# FYS3150 Project 2

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(Dated: September 22, 2021)

*https://github.com/carlfre/FYS3150-Project-2*

## PROBLEM 1

Firstly, we want to show that given $\hat{x} \equiv x/L$ and $\lambda = \frac{FL^2}{\gamma}$, this equation

$$\gamma \frac{d^2 u(x)}{dx^2} = -Fu(x),$$

can be written as

$$\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}) \tag{1}$$

Let us start by replacing $x$ with $\hat{x}$. Since $\frac{d}{dx} = \frac{d\hat{x}}{dx}\frac{d}{d\ \hat{x}} = \frac{1}{L}\frac{d}{d\hat{x}}$, we get

$$\gamma \frac{1}{L^2}\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -Fu(x)$$

If we now insert $F = \frac{\lambda\gamma}{L^2}$, we see that our final expression becomes

$$\begin{aligned} \gamma \frac{1}{L^2}\frac{d^2 u(\hat{x})}{d\hat{x}^2} &= -\frac{\lambda\gamma}{L^2}u(x) \\ \frac{d^2 u(\hat{x})}{d\hat{x}^2} &= -\lambda u(\hat{x}), \end{aligned}$$

which is the same as (**??**), as we wanted to show.

## PROBLEM 2

Next, we want to show an important property of orthogonal transformations. Let us assume that we have a set of vectors $v_i$ that is orthonormal, in other words $v_i^T \cdot v_j = \delta_{ij}$, and that $U$ is an orthogonal matrix, i.e. that $U^T = U^{-1}$. Now, we want to show that the set of vectors $w_i = Uv_i$ is also orthornormal. Saying that it is orthonormal is the same as saying that $w_i^T \cdot w_j = \delta_{ij}$. We insert our definition of $w_i$ into this, and get

$$(Uv_i)^T \cdot (Uv_j) = v_i^T U^T U v_j = v_i^T U^{-1} U v_j = v_i^T \cdot v_j$$

This, we know is equal to $\delta_{ij}$, and thus we have proved the property.
//tridiagonal.print(); //tridiagonal.print();