Modified Multiple Plant Growth Model

Notable differences from previous model:

- Specifies model changes for the different phases of growth

  seedling → vegetative → flowering / fruiting

- Includes a fertilizer model
- Includes an irrigation model
- Includes a solar model
- Leaves represented as walls of cone, rather than a disk
- No ground cover model

Basic principles applied throughout:

① $\frac{d}{dt}$(variable) = growth rate − decay rate

$$\frac{dx}{dt} = a\left(1 - \frac{x}{K}\right)x$$

↓

$$x(t) = \frac{K x_0 e^{ax}}{K + x_0(e^{ax} - 1)}$$

Logistical
- height
- canopy
- fruit
- fertilizer

Gaussian
- leaves
- irrigation

Exponential

In the future, could add
- pesticide model
- soil health model
- energy / manpower / $$ to deploy irrigation and fertilizer

② "Cumulative Temperature" Model for temp, water

Plant Height Model

$$\frac{dh(t)}{dt} = a_h h(t)\left(1-\frac{h(t)}{K_h}\right) - d_h h(t)$$

for water $\Big\{$ $$+ H_w\, a_{hw}\, W_{eff}(\omega_{avg}, t) \exp\left\{-\left(\frac{w_c(t) - b_{hw} W_{eff}}{c_{hw} W_{eff}}\right)^2\right\}$$

for fertilizer $\Big\{$ $$+ H_f\, a_{hf}\, F_{eff}(F_{avg}, t)\left(1-\frac{F_{eff}(F_{avg}, t)}{K_{hf}}\right)$$

Leaf Area Model

$$\frac{dA(t)}{dt} = \begin{cases} L_T\, a_{LT}\, T_{eff}(T_{avg}, t) \exp\left\{-\left(\frac{T_c(t) - b_{LT} T_{eff}}{c_{LT} T_{eff}}\right)^2\right\} \\ + L_w\, a_{Lw}\, W_{eff}(\omega_{avg}, t) \exp\left\{-\left(\frac{w_c(t) - b_{Lw} W_{eff}}{c_{Lw} W_{eff}}\right)^2\right\} \\ + L_f\, a_{Lf}\, F_{eff}(F_{avg}, t)\left(1-\frac{F_{eff}(F_{avg}, t)}{K_{Lf}}\right) \\ - d_{LT}\, T_{SL}(T_{avg}, t)\, A(t) \\ - d_{Lw}\, W_{SL}(W_{avg}, t)\, A(t), & h(t) > h_{thres} \\ 0, & \text{otherwise} \end{cases}$$

Seedling → Vegetative

Photosynthetic Radiation Model

$$\phi_R = \frac{\pi t}{\text{sunset} - \text{sunrise}}$$

$$\phi = \phi_L - \phi_R \cos(\theta_L - \theta_R)$$

$$K = \sin \phi$$

$$R(t) = R_0 \left(1 - \exp\{-K \rho(t) / \rho_{std}\}\right)$$

Canopy Density Model

$$\rho(t) = \frac{3c(t)}{A(t)} \sqrt{\frac{\pi}{A(t)}} \tan \phi_L$$

Canopy Biomass Model

$$\frac{dc(t)}{dt} = c_e R(t) \frac{dA(t)}{dt} c(t) \left(1 - \frac{c(t)}{K_c}\right) - d_c c(t)$$

Fruit Biomass Model

$$\frac{1}{S} \, {}^\circ C$$

$$\frac{dP(t)}{dt} = \begin{cases} a_p T_{sp}(T_{avg}) \frac{dc(t)}{dt} P(t) \left\{1 - \frac{P(t)}{c(t)}\right\} \\ \quad - d_p P(t), & h(t) > h_{thres} \text{ and } c(t) > c_{thres} \\ 0, & \text{otherwise} \end{cases}$$

Vegetative $\rightarrow$ flowering / fruiting

So that fruit biomass does not begin to grow until the fruiting phase has been entered (when height and canopy biomass surpass a threshold).

Cumulative Temperature Model

T_{opt} = optimal temp for fruit growth

T_{ceil} = temp above which fruit will not grow

T_{crit} = temp below which fruit will not grow

$$T_{avg}(t) = \frac{1}{24}\sum_{j=1}^{24} T_j(t)$$ daily average temp

$$T_{eff}(T_{avg}, t) = \frac{1}{24}\sum_{j=1}^{24} \left(T_j(t) - T_{crit}\right)$$ daily effective temp

days after sowing

$$T_c(T_{avg}, t) = \int_0^{DAS} T_{eff}(T_{avg}, t)\, dt$$ cumulative thermal time

$$T_{SL}(T_{avg}, t) = \frac{T_{opt} - T_{crit}}{T_{avg}(t) - T_{crit}}$$ leaf sensitivity to temp

$$T_{sp}(T_{avg}) = \begin{cases} 1, & T_{avg} \leq T_{opt} \\ 1 - \left|1 - \omega_T \frac{T_{avg} - T_{crit}}{T_{opt} - T_{crit}}\right|, & T_{opt} < T_{avg} < T_{ceil} \\ 0 & T_{avg} \geq T_{ceil} \end{cases}$$

tuning param

fruit sensitivity to temp

Cumulative Irrigation Model

W_{opt} = optimal amount of water for fruit growth

W_{ceil} = amount of water above which plant is flooded

W_{crit} = amount of water below which fruit will not grow

$$W_{avg}(t) = \frac{1}{24} \sum_{j=1}^{24} W_j(t)$$

daily average water from irrigation/rain

$$W_{eff}(W_{avg}, t) = \frac{1}{24} \sum_{j=1}^{24} (W_j(t) - W_{crit})$$

daily effective water

days after sowing

$$W_c(W_{avg}, t) = \int_0^{DAS} W_{eff}(W_{avg}, t)\, dt$$

cumulative irrigation time

$$W_{SL}(avg, t) = \frac{W_{opt} - W_{crit}}{W_{avg}(t) - W_{crit}}$$

leaf sensitivity to water

Effective Fertilizer Model

F_{crit} = amt. of fertilizer below which fruit will not grow

$$F_{avg}(t) = \frac{1}{24} \sum_{j=1}^{24} F_j(t)$$

daily average fertilizer from soil/additions

$$F_{eff}(F_{avg}, t) = \frac{1}{24} \sum_{j=1}^{24} (F_j(t) - F_{crit})$$

daily effective fertilizer

Variable Glossary - Farm Location

sunrise time $\in [5, 9]$ (unitless)

sunset time $\in [16, 20]$ (unitless)

θ_R = polar angle at which radiation hits farm (rad)

ϕ_L = azimuthal (conal) angle of leaves w.r.t. stem (rad)

R_0 = solar irradiance at surface (W/m^2)

Variable Glossary - Simulation settings

t_{final} = total simulation time in days $\in \mathbb{Z}^+$ (unitless)

A_{comp} = leaf test point area (m^2)

ρ_{comp} = 1 test point per A_{comp} ($points/m^2$)

Variable Glossary - Time-Dependent Variables

DAS = days after sowing $\in \mathbb{Z}^+$ (unitless)

$h(t)$ = plant height (m)

$A(t)$ = leaf area (m^2)

$R(t)$ = radiation at leaf surface (W/m^2)

$c(t)$ = canopy biomass (kg)

$\rho(t)$ = canopy density (kg/m^3)

$P(t)$ = fruit biomass (kg)

Variable Glossary - Farm Configuration

m = # of plants $\in \mathbb{Z}^+$ (unitless)

$\{(x^{(i)}, y^{(i)}) \mid i = 1, \ldots, m\}$ = set of (x,y) tuples, coordinates of plant roots (m, m)

$\{h_0^{(i)} \mid i = 1, \ldots, m\}$ = initial plant heights (m)

$\{A_0^{(i)} \mid i = 1, \ldots, m\} = 0$ = initial leaf areas (m^2)

$\{c_0^{(i)} \mid i = 1, \ldots, m\} = 0$ = initial canopy biomass (kg)

$\{P_0^{(i)} \mid i = 1, \ldots, m\} = 0$ = initial fruit biomass (kg)

Variable Glossary - Growth Rates

a_h = plant height growth rate $(1/s)$

a_{hw} = plant height growth rate w.r.t. water $(1/s)$

a_{hf} = plant height growth rate w.r.t. fertilizer $(1/s)$

a_{LT} = leaf area growth rate w.r.t. temp $(1/s)$

a_{LW} = leaf area growth rate w.r.t. water $(1/s)$

a_{Lf} = leaf area growth rate w.r.t. fertilizer $(1/s)$

a_p = fruit biomass growth rate $(1/s)$

Variable Glossary - Peak Growth Parameters

b_{hw} = Scaling factor for peak growth of height w.r.t. water (unitless)

b_{LT} = Scaling factor for peak growth of leaf area w.r.t. temperature (unitless)

b_{LW} = Scaling factor for peak growth of leaf area w.r.t. water (unitless)

Variable Glossary - Peak Growth Times

c_L = determines duration of peak growth time (leaves) (unitless)

c_g = determines duration of peak growth time (canopy) (unitless)

c_{hw} = Scaling factor for duration of peak growth time of height w.r.t. water (unitless)

c_{LT} = Scaling factor for duration of peak growth time of leaf area w.r.t. temp (unitless)

c_{LW} = Scaling factor for duration of peak growth time of leaf area w.r.t. water (unitless)

Variable Glossary - Decay Rates

d_h = plant height decay rate $(1/s)$

d_{LT} = leaf decay rate per degree °C $(1/s/°C)$

d_{LW} = leaf decay rate per kg water $(1/s/kg)$

d_c = canopy decay rate $(1/s)$

d_p = fruit decay rate $(1/s)$

Variable Glossary - Carrying Capacities

K_h = maximum plant height (m)

K_c = maximum canopy area (m^2)

K_{hf} = maximum fertilizer absorption that affects height (kg)

K_{Lf} = maximum fertilizer absorption that affects leaf area (kg)

Variable Glossary - Gains from Various Stimuli

H_w = height gained per kg of water (m/kg)

H_f = height gained per kg of fertilizer (m/kg)

L_T = leaf area gained per °C of temperature $(m^2/°C)$

L_w = leaf area gained per kg of water (m^2/kg)

L_f = leaf area gained per kg of fertilizer (m^2/kg)

Variable Glossary - Other Crop-Specific Parameters

c_e = photosynthesis efficiency coefficient (1/W)

ρ_{std} = typical canopy density (kg/m^3)

K = extinction coefficient for photosynthesis (unitless)

α = absorption factor for a single leaf (unitless)

ω_T = tuning parameter for fruit sensitivity to temp (unitless)

T_{opt} = optimal temp for fruit growth (°C)

T_{ceil} = temp above which fruit will not grow (°C)

T_{crit} = temp below which fruit will not grow (°C)

W_{opt} = optimal amount of water for fruit growth (kg)

W_{ceil} = amount of water above which plant is flooded (kg)

W_{crit} = amount of water below which fruit will not grow (kg)

F_{crit} = amt. of fertilizer below which fruit will not grow (kg)

Algorithm

1. Set all time-independent variables in the variable glossaries.
2. If 24 hours have passed since the last update of $\{T_{avg}, T_{eff}, T_c, T_{SL}, T_{SP}, W_{avg}, W_{eff}, W_c, W_{SL}, F_{avg}, F_{eff}\}$. then update them. Otherwise continue to next step.
3. Evaluate $h(t)$ and $A(t)$.
4. For each of the m plants, discretize the canopies $c(t)$ according to A_{comp} and find R_0 for each plant.
5. Use solar model to determine ϕ_R, ϕ, K, $\rho(t)$, and $R(t)$.
6. Evaluate $c(t+\Delta t)$.
7. Evaluate $P(t+\Delta t)$.
8. Update $t \leftarrow t + \Delta t$.
9. Repeat steps 2-8 until t_{final} is reached.