Week 3 / Exercise 1

For all polynomials, there exists a natural number N, such that for all $\lambda > N$:

1. $|f(\lambda)| \leq 1 / |p(\lambda)|$

2. $1/|f(\lambda)| \geqslant |p(\lambda)|$

3. For natural numbers

$$\frac{1}{|f(\lambda)|} \geqslant |\lambda^c|$$

Prove that a function f satisfying one of these properties satisfies all of them.

Assume 1 to be true.

Starting from 1, prove 2 to be true.

$$|f(\lambda)| \leq 1/|p(\lambda)|$$

We can multiply and divide without impacting the inequality because we are using absolute values:

$$|p(\lambda)| \leq 1/|f(\lambda)| = 2$$

Starting from 2, prove 3 to be true.

$$1/|f(\lambda)| \geqslant |p(\lambda)|$$

This statement is true for all polynomials, therefore it is true for λ^c which is a polynomial for all $c \in \mathbb{Z}$