EE 286: Digital Audio Signal Processing

Exercise 1: Short-time Fourier Transform

The validity of spectrum analysis using the Discrete Fourier Transform is based on an implicit fundamental assumption: the amplitudes, frequencies, and phases of the sinusoidal components of the analyzed signal do *not* change with time within the analysis window. Since increasing the length of the window results in better frequency resolution, we would like to use long windows. However, there are two major problems prohibiting the use of very long windows in practical applications. First, waiting to collect all necessary samples introduces long delays and requires the computation of huge DFTs. Second, the frequency content of speech, radar, sonar, and other practical signals changes with time. Thus, the length of the window should be sufficiently short to assure that the spectral content does not vary significantly within the window for practical purposes. If the spectral content changes significantly inside the analysis window, the DFT will provide erroneous frequency analysis results.

The objective of this exercise is to determine the utility of Short-time Fourier Transform (STFT).

Let's try to analyze a *chirp* signal, a linear-swept frequency signal. Suppose we generate a chirp, x, sampled at 8 kHz for 2 seconds. The frequency of the chirp is 500 Hz initially and crosses 750 Hz at 1 sec.

```
t = 0:1/8000:2; %set the time variable
x = chirp(t,500,1,750,'linear'); %generate chirp
```

Q: Given the chirp, what is the expected instantaneous frequency after 2 seconds?

Listen to the chirp.

```
soundsc(x,8000);
```

Q: What can you say about the pitch of the signal?

Now, let's try to analyze the frequency content of the signal using conventional method.

```
X=fft(x); %apply DFT on the signal
F=linspace(0,8000,length(x)); %create x-axis for plotting the spectrum
plot(F,abs(X)./length(x)*2); %plot the magnitude spectrum
```

Q:

a. Based on the plot of the spectrum, describe its symmetry. Explain the reason behind this.
b. At what range of frequencies can you observe the spectrum? Is this expected?
c. Based on your knowledge about the chirp, how would you correlate the observed spectrum?

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Now let's try to analyze the signal using STFT. The process starts by breaking a long signal into small segments and analyze each one with the DFT. Essentially, we extract a windowed segment of the signal and we evaluate the "local" spectrum. This process is also known as *short-time Fourier analysis*. The two-dimensional sequence is called a *spectrogram*. We usually a spectrogram as a grayscale or pseudocolor image where the horizontal axis represents time and the vertical axis frequency. The logarithmic scale helps us to see small amplitude components. To illustrate these ideas, let's implement this in MATLAB:

```
L = 160; %segment size in samples
step=L/2; %hop size of segments in samples, 50% overlap
NFFT=256; %number of FFT points computed per segment
Fs=8000; %sampling rate
N=length(x); K=fix((N-L+step)/step);
w=hanning(L); time=(1:L)';
N2=NFFT/2+1; S=zeros(K,N2);
for k=1:K
   xw=x(time)'.*w; %multiply each segment by Hanning window to reduce spectral leakage
   X=fft(xw,NFFT);
   X1=X(1:N2)';
   S(k,1:N2)=X1.*conj(X1);
   time=time+step;
end
S=fliplr(S)'; S=S/max(max(S));
%colormap(1-gray); %use this for grayscale
colormap(jet);
tk=(0:K-1)'*step/Fs; F=(0:NFFT/2)'*Fs/NFFT;
imagesc(tk,flipud(F),20*log10(S),[-100 0]); axis xy
xlabel('time (sec)')
ylabel('frequency (Hz)')
```

Q:

a. Based on the code, what is the segment size in msec.?
b. Based on the resulting spectrogram, correlate it with the behavior of the chirp.
c. Re-run the STFT code using a segment size of a) 80 samples and b) 320 samples. Based on the results, discuss the trade-offs of varying the frame size.
d. Summarize your insights on the utility of STFT.

Together with your answers to the questions, submit the Matlab script yoursurname_stft.m. Compress your files as a zip file then upload in UVLE. **You are on your honor that your submission is your own work.**

Reference:
Manolakis, Ingle. "Applied Digital Signal Processing", Cambridge University Press, 2011.

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