IMAGE Report 2 - Frequency Analysis

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High and low frequencies of an image

1)

- The low frequencies correspond to areas of zero change in colour, such as solid blocks of a single colour (e.g. the woman's cheeks) and areas across which there is a slow change in the colour values (e.g. the wall in the background). These areas are often found in the large objects in the image.
- The high frequencies correspond to the fast variations in colour. These include small objects and details in the image (e.g. the texture of the woman's hair), and contours of larger objects that contrast well with the background colour (e.g. the rim of the hat).

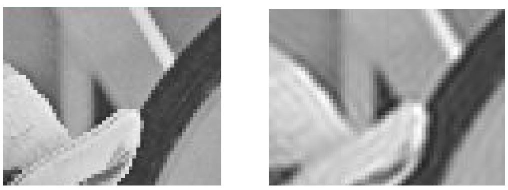
Figure 1: Image filtered to show low and high frequencies separately

2)

We have removed all the high frequencies using a low pass filter, so we have lost all the small details and sharp contours that the high frequencies were responsible for. We are left with only gentle, gradual changes in colour, and large objects whose edges bleed gradually into one another.

As an example, we can examine the edge of the hat. In the original image, the hat is in focus, and its edge contrasts well against the blurred background. In the low-frequency image both the hat and the background are blurred, and the edge of the hat blends into the background, so the depth effect is lost.

Figure 2: Difference in edge contrast between original image (left) and low-frequency image (right)

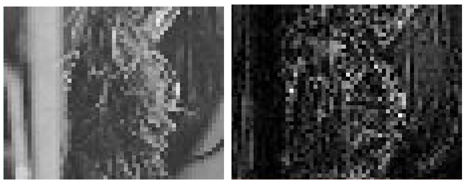


3)

The homogenous areas with slow changes in colour values are represented by the low frequencies which are removed by the high-pass filter, leading to areas of black in the filtered image. On the other hand, the contours and small details in the image are represented by the high frequencies, which are preserved by the high-pass filter. Therefore the filtered image displays only these details and contours against a generally dark background, giving the appearance of bright lines.

As an example, we can examine the woman's hair and the solid white bar to the left of it. In the original image, the hair contains many fast changes in colour due to the many shadows and highlights, while the white bar is a fairly constant colour throughout. In the high-frequency image, the hair retains much of its detail as the many highlights are still well-represented, whereas the white bar in the original image is now shows as a solid black area.

Figure 3: Difference in edge contrast between original image (left) and high-frequency image (right)



Sampling

4)

Spectral aliasing is the phenomenon of overlapping spectra of a sampled image, causing a loss of information that prevents us from faithfully reconstructing the original image from the sampled image.

Spectral aliasing occurs because sampling an image causes its spectrum to repeat an infinite amount of times. If the sampling frequency F_e is less than twice the maximum frequency found in the image F_{max} (i.e. the inverse of Shannon's law is true) then the spectra overlap, and spectral aliasing occurs.

Figure 4: (a) Spectrum of an image.

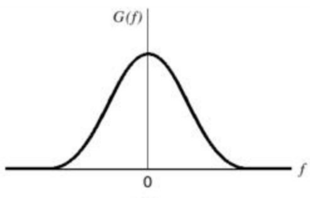
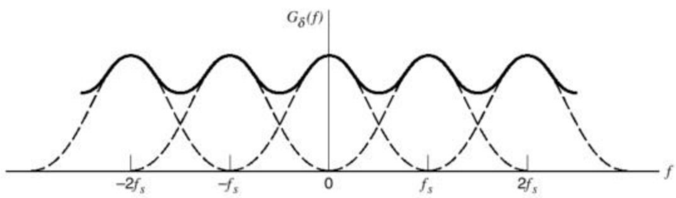


Figure 5: Spectrum of an undersampled version of the image exhibiting aliasing.

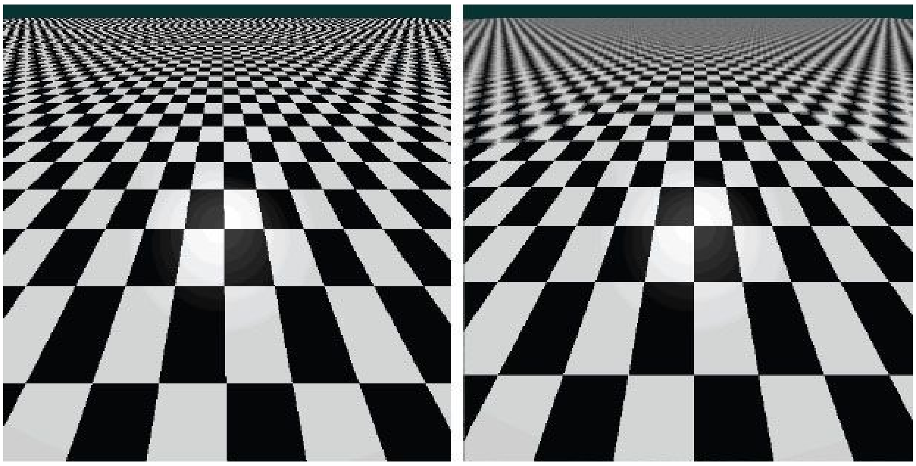


Condition for spectral aliasing to occur:

$$F_e < 2 * F_{max}$$

If an image is reconstructed by taking the inverse Fourier transform of an aliased spectrum, then distortion and/or artefacts will appear in the resulting image. The effect is particularly noticeable when thin parallel lines are present in the original image, as these are represented by very high frequencies which are the frequencies that overlap first with adjacent spectra, and are therefore the most attenuated. One solution to the problem is to ensure F_e is significantly higher than the Nyquist rate, which is termed over-sampling.

Figure 6: Image with aliasing effects (left) and anti-aliased image by over-sampling (right)



5)

Another option to prevent aliasing is to apply an anti-aliasing filter to the original image before sampling it. This removes the high frequencies from the original image, which causing each repeating spectra of the sampled image to be more narrow, and therefore not overlap with the adjacent spectra.

Performing an inverse Fourier transform on this non-aliased spectrum allows us to faithfully reconstruct the original image without loss of information due to aliasing. It should be noted however that loss of information is caused by the anti-aliasing filter itself, due to the loss of the high frequencies, and this is the price we pay.

We use this method when we want to (or have to) use a low (small) sampling frequency, as opposed to the over-sampling method where the sampling frequency must be very high.

The anti-aliasing filter is a low-pass filter with a maximum cut frequency of half the sampling frequency:

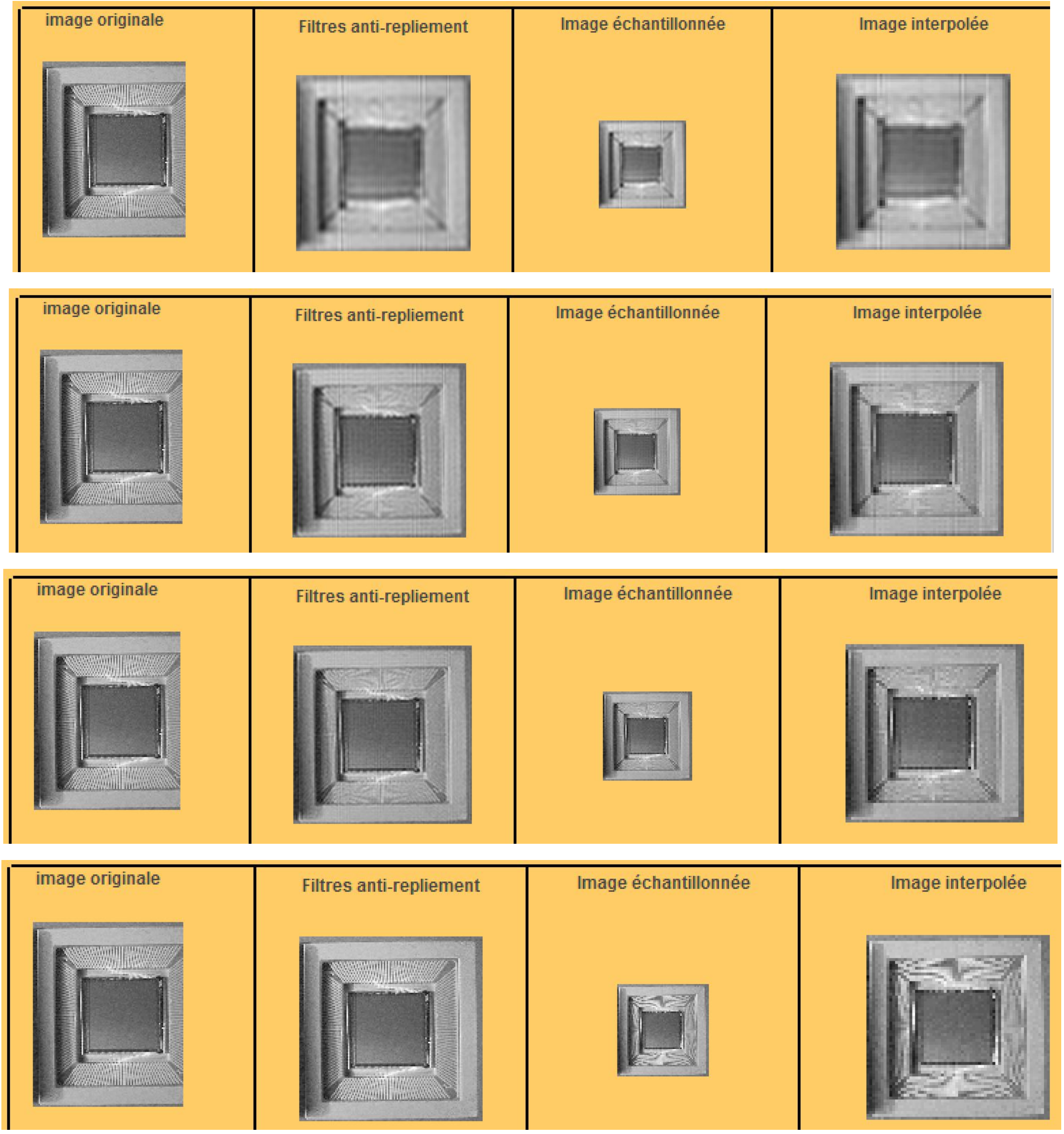
$$F_c \leq F_e / 2$$

Choosing a cut frequency that is less than 50% of the sampling frequency ensures that the new maximum frequency of the filtered image is less than or equal to half the sampling frequency, and therefore obeys Shannon's law:

$$F_{max} \leq F_e / 2$$

As an example, we vary the cut frequency while keeping the sampling frequency fixed. We see in the following series of images that the cut frequencies of 25% and 50% produce anti-alias-filtered images that when sampled and interpolated, result in a reconstructed image that does not contain any aliasing artefacts. The 75% and 100% cut frequencies however does result in information loss, and distortion is clearly visible between the fine lines (high frequency elements) of the reconstructed image.

Figure 7: Anti-aliasing filters applied: F_c = 25%, 50%, 75%, 100% of F_e, fixed sampling step = 2



6)

Bilinear interpolation

- Bilinear interpolation is an extension of linear interpolation for interpolating functions of two variables (e.g., x and y) on a rectilinear 2D grid. Bilinear interpolation takes 4 pixels (2×2) into account, where linear interpolation is first performed in one direction, and then again in the other direction.
- In the time domain, bilinear interpolation is performed as a convolution of a triangular impulse response with the image pixels. A triangular impulse response is itself a convolution of a rectangular impulse response with another rectangular impulse response.
- In the frequency domain, bilinear interpolation is performed as a multiplication of the sinc^2 function with the frequency spectrum. This results in more high frequencies being retained than when multiplying by the standard sinc function.
- Bilinear interpolation is calculation heavy.

Bicubic interpolation

- Unlike bilinear interpolation, which only takes 4 pixels (2×2) into account, bicubic interpolation considers 16 pixels (4×4).
- Images resampled with bicubic interpolation are smoother and have fewer interpolation artifacts than with bilinear interpolation.
- In the time domain, bicubic interpolation is performed as a convolution of a cubic spline impulse response with the image pixels. A cubic spline is itself a convolution of a triangular impulse response with another triangular impulse response.
- In the frequency domain, bicubic interpolation is performed as a multiplication of the $sinc^4$ function with the frequency spectrum.
- Bicubic interpolation is even more calculation heavy than bilinear interpolation.

Figure 8: Bilinear and bicubic interpolation illustrations