Punto 0-2

Para todos los casos se asume que: $h_x = h_y = h$

$$\frac{\partial^2 U}{\partial x^2} = \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{h_x^2} \quad (1)$$

Se tiene que:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = \omega \quad (2)$$

Reemplazando (2) en (1)

$$\left[U_{i+1,j} - 2U_{i,j} + U_{i-1,j} + U_{i,j+1} - 2U_{i,j} + U_{i,j-1}\right]\frac{1}{h^2} = \omega_{i,j}$$

$$-4U_{i,j} + U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} = h^2\omega_{i,j}$$

$$-4U_{i,j} = h^2\omega_{i,j} - U_{i+1,j} - U_{i-1,j} - U_{i,j+1} - U_{i,j-1}$$

$$U_{i,j} = \frac{1}{4}\left(-h^2\omega_{i,j} + U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1}\right) \quad (3)$$

Se tiene también que:

$$V\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) = \frac{\partial U}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial U}{\partial x}\frac{\partial \omega}{\partial y} \quad (\mathrm{I})$$

$$V\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) = \left[-4\omega_{i,j} + \underbrace{\omega_{i+1,j} + \omega_{i-1,j} + \omega_{i,j+1} + \omega_{i,j-1}}_{\lambda}\right] V.\frac{1}{h^2} \quad (4)$$

Se tiene que:

$$\frac{\partial U}{\partial x} = \frac{U_{i+1,j} - U_{i-1,j}}{2h_x} \quad (5)$$

Usando la definición de (5) en (I) se tiene que:

$$\frac{\partial u}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial \omega}{\partial y} = \frac{\overbrace{[u_{i,j+1} - u_{i,j-1}][\omega_{i+1,j} - \omega_{i-1,j}]}^{\beta}}{4h^2} - \frac{\overbrace{[u_{i+1,j} - u_{i-1,j}][\omega_{i,j+1} - \omega_{i,j-1}]}^{\Lambda}}{4h^2} \qquad (6)$$

Volviendo a (I) con (4) y (6) se tiene:

$$\frac{\nu}{\cancel{h^2}}[-4\omega_{i,j} + \lambda] = \frac{\beta}{4\cancel{h^2}} - \frac{\Lambda}{4\cancel{h^2}}$$

$$-4\nu\omega_{i,j} = \frac{\beta}{4} - \frac{\Lambda}{4} - \nu\lambda$$

Finalmente:

$$\omega_{i,j} = \frac{\Lambda}{4}\cdot\frac{1}{4}\cdot\frac{1}{\nu} + \frac{1}{4}\cdot\frac{1}{\cancel{\nu}}\cdot\cancel{\nu}\lambda - \frac{\beta}{4}\cdot\frac{1}{4}\cdot\frac{1}{\nu}$$

se define el número de Reynolds como $\frac{1}{\nu}$

$$\rightarrow \omega_{i,j} = \frac{R}{16}\Lambda + \frac{\lambda}{4} - \frac{R}{16}\beta \qquad (II)$$