

1 Exercise 1

1.1 Cross Entropy

$$H(y,g) = -(0 \cdot \log(0.25) + 1 \cdot \log(0.6) + 0 \cdot \log(0.15)) = 0.222$$

1.2 Mean Squared Error Loss

$$MSE(y,g) = \frac{1}{3}(0.25^2 + (-0.4)^2 + 0.15^2) = 0.0817$$

1.3 Hinge Loss

$$SVM(y,j) = \max(0, 1.25) + \max(0, 0.4) + \max(0, 1.15) = 2.8$$

2 Task A

2.1 Accuracy

$$ACC = \frac{TP + TN}{P + N}$$

, high accuracy of predicting predominant class is easily achieved and misleading. If we have highly unbalanced data the problem of accuracy as a metric is quite apparent: Let's consider an example with medical data: normally only a few patients have a certain disease. If our dataset consists of 200 patients, of which only 8 have cancer, than a classifier which predicts that no patient has cancer would yield 96 percent accuracy; this prediction however would be deadly for the patients. If we also consider false negatives, than we see that in every case of a cancer patient, the classifier predicted no cancer and thus false negatives are 100 percent. The ability to capture misclassifications in a satisfying way is what is lacking in the accuracy metric.

In a multi-class setting, exact matching does not distinguish between partially correct and completely incorrect labeling. The denominator, $P+N$ which is equivalent to number of samples (N) is constant. It is thus sensitive to only how many labels correctly identified with respect to N

$$F1 = \frac{1}{n} \sum_{i=1}^n \frac{2|T_i \cap P_i|}{|P_i| + |T_i|}$$

The harmonic mean of precision $Precision = \frac{1}{n} \sum_{i=1}^n \frac{|T_i \cap P_i|}{|T_i|}$ and recall

$$Recall = \frac{1}{n} \sum_{i=1}^n \frac{|T_i \cap P_i|}{|P_i|}$$

$$J_{acc} = \frac{1}{n} \sum_{i=1}^n \frac{|T_i \cap P_i|}{|T_i \cup P_i|}$$

which can be rewritten as

$$J_{acc} = \frac{1}{n} \sum_{i=1}^n \frac{|T_i \cap P_i|}{|P_i| + |T_i| - |T_i \cap P_i|}$$

$$J_{acc} \leq F1$$

Jaccard metric penalize bad classification more than the F score ”<https://stats.stackexchange.com/questions/27dice-score-vs-iou>”

2.2 Task B

GT Class	#	Freq
B	4	0.5
TD	4	0.5
all other classes($B^*C^*D^*T^*$)\((B, TB)	0	0

Evaluation of class B

$$|T_b| = |4B| = 4$$

$$|P_b| = |2B, 2T, 2D| = 6$$

$$J_{acc_b} = \frac{|4B \cap (2B, 2T, 2D)|}{|P_b| + |T_b| - |(4B \cap 2B, 2T, 2D)|}$$

$$J_{acc_b} = \frac{|2B|}{4 + 6 - |2B|}$$

$$J_{acc_b} = \frac{2}{4 + 6 - 2} = \frac{1}{4}$$

Following the definition :

$$Precision_b = \frac{2}{6} = \frac{1}{3}$$

$$Recall_b = \frac{2}{4} = \frac{1}{2}$$

$$F1 = \frac{2(Precision \times Recall)}{Precision + Recall} = \frac{2}{5}$$

Evaluation of class TD

$$|T_{td}| = |4T, 4D| = 8, |P_{td}| = |3T, 2C, B, D| = 7$$

$$J_{acc_b} = \frac{|(4T, 4D) \cap (3T, 2C, B, D)|}{|P_{td}| + |T_{td}| - |(4T, 4D) \cap (3T, 2C, B, D)|}$$

$$J_{acc_{td}} = \frac{|(3T, D)|}{8 + 7 - |(3T, D)|}$$

$$Jacc_{td} = \frac{4}{8+7-4} = \frac{4}{11}$$

$$Precision_{td} = \frac{4}{7}$$

$$Recall_{td} = \frac{4}{8} = \frac{1}{2}$$

$$F1 = \frac{2(Precision \times Recall)}{Precision + Recall} = \frac{8}{15}$$

Mean Computations

class balance:

$$Jacc_{cb} = \frac{1}{n} \sum_{i=1}^n \frac{|T_i \cap P_i|}{|T_i \cup P_i|} = \frac{1}{2} \left(\frac{1}{4} + \frac{4}{11} \right) = 0.3068$$

$$Precision_{cb} = \frac{1}{n} \sum_{i=1}^n \frac{|T_i \cap P_i|}{|T_i|} = \frac{1}{2} \left(\frac{1}{3} + \frac{4}{7} \right) = 0.4523809$$

$$Recall_{cb} = \frac{1}{n} \sum_{i=1}^n \frac{|T_i \cap P_i|}{|P_i|} = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = 0.5$$

$$F1_{cb} = \frac{1}{n} \sum_{i=1}^n \frac{2|T_i \cap P_i|}{|P_i| + |T_i|} = \frac{1}{2} \left(\frac{2}{5} + \frac{8}{15} \right) = 0.4\bar{6}$$

class frequency:

exact match:

$$ExMatch = \frac{\#correctInstances}{\#instances} = \frac{2}{8} = 0.25$$

Questions

1. Intuitively the Exact Match would be the strictest metric possible. However, it might not be the lowest number: how come?

2. Can you compute the global accuracy on this example? Justify your answer.

3. What is the main issue going from multi-class to multi-label setting?