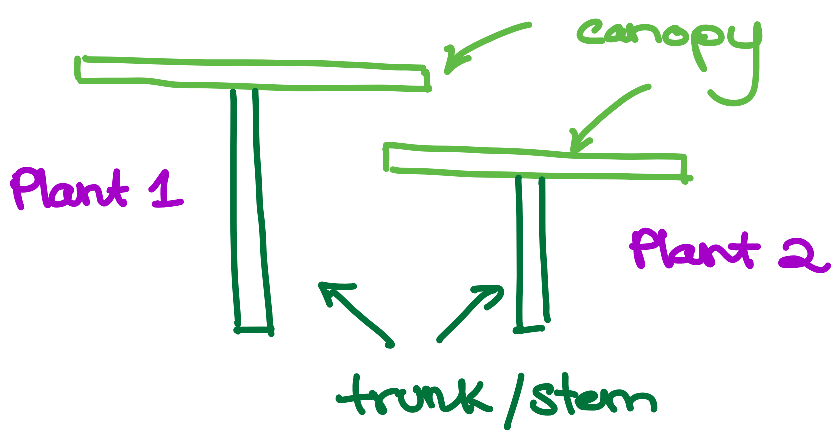
Single Plant Growth Model

Largely taken from "A multiscale mathematical model describing the growth and development of bambara groundnut"

- A nut closely related to the peanut
- Grows to 30 cm high



Plant height model

Logistical growth, exponential decay

$$\frac{dh(t)}{dt} = a_h h(t)\left(1 - \frac{h(t)}{K_h}\right) - d_h h(t)$$

a_h = growth rate $\quad$ K_h = maximum height $\quad$ d_h = height decay rate

Leaf area model

Say change in $A(t)$ = leaf area per plant is given by

$$\frac{dA(t)}{dt} = \text{leaf area growth rate} - \text{leaf area degradation rate}$$

where

Gaussian-like behavior w.r.t. T_c

Leaf area growth rate

$$= L_A\, a_2(T_{avg}, t) \exp\left\{ -\left(\frac{T_c(t) - b_2(T_{avg}, t)}{c_2(T_{avg}, t)} \right)^2 \right\}.$$

In this equation, L_A = leaf area per leaf, a_2, b_2, c_2 are species specific params, but for ease we say

$$a_2(T_{avg}, t) = a\, T_{eff}(T_{avg}, t)$$

$$b_2(T_{avg}, t) = b\, T_{eff}(T_{avg}, t)$$

$$c_2(T_{avg}, t) = c\, T_{eff}(T_{avg}, t)$$

where a, b, c are species specific constants.

$T_c(t)$ is the "cumulative thermal time":

$$T_c(T_{avg}, t) = \int_0^{DAS} T_{eff}(T_{avg}, t)\, dt,$$

where DAS = days after sowing and

$$T_{eff}(T_{avg}, t) = \underbrace{\frac{1}{24} \sum_{j=1}^{24} (T_j(t) - T_{crit})}_{\text{avg diff between temp and critical temp over one day, sampled hourly}}$$

We call T_{eff} the daily effective temperature and T_{crit} the critical temperature above which plants will grow. T_{avg} is given by

$$T_{avg}(t) = \frac{1}{24} \sum_{j=1}^{24} T_j(t)$$

Cumulative thermal temperature can be thought of as the # of temperature units required for plant growth, measured in "degree days"

In the

vegetative → flowering → podding

transition, if temperatures are high, leaf area growth will not decay as fast and pod growth will not increase as quickly.

Vegetative phase: Leaf dev. rate is constant when measured in degree days
(most energy to leaves)

Flowering and podding stages: Leaf dev. continues but decreases
(most energy to pods)

Leaf degradation is thus described by

$$\text{Leaf area degradation rate} = d_L T_{SL}(T_{avg}, t) A(t)$$

where d_L = degradation rate constant and

$$T_{SL}(T_{avg}, t) = \frac{T_{opt} - T_{crit}}{T_{avg}(t) - T_{crit}}$$

the further T_{avg} is from T_{opt}, the more the leaf degradation rate deviates from $d_L A(t)$

Ground cover model

Not all of the leaf canopy will recieve light. To better calculate how much of the canopy is interecepted by light irradiation, we will calculate the ground cover.

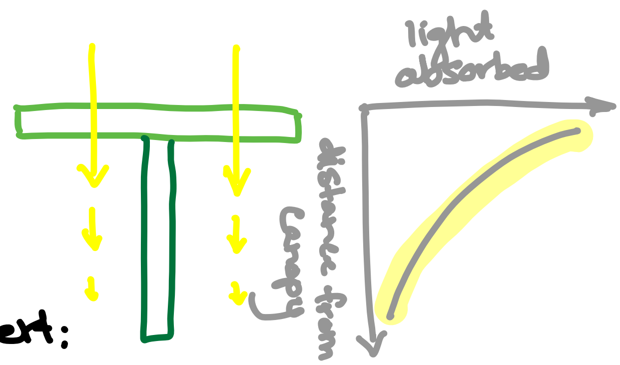
It has been found the a Gaussian relationship with time describes ground cover $G(t)$:

$$\frac{dG(t)}{dt} = a_g A(t) \exp\left\{ -\left(\frac{t-b_g}{c_g}\right)^2 \right\}$$

where $A(t)$ is leaf area per leaf, a_g is the ground cover growth rate, b_g determines the time of peak growth, and c_g determines the rate of time for which peak growth occurs.

Canopy growth rate

First, establish that light absorption falls off according to Beer-Lambert:



$$R(t) = R_0 \left(1 - \exp\{-K\gamma(t)\}\right)$$

where R_0 = available photosynthetically active radiation above canopy

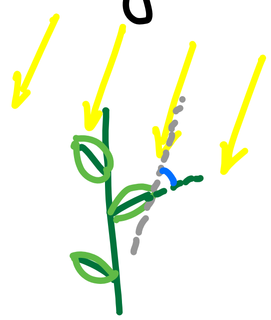
K = extinction coefficient

$\gamma(t)$ = leaf area index

(leaf area per unit ground surface area)

$A(t) / G(t)$

The extinction coefficient depends on the leaf orientation and angle of the light source. $K = 1$ when leaf an light are exactly perpendicular



$0 < \theta < \pi/2 \Rightarrow K < 1$

We then say that the canopy growth rate is related to $R(t)$ by

$$\text{canopy growth rate} = c_e R(t) G(t) \left(1 - \frac{c(t)}{K_c}\right)$$

where c_e = efficiency coefficient for photosynthesis (how well light $\rightarrow$ biomass)

$c(t)$ = canopy biomass

K_c = carrying capacity (b/c plants grow to a max size)

There is also a canopy decay rate due to leaf senescence and pests:

$$\text{canopy decay rate} = d_c\, c(t)$$

where d_c = leaf biomass decay rate.

Altogether: $$\frac{dc(t)}{dt} = \text{canopy growth rate} - \text{canopy decay rate}$$

Pod growth model

It is assumed that an increase in pod mass $P(t)$ is a proportion of the increase in canopy mass $c(t)$.

$$\Rightarrow P(t) = f \circ c(t)$$

It is also assumed that as pod mass increases, it takes more photosynthetic energy than the leaf growth.

Finally, it is assumed that pods have a carrying mass and that pod mass cannot surpass canopy mass.

with this information, the authors construct the model

$$\frac{dP(t)}{dt} = a_p T_{sp}(T_{avg}) \frac{dc(t)}{dt} P(t) \left\{ 1 - \frac{P(t)}{c(t)} \right\} - d_p P(t)$$

where a_p = is the growth rate for pod mass

d_p = pod decay rate

and the stress on the pod due to temperature is

$$T_{sp}(T_{avg}) = \begin{cases} 1, & T_{avg} \leq T_{opt} \\ 1 - \left| 1 - \omega \dfrac{T_{avg} - T_{crit}}{T_{opt} - T_{crit}} \right|, & T_{opt} < T_{avg} < T_{ceil} \\ 0 & T_{avg} \geq T_{ceil} \end{cases}$$

where ω = tuning parameter

T_{opt} = optimal temp for pod growth

T_{ceil} = temp above which pods will not grow

T_{crit} = temp below which pods will not grow

Summary of single Plant Growth model

Plant height $h(t)$

$$\frac{dh(t)}{dt} = a_h h(t)\left(1 - \frac{h(t)}{K_h}\right) - d_h h(t)$$

Leaf Area $A(t)$

$$\frac{dA(t)}{dt} = L_A a_L(T_{avg}, t) \exp\left\{ -\left(\frac{T_c(t) - b_L(T_{avg}, t)}{c_L(T_{avg}, t)} \right)^2 \right\} - d_L T_{SL}(T_{avg}, t) A(t)$$

Ground Cover $G(t)$

$$\frac{dG(t)}{dt} = a_g A(t) \exp\left\{ -\left(\frac{t - b_g}{c_g} \right)^2 \right\}$$

Leaf Area Index $\gamma(t)$

$$\gamma(t) = A(t) / G(t)$$

Canopy Biomass $c(t)$

$$\frac{dc(t)}{dt} = c_e R(t) G(t)\left(1 - \frac{c(t)}{K_c}\right) - d_c c(t)$$

Pod Biomass $P(t)$

$$\frac{dP(t)}{dt} = a_p T_{sp}(T_{avg}) \frac{dc(t)}{dt} P(t) \left\{ 1 - \frac{P(t)}{c(t)} \right\} - d_p P(t)$$

Photosynthetic Radiation $R(t)$

$$R(t) = R_0 \left(1 - \exp\{-K \gamma(t)\}\right)$$

where

a_h = plant height growth rate

a_L = leaf area growth rate

a_g = ground cover growth rate

a_p = pod biomass growth rate

b_L = determines time of peak growth of leaf area

b_g = determines time of peak growth of ground cover

c_L = determines duration of peak growth time (leaves)

c_g = determines duration of peak growth time (canopy)

d_h = plant height decay rate

d_L = leaf decay rate

d_c = canopy decay rate

d_p = pod decay rate

K_h = maximum plant height (height carrying capacity)

K_c = maximum canopy area (canopy carrying capacity)

L_A = leaf area per leaf

c_e = photosynthesis efficiency coefficient

k = extinction coefficient for photosynthesis

ω = tuning parameter for pod temperature sensitivity

R_0 = available photosynthetically active at canopy

DAS = days after sowing

T_{opt} = optimal temp for pod growth

T_{ceil} = temp above which pods will not grow

T_{crit} = temp below which pods will not grow

and

$$T_{avg}(t) = \frac{1}{24} \sum_{j=1}^{24} T_j(t)$$ daily average temp

$$T_{eff}(T_{avg}, t) = \frac{1}{24} \sum_{j=1}^{24} \left(T_j(t) - T_{crit} \right)$$ daily effective temp

$$T_c(T_{avg}, t) = \int_0^{DAS} T_{eff}(T_{avg}, t)\, dt$$ cumulative thermal time

$$T_{SL}(T_{avg}, t) = \frac{T_{opt} - T_{crit}}{T_{avg}(t) - T_{crit}}$$ leaf sensitivity to temp

$$T_{sp}(T_{avg}) = \begin{cases} 1, & T_{avg} \leq T_{opt} \\ 1 - \left(1 - \omega \dfrac{T_{avg} - T_{crit}}{T_{opt} - T_{crit}} \right|, & T_{opt} < T_{avg} < T_{ceil} \\ 0 & T_{avg} \geq T_{ceil} \end{cases}$$

pod sensitivity to temp