DEL OPERATIONS

$\vec{\nabla}$ of a static vector is $\emptyset$:

$$\vec{\nabla}\,[a \;\; b \;\; c] = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} [a \;\; b \;\; c] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f\vec{\nabla} \neq \vec{\nabla} f \qquad f = 2xy$$

$$f\vec{\nabla} = 2xy \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} , \quad \vec{\nabla} f = \begin{bmatrix} 2y \\ 2x \\ \emptyset \end{bmatrix}$$

$$g = x^2 + y^2$$

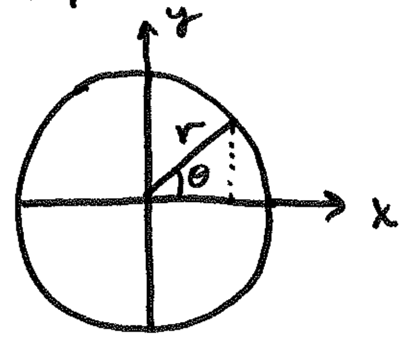
$$f\vec{\nabla} g = 2xy \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} g = 2xy \begin{bmatrix} 2x \\ 2y \\ \emptyset \end{bmatrix} =$$

$$= \begin{bmatrix} 4x^2y \\ 4xy^2 \\ \emptyset \end{bmatrix}$$

$$\vec{\nabla} g = \begin{bmatrix} 2x \\ 2y \\ \emptyset \end{bmatrix}$$

$$V(x,y) = \begin{bmatrix} \frac{y}{x^2+y^2} \\ -\frac{x}{x^2+y^2} \\ \emptyset \end{bmatrix}$$

$$x^2 + y^2 = r^2$$



$$x = r \cdot \cos\theta$$
$$y = r \cdot \sin\theta$$

$$V(x,y) = \begin{bmatrix} \frac{r \sin\theta}{r^2} \\ \frac{-r\cos\theta}{r^2} \end{bmatrix} = \begin{bmatrix} \frac{\sin\theta}{r} \\ -\frac{\cos\theta}{r} \end{bmatrix}$$

$$\vec{\nabla} V = \begin{bmatrix} \frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right) \\ \frac{\partial}{\partial y}\left(-\frac{x}{x^2+y^2}\right) \end{bmatrix} = \begin{bmatrix} \frac{-2xy}{(x^2+y^2)^2} \\ \frac{2xy}{(x^2+y^2)^2} \end{bmatrix} =$$

$$\frac{-2xy}{(x^2+y^2)^2} + \frac{2xy}{(x^2+y^2)^2} = \phi$$

$$\vec{f} : \mathbb{R}^2 \to \mathbb{R}^2 : \vec{f}(x,y) = \begin{bmatrix} xy^2 \\ e^{xy} \end{bmatrix}$$

vector function

$$\vec{\nabla} \vec{f} = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix} = \begin{bmatrix} y^2 & 2xy \\ y e^{xy} & x e^{xy} \end{bmatrix}$$

$\Downarrow$

This is a Jacobian and represents a change of basis

$g(x,y) = e^{2x} + 3yx^2 \Rightarrow$ scalar function

$h(x,y) = x^2 + 2xy + y^2$

$$h \vec{\nabla} = (x^2 + 2xy + y^2) \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix}$$

$$\vec{\nabla} h = \begin{bmatrix} 2x + 2y \\ 2x + 2y \end{bmatrix}$$

DEL ($\vec{\nabla}$) OPERATOR IS A VECTOR OF PARTIAL DERIVATIVES:

$$\vec{\nabla} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \vdots \\ \frac{\partial}{\partial x_n} \end{bmatrix}$$