# General Solar Position Calculations

From NOAA Global Monitoring Division

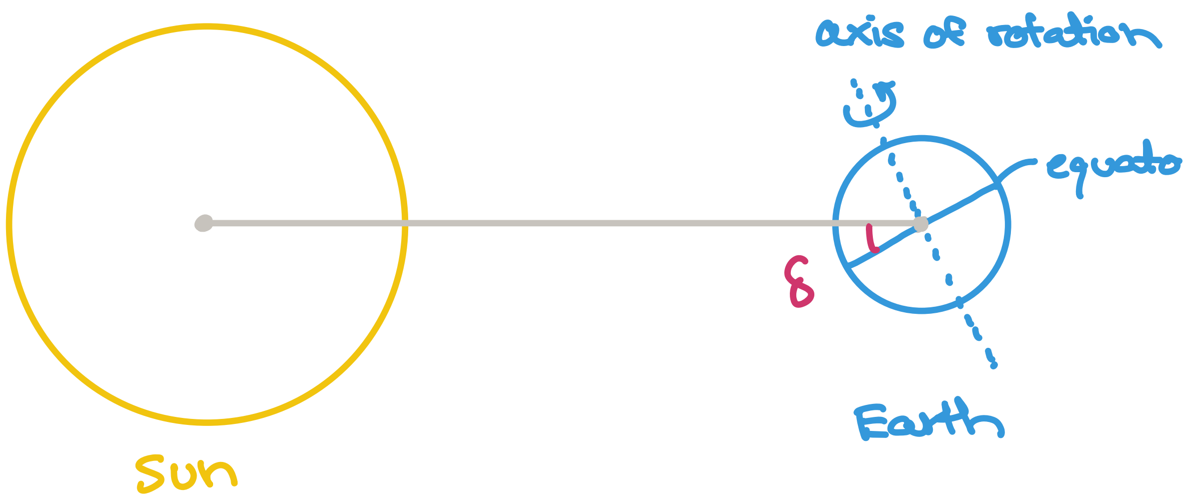
## Fractional Day of the Year

The fractional day of the year $\gamma$ is given by

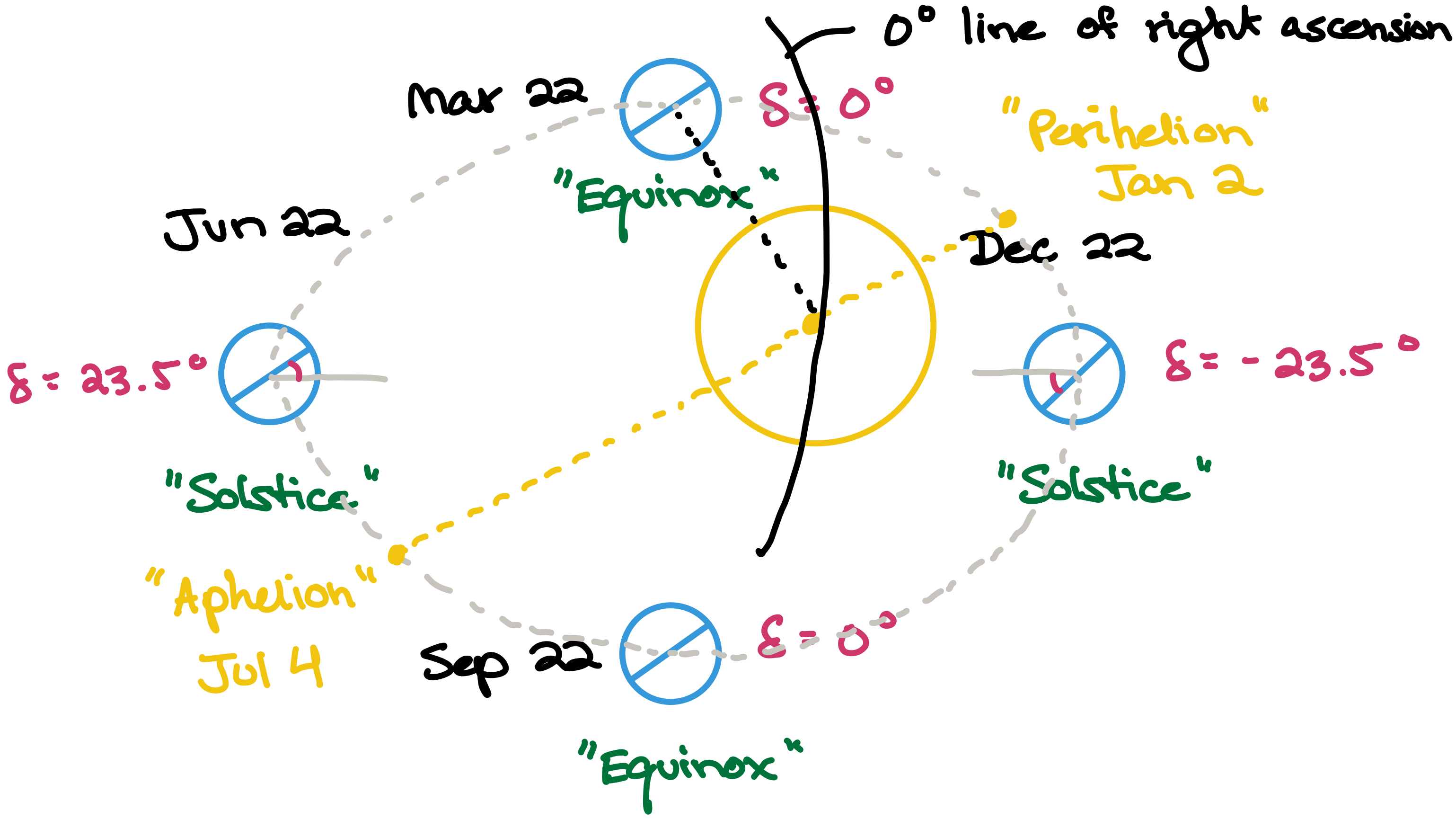
$$\gamma = \frac{2\pi}{365}\left(d - 1 + \frac{h-12}{24}\right)$$

where $d$ is the day of the year (Jan 1 = 1, Dec 31 = 365) and $h$ is the hour of the day (noon = 12, 1 pm = 13).

## Solar Declination Angle




The solar declination angle is $\delta$, is the angle between the rays of the sun and the plane of the Earth's equator




Solar declination angle can be calculated as

$$\sin \delta = \sin(23.5°) \sin\left(\frac{360}{365} \times (d+10)\right)$$

## Equation of Time

apparent solar time = directly tracks the diurnal motion of the sun (what is measured by a sundial)

mean solar time = based on a day being 24 hours as measured by a strict clock

equation of time = apparent solar time − mean solar time

↳ more like a reconciliation

## Causes of equation of time:

- Earth moves in an elliptical orbit
  - faster in Jan
  - slower in July
- The Earth's rotational axis is tilted

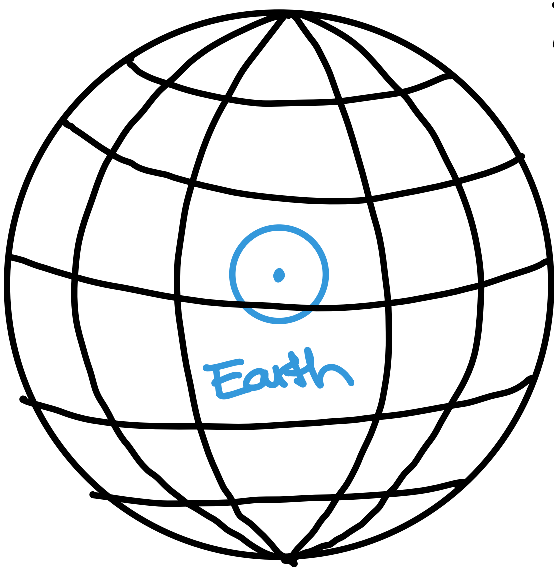
To calculate the equation of time (EOT):

$$EOT = 229.18\left[7.5\times10^{-5} + 1.868\times10^{-3}\cos\gamma - 3.2077\times10^{-2}\sin\gamma - 1.4615\times10^{-2}\cos(2\gamma) - 4.0849\times10^{-2}\sin(2\gamma)\right]$$

result in minutes

## Celestial sphere

An imaginary sphere surrounding the Earth




Longitudes are "lines of right ascension"

Latitudes are "lines of declination"

Line of zero right ascension is the sun's position at the March equinox.

⇒ Longer days near solstices because sun moves more quickly

# True Solar Time (Apparent Solar time)

$$\underbrace{\text{time offset}}_{\text{minutes}} = \underbrace{EOT}_{\text{minutes}} + 4 * \underbrace{\text{longitude}}_{\text{degrees}} - 60 * \underbrace{\text{timezone}}_{\substack{\text{offset from} \\ \text{UTC in hours}}}$$

$$\text{true solar time} = 60 * \text{hour} + \text{minute} + \frac{1}{60}\,\text{second} + \text{time offset}$$

at current location

# Solar Zenith and Azimuth

The solar hour angle (HA) is given by

$$\underbrace{HA}_{\text{in degrees}} = \frac{1}{4}(\text{total solar time}) - 180$$

The solar zenith angle $\phi_R$ is found via

$$\cos \phi_R = \sin(\text{latitude}) \sin \delta + \cos(\text{latitude}) \cos(HA)$$

The solar azimuth angle $\theta_R$ (measured clockwise from north) is given by

$$\cos(180 - \theta_R) = - \frac{\sin(\text{latitude})\cos\phi_R - \sin\delta}{\cos(\text{latitude})\sin\phi_R}$$

## Sunrise / Sunset Times

Set the zenith angle to $\phi_R = 90.833^\circ$ (corrected from $90^\circ$ for atmospheric refraction). Then

$$HA = \pm\cos^{-1}\left\{\frac{\cos\phi_R}{\cos(\text{latitude})\cos\delta} - \tan(\text{latitude})\tan\delta\right\}$$

where + corresponds to sunrise
- corresponds to sunset.

The UTC time of sunrise (or sunset) is then

$$\underbrace{\text{sunrise}}_{\text{minutes}} = 720 - 4(\text{longitude} * HA) - EOT$$

Solar noon is given by

$$\text{solar noon} = 720 - 4 * \text{longitude} - EOT$$