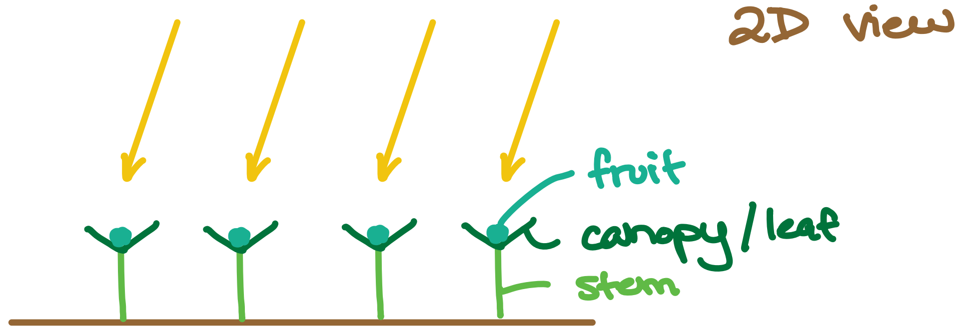
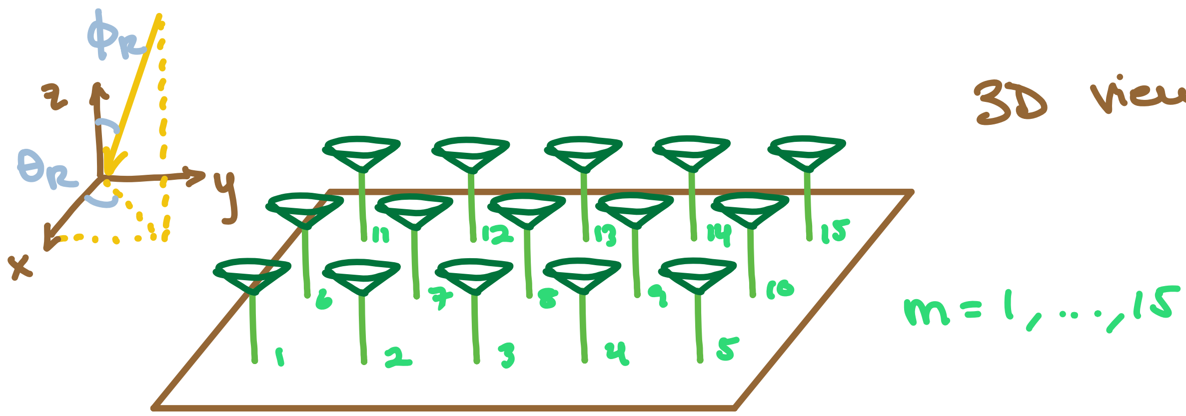
Solar Model





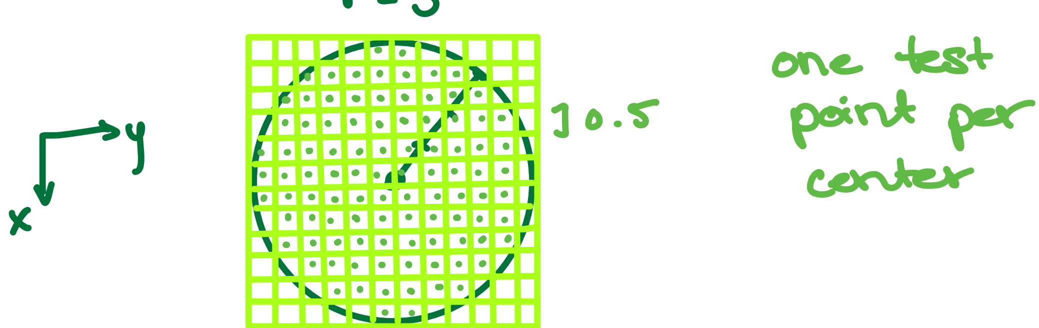
To Define All the Test Points

Given center coordinates and radius (e.g. 3) along with

ρ_{comp} = 1 test points/unit area

A_{comp} = unit area e.g. 0.5 x 0.5

We can define the (x,y) coordinates of the test points according to



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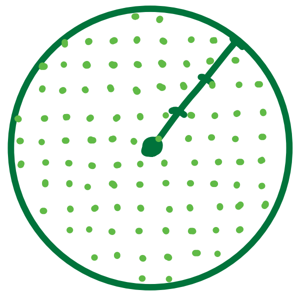
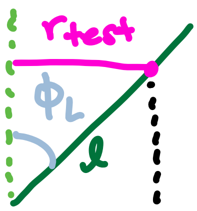


To find the z-coordinates of the test points, we find the distance from the point to the center:

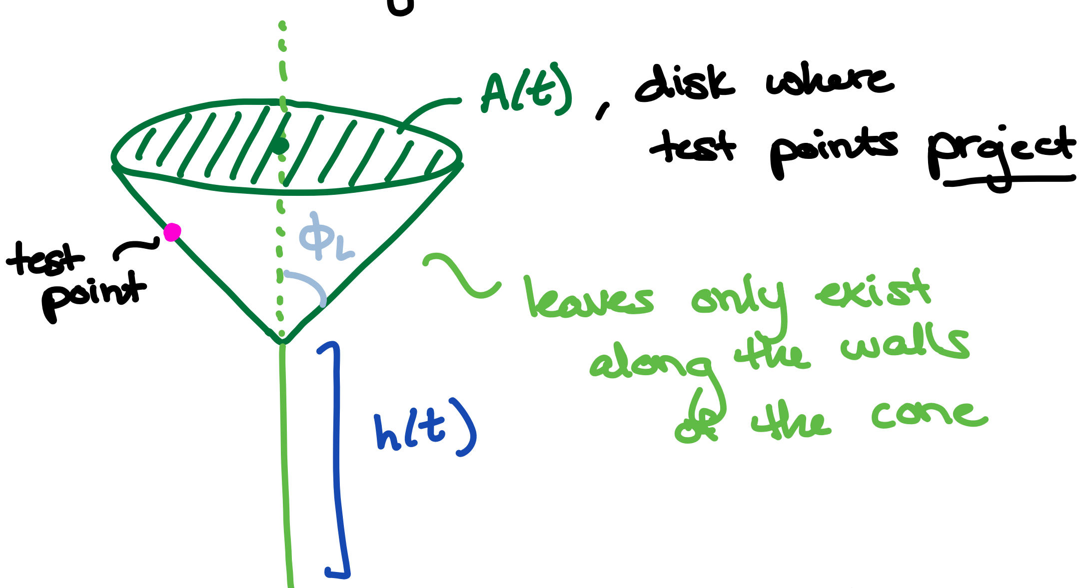
$$r_{test} = \sqrt{x_{test}^2 + y_{test}^2}$$

Then use the leaf angle to get the z-coordinate:



$$r_{test} = l \sin \phi_L$$
$$z_{test} = l \cos \phi_L$$
$$\Longrightarrow z_{test} = r_{test} \cot \phi_L$$

For light incident on a single test point in plant i,



We need to find the angle between the leaf and incident light in order to find the extinction coeff.

To do this, we use the formula

$$\kappa = \sin \phi$$

where

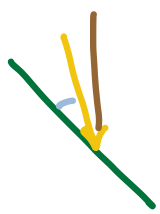
$$\phi = \phi_L - \phi_R \cos(\theta_L - \theta_R)$$

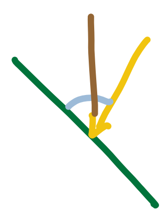
But why?

Well, light only hits test points when $\theta_R < \theta_L$

and, consider the two edge cases

Case 1: $\theta_L - \theta_R = 0$ Case 2: $\theta_L - \theta_R = \pi$





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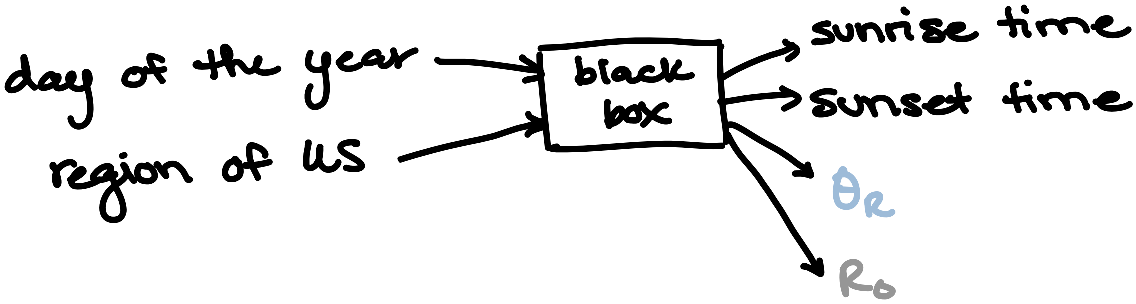
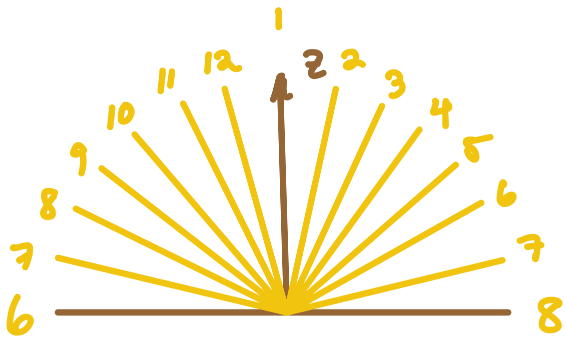
Use $\phi_L - \phi_R$ Use $\phi_L + \phi_R$

$$\phi_L - \phi_R \cos(\theta_L - \theta_R)$$

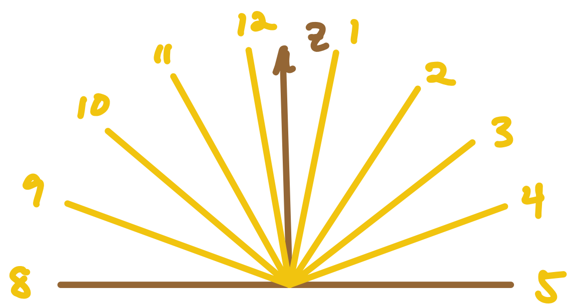
Use real-world data on trajectories of sun over the Earth at different times of the year to obtain:



Using this information, our job is to obtain hourly values of ϕ_R.



Summer



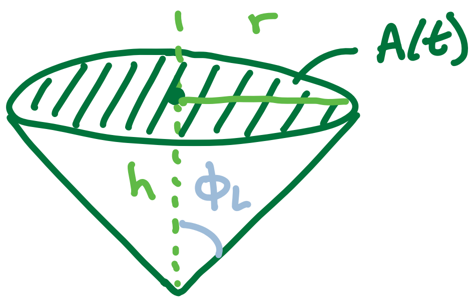
Winter

sunrise = 6
sunset = 8 (20)

$$\Rightarrow \phi_R = \frac{\pi t}{\text{sunset} - \text{sunrise}}$$

where t is the current time of the day on the 24 hour clock.

How do $A(t)$ and $c(t)$ together define the leaves?



Well, given A, we know the radius of the conical canopy is

$$A = \pi r^2 \Rightarrow r = \sqrt{A/\pi}$$

Given the cone angle ϕ_L, we know the height of the cone is

$$\tan\phi_L = r/h \Rightarrow h = r/\tan\phi_L$$

The volume of the cone is then

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{A}{\pi}\frac{\sqrt{A/\pi}}{\tan\phi_L} = \frac{1}{3}A\sqrt{\frac{A}{\pi}}\cot\phi_L$$

The density of the canopy is then

$$\varphi = \frac{c}{V} = \frac{3c}{A}\sqrt{\frac{\pi}{A}}\tan\phi_L$$

The density of the canopy determines how well light passes through:

higher $K \Rightarrow$ stronger radiation

$$R(t) = R_0\left(1 - \exp\{-K\varphi(t)/\varphi_{std}\}\right)$$

less dense $\Rightarrow$ stronger radiation