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**Solution for Project 4**Due date: Monday 29 April 2024, 23:59 (midnight).

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**1. Ring sum using MPI [10 Points]**

The idea of this task is to implement a ring sum using MPI. The ring sum is a simple algorithm to compute the sum of all elements in an array. To avoid any deadlocks, I decided to use the MPI\_Sendrecv function to send and receive the data. The MPI\_Sendrecv is specially useful when you want to send and receive data between two processes. The function is non-blocking, which means that the function returns immediately after the data is sent and received. This avoids any potential deadlocks. The destination and source ranks are calculated as follows:

```
int rank_dest, rank_source;
rank_source = (rank + size - 1)
rank_dest = (rank + 1)
```

The messages are chosen to be the sum of the current rank and the previous received message, which is done repeatedly over the number of ranks so to calculate the whole sum. The code block shows the logic used;

```
int out_msg, in_msg;
if (i == 0)
{
    out_msg = rank;
}
sum = sum + in_msg;
out_msg = in_msg;
```

Finally, the code block below aims to give an idea of the function parameters, data types and assigned values (it is not meant to represent the actual syntax used in the implementation):

```
MPI_Sendrecv(
    const void *sendbuf = &out_msg,
    int sendcount = 1,
    MPI_Datatype sendtype = MPI_INT,
    int dest = rank_dest,
    int sendtag = 0,
    void *recvbuf = &in_msg,
    int recvcount = 1,
    MPI_Datatype recvtype = MPI_INT,
    int source = rank_source,
    int recvtag = 0,
```

```

    MPI_Comm comm = MPI_COMM_WORLD,
    MPI_Status *status = &status
)

```

The code was tested with 5 processes and the result is 10.

## 2. Cartesian domain decomposition and ghost cells exchange [20 Points]

The objective of this task is to implement a 2D Cartesian domain decomposition and exchange ghost cells between neighboring processes. The idea is to divide a 2D grid into smaller subgrids, where each subgrid is assigned to a process. The borders are assigned the value of the neighboring process. To implement this, I used the `MPI_Cart_create` and `MPI_Cart_shift` functions to create a 2D Cartesian topology and to get the neighboring processes. To communicate the column ghost cells, I created a new MPI datatype using the `MPI_Type_vector` function using the domain and subdomain sizes, defined as follows:

```

MPI_Type_vector(SUBDOMAIN, 1, DOMAINSIZ, MPI_DOUBLE, &data_ghost_col);
MPI_Type_commit(&data_ghost_col);

```

Afterwards, I used the `MPI_Send` and `MPI_Recv` function to communicate the messages because it is sufficient and easier to implement. This implementation seemed to work well for the given test case, although it may not be thread safe for other cases. For the given test case, the buffer was enough to store the data and the communication was successful. An example of the implementation regarding the first row and interaction with the top block is added below:

```

// SEND
// to top
MPI_Send(&data[first_row_index], 1, data_ghost_row, rank_top, tag, MPI_COMM_WORLD
);

// RECEIVE
// from top
MPI_Recv(&data[first_row_index], 1, data_ghost_row, rank_top, tag, MPI_COMM_WORLD
, &status);

```

Some auxiliary variables were used to store the ranks of the desired cells (rows and columns). The complete implementation can be found in the attached code. The code was tested with a 6x6 matrix, 8x8 considering ghost cells. We report the boundary exchange on rank 9.

```

9.0 5.0 5.0 5.0 5.0 5.0 5.0 9.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
9.0 13.0 13.0 13.0 13.0 13.0 13.0 9.0

```

## 3. Parallelizing the Mandelbrot set using MPI [30 Points]

The goal of this task is to parallelize the computation of the Mandelbrot set using OpenMP. We use the base code of the previous project and modify it to use MPI. The idea is to divide the image into smaller subimages and assign each subimage to a process. This should be done

automatically, without any manual input. This is done using *Partition* and *Domain* structs, which contain information such as index of the pixels, size, and coordinated of the subimages. These data structures are based on the functions `MPI_Dims_create`, `MPI_Cart_create`, `MPI_Cart_coords` to create a 2D Cartesian topology and to get the coordinates, similar to the previous task. In this implementation, we are also using the `MPI_Send` and `MPI_Recv` functions to communicate the data between the processes. The data is sent and received a contiguous vector of pixels. Each subimage is computed in separate processes and sent to the root process, which is responsible for storing and creating the final image. Figure 1 shows the result of the computation of the Mandelbrot set using 4096x4096 pixels distributed over 16 processes. We can see that the image reproduces the

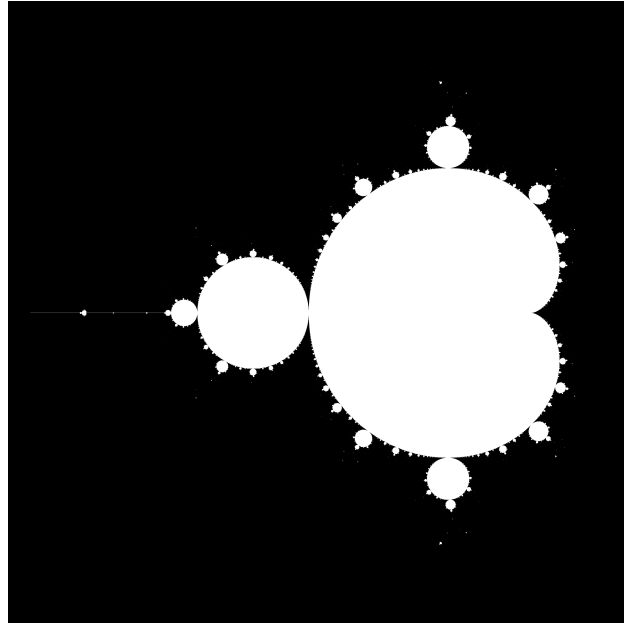


Figure 1: Mandelbrot set

correct result, as we obtained in a previous project. The performance of the parallel Mandelbrot set using this implementation is shown in Figure 2. It reflects how the work is distributed among the processes and the time taken by each. We can see that as the number of processes increases, the time taken per process to compute their part of the image decreases in comparison to the single processor implementation but not equally among all processes. This is because the work of the Mandelbrot set is not evenly distributed spatially, that is to say, some parts of the image are more complex to compute than others. The graph reflects that the partition between the processes is not optimal, as some of the processes do not reduce the time significantly as the number of processes increases. A more efficient partitioning algorithm could be implemented to solve this issue, but this would require previous knowledge of the desired solution. Another approach could be to use a dynamic partitioning algorithm, which would distribute the work among the processes according to the complexity of the computation.

#### 4. Parallel matrix-vector multiplication and the power method [40 Points]

The goal of this task is to complete a code trying to parallelize the matrix-vector multiplication and the power method using MPI. This was achieved using `MPI_Bcast` and `MPI_Allgatherv` functions. The idea is to broadcast the original vector solution  $y$  of size  $n$  and then gather the results from all processes which were computed in  $y_{local}$ , of size  $nrows_{local}$ . The implementation of this specific part is shown below:

```
MPI_Bcast(y, n, MPI_DOUBLE, 0, MPI_COMM_WORLD);
```

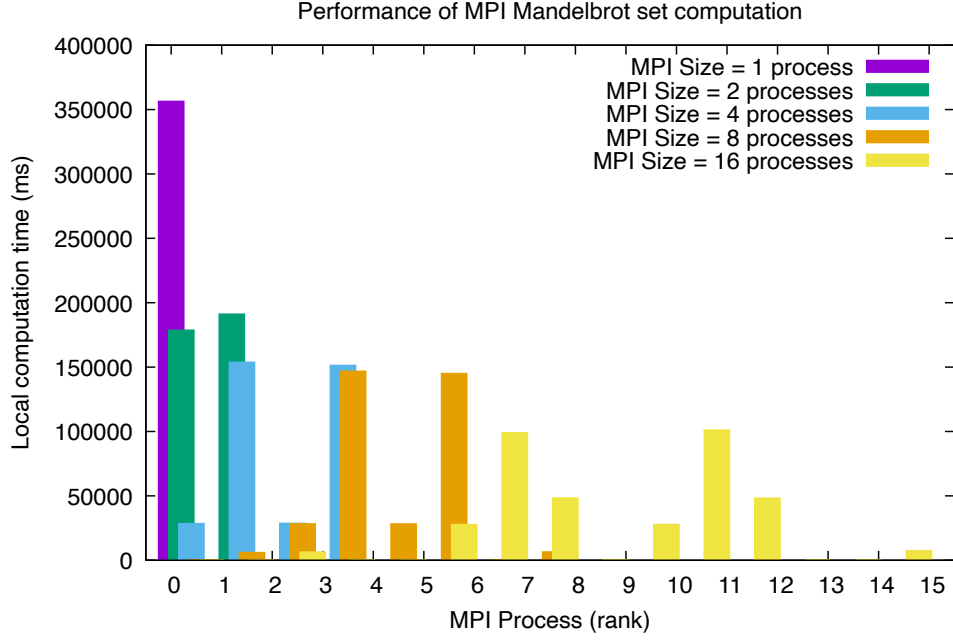


Figure 2: Parallel Mandelbrot set

```
...
MPI_Allgather(y_local, nrows_local, MPI_DOUBLE, y, recvcounts, displs,
MPI_DOUBLE, MPI_COMM_WORLD);
```

The variable *recvcounts* is an array that contains the number of elements to be received from each process. The *displs* array contains the displacement of the data to be received. The code was tested with a matrix of  $10,000 \times 10,000$ , with a maximum of 3,000 iterations, using test number 3, which resulted in a eigenvalue of 9998.059. The strong and weak scaling of the matrix-vector multiplication and the power method are shown in Figures 3 and 4, respectively. The scaling was done using one core as well as multiple cores. The time used as reference was the parallel implementation with 1 process. The reported value is the mean of fifty runs. We can see that the strong scaling presents a more than linear speedup, which is odd. This could be attributed to the overhead of the communication between the processes when not comparing against the naive implementation. For some reason, this is not present in the run over multiple cores. Nevertheless, the weak scaling shows a good performance in all number of processes.

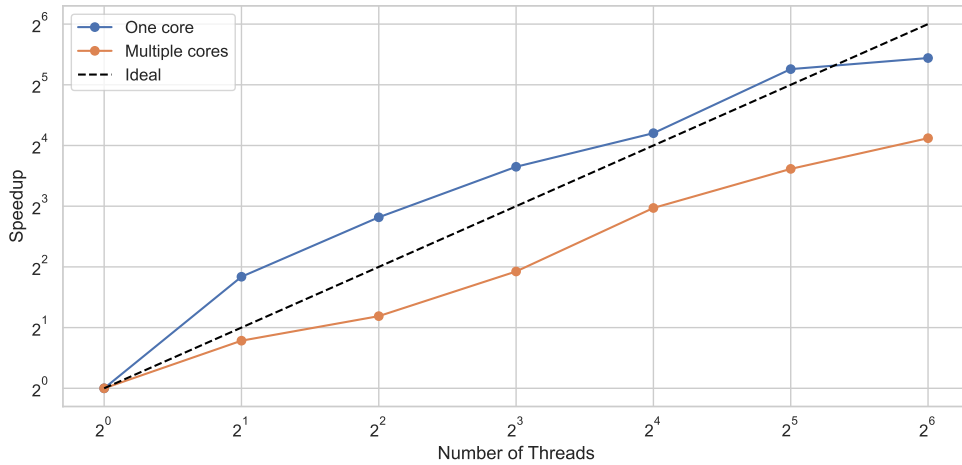


Figure 3: Strong scaling of the matrix-vector multiplication and the power method.

The weak scaling shows a good performance in the low number of processes, but then it starts to decrease. This can be attributed to the overhead of the communication. Contrary to the strong

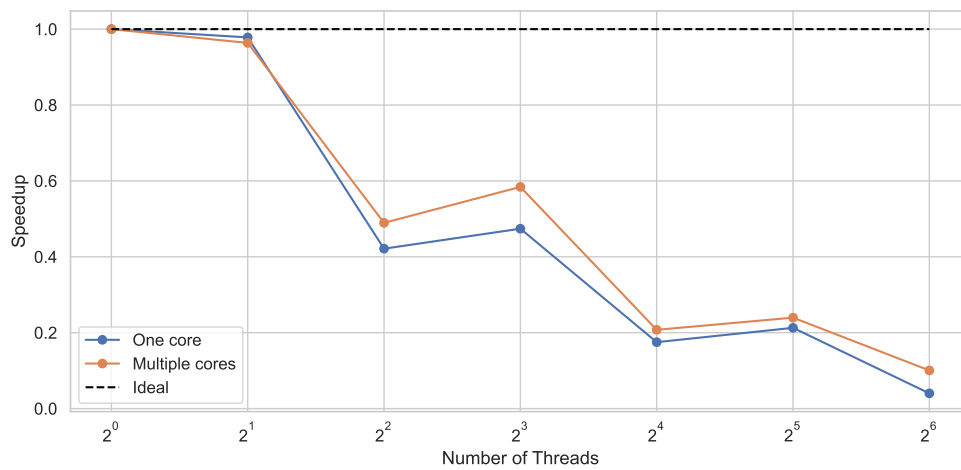


Figure 4: Weak scaling of the matrix-vector multiplication and the power method.

scaling, the weak scaling shows similar performance for both runs over one and multiple cores.