PREDICTING LANDING TIMES AND INITIAL VELOCITIES FOR A LANDER IN A TWO-BODY SYSTEM

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Parameter Study

1. Set up the model.
2. Validate the case study.
3. Gather parameter study data.
4. Share results and analysis.

Systems of Equations: Physics

Position: $r(t)$ for $r(t) = x_0$

Velocity: $\frac{dr}{dt} = v(t)$ for $v(t) = v_0$

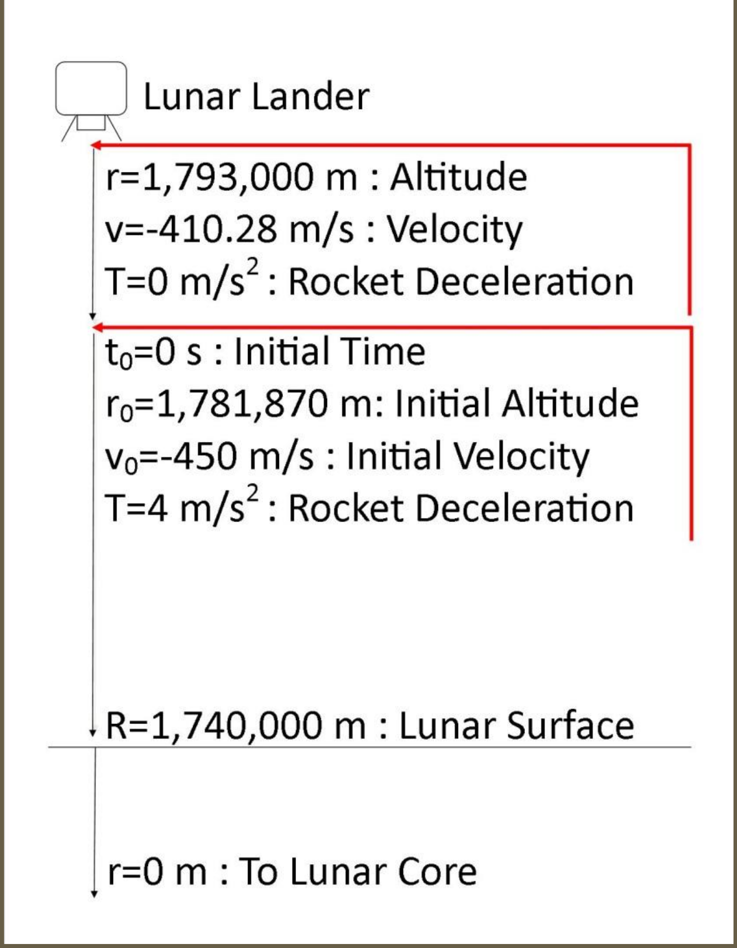
Acceleration: $\frac{d^2r}{dt^2} = \frac{dv}{dt} = a(t)$

General Euler Equation: $y_{i+1} = y_i + F(x_i, y_i)h$

Position Euler Equation: $r_1 = r_0 + v(t) \times h$

Case Study: Lunar Lander

Diagram:



The Differential Equation:

$$F = ma = \frac{GMm}{r^2} \qquad a = \frac{GM}{r^2}$$

$$a(t) = T - \frac{GM}{r^2}$$

Note:

1. The lander's altitude [r] is measured from the center of the moon.
2. To calculate initial conditions
   1. Use a chain rule substitution.
   2. Integrate using separation of variables.
   3. Evaluate at T=0 and T=4.
   4. Set the equations equal and solve.

System of Equations: Lunar Lander

Altitude: $r = x(t)$ for $x(0) = 1{,}781{,}870\ m$

Velocity: $\frac{dr_1}{dt} = v(t)$ for $v(0) = -450\ m/s$

Acceleration: $\frac{dr_2}{dt} = T - \frac{GM}{x^2}$

General Euler Equation: $y_{i+1} = y_i + F(x_i, y_i)h$

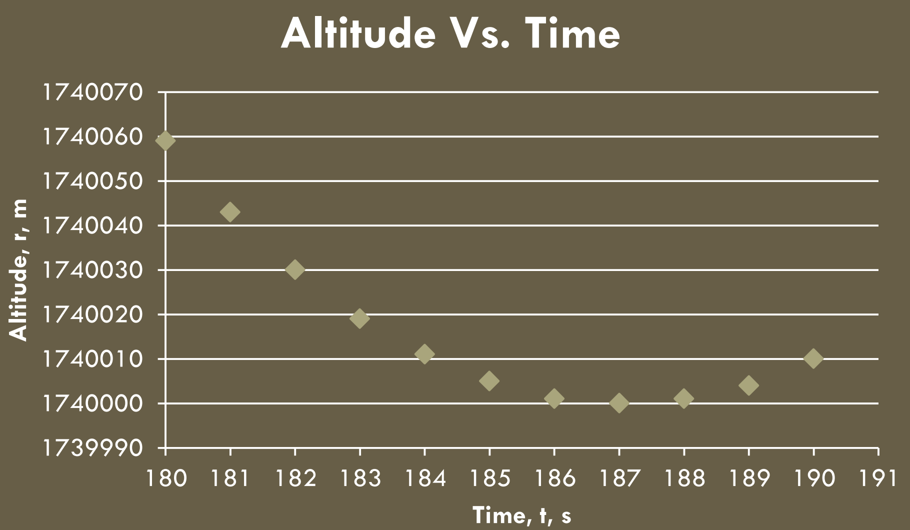
Position Euler Equation: $r_1 = 1{,}781{,}870 + (-450)h$

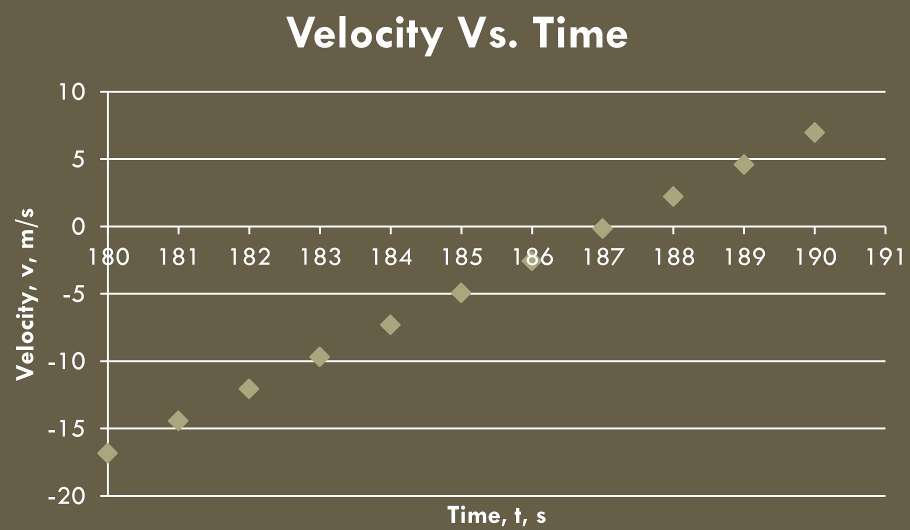
Case Study Values

- Constants
  - Deceleration from thrust: $T = 4\ m/s^2$
  - Gravitational constant : $G = 6.6726 \times 10^{-11}\ N*m^2/kg^2$
  - Lunar mass: $M = 7.35 \times 10^{22}$ kg
  - Lunar radius: $R = 1{,}740{,}000$ m
- Inputs
  - Initial time value: $x_0 = 0$ s
  - Initial altitude value: $y_0[0] = 1{,}781{,}870$ m
  - Initial velocity value: $y_0[1] = -450$ m/s
  - Upper limit of integration: $x_n = 200$ s
  - Number of steps: $n = 200$
- Calculations
  - Step size: $h = 1$ s

Case Study Results

Step	Time	Case Study Altitude	Simulation Altitude	Case Study Velocity	Simulation Velocity
180	180	1740059	1740059	-16.83	-16.83
181	181	1740044	1740043	-14.45	-14.45
182	182	1740030	1740030	-12.07	-12.07
183	183	1740019	1740019	-9.69	-9.69
184	184	1740011	1740011	-7.31	-7.31
185	185	1740005	1740005	-4.93	-4.93
186	186	1740001	1740001	-2.55	-2.55
187	187	1740000	1740000	-0.17	-0.17
188	188	1740001	1740001	2.21	2.21
189	189	1740004	1740004	4.59	4.59
190	190	1740010	1740010	6.97	6.97



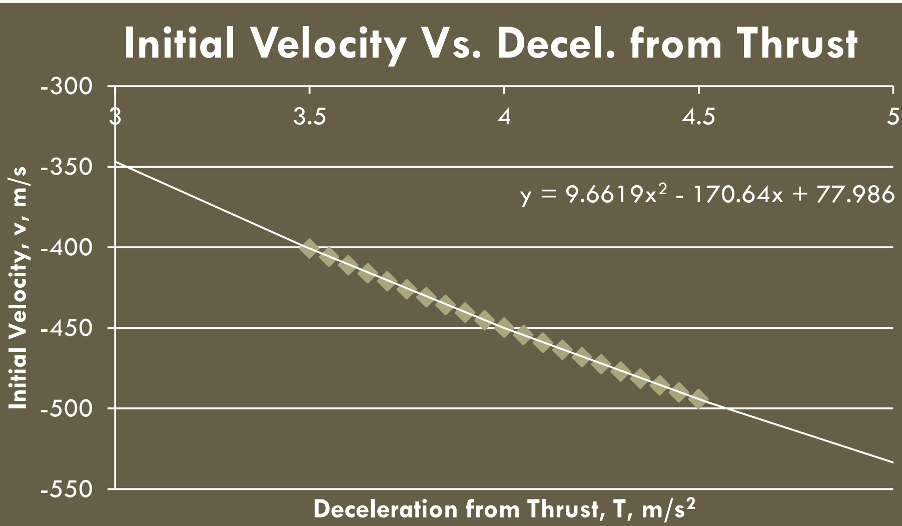


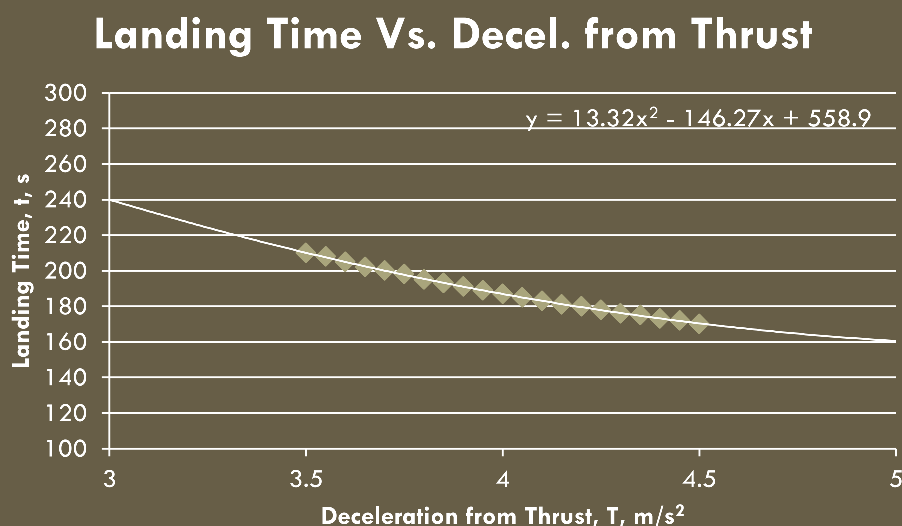
Parameter Study

- Fixed retro-rocket ignition altitude
- Independent variable:
  - Acceleration from Thrust
  - Domain $3.5 \leq T \leq 4.5$
  - Steps size: 0.05
- Dependent variables:
  - Landing Time
  - Starting velocity at fixed ignition point
  - Tolerances: $r = \pm 5m$ and $v = \pm 1m/s$

Landing Solutions

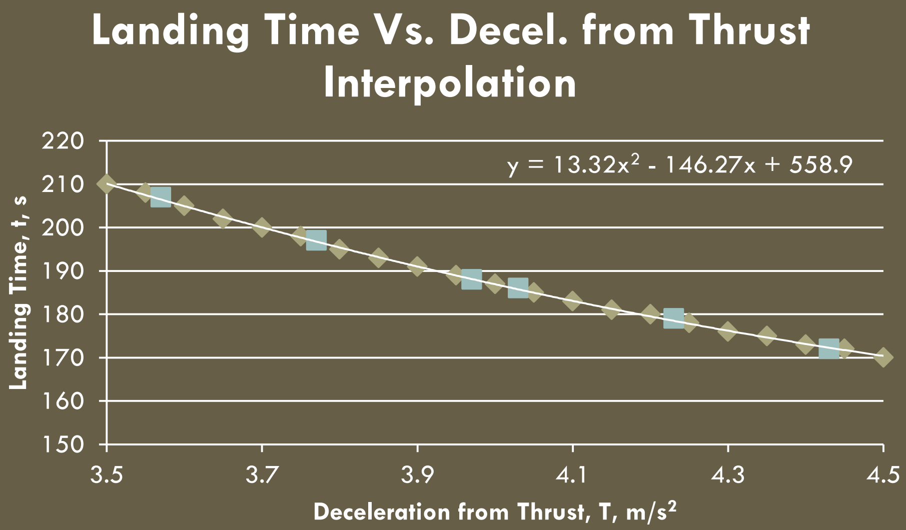
Thrust (T)	Time (t)	Start Velocity (v_0)
3.5	210	-400.8
3.55	208	-406
3.6	205	-411.12
3.65	202	-416.18
3.7	200	-421.18
3.75	198	-426.1
3.8	195	-431
3.85	193	-435.8
3.9	191	-440.6
3.95	189	-445.3
4	187	-450
4.05	185	-454.6
4.1	183	-459.2
4.15	181	-463.72
4.2	180	-468.26
4.25	178	-472.66
4.3	176	-477.1
4.35	175	-481.44
4.4	173	-485.8
4.45	172	-490.1
4.5	170	-494.31

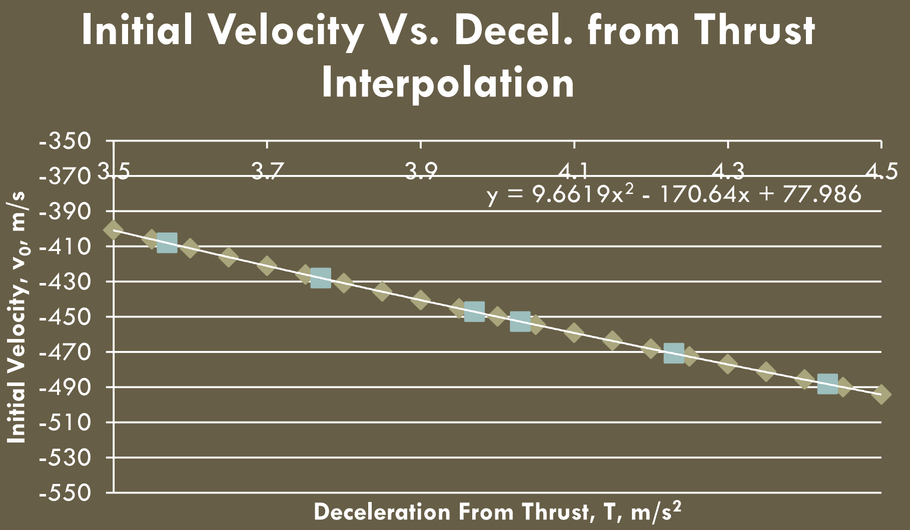




Interpolation

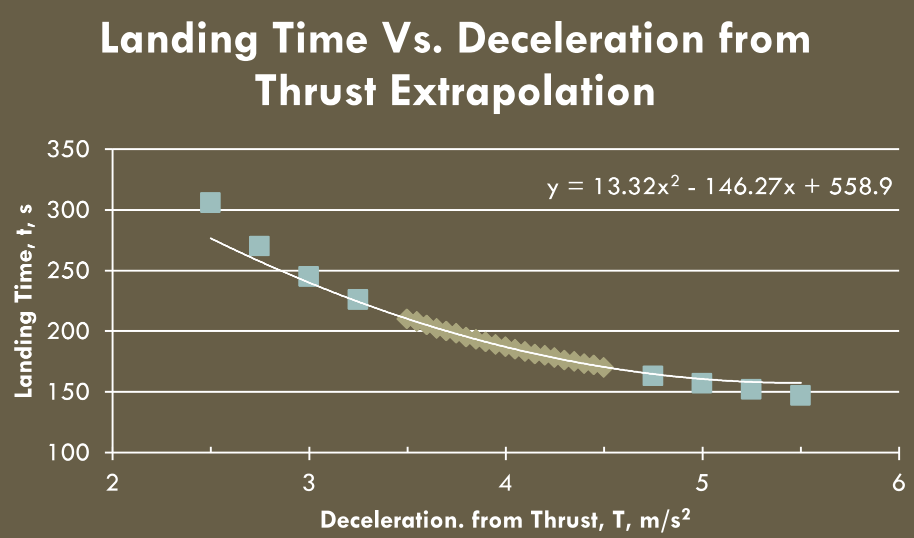
Thrust	Estimated Time	Estimated Velocity	Numerical Time	Numerical Velocity	% Diff. Time	% Diff. Velocity
3.57	206.478168	-408.0588507	207	-408.05	0.25%	0.00%
3.77	196.777928	-428.0031815	197	-428.08	0.11%	0.02%
3.97	188.143288	-447.1745603	188	-447.22	0.08%	0.01%
4.03	185.760688	-452.7752483	186	-452.76	0.13%	0.00%
4.23	178.511328	-470.9417895	179	-470.89	0.27%	0.01%
4.43	172.327568	-488.3353787	172	-488.35	0.19%	0.00%

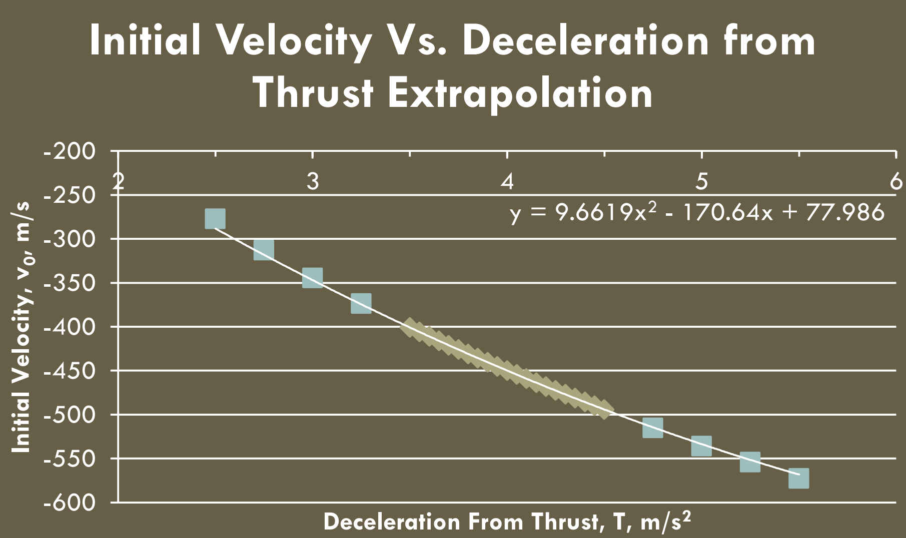




Extrapolation

Thrust	Estimated Time	Estimated Velocity	Numerical Time	Numerical Velocity	% Difference Time	% Difference Velocity
5.5	157.35	-568.26	147	-572.78	7.04%	0.79%
5.25	158.12	-551.57	152	-554.2	4.03%	0.47%
5	160.55	-533.67	157	-535.98	2.26%	0.43%
4.75	164.65	-514.56	163	-515.05	1.01%	0.10%
3.25	224.22	-374.54	226	-373.78	0.79%	0.20%
3	239.97	-346.98	245	-344.63	2.05%	0.68%
2.75	257.39	-318.21	270	-312.78	4.67%	1.74%
2.5	276.48	-288.23	306	-277.3	9.65%	3.94%





Summary

- The quadratic curve is reliable for interpolation and close extrapolations.
- Changes to best-fit curve
  - Interpolation shows no change.
  - A cubic best-fit curve increases accuracy for extrapolation within short increments of the data set.
  - Higher order polynomials decrease the reliability of extrapolations.
  - Power curve fit improves extrapolations for Landing Time vs. Deceleration from Thrust.
    - $t = 600.14x^{-0.84}$
  - Logarithmic curve fit improves extrapolations for Initial Velocity vs. Deceleration from Thrust.
    - $v = -372\ln(x) + 75.565$