Exercises 3

1. A one-dimensional classification problem for three classes, A, B and C and pdf-s $f_A(x)$, $f_B(x)$, $f_C(x)$ is defined as:

   $p_A = 1/2, f_A(x) = 1/8$ for $x \in [-4, 4]$

   $p_B = 1/3, f_B(x) = 3(1 - x^2)/4$ for $x \in [-1, 1]$

   $p_C = 1/6, f_C(x) = x/8$ for $x \in [0, 4]$

   Estimate optimal classification boundaries and decision rules for the system. Explain the general estimation steps and draw a figure that describes the estimation process and the results.

2. For a classification problem with two classes C_A and C_B, are the a priori probabilities $p_A = 3/4$ and $p_B = 1/4$. Assume the following pdf-s

   $$p(\bar{z}|C_k) = \frac{1}{2\pi|det\Sigma_k|^{1/2}} e^{-(\bar{z}-m_k)^T\Sigma_k^{-1}(\bar{z}-m_k)/2}$$

   with

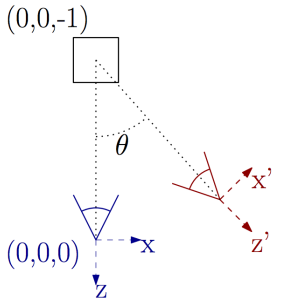
   $$m_A = m_B = 0,$$

   and

   $$\Sigma_A = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}, \Sigma_B = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}.$$

   Estimate decision boudaries for the problem!

3. A robot is trying to gather 3D information from an object. Since it has only a single camera, it rotates around the object to obtain multiple views from it, as seen in the figure below.



   a) What is the relation between points P in 3D space and their image projections p on the image camera at $(0, 0)$? Consider unit focal length, $f = 1$, and centered image origin, with x increasing to the right and y increasing up.

b) What is the relation between 3D points $P = (x, y, z)$ in the original coordinate frame and $P' = (x', y', z')$ in the new coordinate frame after rotating θ radians?

c) What is the relation between image points p and p'?

d) At some point the encoders of the motors fail and the robot doesn't know how large θ is. Estimate θ given point correspondences between the two images. How many point do you need?

4. Assume that you have a robot translating in space. For every point in time a camera (with focal length f), mounted on the robot, observes two points A and B by measuring the image coordinates (x_A, y_A) and (x_B, y_B), as well as the motion field (x'_A, y'_A) and (x'_B, y'_B). Set up a linear system of equations from which is possible to determine the translational direction of the robot (T_X, T_Y, T_Z) up to an unknown scale factor, including the depths of the two points Z_A and Z_B. Explain how it is possible to determine the direction, even if the scale factor is unknown.