



Some Benefits and Limitations of Modern Argument Map Representation

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Abstract

Argument maps represent some arguments more effectively than others. The goal of this article is to account for that variability, so that those who wish to use argument maps can do so with more foresight. I begin by identifying four properties of argument maps that make them useful tools for evaluating arguments. Then, I discuss four types of argument that are difficult to map well: *reductio ad absurdum* arguments, charges of equivocation, logical analogies, and mathematical arguments. The difficulties presented by these four types appear unrelated to one another, but I show that, in each case, the difficulty can be traced back to the use of metalinguistic reasoning. The need to represent a transition between object language and metalanguage can undermine one or more of the benefits that argument map representation would otherwise confer.

Keywords Argument map · Informal logic · Metalanguage · Argument representation · Paul Grice · Inferential justification

1 Benefits Both Logical and Psychological

Argument maps can help us achieve a variety of intellectual goals. One goal is improving critical thinking skill. Protracted training in argument map construction appears to improve critical thinking skill by a greater margin than training in other disciplines (Harrell 2011; van der Brugge 2018; Cullen et al. 2018). The relationship between argument mapping and critical thinking has been the primary target of the academic literature in this area, but my interest is somewhat different. My interest is not in how (or whether) argument maps improve durable cognitive skills, but in how argument maps facilitate the process of argument evaluation in real time.

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To make a defensible judgment about the quality of an argument, we cannot simply consult intuition. Instead, we must follow a rational procedure that is capable of delivering a judgment about the quality of the argument. The question I want to ask is how argument maps facilitate this procedure. Or, to put the question more perspicuously, what is it about *the design* of argument maps that facilitates this rational procedure? A complete answer to this question can't be restricted to the manner in which visual cues guide attention, even if, as is likely, the attention-guiding properties of argument maps are vital (Wolfe and Horowitz 2004). To facilitate a rational evaluation procedure, the map must not only make relevant information visually salient, but also group informational elements in a manner that facilitates the individual cognitive operations that make up the evaluation procedure.

To articulate the relationship between argument map design and argument evaluation, I analyze an example in some detail, and draw from it four properties that enable argument evaluation to be carried out efficiently. These four properties are unlikely to surprise seasoned argument mappers, who, I suspect, will already have an implicit awareness of them. Nevertheless, there is value in making the list explicit. Once these properties have been identified, we can investigate the unique epistemic benefits of each. Individualized knowledge of these benefits will, in turn, enable us to craft our maps with a greater degree of precision.

There is another reason to make the list explicit. In light of the list, we can better understand the limits of argument map representation. In particular, the four properties on the list help explain why argument maps represent some arguments types more effectively than others. This variability has not been discussed in the academic literature, but it deserves to be. Understanding why some argument types are harder to mold into argument map format will help us to manage the tradeoffs involved in mapping them. I examine four such argument types: *reductio ad absurdum* arguments, charges of logical equivocation, logical analogies, and mathematical arguments. At first glance, the difficulties presented by each type seem idiosyncratic. I show, however, that in each case, the difficulty can be traced back to the use of metalinguistic reasoning. In each case, the need to represent a transition between object language and metalanguage can undermine one or more of the benefits that argument map representation would otherwise confer. The article concludes with a practical evaluation of strategies for mapping metalinguistic arguments.

2 Narrowing the Focus: Modern Beardsley-Freeman Maps

The term “argument map” has been used to describe a great variety of representational devices. In this article, I am concerned exclusively with a particular kind of argument map that has recently become popular in English-speaking philosophy departments, and which were used in the aforementioned publications to demonstrate the cognitive benefits of argument mapping. These maps are typically constructed by means of software programs like *MindMup*,¹ *Rationale*,² and Carnegie

¹ Available at <https://www.mindmup.com>.

² Available at <https://www.rationaleonline.com>.

Mellon's free *Argument Diagramming* program.³ Each of these software tools enables users to construct some variation on what Harrell and Wetzel (2015) call Beardsley-Freeman (BF) maps, after James Freeman, whose first made the case for using diagrams of this kind for argument analysis in his book *Dialectics and the Macrostructure of Arguments*, and Monroe Beardsley, whose (1966) book *Thinking Straight* inspired Freeman's own.

Freeman's book is over 30 years old now. When it was published, Freeman assumed that most people would construct argument maps with pencil and paper. Writing out a complete argument map with pencil and paper is painstaking. If you make a mistake, it is not easy to re-arrange the existing map into a new configuration without having to start from scratch. To streamline the process, Freeman's method labels each statement in the text with a number, and constructs the diagram as a configuration of those numbers. This method cuts down on writing time, but also forces the reader to move back and forth between the map and the passage of prose to which it corresponds. With the advent of argument mapping software, this numbering method is no longer necessary. You can type each sentence into a box, and then drag the boxes into whatever configuration you like. The resulting map can then serve as a standalone representational device. In this article, I am concerned exclusively with such standalone maps. My use of the qualifier "modern" in the title of this subsection is intended to distinguish these standalone maps from their numerical forbears, which cannot be evaluated without an accompanying text.

Modern BF maps are perhaps most usefully contrasted with those developed in Stephen Toulmin's (1958) well-known book *The Uses of Argumentation*. The most distinctive feature of modern BF maps is the simplicity of their ontology. Unlike Toulmin's system, which categorizes each statement in an argument into one of 6 functional kinds, BF maps are composed of only two kinds of statement: premises and conclusions. Freeman argues that the simplicity of his system results in greater generality. It is possible, in other words, to represent a wider variety of arguments in the BF system than in Toulmin's. While I think that Freeman was right to consider his own system more general than Toulmin's, the message in this article is that generality comes at a cost.

Some of what I say here might have implications for other argument mapping systems. Nevertheless, I want to emphasize that my claims are directed exclusively toward modern BF maps. While other argument mapping systems may also run into trouble with metalinguistic arguments, I am concerned with the tradeoff involved in choosing the modern BF system. That system delivers a set of particular epistemic benefits and these must be weighed up against the limitations I intend to point out. Insofar as the epistemic benefits of alternative argument mapping systems deviate from those of the modern BF system, the tradeoffs implicit in those systems will be different as well.

Some variant of BF argument mapping is currently taught in philosophy departments at flagship universities such as Princeton, Harvard, New York University, Rutgers, Carnegie Mellon, Notre Dame, Ohio University and UC San Diego, along with many smaller but nevertheless influential colleges like Amherst

³ Available at <https://oli.cmu.edu/courses/argument-diagramming-open-free/>.

and Williams. It is also noteworthy that Harvard's philosophy department has entered into a partnership with a company called *ThinkerAnalytix*,⁴ which has been promoting the BF argument mapping technique across the English-speaking world, at both the high school and college level. BF maps appear to be the ascendant argument mapping format, and it is for this reason that I have chosen to focus exclusively on them.

3 How Maps Facilitate Argument Evaluation

The best way to show how argument maps facilitate argument evaluation is by way of example. The following passage presents my own rendition of a famous argument given originally by Lucretius, the Roman poet and Epicurean philosopher.

The state of being not-yet-born is not bad for you. But being dead is just like being not-yet-born, in all relevant respects. Death, therefore, is not bad for you. And since it is irrational to fear what is not bad for you, you should not fear death.⁵

This argument is composed of only five claims, including the conclusion. To map it, we find the conclusion, isolate any additional claims asserted by the author, and then arrange them underneath the conclusion in an inverted tree diagram that is designed to convey the inferential structure of the argument. Figure 1 presents the mapped version of this passage.

I will now describe four properties of modern BF argument maps that make them such beneficial tools for argument evaluation.

3.1 Uniformity of Argument Units

My prose rendition of Lucretius' argument was crafted to accentuate its inferential structure, and to minimize the need for editing when being repackaged in argument map format. Nevertheless, as careful readers will have noticed, the sentences in the map *have* been edited slightly. For the purpose of understanding argument map design, the most significant change is that the words "therefore" and "since" have been removed. In the prose version of the argument, these words serve to indicate the author's intention to draw an inference. In a map, inference indicators are removed, and inferences are represented instead by means of a line-like graphic. Despite its simplicity, the replacement of lexical with graphical inference indicators is perhaps the most important principle in argument map design,⁶

In the context of argument mapping, it is useful to conceptualize inference in terms of epistemic justification. Think of an inference as a relation between claims

⁴ Their software can be found at <https://thinkeranalytix.org>.

⁵ Adapted from Smith (2001).

⁶ In this case, inferences are represented by a inverted T-shape depicted in green. All maps in this article were produced using a software package called *Mindmup* which can be found at <https://www.mindmup.com>.

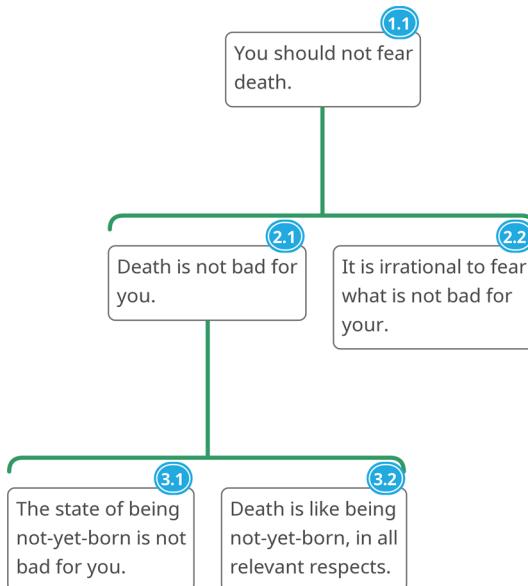


Fig. 1 A simple map composed of two argument units. The first argument unit is itself composed of claims 1.1, 2.1, and 2.2. The second is composed of claims 2.1, 3.1, and 3.2. The convention for numbering claims is maximally simple. The numeral before the decimal is determined by the vertical position of the claim, starting with 1 at the top and increasing as the map expands below. The numeral after the decimal is determined by the horizontal position of claim, beginning with 1 at the left and increasing as the map expands to the right. Alternative numbering conventions are of course possible. What matters is only that each claim receive a unique numerical label so that we can refer to it without confusion

such that two or more claims, which we call *co-premises*, will strengthen the reader's justification for believing some additional claim, which we call the *conclusion*. The subsection of an argument map that represents an inference is called an *argument unit*. We can define an argument unit as a set of claims that includes one target claim, and two or more co-premises which, together, constitute a reason either to believe, or to disbelieve the target claim.⁷ Using the argument unit concept, we can express the content of the convention governing the representation of inferences more precisely. It is that each argument unit represents exactly one inference, and each text box represents exactly one claim.⁸

This convention entails that, from a sufficiently abstract point of view, the structure of every argument unit is the same. Let's call this property *uniformity*.

⁷ One might wonder why an argument unit cannot have just one premise. The insistence on two premises is merely a practical convention that forces the mapper to consider the role of hidden premises even in the simplest case of one-premise arguments.

⁸ Counting claims can be tricky, because the number of claims can be more or less than the number of propositions expressed. When you assert a conditional, for example, you make one claim, but convey at least two propositions. Counting inferences can also be tricky, because sometimes two claims that can be interpreted as standing in a justificatory relation, despite the fact that the author's intention was to offer them as a pair of co-premises. Heuristics are available to help make such judgment calls, but they are beyond the scope of this article.

As a result of the uniformity of (well-constructed) argument maps, every argument unit can be evaluated using the same test, which we can think of as an oft-repeated sub-routine in the procedure for evaluating the argument as a whole. The best way to characterize the test for individual argument units will depend on whether the argument map was designed to be deductively valid, or whether it was designed instead to satisfy some weaker inferential standard. There are cases in which the language necessary to achieve deductive validity is so cumbersome, and so far removed from the original formulation, that it is best to settle for a weaker standard, such as *beyond a reasonable doubt*.

In any case, the ultimate goal of argument analysis is to work out the degree to which the premises provide justification for believing that the conclusion is true, and that is what our test is intended to establish. The test is composed of two steps. The first step is designed to assess the inferential strength of an argument unit, or what Freeman called the “connection adequacy.” The second step is designed to assess either the probability that the premises are true, or what Freeman called the “premise acceptability.”

1. *Step one.* Try to think of a situation in which all of the premises in the unit are true and the conclusion is false. If you find it impossible to think of such a situation, the inference is likely to be valid, and may be regarded as such. If the only situations you can think of are so remote from actuality that they can reasonably be ignored, then the inference is beyond reasonable doubt.
2. *Step two.* Ask whether the *unsupported* premises are in fact likely to be true, given your background knowledge. Skepticism about one or more of the unsupported premises will mitigate the degree to which the inference strengthens your justification for believing the conclusion. (Why only the unsupported premises? I’ll answer that question below.)⁹

In order to make a final judgment about the degree of justification conferred upon the conclusion by the premises, the results of these two steps of the test must be combined. Doubt about either step will diminish the degree of justification conferred.¹⁰ For our purposes, the most important fact about this two-part test is that it can be deployed for *every* argument unit. This fact eliminates uncertainty about how to proceed, and requires that we memorize only one strategy.

⁹ As I argue below, the ability to rely on this test is part of what makes argument mapping so helpful. Nevertheless, this particular formulation is probably too complicated to present to students who are just starting to think about argument structure. My own practical solution to this problem is to stick to deductive arguments at the outset, and expand to non-deductive arguments only after they have mastered the basics. If you are only dealing with deductive arguments, only the first part of step one is relevant, and that part is considerably easier to apply.

¹⁰ There are presumably epistemological norms that dictate whether the process of combination is performed well (or poorly, as the case may be). One could look, for example, to the various principles of conditionalization in the literature on Bayesian epistemology. That literature, however, tends to remain at an abstract level of analysis, and is unlikely to be of much practical help here. It is also, in any case, well beyond the scope of this article.

3.2 Informational Encapsulation

In order to appreciate the other ways that argument maps facilitate argument evaluation, we need another example. Our first map contains only one vertical branch. That is, neither of the two target claims has more than one reason attached to it. Often, however, target claims *do* have more than one reason attached. To demonstrate this, I'll add another argument unit to the map, which I draw, with considerable interpretive liberty, from Epicurus.

You should not fear death because when death comes, you won't be there to perceive it. And you should not fear that which you shall never perceive.¹¹

Figure 2 shows that Epicurus's argument should be represented as a separate branch in the tree diagram. This convention represents the fact that, although Epicurus' argument is designed to support the same conclusion as Lucretius' argument, we must not allow ourselves to pre-judge the quality of the reasoning in one argument on the grounds of having identified a flaw in the other. After all, having been presented with a bad argument for some conclusion is no reason to believe that the conclusion is false. If we are to evaluate an argument map charitably, therefore, we must ensure that the epistemic fallout of any flaw we may find be confined to the branch in which the flaw is found. As a terminological reminder of this principle, we will say that, with respect to one another, argument units located in distinct vertical branches are *independent*, or that they provide *independent reasons* for believing in the truth of the conclusion.

When we look at the relations between argument units within one vertical branch, independence in this sense clearly fails because the conclusion of one unit serves as a premise in the unit above. Nevertheless, for the purposes of argument evaluation, even argument units within a given vertical branch stand in a weaker relation of semi-independence, which is important for understanding how argument maps facilitate argument evaluation. This semi-independence can be defined as follows: all of the information in the map that is relevant to the evaluation of any given unit is located within that unit itself. I call this property *informational encapsulation*.

To see that informational encapsulation really holds, recall the two-step strategy for evaluating an argument unit. In step one, you assume the truth of all premises within the unit. In step two, you assess the probability of the unsupported premises only. This strategy advises you to assume that all *supported* premises are true. Given that assumption, the actual truth values of supported premises have no bearing on the evaluation of the current unit. As a result, you never need to venture outside the unit in order to locate the information needed to evaluate it.

The justification behind this two-step strategy is that it avoids double-counting the flaws in the map. If the supported premises are knowably false, the problem will be detected during the evaluation of the lower argument unit for which it serves as the conclusion.

The informational encapsulation of argument units is responsible for the second way that argument maps facilitate argument evaluation. Informational encapsulation

¹¹ Adapted from Bailey (1926).

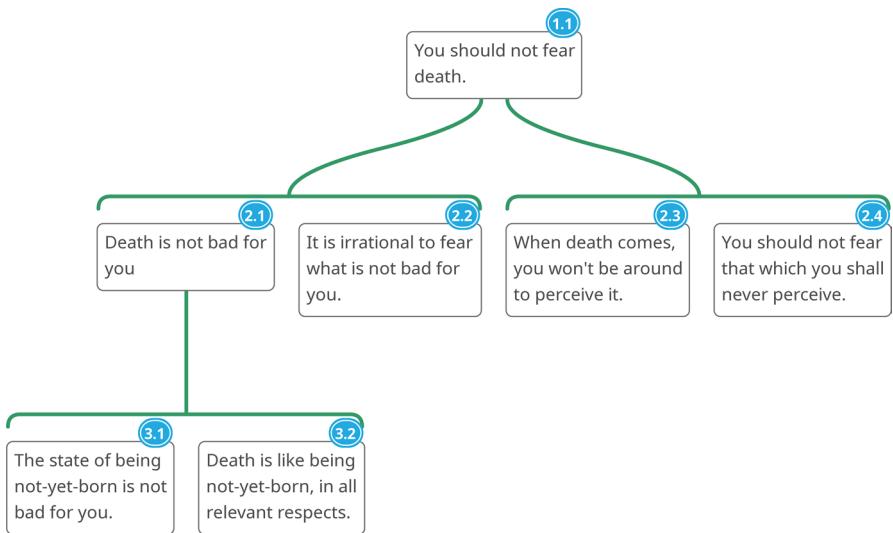


Fig. 2 This map is slightly more complex than the one depicted in Fig. 1. In this map, the conclusion of the argument (claim 1.1) is supported by two independent reasons, each of which is composed of two premises. Each reason is separated from the other by means of a green bracket, which is itself connected to the conclusion. Because claim 2.1 is provided with support of its own, this map includes a total of 3 argument units

allows the reader to focus their attention exclusively on the unit under evaluation, and confidently ignore every other unit on the map. If argument units were unbounded in size, informational encapsulation might turn out to have little effect on argument evaluation, because working memory capacity might be overwhelmed even when attention is confined to a single unit. However, it is an interesting and under-theorized feature of natural language reasoning that inferences are bounded in size. If the examples used in the many books on natural language reasoning are anything to go by, natural language arguments typically consist of argument units with no more than four or five premises. Consequently, argument units are typically *compact*; containing a maximum of five or six claims. When we combine informational encapsulation with this empirical observation about natural language arguments, we have the result that at any given point in time, you need to evaluate at most five or six claims. As a result, informational encapsulation imposes a useful limit on the degree to which argument evaluation can tax your cognitive resources.

3.3 Arborescence

The third way that argument maps facilitate argument evaluation has to do with their tree-like organization. If we think of each argument unit as a node, and each inference as a vertex, argument maps can be viewed as a special kind of directed graph in which there is only ever one path from one of the unsupported nodes at the bottom of the map to any other node. A graph of that kind is sometimes called

an *arborescence* (Weisstein 2020). The advantage of arborescent organization is that, if you do find a flaw in some part of the map, you can immediately see the consequences of that flaw for the whole. Justification will be undermined for all units above the one in which the flaw is located, until you reach either the global conclusion of the argument, or a claim that is supported by an independent reason. Moreover, argument maps are composed of nothing but argument units. So if you do not find a flaw in any particular unit, you know there is no flaw in the map. To put the point more generally, we can say that, because of the arborescent organization of the map, your evaluation of the whole map follows automatically from your evaluation of the parts. There is no extra step in which you must, for example, check to make sure that the various subsections of the map cohere with one another appropriately.¹²

3.4 Scalability

Finally, the fourth property of argument maps that makes them good at facilitating argument evaluation has to do with their *scalability*. Scalability refers to the fact that a map can display structural information effectively, even for unusually large arguments. For the sake of saving space, I will refrain from giving an example of a very large argument, but the point is easy enough to appreciate without an example. When large, complex arguments are presented in prose, it can take a lot of thinking to figure out which claims are most important. If you present the same argument in map format, however, you can immediately see which claims provide the most direct support to the global conclusion, and which claims are situated at the top of a dense bush. The fact that the relative importance of claims can be viewed at a glance is helpful especially in cases in which maps are lop-sided. Sometimes, you can have a conclusion supported on the one side by a two-premise argument, and on the other side by an argument with 16 claims. Seeing this kind of organization at a glance helps you distribute your evaluative efforts appropriately. In particular, it helps you assess how the justification undermining effects of a successful objection propagate through the map.

3.5 Section Summary

For ease of reference, I have summarized the four properties we've just reviewed into a table (See Table 1).

These four properties, along with their corresponding benefits, are realized by most argument maps, but not all. In what follows, I describe four different argument types, and show that in each case, it is not possible to build a map that both

¹² When I claim that the evaluation of the whole map follows automatically from the evaluation of the parts I do not mean to imply that the all possible information relevant to assessing the truth of the conclusion is represented in a map. Particularly in cases in which the argument depicted is not deductively valid, every map will inevitably fail to represent some considerations that are relevant to assessing the truth of the conclusion.

Table 1 Four properties of argument map design, and the manner in which each property facilitates the evaluation of arguments

Design property	Manner of facilitation
Uniformity	Universal evaluation strategy
Informational encapsulation	No informational overload
Arborescent organization	Part-whole derivability
Scalability	Global structure viewable at a glance

represents the argument accurately, *and* realizes all four benefits. The maps still facilitate argument evaluation, but not quite as elegantly as they do in other cases.

4 Some Unavoidably Inelegant Maps

4.1 Reductio Ad Absurdum

In reductio ad absurdum arguments, you assume the truth of a claim that is in doubt and add to it some additional claims that are not. From this original set of claims, you proceed to derive, by way of valid inference, a new claim that clearly contradicts one of the members of the original set. Since truth permits no contradiction, and since validity preserves truth, you can conclude that at least one claim in the original set must be false. Moreover, since only one claim in the original set was ever open to doubt, you can conclude that, in particular, it is the original dubious assumption that must be false. To see the difficulty with mapping reductio ad absurdum arguments, consider the following example.

Assume that time travel is possible. Then, my friend Destiny could travel back to 1960 to prevent her grandfather from doing something terrible he did that year. She could even kill him. Then again, if Destiny killed her grandfather in 1960, Destiny herself could never have come into existence. Obviously, you can't kill someone if you don't exist. So we have a contradiction: Destiny both can and cannot kill her grandfather. We should, therefore, reject our initial assumption, and conclude that time travel is impossible.

How should this argument be mapped? The conclusion of the argument is that time travel is impossible. To reach that conclusion, we must explicitly reject our initial assumption. But the reason for rejecting that assumption is that it leads to a contradiction. In order to demonstrate how the contradiction arises, we must employ the assumption as a premise, and derive the contradiction from it. According to the principle of explosion (*ex falso quodlibet*), anything follows trivially from a contradiction. By appealing to this principle, we could, technically speaking, construct a valid argument in which our desired conclusion is derived directly from the pair of premises that constitute the contradiction. But this is a bad idea because such a derivation would do nothing to demonstrate *why* the conclusion is justified. If we are committed to using an argument map to derive the contradiction, a better way to proceed is to ascend to the metalanguage as soon as the contradiction has been

reached, and proceed to the conclusion from there. This strategy will yield a map like the one in Fig. 3.

The primary difficulty with this map is found in the transition between object language and metalanguage reasoning, which occurs in claim 2.1. Claim 2.1 makes reference to claims 3.1 and 3.2, but follows from them only in the trivial sense captured by the principle of explosion. The argument unit supporting claim 2.1 does not strengthen the justification for believing that it is true, and therefore does not represent an inference, in the epistemic sense defined above. Recall that one of the conventions involved in argument map design is that argument units are uniform, in the sense that they do not represent anything other than inferential relations.

Because this map is not uniform, our default evaluation strategy is not universally applicable. To evaluate this map well, you need to treat this argument as a special case, and bring an alternative evaluative strategy to bear. The map is also not organized as an arborescence, in the sense defined above. If there is a flaw in the lower portion of the argument, it is unclear how the consequences of that flaw propagate from level 3 up to level 2.

How can this map be improved? You might think that the bulk of the difficulty is caused by making explicit reference to the numerical labels for claims elsewhere in the map, as claim 2.1 does. But this is not the heart of the matter, for the problem does not diminish if we instead restate the content from level 3 within claim 2.1, and thereby avoid the need to refer to labels. If we did that, claim 2.1 would read something like this: “It is a contradiction to assert both that Destiny can and that she cannot kill her grandfather in 1960.” This alternative formulation derives as little justification from the contradiction at level 3 as the original claim did. We should be willing to accept the claim of course, but our acceptance is justified by the fact that it is a tautology, and not by any kind of evidential support that derives from information presented by the claims in level 3.

A useful way to handle *reductio ad absurdum* arguments in an argument mapping setting is to create two distinct maps. One map represents the object language content, and shows that it yields a contradiction. This is illustrated by Fig. 4, where I have represented the trivializing effect of deriving contradiction, perhaps a bit whimsically, with an exclamation point. The other map presents the metalanguage reasoning about the significance of having reached the contradiction, which is illustrated in Fig. 5. Claim 2.1 in Fig. 5 does make a metalinguistic claim, but this instance of metalinguistic reasoning does not have the same undermining effect as it did in the larger, original map, because in this case, it refers to material wholly outside the map itself.¹³

¹³ Another approach to representing *reductio* arguments, known as conditionalization, is articulated in Thomas (1996). Call the target of the *reductio* P . According to the conditionalization strategy, an *inference* from claim P to claim Q (rather than the set of claims, P and Q) is taken as a premise for the conditional claim $P \rightarrow Q$. That conditional claim is combined with the claim that $\neg Q$, which is supported along an independent branch. Finally, by appeal to *modus tollens*, the negation of P is derived. Although Thomas’ maps look like BF maps, they are not. The conditionalization strategy can’t be represented in a BF map because BF maps take only claims (or sets of claims) - and never inferences between claims - as the basis of a mappable inference. Moreover, there is a good reason that BF maps do not allow conditionalization, and the discussion in Sect. 2 of this article helps to articulate what that reason is: conditionalization

4.2 Objections

Generally speaking, argument maps are very well designed for representing the logical impact of objections. When we construct, evaluate, and respond to objections in argument map format, we are forced to think both creatively and carefully. Working out how to map objections and rebuttals nearly always sheds light on subtle properties of the target claim that you might not otherwise have noticed. Nevertheless, maps are not equally suited to all kinds of objection. I'll discuss two particular objection-types that are difficult to map elegantly. But to show what makes them difficult, I must first explain how objections are handled in argument map format. For this purpose, it is useful to sort objections into two large classes: direct objections and inference objections. A direct objection aims to show that the claim it targets is false. An inference objection aims to show that some claim is not well supported by the proffered set of premises.

Let's examine a direct objection first. Here is a claim that many people find objectionable.

The National Security Agency should adopt a policy of monitoring everyone's digital communication.

A direct objection to this claim will be a reason to think that it is false. The most transparent way of showing that a claim is false is to take its negation, and then to build a positive argument for *that*. One reason to think that the NSA should *not* monitor everyone's communication is that the information they gather can be abused.¹⁴ Figure 6 shows a direct objection that expresses a concern of that sort.

The red lines in the argument unit in Fig. 6 indicate that it is an objection. Together, claims 2.1 and 2.2 entail the implicit claim that a policy of monitoring everyone's digital communication should be avoided. Moreover, since this claim is one way of expressing the negation of the target claim, the two premises that entail it count as an inferentially strong objection to the original target claim. Notice that direct objections do not involve metalanguage. Neither claim 2.1 nor claim 2.2 says anything like "Claim 1.1 is false because..." Instead, they talk directly about the content of claim 1.1. This is important because in many settings, and in debate-like settings in particular, the act of objecting to a claim can sound like a commentary on that claim. If objections *were* necessarily commentaries, they would be inherently metalinguistic, and would force us to give up on at least one of the four properties that make argument maps such beneficial tools for argument evaluation.

Footnote 13 (Continued)

zation undermines the informational encapsulation property. The more steps required to get from P to Q , the less encapsulated the map becomes. In order to derive the conditional $P \rightarrow Q$, you need to make an assessment of all such steps at once. It is a virtue of BF maps that they enable one to assess an entire map while working one argument unit at a time.

¹⁴ In the context of argument mapping, it is convenient to talk as if value judgments are simply true or false, because it allows me to express the evaluation procedure for argument units in a way that covers both statements of fact and statements of value. This way of talking should not be interpreted as an endorsement of meta-ethical cognitivism. Everything I say here can be translated into non-cognitivist terms.

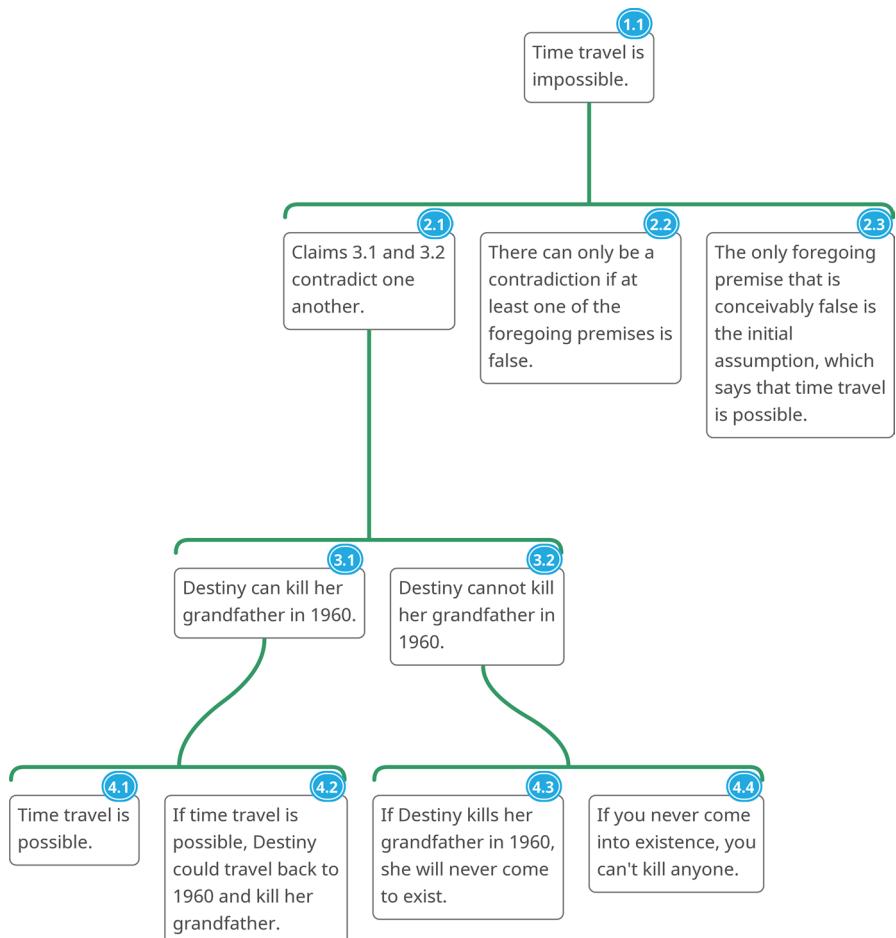


Fig. 3 A map that ascends to the metalanguage to point out a contradiction

The way of conceptualizing direct objections that I have suggested, according to which they are equivalent to positive arguments for the negation of their target, has at least two significant benefits. First, the objection can be counted as a deductively valid argument. Second, the objection can be counted as an argument unit, which was defined as *a subsection of map composed of one target claim and a set of two or more co-premises that collectively provide a reason either to believe or disbelieve the target claim*. Inference objections are not like this. They take aim at the quality of an inference, rather than the truth (or falsity) of a given claim, and do not, therefore, count as argument units. Because inference objections are not argument units, maps that include them will violate the uniformity property. They will have to be evaluated on the basis of something other than the two-step evaluation strategy described above.

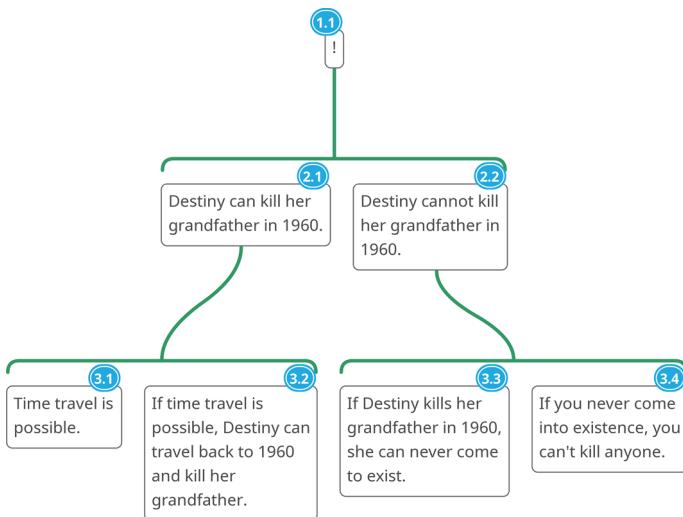


Fig. 4 A map that leads to a contradiction without expressing it

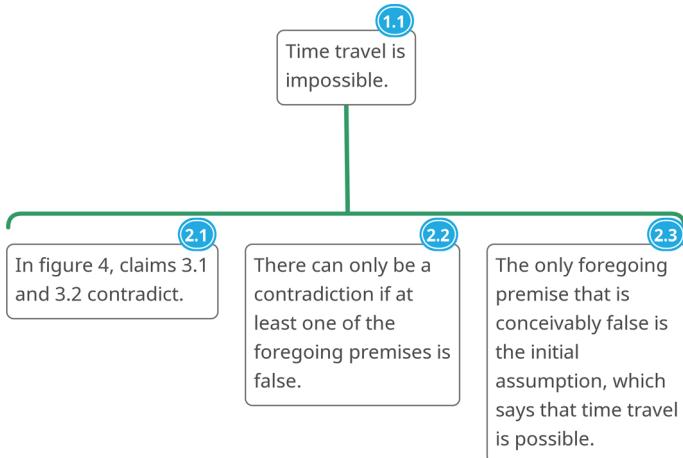


Fig. 5 A map that not only includes metalinguistic content, but also refers to claims made in another map

Moreover, in order to assess the quality of an inference, inference objections must describe a logical relation between two or more claims. In order to describe logical relations between claims, however, you must ascend to the metalanguage. Because inference objections include metalinguistic content, they cannot be evaluated without examining object-level content of the inference, in addition to the information in the objection itself. As a result, the inference objection will

violate the informational encapsulation property.¹⁵ The most important consequence of these representational difficulties is that it will always be more informative to formulate objections as direct objections to hidden premises, rather than as inference objections. This heuristic ensures the preservation of both uniformity and informational encapsulation.

To see how this works, consider another variation on our previous example. In that example, the conclusion was “The National Security Agency should adopt a policy of monitoring everyone’s digital communication.” Imagine that someone offers the following claim in support of that conclusion.

Adopting a policy of monitoring everyone’s digital communication is the best way to prevent terrorism.

What would constitute a good objection to this single-premise argument? Here is one intuitive option.

The conclusion does not follow from the premise because, even if the premise is true, preventing terrorism is not the only factor we have to take into account when choosing security policy.

This response is reasonable. However, in addition to violating both informational encapsulation and uniformity, it is not (yet) a deductively valid argument, because we have yet to identify the conclusion it should be interpreted as supporting. To build a deductively valid argument from the insight contained in this intuitive response, we must first draw out the hidden premise that our interlocutor had been relying on in order to have interpreted the premise as having any relevance to the conclusion.

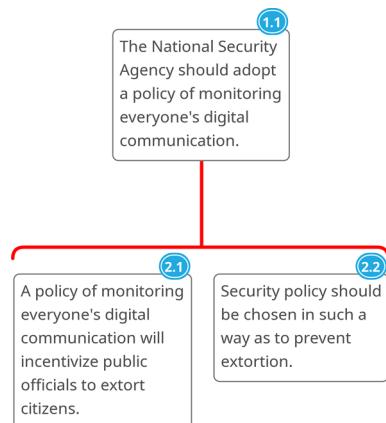
There will always be more than one way to formulate hidden premises, and the considerations involved in choosing a good formulation are beyond the scope of this article. Given the simplicity of our example, however, it should be possible to construct a relatively uncontroversial formulation. In this case, the hidden premise will have to convey the idea that terrorism prevention ought to take overwhelming priority in choice of security policy. Here is one explicit formulation.

We should choose to implement whichever security policy most effectively prevents terrorism.

This claim, when added to the original premise, yields a deductively valid argument. But it also yields a claim against which we can build a direct objection. Figure 7 depicts a map with this configuration.

The yellow background in claim 2.2 in Fig. 7 is my idiosyncratic convention for indicating that claim 2.2 is a hidden premise, that was not explicitly represented in the original text. Beneath that claim is a reason to resist it, formulated here as a

¹⁵ Despite these difficulties, I do not include the entire class of inference objections as an item in my list of four argument types that resist argument map representation. This is simply because inference objections are such a broad class that they don’t have anything approaching a shared logical form. At the risk of introducing needless terminology, we might say that inference objections constitute more of an *argument strategy* than an *argument type*.

Fig. 6 A direct objection

direct objection, despite its initial guise as an inference objection. In this version of the map, claims 2.2, 3.1, and 3.2 clearly constitute an argument unit. Moreover, if we keep in mind the negation convention, according to which a direct objection is to be interpreted as an argument for the negation of its target claim, the argument unit also expresses a deductively valid argument. Finally, this version of the argument also preserves the uniformity and informational encapsulation of the map as a whole.

To summarize the results of this section, we have reviewed two good strategies for representing objections. Objections to a claim can be represented as valid arguments for the negation of that claim, and objections to an inference can be converted into direct objections aimed at a hidden premise. Most natural language objections can be represented well using one of these two strategies. In some cases however, neither strategy works without comprising at least one of the four benefits arguments maps usually offer, and it is to these special cases to which we now turn.

4.3 Charges of Equivocation

To equivocate is to make a bad inference look good by relying on a subtle shift in the meaning of an important term. Here is a well-known example.

Everyone agrees that evolution by natural selection is a scientific theory. Unlike facts, mere theories cannot provide genuine explanations. Therefore, evolution by natural selection cannot explain the apparent design in nature. The only alternative explanation is that nature was designed by God. God, therefore, must exist.

A reasonable diagnosis of the problem in this argument is that the meaning of the term “theory” changes from the first sentence to the second. To render the first sentence plausibly true, the term “theory” will have to mean something like “a claim that purports to describe some cause that gives rise to the data.” Crucially, this interpretation of the term “theory” is compatible with the fact that some theories that are so well-supported by empirical evidence as to place them beyond reasonable doubt. To make the second sentence plausibly true, the term “theory” will have to mean

something like “a merely speculative hypothesis.” This latter interpretation is clearly incompatible with the fact that some theories are beyond reasonable doubt. Nevertheless, anyone who would assert the claims in this passage, in the order in which they appear, clearly intends the first two claims to serve as co-premises.

Now let’s consider how best to represent an objection to this argument on the grounds that it equivocates. The first thing to notice is that the charge of equivocation is an inference objection. The worry is not that one of the claims that uses the term “theory” is false (although that may be the case.) Rather, the worry is that the two sentences do not yield any interesting inference, because, although they do employ the same word, they do not appeal to the same concept.

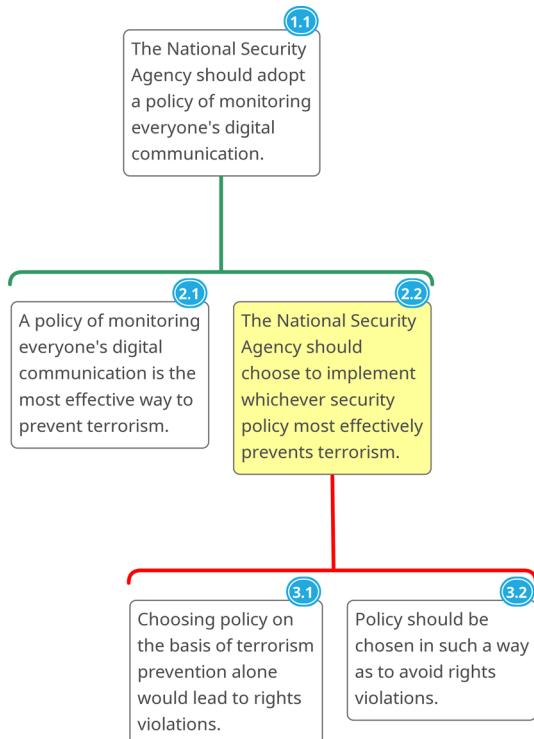
In this case, our hidden premise strategy for representing inference objections looks promising. As we saw, the argument relies on the implicit claim that the meaning of the term “theory” does not change from one premise to the next. Once that implicit claim is brought out in the open, an objection follows naturally, and is depicted in Fig. 8. Notice that the hidden claim in Fig. 8 - claim 3.3 - is already metalinguistic. Fortunately, this instance of metalinguistic reasoning is less problematic than the metalinguistic reasoning required for the *reductio ad absurdum* example depicted in Fig. 3, because the claims to which it refers are located within the same argument unit. It does not, therefore, undermine the informational encapsulation of the map.

Informational encapsulation *is* undermined, however, by the references to claims 3.1 and 3.2 that appear in claims 4.1 and 4.2, respectively. In order to evaluate the objection, you must simultaneously consider the meanings of the claims at two levels.

Are there other strategies for representing the charge of equivocation? There are. Notice that, if we presume, despite the evidence to the contrary, that every term in the map is used consistently, then at least one of the premises would turn out false. Is such a presumption reasonable? Perhaps. Paul Grice’s maxim of manner says that in general, communicators avoid ambiguity, and interpreters presume that ambiguity has been avoided (Grice 1989). If we want to map the argument in a way that respects the Gricean presumption against ambiguity, we will have to construct two distinct maps - one for each possible meaning of the term “theory.” If we interpret the term “theory” as a merely speculative hypothesis, we can build a direct objection like the one in Fig. 9. If we interpret the term “theory” instead as a claim that purports to describe some cause that gives rise to the data, we can build a direct objection like the one in Fig. 10.

Figures 9 and 10 are both insightful maps, and the objections in each are inferentially strong. However, because each map suggests, without qualification, that the term “theory” has a single meaning, they are both misleading. Moreover, they contradict one another about what that meaning is. (After all, the Gricean presumption against ambiguity is just an idealization that misdescribes many actual cases. Moreover, our habitual willingness presume non-ambiguity leaves us vulnerable to various kinds of cognitive exploitation, such as being duped into accepting an equivocating argument.) If you have already accepted

Fig. 7 An objection appended to a hidden premise



that the problem with the argument is equivocation, this pair of objections is likely to be helpful. But, precisely because they are aimed at truth-value rather than inferential quality, they cannot articulate the diagnosis of the underlying problem in the argument as one of equivocation.

4.4 Logical Analogies

A counterexample by logical analogy, also known simply as a logical analogy, is designed to highlight a problem in the structure of one argument by offering a second argument that has the same structure, but that also makes the failing of that structure plain to see.

In the following example, skillfully discussed by André Juthe (2009), philosopher Bryan Wilson (1988) uses a logical analogy to object to an influential Kantian argument for the wrongness of abortion, originally due to Harry Gensler (1986). Here is a slightly edited version of Wilson's presentation of Gensler's argument.

If you are consistent and think that abortion is normally permissible, then you will consent to the idea of your having been aborted in normal circumstances. But of course you do not consent to the idea of your having been aborted in

normal circumstances. So, if you are consistent, you will not think that abortion is normally permissible.

Wilson then offers the following argument as a parallel to Gensler's.

If you are consistent and think that contraception is normally permissible, then you will consent to the idea of having had your conception prevented. But of course you do not consent to the idea of having your conception prevented. So, if you are consistent, then you will not think that contraception is normally permissible.

Wilson assumes that the conclusion of his parallel argument, which states that if you are consistent, you will not think contraception is normally permissible, will strike most readers as unacceptable. Using contraception is just not that bad. And if you agree that contraception is not that bad, you must also think that Wilson's argument is flawed in some way. Finally, since the structure of the reasoning carefully mimics the structure of the reasoning in Gensler's argument, whatever that flaw is, it must be present in Gensler's anti-abortion argument as well.

If there is a flaw, what kind of a flaw could it be? Often, logical analogies are designed to point out a failed inference. In this case, however, the logical form that licenses the inference is, quite unambiguously, modus tollens. Because modus tollens is transparently valid, the flaw must be attributable to a false premise. Perhaps the most vulnerable premise is the one formulated as a conditional. Figure 11 shows one way of constructing an objection to that claim.

The trouble with this way of constructing the objection is most visible in claim 3.1. Any claim of the form “if this premise were true, then it would also be true that...” involves metalinguistic reference. To evaluate 3.1, you must mentally toggle back and forth between 3.1 and 2.1. Since claim 2.1 is located in a distinct argument unit, this toggling undermines both the compactness of the map and its informational encapsulation.

As I suggested for the case of reduction ad absurdum, the best solution in this case might be to create multiple maps, so as to separate out the object language content from the metalanguage content. In this case, however, the two object-level arguments do not contribute to a single conclusion. We therefore need to create three maps rather than two: one for Gensler's original argument, one for Wilson's parallel argument, and one for the metalinguistic reasoning that links the two. The difficulty with this strategy is that it undermines scalability. The more you break up a piece of reasoning into small maps and then string those maps together with commentary, the harder your reader will have to work to retain the global structure of the argument in working memory, while simultaneously deducing the logical implications of any potential flaws.

Once again, we see that metalinguistic structure prevents us from constructing a map with the full suite of evaluation-facilitating benefits described above.

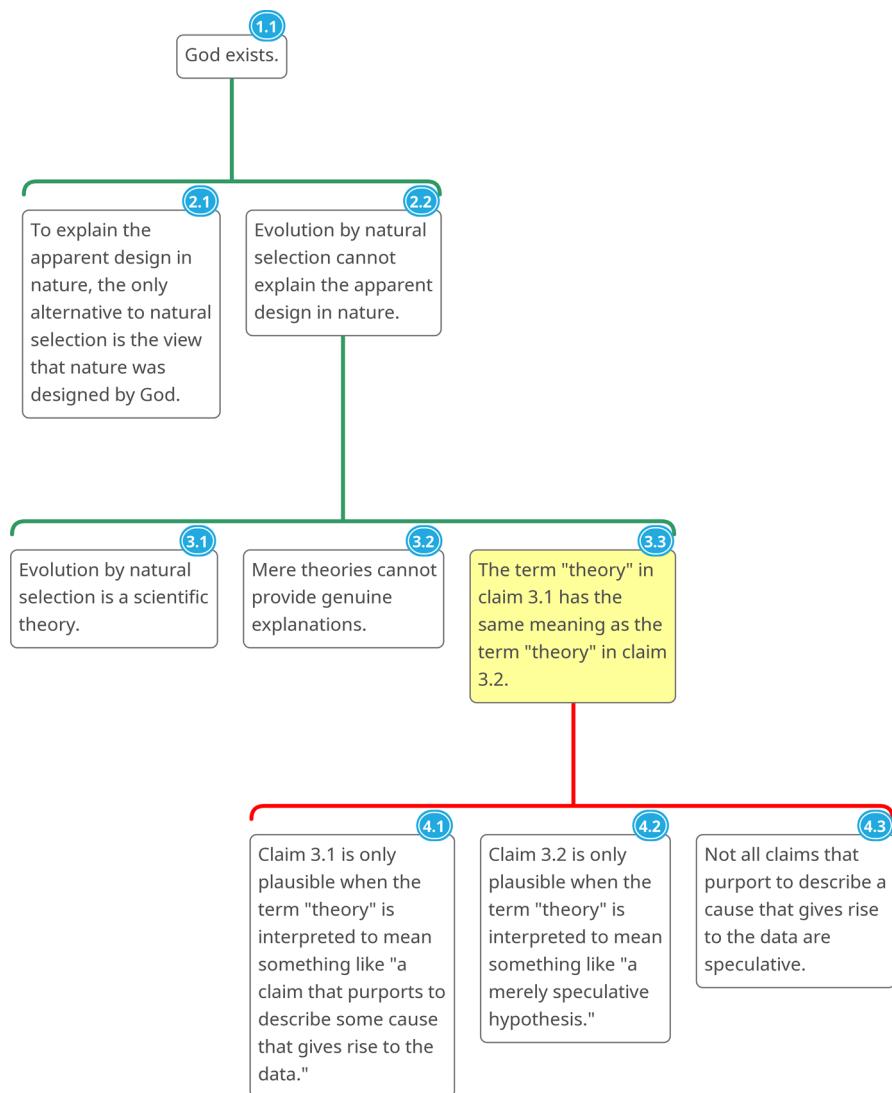
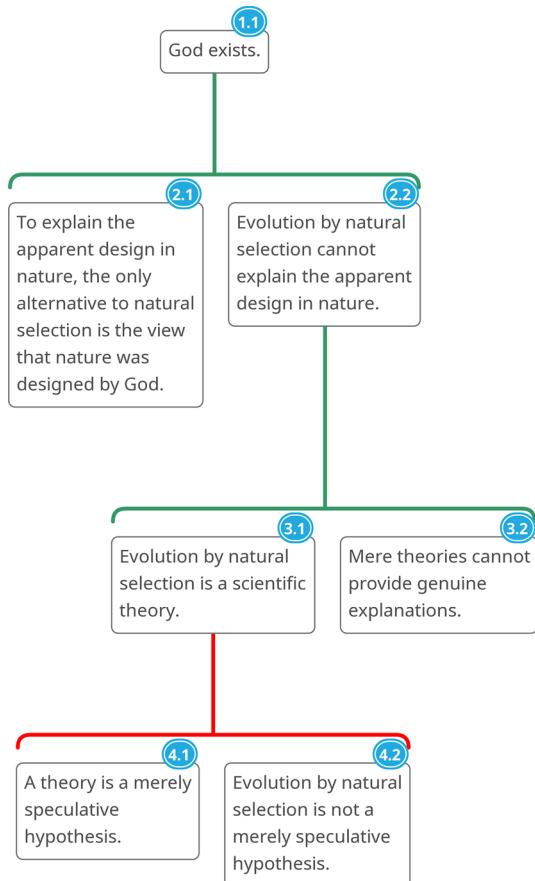


Fig. 8 A map that explicitly ascends to the metalanguage in order to point out the equivocation on the term “theory.”

4.5 Mathematical Arguments

It is uncommon to see argument maps being used to represent mathematical content. But mathematics is full of argumentation, so why wouldn’t argument maps help us evaluate mathematical content? In this section, my suggestion is that one reason for the near absence of mathematical content is that, in mathematical

Fig. 9 A map that relies on only one of two interpretations of the term “theory.”



reasoning, the transition between object language and metalanguage plays a particularly critical role. To illustrate this, let's consult an example.

Imagine that we are sitting in a typical box-shaped classroom. Now imagine the diagonal line that runs from the upper right corner of the room to the lower left corner at the opposite side, crossing the center. Now consider the following claim: the length of that line is equal to the square root of the sum of the squares of the width, the length, and the height of the room.

The truth of this claim is not obvious. To convince someone that it is true, we need an argument. The argument proceeds by pointing out that the line in question forms the hypotenuse of a right triangle. One of the legs of that triangle is the height of the classroom, and the other leg is the diagonal line running across the classroom floor. Call the width of the room a , the length b , and the height c . Furthermore, call the diagonal running through the center of the classroom x , and the diagonal of the floor beneath it, y . The situation then looks like this (Fig. 12).

With the help of this diagram, you can see that y can be described as a Pythagorean function of a and b . Moreover, x can be described as a Pythagorean

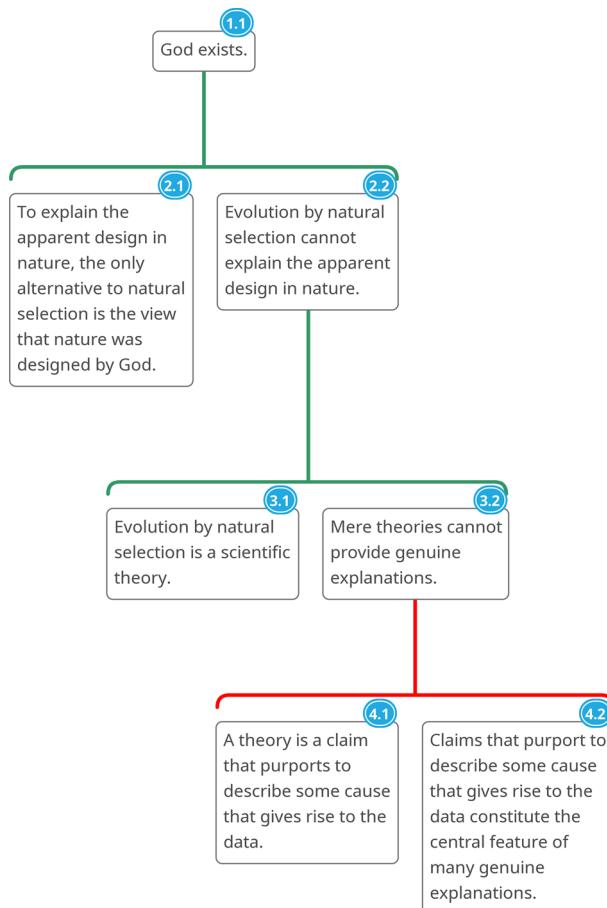


Fig. 10 A map relies on the other interpretation of “theory,” which was ignored in Fig. 9

function of y and c . Assuming that Pythagoras’ theorem can be drawn upon as common background knowledge, we have

$$\begin{aligned}y^2 &= a^2 + b^2 \\x^2 &= y^2 + c^2\end{aligned}$$

By substitution, therefore:

$$x^2 = a^2 + b^2 + c^2$$

Take the square root of each side, and the resulting statement is equivalent to the asserted claim. We have arrived at it by reasoning deductively from unproblematic premises. It is, therefore, a paradigm of good argumentation. So how do we represent the argument in a map? If we focus on the algebra, we’ll have a map of the following sort.

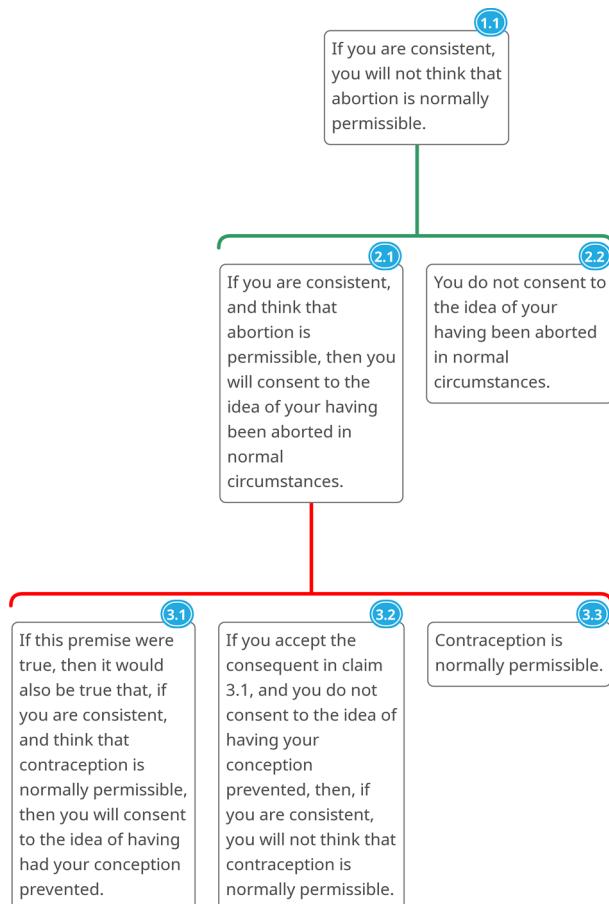
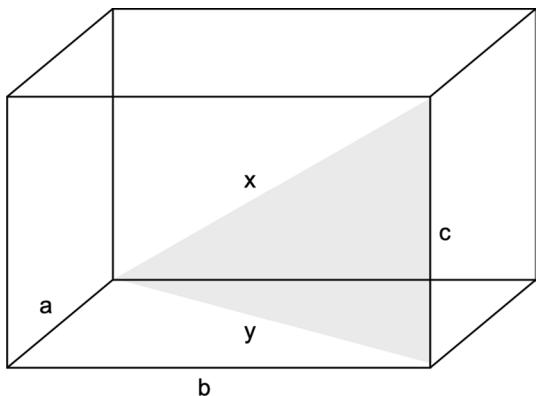


Fig. 11 A map with an explicit metalingusitic claim in box 3.1

This map is trivial for three reasons. First, the most insightful part of the prose form of the argument is the suggestion that the problem can be seen as a relation between right-triangles. This fact is not itself justified in terms of explicit premises. It is justified by geometrical intuition, and by the diagram that depicts those two triangles. The map in Fig. 13 captures none of this. The second reason that the map in Fig. 13 is trivial is that the logic of the substitution operation was already transparent. This is not only because the logic is simple, but also, and more importantly, because when dealing with variables represented as single letters, there is no complex web of semantic association to be disentangled. Once you have the algebra, you can manipulate symbols without much regard to their referents.

Nevertheless, in order to serve as variables in the first place, the letters must be assigned meanings. This fact highlights the third, and perhaps most important reason that the map in Fig. 13 is trivial, which is that information about the meanings of the variables is not represented at all.

Fig. 12 As an aid to geometric intuition, a picture is often worth more than a paragraph



If we do try to represent the assignment of variables explicitly, we must ascend to the metalanguage, and will have to confront difficulties of the same general sort we have already seen. One difficulty that stands out in this case is the fact that, when you include claims that assign variable meanings, the order in which you assess each argument unit comes to matter. If you go out of order, you won't understand the meanings of the algebraic claims. This is another way of saying that, in any map that includes variable assignment, informational encapsulation breaks down. The reason order matters is that you have to retain information from one unit and carry it with you, mentally, to the next. This is exactly what informational encapsulation says you don't have to do.

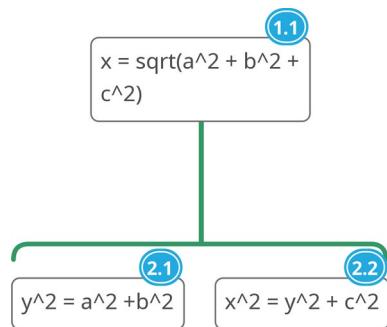
Unlike I have done in the three foregoing cases, I offer no alternative representative strategy here. For mathematical content, it seems that linear, rather than tree-like organization of information is preferable, because it allows for a more fluid transition between object level claims, metalinguistic claims, and references to diagrams or other cognitive aids.

5 How to Handle Metalinguistic Arguments

In all four argument types discussed above, at least some of the benefits usually associated with argument map representation are either diminished or absent. These limitations derive from the fact that, in an argument map, it is difficult to represent the transition between object language reasoning to metalanguage reasoning without violating the restrictions implicit in the map's modular format. You might conclude that, from a practical perspective, the best response to this observation is that you should just avoid trying to map arguments with metalinguistic content. But that is not the lesson I take from these observations.

Argument maps are wonderful tools for understanding arguments. One reason not to be disheartened by the limitations discussed here is that they apply primarily to the process of argument evaluation. The benefits associated with the process of argument map construction are left relatively undisturbed. Mapping an argument invariably leads you to notice things about it that you had not noticed before. So,

Fig. 13 An argument map with algebraic content



even if an argument has metalinguistic content, the process of trying to construct a good map will almost certainly be fruitful. My hope is that the process of constructing such maps will be even more fruitful if done in full knowledge of the difficulties I have discussed here.

Another reason not to be disheartened by the limitations discussed here is that you can often find ways to convert arguments with metalinguistic content into other argument types that can be expressed with object-language claims alone. For example, a *reductio ad absurdum* argument can usually be converted into a *modus tollens* argument, by replacing the false assumption with a conditional claim that conjoins that false assumption to one of its implications. In some cases, conversions of this sort may preserve everything worth preserving in the original prose. In other cases, the persuasive force of the argument will suffer. For example, the conditional you build may not be as self-evident as the original assumption (Dutilh Novaes 2016). Alternatively, in cases in which the false assumption supports two distinct inferences, conversion to *modus tollens* may undermine the symmetry of the argument. If our only goal were to decide whether the conclusion of the argument is true, these pragmatic factors might not matter much. However, argument maps are valuable not only because they help us assess particular conclusions, but because they help us see and understand the complex internal structure of an argument. Argument evaluation can teach you a lot, even if, at the end of the process, you remain undecided about whether the conclusion is true. So, in at least some cases, there is value in representing a *reductio* as a *reductio*. In other cases, perhaps not. This potential diversity points to a new area that is ripe for future research. Which argument types are well-suited to conversion from metalanguage to object-language format? How does the semantic content of the argument influence the quality of the conversion? How does conversion influence the persuasive force of the argument? These questions are wide-open, and, as argument mapping becomes increasingly popular, they become increasingly worthy of scholarly attention.

Another area for future research is the question of what tradeoffs we must confront when attempting to map metalinguistic arguments with alternative argument mapping systems. Dale Jacquette (2011), for example, has developed a system that includes conventions specifically designed to handle metalinguistic reasoning. That system is considerably more complex than the system developed here, and it would be interesting to see whether working with Jacquette's system has a positive

influence on critical thinking ability comparable to the effect that working with modern BF maps has been shown to have.

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Declarations

Ethical approval Neither human nor non-human animal subjects were involved in this research.

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