Number Theory (Interesting Facts and Algorithms)

Questions based on various concepts of number theory and different types of number are quite frequently asked in programming contests. In this article, we discuss some famous facts and algorithms:

Interesting Facts :

1. All 4 digit palindromic numbers are divisible by 11.
2. If we repeat a three-digit number twice, to form a six-digit number. The result will will be divisible by 7, 11 and 13, and dividing by all three will give your original three-digit number.
3. A number of form 2^N has exactly N+1 divisors. For example 4 has 3 divisors, 1, 2 and 4.
4. To calculate sum of factors of a number, we can find the number of prime factors and their exponents. Let p_1, p_2, ... p_k be prime factors of n. Let a_1, a_2, .. a_k be highest powers of p_1, p_2, .. p_k respectively that divide n, i.e., we can write n as **$n = (p_1^{a_1})*(p_2^{a_2})* \ldots (p_k^{a_k})$**.

$$\text{Sum of divisors} = (1 + p_1 + p_1^2 \ldots p_1^{a_1}) * (1 + p_2 + p_2^2 \ldots p_2^{a_2}) * \ldots\ldots (1 + p_k + p_k^2 \ldots p_k^{a_k})$$

   We can notice that individual terms of above formula are Geometric Progressions (GP). We can rewrite the formula as.

$$\text{Sum of divisors} = (p_1^{a_1+1} - 1)/(p_1 -1) * (p_2^{a_2+1} - 1)/(p_2 -1) * \ldots\ldots (p_k^{a_k+1} - 1)/(p_k -1)$$

5. For a product of N numbers, if we have to subtract a constant K such that the product gets its maximum value, then subtract it from a largest value such that largest value-k is greater than 0. If we have to subtract a constant K such that the product gets its minimum value, then subtract it from the smallest value where smallest value-k should be greater than 0

6. Goldbech`s conjecture: Every even integer greater than 2 can be expressed as the sum of 2 primes.
7. Perfect numbers or Amicable numbers: Perfect numbers are those numbers which are equal to the sum of their proper divisors. Example: 6 = 1 + 2 + 3
8. Lychrel numbers: Are those numbers that cannot form a palindrome when repeatedly reversed and added to itself. For example 47 is not a Lychrel Number as 47 + 74 = 121
9. Lemoine's Conjecture : Any odd integer greater than 5 can be expressed as a sum of an odd prime (all primes other than 2 are odd) and an even semiprime. A semiprime number is a product of two prime numbers. This is called Lemoine's conjecture.
10. Fermat's Last Theorem : According to the theorem, no three positive integers a, b, c satisfy the equation, $a^n + b^n = c^n$ for any integer value of n greater than 2. For n = 1 and n = 2, the equation have infinitely many solutions.

Number Theory Algorithms

GCD and LCM

1. GCD and LCM
2. LCM of array
3. GCD of array
4. Basic and Extended Euclidean algorithms

Recent Articles on GCD and LCM!

Prime Factorization and Divisors :

1. Prime factors
2. Pollard's Rho Algorithm for Prime Factorization
3. Find all divisors of a natural number
4. Sum of all proper divisors of a natural number
5. Prime Factorization using Sieve O(log n) for multiple queries
6. Find politeness of a number
7. Print prime numbers in a given range using C++ STL
8. k-th prime factor of a given number
9. Smith Numbers

Recent Articles on Prime Factors!

Fibonacci Numbers:

1. Fibonacci Numbers
2. Interesting facts about Fibonacci numbers