Errata, Clarifications, & Additional Material

December 16, 2024

For *Applied Numerical Methods for Partial Differential Equations* by Carl L. Gardner, Springer, 2024

Errata

Clarifications

p 130, 2nd paragraph: Requiring $\Delta t \leq h/c$ for stability is an example of the CFL condition: ...

p 132, 2nd paragraph: *FTCS for* $u_t + cu_x = 0$ satisfies the CFL condition for $r \leq 1$ but is unconditionally unstable: ...

Add after sentence with (8.149) on p 160: In (8.149), the forward-in-time $\Delta w/\Delta t$ is a shorthand for any consistent and stable (explicit) timestepping scheme like RK3.

p 169, 3rd paragraph: ...two copies of the 1D code (see (8.149)), one for the x sweep for evaluating $f(w)_x$ and one for the y sweep for evaluating $g(w)_y$.

p 172, 3rd paragraph: ...two copies of the 1D WENO3 method (see (8.149)): an x sweep for calculating $f(w)_x$ and a y sweep for calculating $g(w)_y$.

p 192 after (9.40): ...$\Delta \mathbf{u}/\Delta t$ is a shorthand for any consistent and stable (explicit) timestepping scheme ...

Additional Material

Space-Time Stencils for Classical Parabolic Methods

The stencils for classical methods for time-dependent PDEs are shown in Figs. 1–8. In these diagrams, space is horizontal and time is vertical.

Figures 1–3 display the stencils for the classical parabolic methods, with Fig. 3 annotated to indicate spatial grid points $i-1$, i, and $i+1$ and time levels n and $n+1$. To compute the new solution u_i^{n+1} with the trapezoidal rule method, for example, the old solution values u_i^n and $u_{i\pm1}^n$ are coupled with the new solution values u_i^{n+1} and $u_{i\pm1}^{n+1}$.

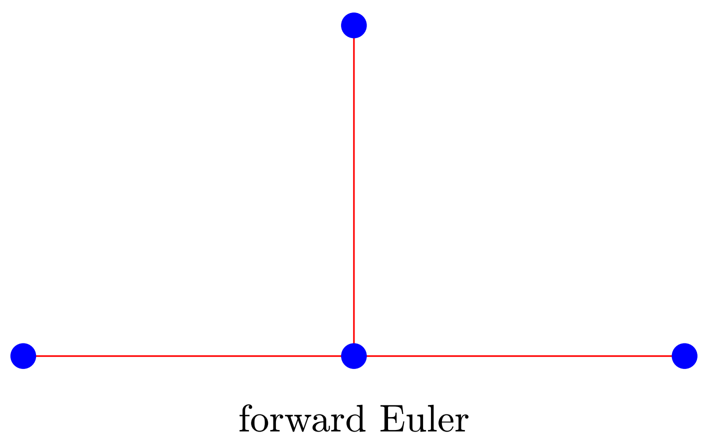


Figure 1: Space-time stencil for the forward Euler parabolic method.

Space-Time Stencils for Classical Hyperbolic Methods

Figures 4–8 display the stencils for the classical hyperbolic methods, where again space is horizontal and time is vertical. The cyan line is the characteristic for the first-order wave equation $u_t + cu_x = 0$ flowing into the new solution point u_i^{n+1}. For stability, the foot of the characteristic must lie within the domain of dependence of the discrete scheme (by virtue of the CFL condition), which implies $c\Delta t \le \Delta x$ (see Figs. 4 and 5).

These ideas are easily extended to general hyperbolic PDEs, where all the characteristic information determining the new solution must be included in the domain of dependence of the discrete scheme.

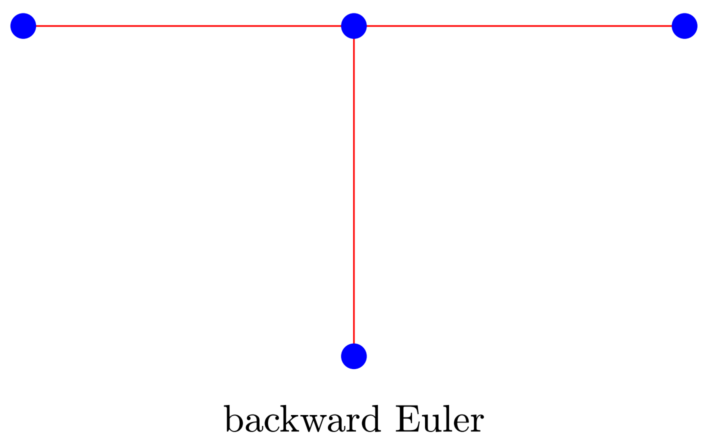


Figure 2: Space-time stencil for the backward Euler parabolic method.

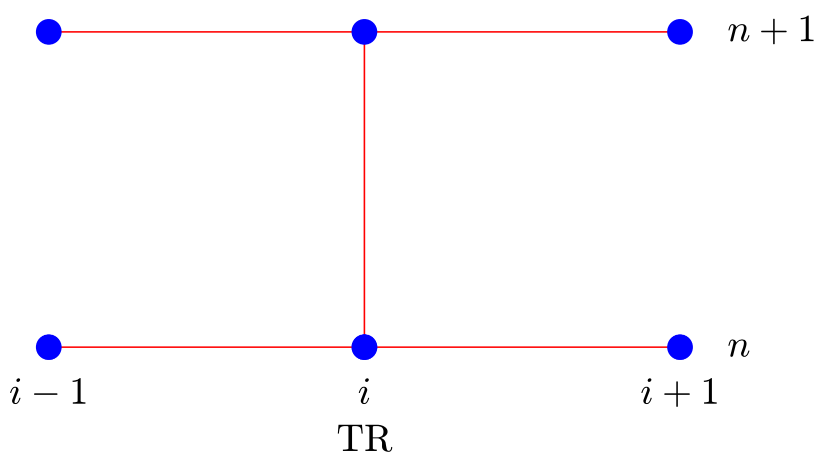


Figure 3: Annotated space-time stencil for the trapezoidal rule parabolic method.

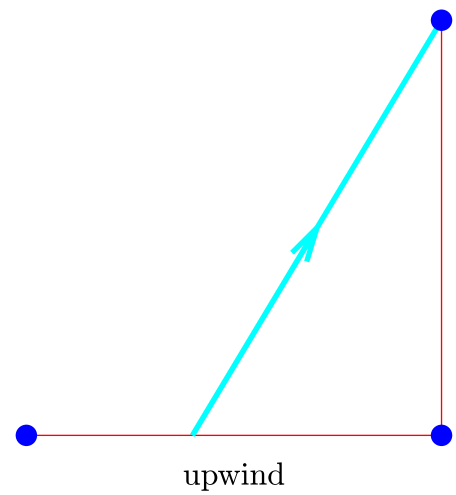


Figure 4: Stable space-time stencil for the upwind hyperbolic method with $c\Delta t \leq \Delta x$, satisfying the CFL condition.

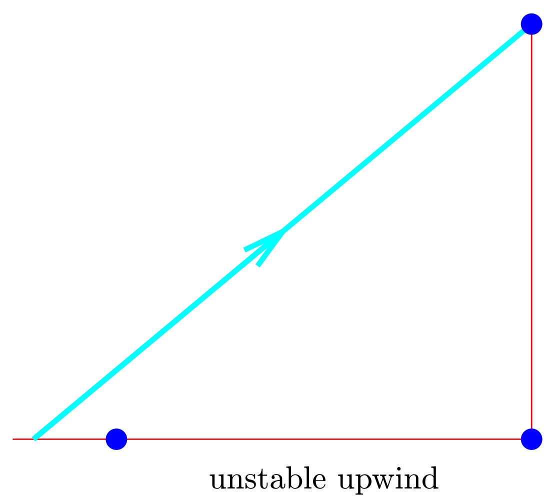
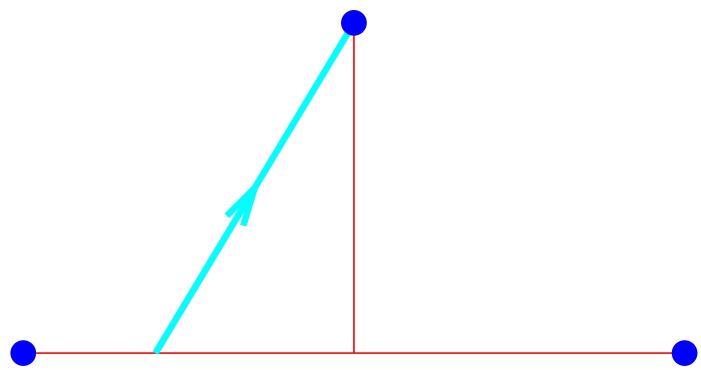


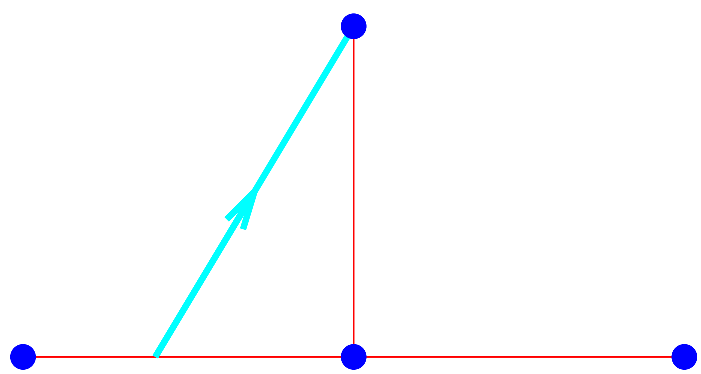
Figure 5: Unstable space-time stencil for the upwind hyperbolic method with $c\Delta t > \Delta x$, violating the CFL condition.



LF

Figure 6: Space-time stencil for the Lax-Friedrichs hyperbolic method with $c\Delta t \leq \Delta x$.

The CFL condition is necessary but not sufficient for stability: the FTCS method in Fig. 7 satisfies the CFL condition for $\Delta t \leq \Delta x/c$, but is always unstable for hyperbolic PDEs.



FTCS

Figure 7: Space-time stencil for the *always unstable* forward time central space method when applied to hyperbolic PDEs.

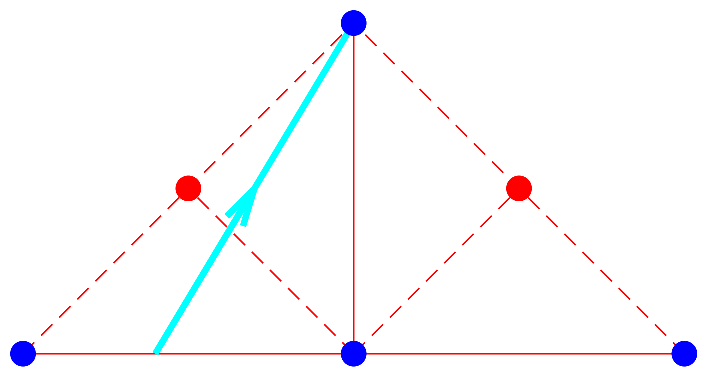


Figure 8: Space-time stencil for the Lax-Wendroff hyperbolic method with $c\Delta t \leq \Delta x$. The red dots are at time level $n + \frac{1}{2}$ and grid points $i \pm \frac{1}{2}$.