# Errata, Clarifications, & Additional Material

#### December 12, 2024

For Applied Numerical Methods for Partial Differential Equations by Carl L. Gardner, Springer, 2024

#### **Errata**

#### Clarifications

p 130, 2nd paragraph: Requiring  $\Delta t \leq h/c$  for stability is an example of the CFL condition: ...

p 132, 2nd paragraph: FTCS for  $u_t + cu_x = 0$  satisfies the CFL condition for  $r \leq 1$  but is unconditionally unstable: ...

Add after sentence with (8.149) on p 160: In (8.149), the forward-in-time  $\Delta w/\Delta t$  is a shorthand for any consistent and stable (explicit) timestepping scheme like RK3.

p 169, 3rd paragraph: ...two copies of the 1D code (see (8.149)), one for the x sweep for evaluating  $f(w)_x$  and one for the y sweep for evaluating  $g(w)_y$ .

p 172, 3rd paragraph: ... two copies of the 1D WENO3 method (see (8.149)): an x sweep for calculating  $f(w)_x$  and a y sweep for calculating  $g(w)_y$ .

p 192 after (9.40): ... $\Delta \mathbf{u}/\Delta t$  is a shorthand for any consistent and stable (explicit) timestepping scheme ...

### **Additional Material**

## Space-time Stencils for Classical Parabolic Methods

The stencils for classical methods for time-dependent PDEs are shown in Figs. 1–7. In these diagrams, space is horizontal and time is vertical. Figure 3 is annotated to indicate spatial grid points i-1, i, and i+1 and time levels n and n+1. To compute the new solution  $u_i^{n+1}$  with the trapezoidal rule method, the old solution values  $u_i^n$  and  $u_{i\pm 1}^n$  are coupled with the new solution values  $u_i^{n+1}$  and  $u_{i\pm 1}^{n+1}$ .

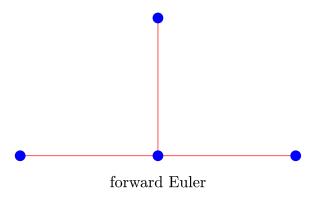


Figure 1: Space-time stencil for the forward Euler parabolic method.

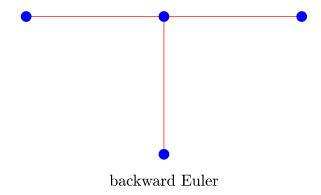


Figure 2: Space-time stencil for the backward Euler parabolic method.

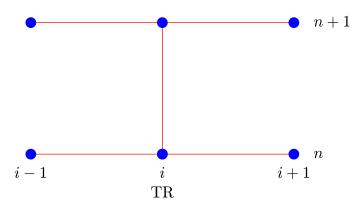


Figure 3: Annotated space-time stencil for the trapezoidal rule parabolic method.

## ${\bf Space-time\ Stencils\ for\ Classical\ Hyperbolic\ Methods}$

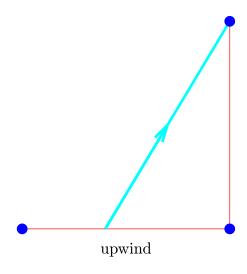


Figure 4: Space-time stencil for the upwind hyperbolic method for  $u_t + cu_x = 0$  with  $c\Delta t < \Delta x$ .

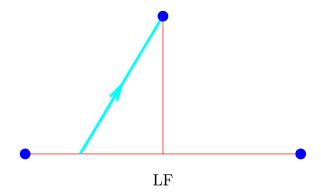


Figure 5: Space-time stencil for the Lax-Friedrichs hyperbolic method for  $u_t + cu_x = 0$  with  $c\Delta t < \Delta x$ .

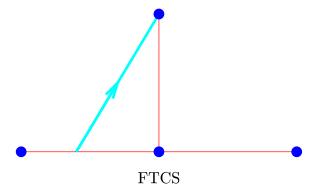


Figure 6: Space-time stencil for the *always unstable* forward time central space method when applied to hyperbolic PDEs.

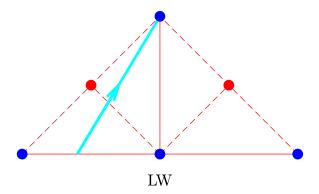


Figure 7: Space-time stencil for the Lax-Wendroff hyperbolic method for  $u_t + cu_x = 0$  with  $c\Delta t < \Delta x$ .