One-hit wonders and overnight successes

Using Weibull models to examine the adoption of music albums in the 1990s

Executive Summary

In this paper, I model the adoption of two music albums in the 27-weeks following their release in the 1990s using different variations of the exponential model. The final two models use a Weibull model with lagged airplay data as covariates achieving out of sample mean percentage errors of 14-16%. Other models considered included an Exponential, an Exponential-Gamma, a Weibull, a 2-segment Weibull with a lagged airplay covariate.

In examining the results of the two models, I found that both models follow a similar pattern of adoption according to the Weibull. However, the adoption of the two albums is also heavily impacted by external events which impact airplay. Disregarding the impact of significant external events, the adoption behavior of the underlying population of music albums can generally be characterized by a Weibull model with a low lambda and positive duration-dependence.

Introduction

Within the first 27 weeks of releasing their debut albums, two American alternative rock bands experienced what seemed to be two very different adoption patterns. While the album Dink experienced quickly rising sales in the first few weeks, Sparklehorse's album "Vivadixiesubmarinetransmissionplot" did not see an uptick in sales until about 24 weeks after its release (see figure 1 and 7).

Objectives

What drove the difference in the purchasing patterns? Was it simply a matter of musical talent or did external events affect the outcome? By examining sales data, airplay data, and related events, the objective of this paper was to better understand what drove the underlying purchases processes of these two music albums and to uncover any potential similarities between the two adoption patterns. To do so, I created two Weibull models with covariates and analyzed summary statistics to describe the two processes.

The data

The data I used contains U.S. sales and airplay data from the first 27 weeks of the two albums. The sales data describes the number of adoptions in any given week based on a population of N=100,000. The airplay data describes the extent to which songs from the two albums were played by radios weighted by each radio's number of listeners. Other data, such as related events I will describe in my creation and validation of each model.

Since the data is more than 20 years old, there are a few concerns I want to raise: First, the conclusions drawn from these two datasets may not be generalizable to the adoption of music albums today. Second, there is a low chance that some of the numbers were inaccurately recorded.

The Story of Sparklehorse

A closer look at the data

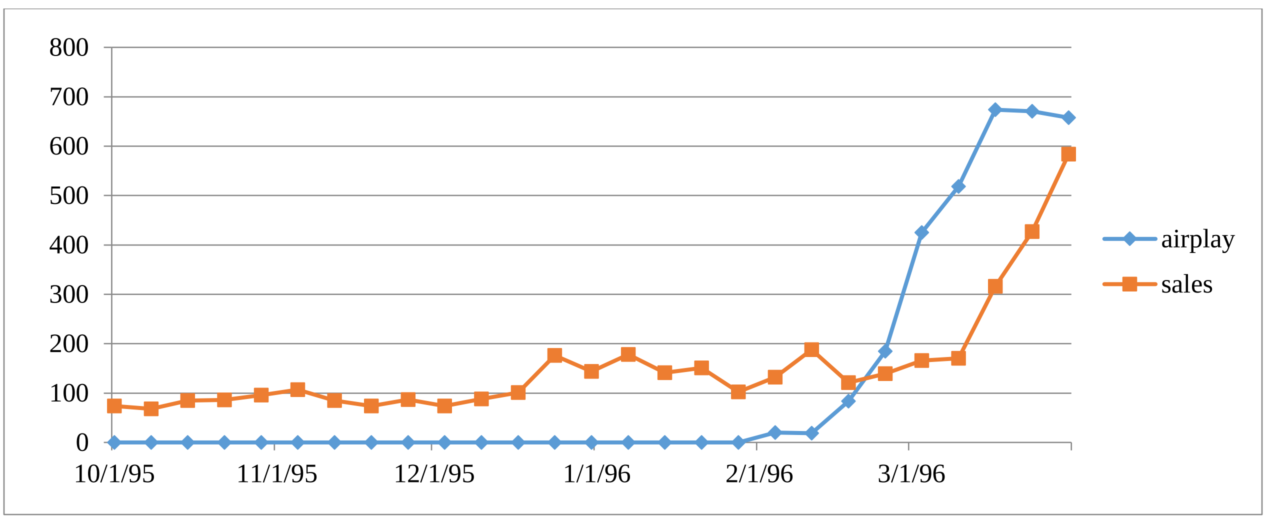
In examining a scatter plot of the sales and airplay data over time, I make three observations:

1) Unsurprisingly, airplay data is closely correlated with sales data with an $R^2 = 0.67$. As more people hear their songs, more people are interested in buying their albums.

2) There is a slight lag between the two variables, which also makes sense given that most listeners don't purchase the albums immediately upon listening to them.

3) In February, there is a sudden uptick of airplay followed by a sudden uptick in sales.

Figure 1: Sales and Airplay of Sparklehorse's debut album (October 1, 1995 – March 31, 1996)

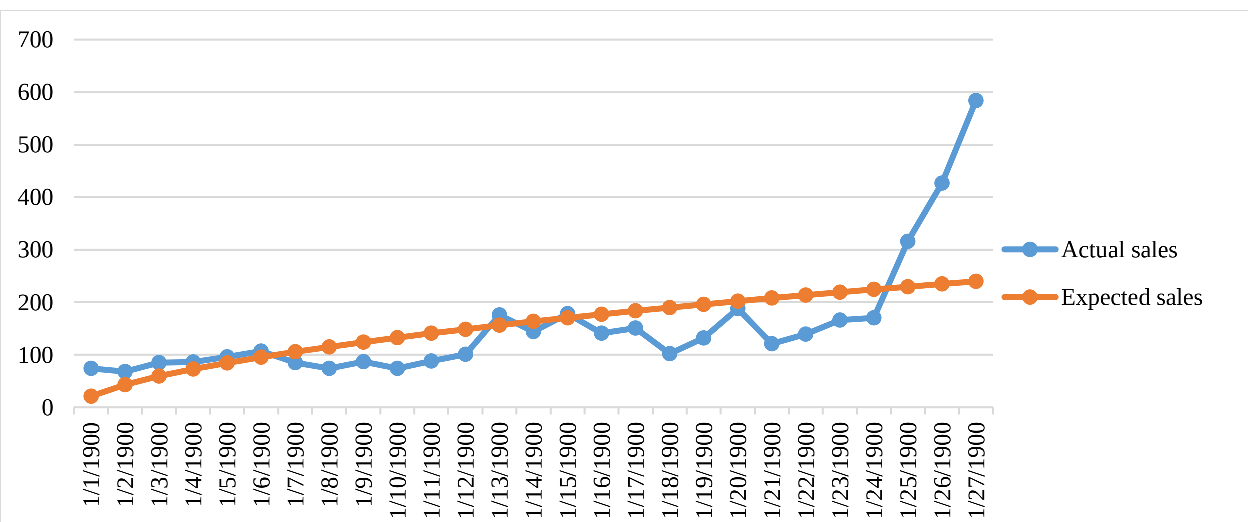


Creating the model

To create the model that best described the adoption pattern of the album, I first fitted an Exponential-Gamma model to the entire dataset using maximum likelihood estimation. The objective was to uncover any possible heterogeneity in the adoption behavior of the underlying population. Given that I was dealing with a population of 100,000 and incremental sales data ranged between 74 to 584 (i.e. a large majority of the population never purchased the album**)**, I found, as expected, that the underlying population was very homogenous (r = 3,008).

Given the observed extreme homogeneity, I proceeded with the assumption that each individual had a lambda (underlying propensity to purchase) that was identical to the sample mean and fitted an Exponential and a Weibull model. In comparing the two models, I expected to find positive-duration dependence; in the few first weeks of hearing songs from the album on the radio, people become increasingly interested in buying the album as they hear more and more of the songs. The Weibull model confirmed my hypothesis with a positive duration-dependence of c = 1.61 and a p-value = 1E-175, demonstrating that the data strongly supported the additional parameter. At this point, however, my model still did not explain the sudden uptick in sales (see below).

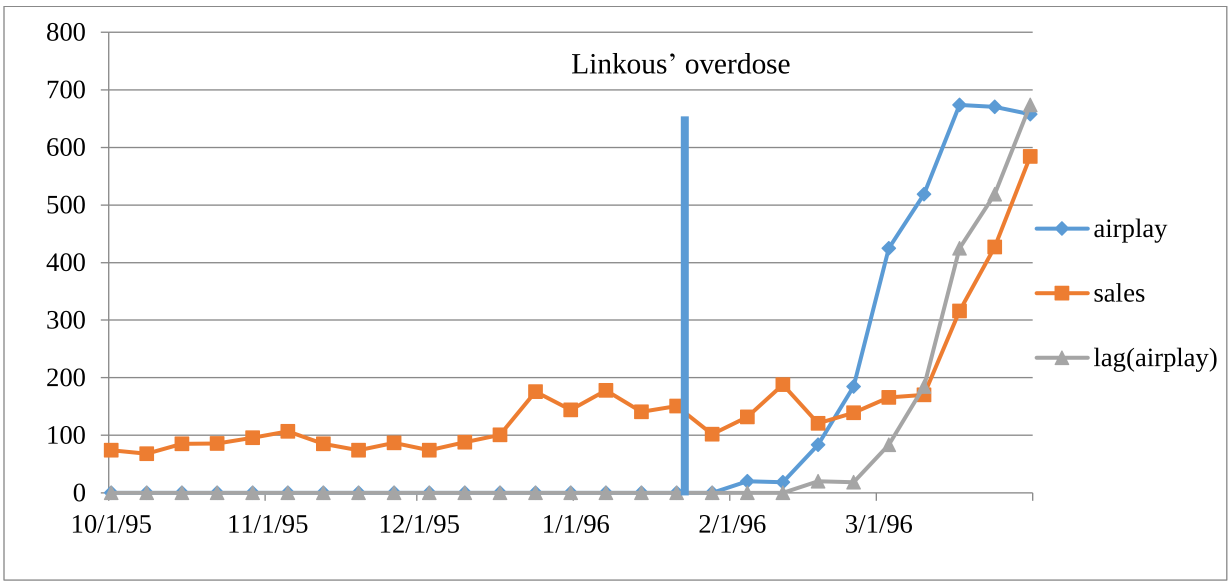
Figure 2: Incremental sales (Weibull model)



Through research, I uncovered that on January 23rd (just before the uptick in sales), lead singer Mark Linkous overdosed on alcohol and anti-depressants during a tour with Radiohead in London. He was declared clinically dead for two whole minutes and subsequent surgeries to save his life left him wheelchair-bound for 6 months.[1] With the appearance at the opening of Radiohead (a very popular band at the time) and the dramatic event that followed, Sparklehorse gained significant media attention and its songs began playing on the radio.

To model the effect of the event, I included added a 2-weeks lagged Airplay covariate, seeing as the uptick in sales followed approximately two weeks after the uptick in airplay (see figure 3).

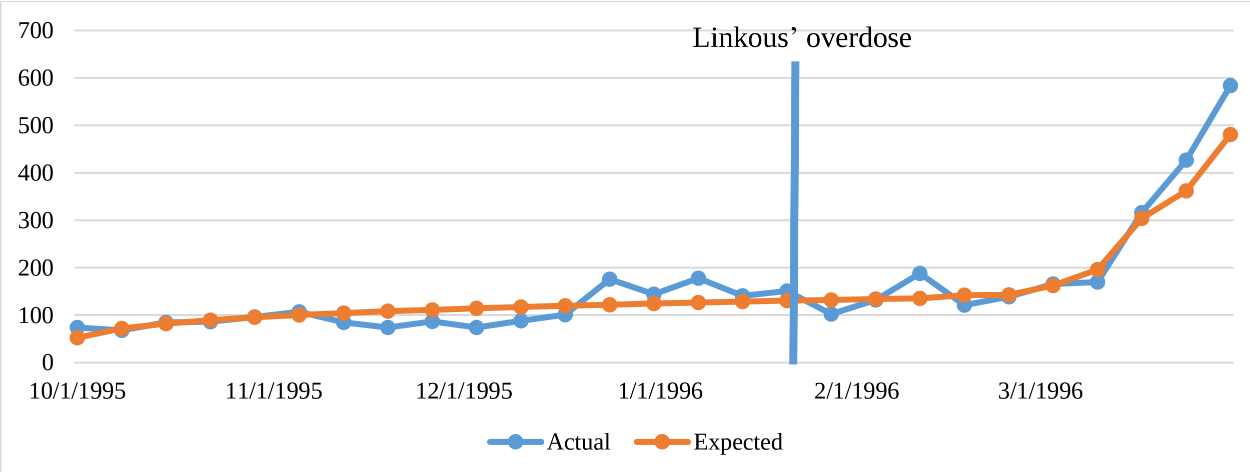
Figure 3: Comparison of Sales and Lagged Airplay Data



To avoid overfitting, I kept a small holdout sample of 8%. The result was a Weibull model with one covariate ($\beta_{lag(AP)}$) that closely followed album sales and with an out-of-sample mean absolute percentage error (OOS MAPE = 16%) similar to that of the in-sample (IS MAPE= 18%).

[1] Gilbey, Ryan. "Life begins after hitting rock bottom; Sparklehorseman Mark Linkous died last year. But it didn't stop his band from making a classic album. By Ryan Gilbey." Independent [London, England], 6 Sept. 1996, p. 8. General OneFile, http://proxy.library.upenn.edu:2084/apps/doc/A67087057/ITOF?u=upenn_main&sid=ITOF&xid=2ddd7c9f. Accessed 2 Apr. 2018.

Figure 4: Incremental sales (Weibull model with lagged Airplay covariate)



Validating the model

At each step of the way I closely monitored the movement of BIC and performed Log-Likelihood Ratio (LRT) tests to confirm the validity of each additional parameter (see Table 1 and 2). Even though I did not keep a holdout sample for the first three model (EG, Exponential, and Weibull), both declining BICs and IS MAPEs convinced me that the Weibull model correctly characterized the underlying process (see Table 1).

Table 1: Summary statistics

Models (No holdout)	Parameter Est.					LL	IS MAPE	OOS MAPE	BIC
	r	α	Γ	c	$\beta_{lag(AP)}$				
EG	3,008	1,923,534	-	-	-	-31,037	51%	-	62,108
Exponential	-	-	2E-03	-	-	-31,037	51%	-	62,085
Weibull	-	-	2E-04	1.61	-	-30,638	38%	-	61,298
With holdout samples of 8%									
Weibull	-	-	4E-04	1.36	-	-24,005	22%	66%	48,032
WB+Cov(lagAP)	-	-	5E-04	1.25	0.0018	-23,944	18%	16%	47,910

In performing an LRT test comparing the Weibull with and without the lagged airplay covariate, I obtained a p-value = 3E-28, showing overwhelming support for the inclusion of the variable. Furthermore, the inclusion of the covariate brought about a decrease in OOS MAPE from 66% to only 16%, also providing strong support for the covariate (see Table 1 and Table 2).

Table 2: Log-likelihood Ratio Tests

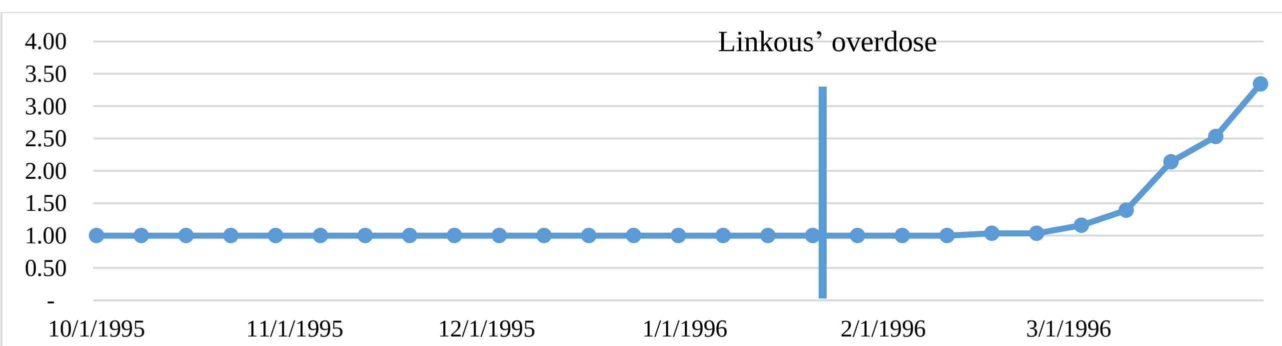
	Additional Parameter Included	
	Duration-Dependence (c)	**Airplay Covariate**
2LR	798	122
Df	1	1
p-value	1E-175	3E-28

I also ran a 2-segment Weibull model with the Airplay covariate, but decided against the model since the additional segment was very small (only 25 out of 100,000) and the p-value of the LRT test not small enough (0.035).

Discussing the model

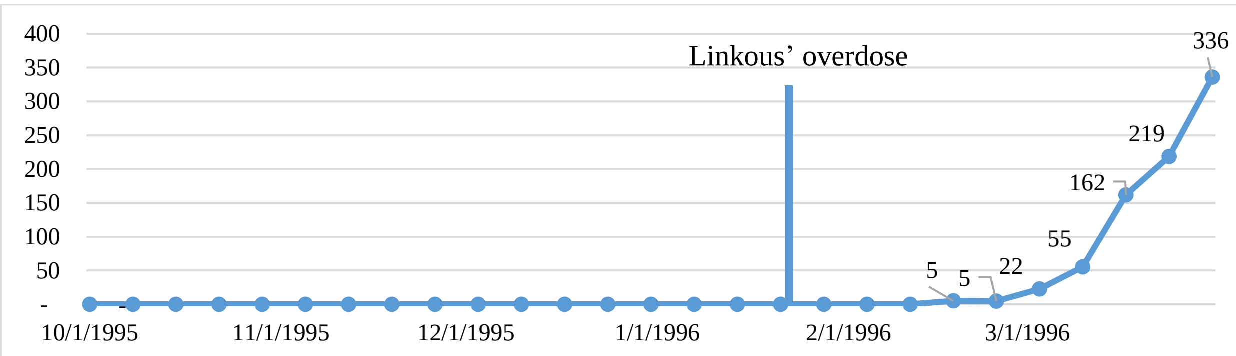
In analyzing the impact of the lagged airplay covariate ($\beta_{lag(AP)}$ = 0.0018) on the proportional hazard, we see that the increase in airplay has a highly positive impact on the baseline propensity of the underlying population to purchase the album (see figure 5), and ultimately, more than tripling the probability of purchase in the given weeks with respect to the baseline.

Figure 5: The impact of lagged airplay on the proportional hazard, $h(t)/h_0(t)$



Examining the difference in sales between and the ultimate model chosen and a baseline model that does not account for the effect of the lagged airplay covariate, we see a total incremental increase of 808 in sales over the 10 week period following Linkous' overdose (see Figure 6). Assuming that the increase in airplay was due to media coverage about Mark Linkous, chances are that Sparklehorse would have stayed undiscovered had Linkous not overdose. Instead, Sparklehorse became *overnight* success.

Figure 6: Covariate Effect of Lagged Airplay on Incremental Sales



Concerns and Improvements

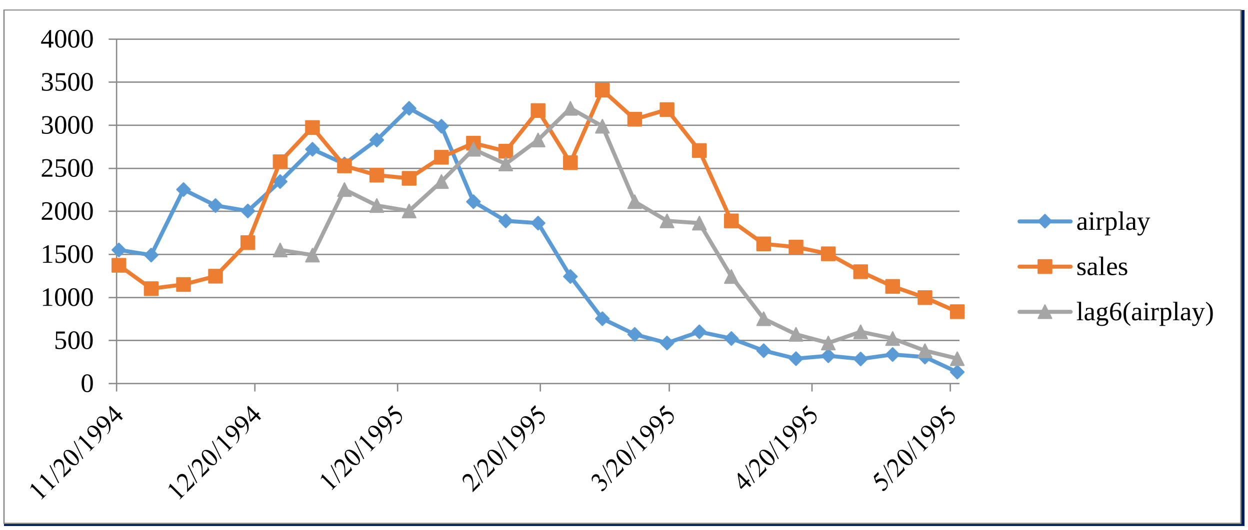
The one concern I had about the final model was that the holdout sample was too small (8%), risking overfitting. The concern became even greater when I witnessed that the OOS MAPE (16%) was lower than the IS MAPE (18%). However, without more airplay and sales data, using a larger holdout set made it very difficult for the lagged airplay covariate to accurately capture the effect of the increase in airplay. To improve and further the model, I would therefore run the model on additional sales and airplay data to confirm the validity of the airplay covariate.

The Story of Dink

A closer look at the data

The story of Dink differs from that of Sparklehorse and we begin to see why in figure 7 below. Again, we see that airplay is correlated with sales. However, for Dink, there seems to be a longer lag between airplay and sales (6 vs. 2 weeks). Perhaps the lag is greater because a larger portion of the weeks directly follow the holiday season, the time when people generally significant amount of money; if people have already made a large number of purchases, chances are it will take them longer to make more purchases following a rise in interest. We also note that after steadily increasing sales, sales begin to drop following a drop in airplay.

Figure 7: Sales, Airplay, and 6-week Lag Airplay of Dink's debut album (November 20, 1994 – May 21, 1995)



Creating and validating the model

In creating a model for Dink, I made the same initial models to examine the presence of homogeneity and duration-dependence and found once again a steady decline in both BIC and IS MAPE (see Table 3), providing support for the assumption of homogeneity and positive-duration dependence that was observed.

Table 3: Summary statistics of initial model runs

Models (no holdout)	Parameter Est.					LL	IS MAPE	OOS MAPE	BIC
	r	α	Γ	c	β_{lagAP}				
EG	5,726	193,964	-	-	-	-255,511	863%	-	511,045
Exponential	-	-	0.029	-	-	-255,511	40%	-	511,033
Weibull	-	-	0.010	1.34	-	-253,012	32%	-	506,048
WB+Cov(AP)			0.010	1.32	0.0006	-	31%	-	506,018

Since including the original Airplay covariate (divided by 10) only provided a slightly lower IS MAPE and BIC than the Weibull (see Table 3), I believed the *lagged* airplay covariate would be a better fit to characterize the purchasing process. The issue in including it was that, in contrast to Sparklehorse which had had zero Airplay prior to Linkous' overdose (which made it easy to impute prior values), it would not be possible to impute the Airplay values prior to November 10, 1994.

Therefore, I fitted two separate models which only considered sales data from January 1, 1995 and onward. This enabled me to include a 6-week lagged airplay covariate and examine the potential improvement in model accuracy.

Indeed, the inclusion of the lagged covariate brought down OOS MAPE from 73% to 15% (see Table 4) yielding a p-value of 0 when performing an LRT test (see Table 5). Given the significant improvement in model accuracy, I decided to add a 3^{rd} covariate denoting the effect of one of Dink of song's ("Green Mind") six week presence on the Billboard Modern Rock Top 40 (January 7, 1995 – February 11, 1995), hoping that it would improve the accuracy of the model.[2,3] I modeled the covariate as an exponential, where $X(t)_{BB} = a*b^{t-1}$, expecting that the covariate would have a positive beta with a declining X(t) over time, giving the baseline proportionate hazard an initial and slowly declining boost. However, since the resulting beta parameter was negative (-0.73), I decided it didn't fit well with the story – the billboard presence should not have a negative effect on adoption. Thus, I ultimately decided to go with the Weibull model with the lagged airplay covariate as my final model (even though including the billboard covariate resulted in a lower OOS MAPE and BIC) (see Table 4). Not only did it have a similar OOS MAPE (15%), it also had a story that fit intuition – positive duration-dependence rather than negative. Figure 8 on the next page provides a visual comparison of the two models along with the actual sales data.

Table 4:

With holdout samples of approximately 20%	**Γ**	**c**	**Lagged Airplay**	**Billboard Presence (BB)**	**IS MAPE**	**OOS MAPE**	**BIC**
Weibull	0.03	1.03	-	-	17%	73%	382,979
WB+Cov(Lag(AP))	0.02	1.03	0.003	-	13%	15%	381,082
WB+Cov(Lag(AP)+BB))	0.08	0.66	0.003	-0.73	11%	14%	380,754

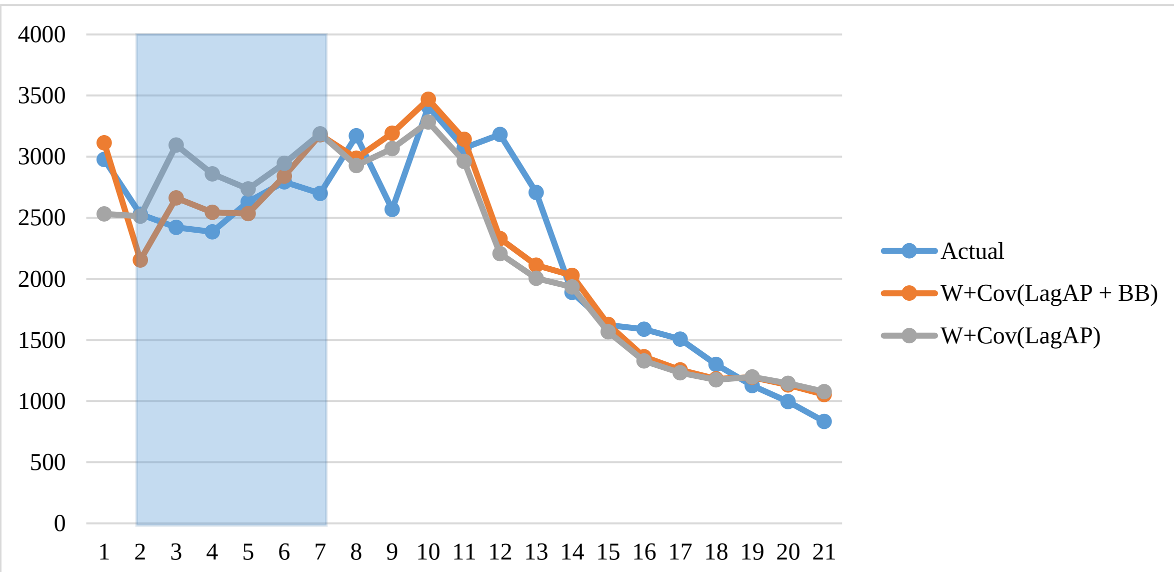
[2] https://books.google.com/books?id=tQsEAAAAMBAJ&pg=PA63&lpg=PA63&dq=dink+billboard+1995&source=bl&ots=kQOuQkGA7D&sig=dV-zKwCNsJde0gFjSCBcMHiswBc&hl=en&sa=X&ved=0ahUKEwiel6Su3J7aAhVokeAKHcxJBmkQ6AEITTAF#v=onepage&q=dink%20billboard%201995&f=false

[3] https://books.google.com/books?id=yAsEAAAAMBAJ&pg=PT86&lpg=PT86&dq=dink+billboard+1995&source=bl&ots=STDomG-qBu&sig=eaH_uxVxFvkET4qUtSJ8ytsW9bI&hl=en&sa=X&ved=0ahUKEwiel6Su3J7aAhVokeAKHcxJBmkQ6AEITzAG#v=onepage&q=dink%20billboard%201995&f=false

Table 5

	Additional Parameter Included	
	Lagged Airplay	Billboard Covariate
2LR	1908	363
Df	1	3
p-value	0	2E-78

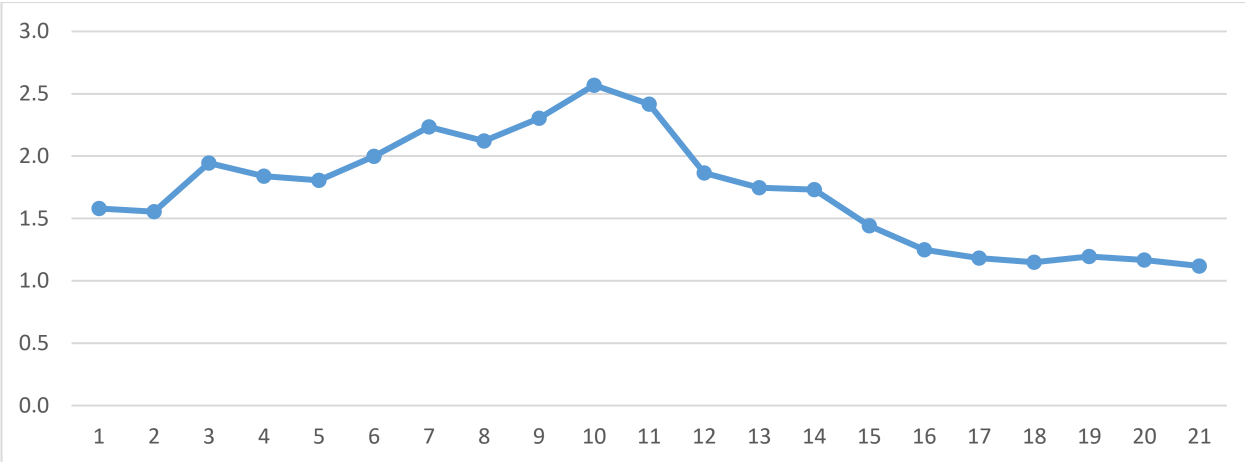
Figure 8: Incremental sales – Comparison of the two models with covariates



Discussing the model

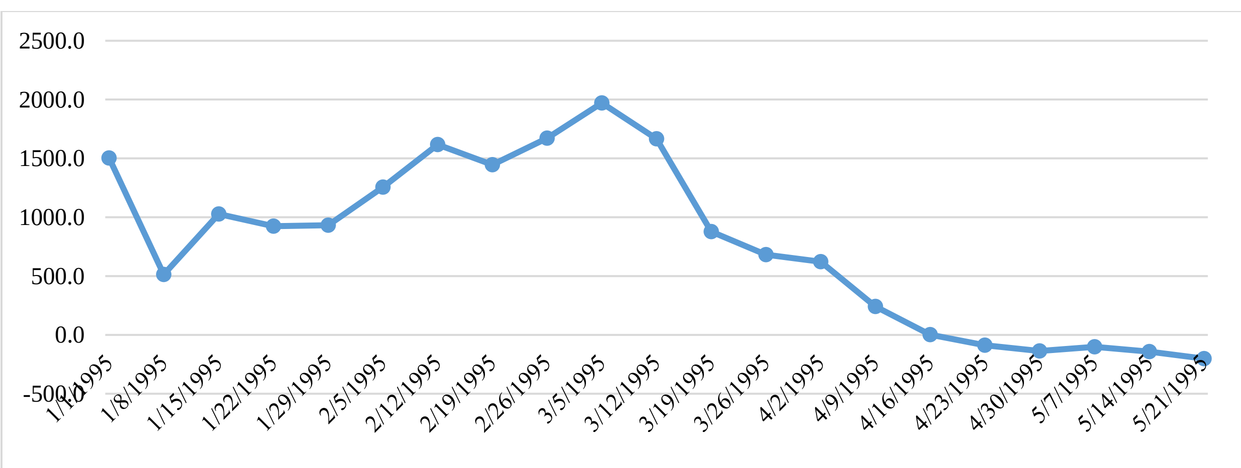
In analyzing the impact of the lagged airplay covariate ($\beta_{lag(AP)} = 0.003$) on the proportional hazard, we see that the increase in airplay has a highly positive impact on the baseline propensity of the underlying population to purchase the album (see figure 9) before returning close to the baseline in week 21.

Figure 9: The impact of lagged airplay on the proportional hazard, $h(t)/h_0(t)$



Examining the difference in sales between and the ultimate model chosen and a baseline model that does not account for the effect of the lagged airplay covariate, we see a total net increase of 16,957 in sales over the 6 week period that Dink is present on the Billboard and the 8 weeks following its removal from the Billboard (see Figure 10). Assuming that the increase in airplay was solely due to Dink's song's ("Green Mind") presence on the Billboard charts, the return to the proportional hazard baseline 8 weeks after its removal suggests that the reason for Dink's success was its *one hit wonder* "Green Mind."

Figure 10: Covariate Effect of Lagged Airplay on Incremental Sales



Concerns and Improvements

To further improve and verify the model, I would attempt to impute the airplay using sales as the predictor and re-run the model again to try to improve the accuracy of the model and to verify the legitimacy of the lagged covariate.