Answers to questions in
Lab 1: Filtering operations

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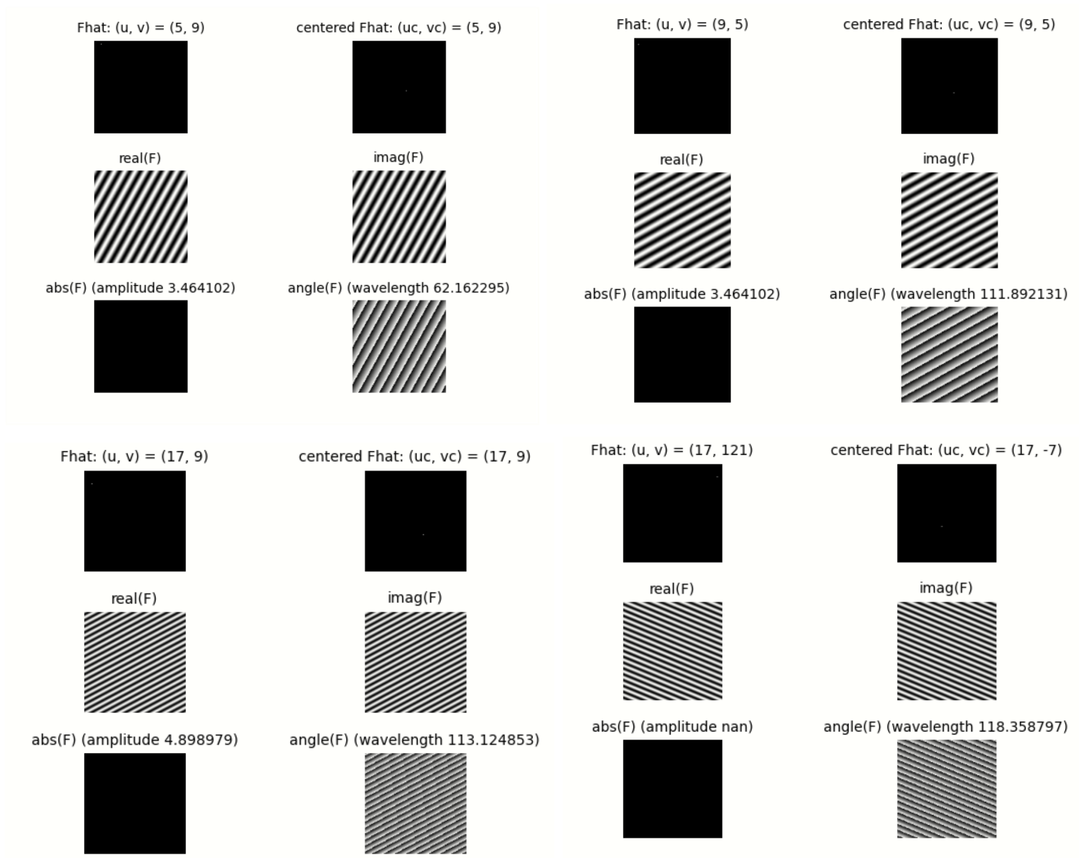
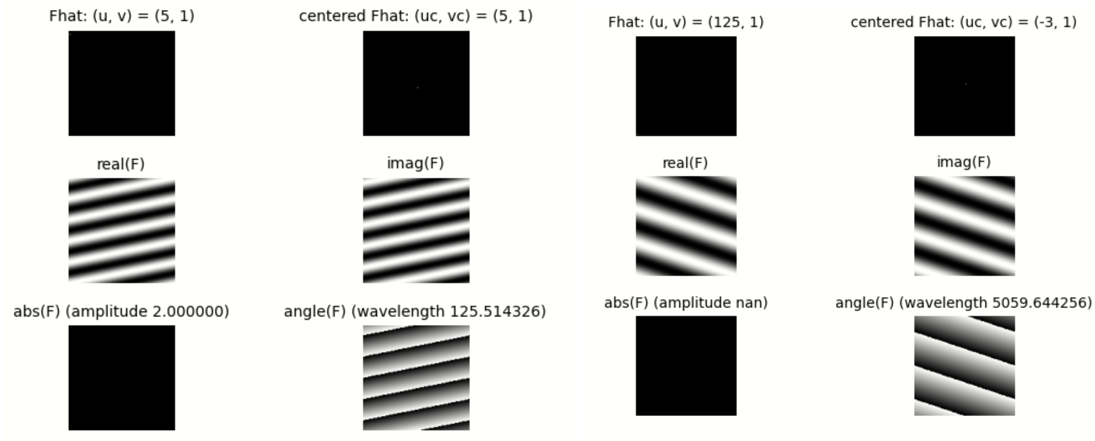
Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?





Answers:
The direction of the waves are changing. Higher p than q means they are more vertical, and vice versa. If the dot is further from origin it results in a higher frequency.

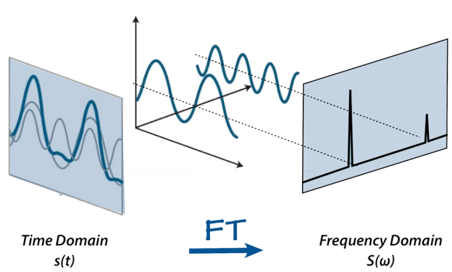
Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

The position of the point in the Fourier domain tells us about the vertical and horizontal frequency of a sine wave in the spatial domain. The p is the horizontal frequency and q is the vertical frequency. So if p=q we have the same amount of horizontal and vertical ”waves” in our image. If q is 0 we only have a horizontal wave and vice versa for p. The brightness of the point tells us the amplitude of that wave.

$$Angular frequency = \frac{2p}{N}(p, q)$$

Below is an image which graphically shows the correlation between the two domains:



Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

The amplitude is the highest value of the sinus wave in the spatial domain. Since the highest value of our sinus wave is 1 at (p,q) with the rest of the points being 0:

$$F(x) = \frac{1}{N} * \sum_{u \in [0..n-1]^2} (\hat{F}(u) e^{\frac{2\pi u^T x}{N}}])) \rightarrow \{max[\sum_{u \in [0..n-1]^2} (\hat{F}(u) e^{\frac{2\pi u^T x}{N}}]))] = 1$$

$$a = max(F(x)) = \frac{1}{N} 1 = \frac{1}{N}, N = 128$$

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

As per slide lecture 3, slide 19/42 the direction/angle is given by (course book eq 4-88) , (eq 5)

$$\lambda = \frac{1}{\sqrt{u^2 + v^2}}$$

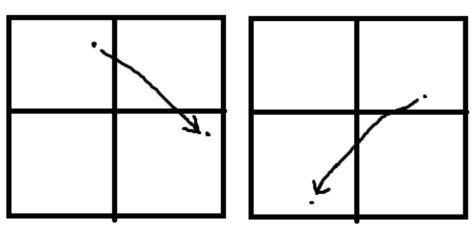
$$\phi = \tan^{-1}[\frac{I(\omega_1, \omega_2}{R(\omega_1, \omega_2)}]$$

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:

When the point exceeds half the image size, either in x or y axis, the point gets shifted back the size of the image, thereafter the point gets shifted with fftshift, which swaps the first and third quadrant, and swaps the second and fourth quadrant. The result is that the point is centered so origo is in the center instead of the left-top corner.

Here are two examples of how a point moves when shifted this way:



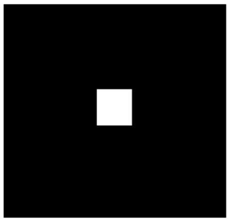
Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:
Just as explained in Question 5, we must calculate the correct new coordinates when changing the origin to the center. In detail, those lines of code check if the point exceeds half the image, and if so, the point is shifted back with the width of the image, which is done both for the x- and y-axis.

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

We discussed two intuitive explanations for this behaviour. First, let's for cimplicity consider a white square as a picture in the middle of the spectra domain as illustrated below.

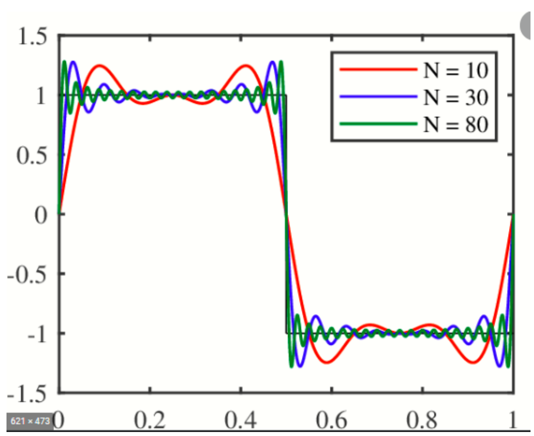


When looking vertically through the square, the signal changes black-white-black, which can be seen as a Rectangle function. We know that the Fourier Transform of a Rectangle function is a Sinc function

$$Rect(\frac{x}{T}) \rightarrow FT \rightarrow T sinc Tu$$

Sinc has increasingly higher frequency further away from the origin (towards the edges), which would explain why the fourier Fourier spectra in this case is concentrated towards the edge of the image.

The second intuitive explanation is that, in order to approach a square wave with a sum of sinusoids, we must add increasingly more sinusoids with increasingly smaller frequency. As we approach the square function, the amount of sinusoids added with small frequencies far exceeds the sinusoids with large frequencies in the sum. Hence, when considering all sinusoids in the sum, when expressed in Fourier domain, we can distinguish a higher concentration of high frequencies, which in Fourier domain is plotted towards the edges if the images, as high frequencies lie far from the origin and low frequencies lie near the origin.



Question 8: Why is the logarithm function applied?

Answers:

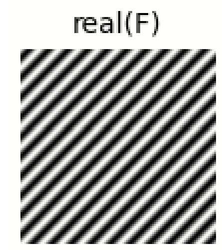
The logarithm function is added to better visualize the result, as to allow for low values to be visible instead of being overshadowed by large amplitudes.

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

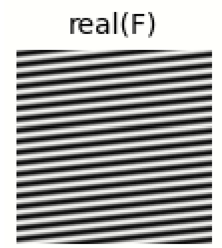
Answers:

Linearity can be shown by computing the following Fourier transforms (we only consider the real part for simplicity, but the same holds for the imaginary part).

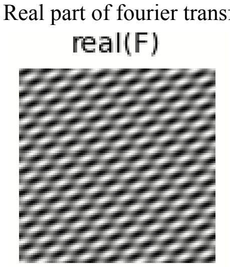
Real part of fourier transform of point (10,10) by running fftwave(10,10):



Real part of fourier transform of point (18,2) by running fftwave(18,2):



Real part of fourier transform of point (10,10) and point (18,2) by modifying fftwave with custom points:



We can see the sum of the fourier transforms for each point, since: FT(sum of points) = FT(point) + FT(point), which shows the property of linearity.

Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

Yes, we can generate this by convoluting in the Fourier domain. This is because convolution in the Fourier domain corresponds to multiplication in the spatial domain.

Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

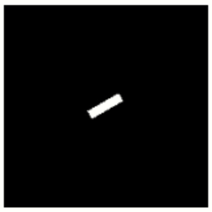
Answers:

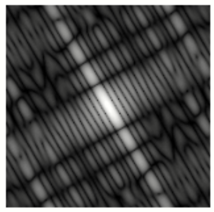
From comparing the original source image to the new we can see a scaling in both x and y domains. In the Fourier domain we observe that an enlargement on the width (x-direction) in the spatial domain corresponds to an enlarged height (y-direction) in the Fourier domain and a reduction in the spatial's y-domain corresponds to a reduction in the Fourier's x-domain.

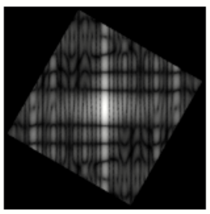
Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

We noticed that when rotating the image by x degrees the image in the Fourier domain is rotated by the same amount. Just rotating an image does not change any frequencies (which is shown visually by the frequencies shown in the Fourier domain having the same distance to the origin), it only affects the direction. Although for the angles between 1-44 and 46-89 we notice a lot of distortions both in the rotated image's Fourier domain but also in the Fourier domain of the image which is rotated back. This could be due to it not properly representing the figure when it is rotated at those angles.







Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

Applying the pow2image() makes the image foggy but we can still see the general shape although the edges are weakened. The magnitude contains information about intensity of each pixel which is shown by a distorted luminosity.
Applying the randphase causes a total distortion of the shapes, but it seems like we can still see the general luminosity, for instance for the mountain image seems to be darker than the others which have a generally lighter original image.

Question 14: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and
100.0?

Answers:

T	**0.1**	**0.3**	**1.0**	**10.0**	**100.0**
Var	[1.33e-02 0 ;0 1.33e-02]	[2.81e-01 0 ;0 2.81e-01]	[1 0 ;0 1]	[10 0 ;0 10]	[100 0 ;0 100]

Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

The differences in the continuous case and discrete case to our results are very similar. They show a trend of having larger differences for t = 0.1 and 0.3 and then at 1.0, 10.0 and 100.0 they have roughly the same small difference (or rather no difference at all).

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

With higher t values the image gets increasingly blurry until barely detail at all is shown for t = 256.0. Since the higher frequencies are responsible for the edges/details and those are being muted this is an expected outcome

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

Ideal: For the gaussian noise the ideal filter did a decent job at making it less grainy for cutoff = 0.6 and cutoff = 0.3. The cutoff seems to be highly sensitive, where for instance cutoff = 0.1 causes the image to become distorted with a bubbly pattern.
Median: filling areas, smooth blobs with sharp edges
Gauss: removes edges, just a big blur, constant blurred filter over entire image

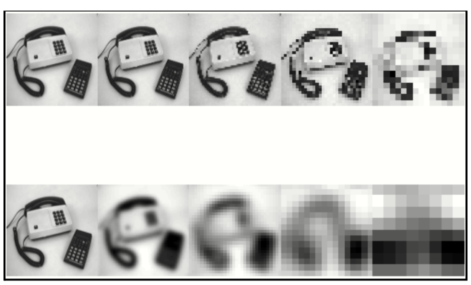
Question 18: What conclusions can you draw from comparing the results of the respective methods?

Answers: We conclude that the median filter worked best at removing noise and resulted in a smooth color throughout the blob, while also preserving edges well.

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

Gauss t=10

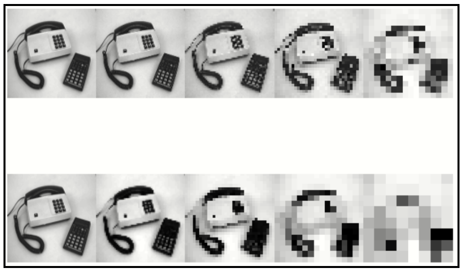


This method of blurring makes it difficult to see the blob's shape because it's smoothed into the background and then further distorted.

Ideal t=0.3

Keeps the image decently sharp until the very last image. Looks like it simply loses some resolution like a highly compressed JPEG.

Median t = 3


Loses the details but keeps the big blob’s shape well.

Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:
We conclude that when smoothing before subsampling, we can better preserve the overall shape of the blob in the image. When smoothing first, we keep the large properties and shapes, but lose the details and noise. By smoothing before subsampling, we have the benefit that the noise doesn’t grow and interfere with the subsampling.