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**Solution for Project 3**Due date: Monday 15 April 2024, 23:59 (midnight)

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**HPC Lab for CSE 2024 — Submission Instructions**(Please, notice that following instructions are mandatory:  
submissions that don't comply with, won't be considered)

- Assignments must be submitted to Moodle (i.e. in electronic format).
- Provide both executable package and sources (e.g. C/C++ files, Matlab). If you are using libraries, please add them in the file. Sources must be organized in directories called:  
*Project\_number\_lastname\_firstname*  
and the file must be called:  
*project\_number\_lastname\_firstname.zip*  
*project\_number\_lastname\_firstname.pdf*
- The TAs will grade your project by reviewing your project write-up, and looking at the implementation you attempted, and benchmarking your code's performance.
- You are allowed to discuss all questions with anyone you like; however: (i) your submission must list anyone you discussed problems with and (ii) you must write up your submission independently.

In this report, we are solving Fisher's equation using the finite difference method. The equation is given by:

$$\frac{\partial s}{\partial t} = D \nabla^2 u + R s (1 - s) \quad (1)$$

where  $s$  is the concentration of a species,  $D$  is the diffusion coefficient, and  $R$  is the reaction rate. The equation is solved on a 2D grid with Dirichlet boundary conditions,

$$s(x, y, t) = 0.1 \quad \text{for} \quad x = 0, x = 1, y = 0, y = 1 \quad (2)$$

and initial conditions is a circle of radius  $1/8$  at the lower left quadrant the domain.

## 1. Task: Implementing the linear algebra functions and the stencil operators [40 Points]

The first part of the project consists in implementing the linear algebra functions and the stencil operators. The linear algebra functions are implemented in the `linalg.cpp` file. I implemented all the functions using simple loops. Similarly, the stencil operators were implemented in the `operators.cpp` file. Writing the interior grid points was very simple since we already had the implementations for the boundary conditions.

Afterwards, I ran the code following the indications on the project description and I got the following results in the terminal

```

=====
                        Welcome to mini-stencil!
version :: C++ Serial
mesh :: 128 * 128 dx = 0.00787402
time :: 100 time steps from 0 .. 0.005
iteration :: CG 300, Newton 50, tolerance 1e-06
=====

-----
simulation took 0.203392 seconds
1513 conjugate gradient iterations, at rate of 7438.82 iters/second
300 newton iterations
-----

### 1, 128, 100, 1513, 300, 0.203392 ###
Goodbye!

```

Which are the matching with the expected results. Additionally, I got the following content for the output.bin file,

```

TIME: 0.005
DATA_FILE: output.bin
DATA_SIZE: 128 128 1
DATA_FORMAT: DOUBLE
VARIABLE: phi
DATA_ENDIAN: LITTLE
CENTERING: nodal
BRICK_ORIGIN: 0. 0. 0.
BRICK_SIZE: 1 1 1.0

```

When plotting the results, they seem to be as expected; the population concentration is higher at the lower left quadrant of the domain, which is matching with the initial condition. Fig. 1 shows

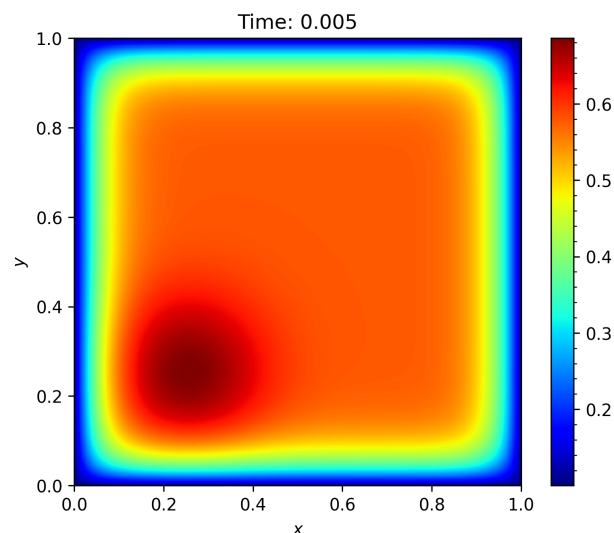


Figure 1: The population concentration at time  $t = 0.005$ .

the results corresponding to a run with a  $1024 \times 1024$  grid.

## 2. Task: Adding OpenMP to the nonlinear PDE mini-app [60 Points]

This section is dedicated to parallelizing the code using OpenMP and analyzing the performance of the code, as well as some discussion regarding the results and differences between the serial and

parallel versions.

## Welcome message and serial compilation

We want to have different welcome messages for the serial and parallel versions. This was done using the macro `_OPENMP` which is defined when OpenMP is enabled. The code is shown below.

```
#ifndef _OPENMP
#include "omp.h" // This line won't add the library if you don't compile with -
                fopenmp option.
#endif

...

#ifdef _OPENMP
    std::cout << "version :: C++ OpenMP" << std::endl;
#pragma omp parallel
    {
        threads = omp_get_num_threads();
    }
    std::cout << "threads :: " << threads << std::endl;
#else
    std::cout << "version :: C++ Serial" << std::endl;
#endif
```

This seems to work as expected. The code is compiled with the `-fopenmp` flag to enable OpenMP.

## Linear algebra kernel

Afterwards, the first step to parallelize the code was to work on the linear algebra functions. I used simple `pragma omp parallel for` and `pragma omp parallel for reduction(+:result)` directives to parallelize the loops.

## The diffusion stencil

Then, I parallelized the diffusion stencil using the `pragma omp parallel for` directive. This was really straightforward since the stencil is mainly composed of simple loops. The boundary loops are important since they are implementing the Dirichlet boundary conditions. They take the data stored in the `bndW` data structure and apply it to the boundary points of the grid to calculate the diffusion.

## Bitwise identical results

I converted the `bin` files to `hex` files to be able to see clearly if the parallel implementation was producing bitwise identical results. The results were that they were not identical. The main reason is that floating point operations are not associative, and the order of the operations can change the results. This influences some calculations such as the norm and the dot product between two vectors. Another reason I thought initially may interfere was the implementation of the diffusion stencil, but upon further inspection, I realized that the update is using the `s_new` and `s_old` data structures, which are not updated in the stencil operation and therefore should not be affected by the parallelization.

In my opinion, in order to get bitwise identical results, one would need to parallelize instead by splitting the grid into chunks depending on the number of threads and then use ghost cells to communicate the boundary conditions. This would be a more complex implementation, but it

should be able to produce bitwise identical results. This would also require for some synchronization between the threads to make sure that the boundary conditions are correctly applied. Also, directives like the `reduction` should be avoided.

## Strong scaling

Next, we want to analyze the performance of the code. We will analyze the strong and weak scaling of the code. I repeated the runs 100 times for the serial and 50 for the parallel versions to get better timing results. It was a bit challenging to be able to determine the run time for certain using only one run, since there seemed to be a lot of variability in the run times. The codes to compile and run the experiments are included in the bash scripts accompanying this report. We can see the distribution of the run times in Fig. 2. Since the run times are so variable, I decided to use the

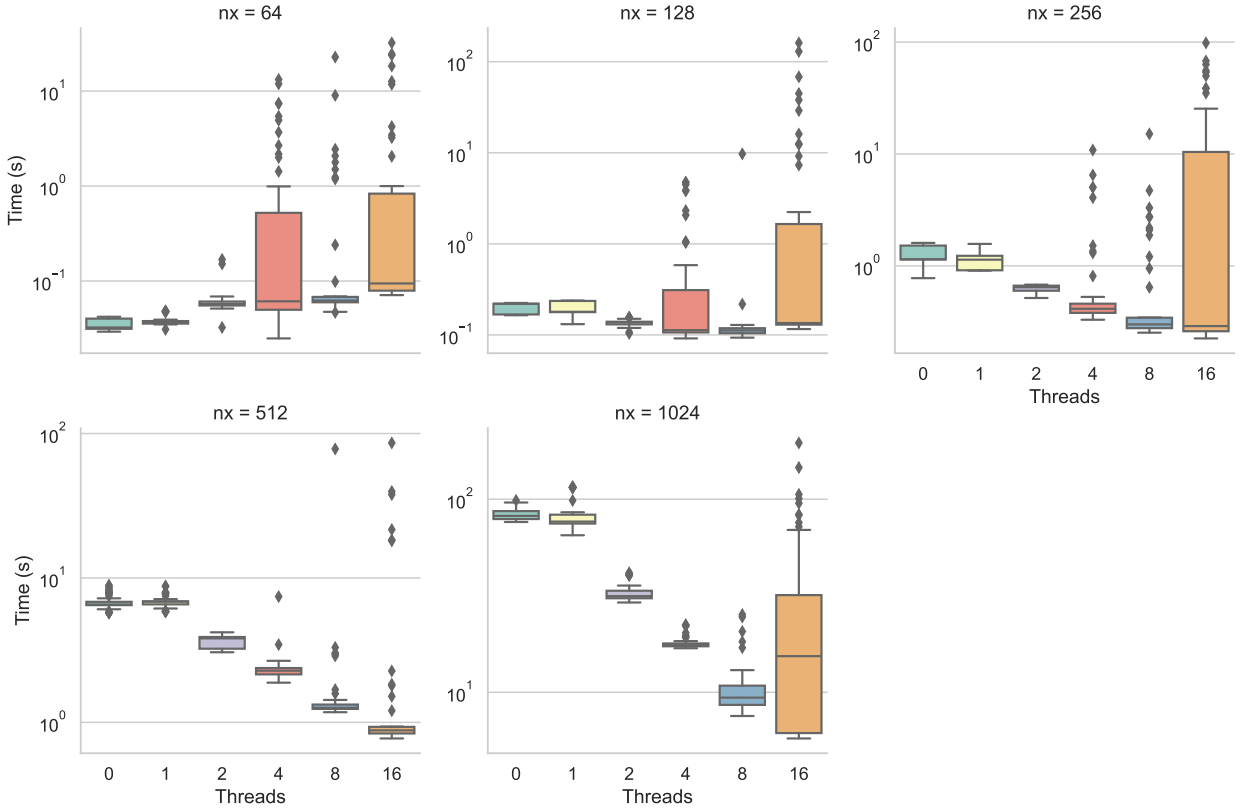


Figure 2: Time taken for the serial and parallel versions of the code; zero threads indicates serial version.

median of the run times to determine the run time for the strong and weak scaling experiments. We can see that specially when using 16 threads, the variability is very high and there is a large number of outliers. This puts into question how reliable is benchmarking on the Euler cluster. There is also the interesting case of 4 threads on a  $64 \times 64$  grid which seems to have very high run times compared to the other cases. I wonder if this has something to do with hardware.

The results of the strong scaling are shown in Fig. 3 The beyond optimal performance can be explained by the chosen representative of the time in the serial and parallel versions. The plot shows that the parallel implementation is doing a good job at scaling with the number of threads when the grid size is at a certain level that is not too small. At first, running it with a small grid size, the parallel version does not show an improvement in performance. This can be explained with the overhead of parallelizing the code. But we can see that as the grid size increases, the parallel version is able to take advantage of the multiple threads and the performance improves.

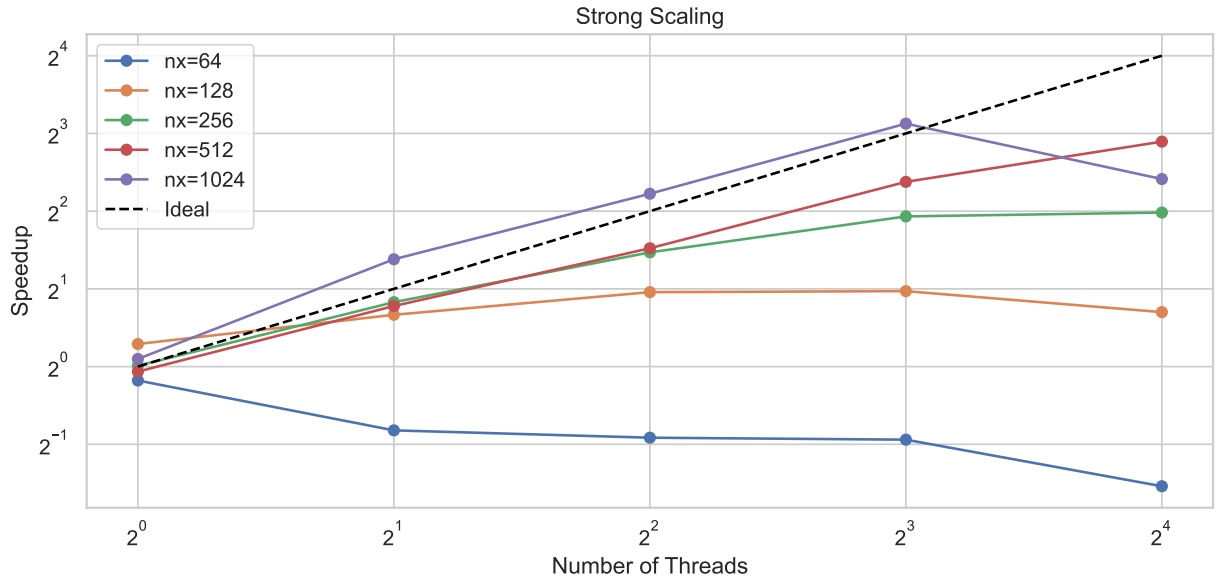


Figure 3: Strong scaling of the code.

### Weak scaling

For the weak scaling, I ran the code for the parallel version 100 times for each combination of threads and grid size. Following the previous analysis of run times, I again used the median as representative. The results are shown in Fig. 4.

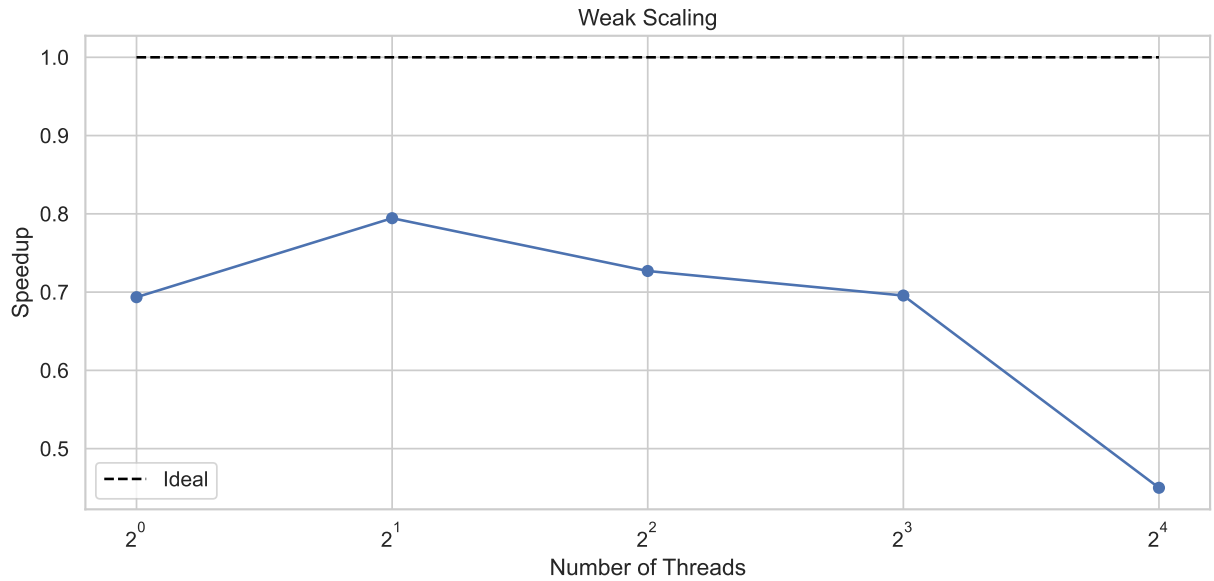


Figure 4: Weak scaling of the code.

The figure shows that the parallel version is able to scale well with the grid, although not reaching the ideal performance, which is expected. The performance is still good, reaching mostly around 0.7, except for the case of 16 threads. As discussed before, this configuration seems to have a large variability in the run times.