeks, you have the following:

$$| \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; | \; |$$

If your number system only has one digit, it will become cumbersome to denote large numbers. A numbering system with only one digit is known as ***unary***.

Introducing a new symbol

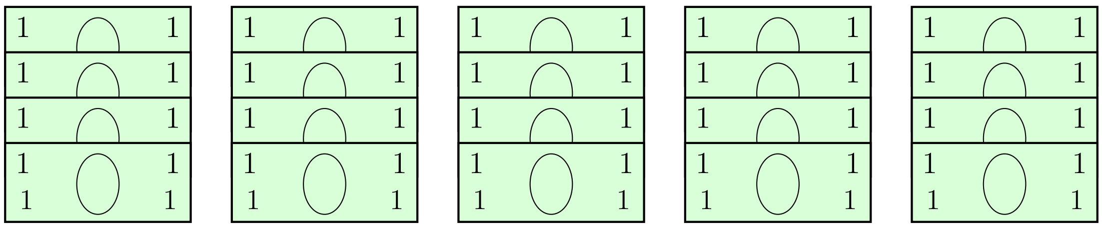
You might decide to make a new mark to collect a group of marks to make the quantities easier to read.

𝍸 𝍸 𝍸 𝍸 𝍸 𝍸

This new symbol 𝍸 represents a separate quantity from | and expands the versatility of the numbering system.

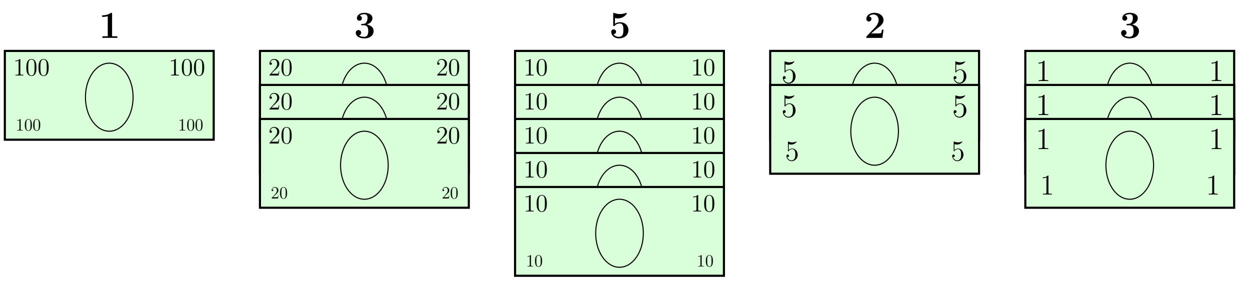
Singles only

A **unary system** is like walking around with a wallet full of one-dollar bills.



Ones, fives, tens, twenties, and hundreds

Luckily for your wallet, there are other bills to represent larger quantities of money. Instead of large stacks of one-dollar bills, you might have a wallet with this:



How much money is in the wallet?

$$(\mathbf{1} \times 100) + (\mathbf{3} \times 20) + (\mathbf{5} \times 10) + (\mathbf{2} \times 5) + (\mathbf{3} \times 1) = 223$$

- ❐ Count the number of each type of bill.
- ❐ Multiply that number by the denomination of the bill.
- ❐ Add to find the total quantity.

This is the same process used to find quantities within a certain number system. It creates the expanded form of a number.

Place Value

Place value is a way to assign a specific quantity to a **numeral** based on its position in the number.

Just like a specific bill denomination represents a certain dollar amount, a place value will tell you what quantity a **number** represents.

Position of a Digit

Place value is given by the position of each **digit**. It determines what quantity that digit represents.

For **decimal numbers**, the values increase by a factor of 10 as the place values move to the left.

ten thousands	thousands	hundreds	tens	ones
10,000	1,000	100	10	1

Base 10

Each place value can be represented in terms of **10** by using exponents. **10** is the base and each place value is a **power of 10** The table below shows the place values represented in terms of **10**.

10^4	10^3	10^2	10^1	10^0
10,000	1,000	100	10	1

Expanded Form

Consider the decimal number **123**. Recall that the **digit** and **place value** determine what that number represents. We will write the numbers in expanded form based on the place value of each digit.

100s place	10s place	1s place
1	**2**	**3**
(1×100)	(2×10)	(3×1)
100	20	3

$$\mathbf{123} = (\mathbf{1} \times 100) + (\mathbf{2} \times 10) + (\mathbf{3} \times 1)$$

$$\mathbf{123} = 100 + 20 + 3$$

Quick Practice

Write the following numbers in their expanded form like the example above.

$432 =$

$76 =$

$8632 =$

$154 =$

$90 =$

$5120 =$

$801 =$

$43 =$

$2922 =$

Base 2 or Binary Numbers

A **binary** number system follows the same structure as the **decimal** number system, but instead of powers of 10, place values are **powers of 2**.

Place Values

Binary

2^3	2^2	2^1	2^0
8	4	2	1

Decimal

10^3	10^2	10^1	10^0
1,000	100	10	1

Binary Digits

There are 2 binary digits **0** and **1**

Examples of Binary Numbers

The following are examples of **binary numbers** alongside their corresponding **decimal numbers**.

Binary	Decimal
00101010	42
101	5
11111	63
000001	1
10	2

We will convert from **binary** to **decimal** by expanding the number.

Expanded Form

Consider the binary number **1110**.

Add the quantities each digit represents to express **1110** as a decimal number.

$$\mathbf{1110} = (\mathbf{1} \times 8) + (\mathbf{1} \times 4) + (\mathbf{1} \times 2) + (\mathbf{0} \times 1)$$

$$\mathbf{1110} = 8 + 4 + 2 + 0$$

$$\mathbf{1110} = 14$$

8	4	2	1
1	**1**	**1**	**0**
$(\mathbf{1} \times 8)$	$(\mathbf{1} \times 4)$	$(\mathbf{1} \times 2)$	$(\mathbf{0} \times 1)$
8	4	2	0

Shortcut

Since there are only two digits, **0** and **1**, you can simply add every place value with a digit of 1.

8	4	2	1
1	**1**	**1**	**1**

$8 + 4 + 2 + 1 = 15$

8	4	2	1
1	**0**	**0**	**1**

$8 + 1 = 9$

8	4	2	1
0	**1**	**1**	**0**

$4 + 2 = 6$

Practice

Convert the following binary numbers to decimal numbers. You may use a table with binary place values.

2^3	2^2	2^1	2^0
8	4	2	1

$0\ 0\ 1\ 0 =$	$0\ 1\ 1\ 0 =$	$0\ 0\ 1\ 1 =$
$1\ 1\ 1\ 1 =$	$0\ 1\ 0\ 0 =$	$0\ 0\ 0\ 1 =$
$0\ 1\ 1\ 0 =$	$1\ 0\ 0\ 0 =$	$0\ 1\ 1\ 1 =$
$1\ 0\ 0\ 1 =$	$0\ 0\ 1\ 1 =$	$1\ 1\ 0\ 1 =$