Introduction

This simulation project is concerned with analyzing the structure formation in the universe. For this purpose, the \Lambda CDM model is utilized for creating a numerical approximation of our universe.

Lambda CDM

The \Lambda CDM model is considered the standard model of cosmology, as it is the simplest theory that reproduces the most important results, such as \cite{ref:lambdacdm}:

\begin{itemize}

\item the existence and structure of the CMB

\item the large-scale structure in the distribution of galaxies

\item the observed abundances of the lightest atoms

\item the accelerating expansion of the universe

\end

In this model, the energy of the universe is divided in three components: dark energy, cold dark matter, and ordinary matter.

To account for the scale of the universe, a dimensionless factor is used $a$. This factor is used together with the comoving position $\vec{x}$ to find the physical position $\vec{r}$, related by $\vec{r} = a \vec{x}$. Using the comoving position, two points will always have the same distance between them at any moment in time. The equations of motion can then be solved by taking derivatives of the physical position $\vec{r}$.

Then, using Poisson’s equation for gravity, we can relate the gravitational potential to the matter distribution.

\begin{equation}

\Lambda\_{r} \phi = 4 \pi G \rho\_{m}

\end{equation}

By also considering General Relativity, one can extend the energy distribution to consider other components of the universe. For this project, dark matter is ignored, ordinary matter has no pressure, and dark energy has a negative pressure (accounting for the accelerating expansion rate of the universe).

\begin{equation}

\Lambda\_{r} \phi = 4 \pi G \sum\_{i} \rho\_{i} (1 + 3w\_{i}) = 4 \pi G \lcurl \rho\_{m} - 2 \rho\_{\Lambda} \rcurl

\end{equation}

Problem and modelling

This project will study the structure formation in this universe. This will be done through an N-body simulation. An $N\_{p}$ number of particles are placed inside a periodic box of size $L$. These particles can represent galaxies or other structures in the universe, and by numerically solving for their positions, it is possible to study how structures are formed.

In order to have a structure formation similar to the one in our universe, we need to have the right initial conditions. For this, we can use the power spectrum. The theoretical power spectrum can be used as a reference. If we set the particles such that they are equidistant from each other, we can then add small random displacements such that the power spectrum matches the theoretical power spectrum.

Analysis of the numerical results

Growth of the linear part of the power spectrum

To confirm that the power spectrum grows proportionally with the square of the growth factor, as implied by:

\begin{equation}

P(k) = \frac{ D(a)^{2}}{ D(a\_{i})^{2}} P\_{i}(k)

\end{equation}

This was done by performing a simulation of 50 realizations, with 11 saved snapshots, and an #\Omega\_{m,0}# value of 0.32. After averaging over the realizations, the power spectrum was plotted, as shown in fig~. On the left, the calculated power spectrum from the simulation is seen. The initial and final power spectrum are shown in blue and red, and the snapshots in between are shown in green, with an increasing opacity over time. On the right, the initial theoretical spectrum can be seen, as well as the intermediate spectrums for the different values of the growth factor $D(a)$, and the final spectrum in red again. The plot of the initial and final spectrums from the simulation are overlayed to show indeed that they agree on the linear part, i.e., at the lower frequencies.

######

Include fig

#######

Conclusion