A Note on Portal-based Bidirectional Path Construction

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1 Introduction

This document will introduce the formulation of *portal-based bidirectional path construction,* where the path construction involves in the generation of *intermediate subpaths* generated from the portal in the scene. We will start the discussion by the general introduction of the light transport simulation and existing bidirectional approaches, followed by the extension of the approaches using intermediate subpaths.

2 Preliminaries

Path Integral. Bidirectional approaches are often discussed with path integral formulation of the light transport [5]. According to the formulation, a pixel intensity I is defined as an integral of the *measurement contribution function* $f_j(\bar{x})$ over the domain of *path space* Ω, with respect to the product area measure $d\mu$:

$$I = \int_{\Omega} f(\bar{x})\, d\mu(\bar{x})\,. \tag{1}$$

The path space Ω is defined as a space of paths of all possible length, specifically $\Omega := \cup_{k=2}^{\infty} \Omega_k$ where Ω_k is a set of paths with k vertices ($k \geq 2$). We call an element of the path space $\bar{x} \in \Omega$ as a *path* and defined as a sequence of points on the scene surface: $\bar{x} := \mathbf{x}_1 \mathbf{x}_2 \cdots \mathbf{x}_k \in \Omega$, where $\mathbf{x}_i \in \mathcal{M}$. Given a path $\bar{x}$ with vertices k, the measurement contribution function is defined as

$$f(\bar{x}) = L_e(\mathbf{x}_1, \mathbf{x}_2)\, G(\mathbf{x}_1, \mathbf{x}_2) \left[\prod_{i=2}^{k-1} f_s(\mathbf{x}_{i-1}, \mathbf{x}_i, \mathbf{x}_{i+1})\, G(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] \cdot W_e(\mathbf{x}_k, \mathbf{x}_{k-1})\,,$$

where $G(\mathbf{x},\mathbf{y})$ is the *geometry term* defined by

$$G(\mathbf{x},\mathbf{y}) = \frac{\left|\mathbf{N}_{\mathbf{x}} \cdot \omega_{\mathbf{x}\to\mathbf{y}}\right| \cdot \left|\mathbf{N}_{\mathbf{y}} \cdot \omega_{\mathbf{y}\to\mathbf{x}}\right|}{\left\|\mathbf{x}-\mathbf{y}\right\|^2} V(\mathbf{x},\mathbf{y})$$

and $V(\mathbf{x},\mathbf{y})$ is a visibility function that equals to 1 when $\mathbf{x}$ and $\mathbf{y}$ is mutually visible, otherwise 0.

Bidirectional Path Tracing. Bidirectional path tracing (BDPT) [6, 3] constructs a path by combining two *subpaths* traced from both a camera and a light source, each called *eye subpath* and *light subpath* respectively. BDPT constructs the complete paths by connecting the vertices between eye and light subpaths. This implies we have multiple sampling strategies to generate a single path. BDPT combines multiple sampling strategies by multiple importance sampling (MIS) [6].

We denote a light subpath by $\bar{y} = \mathbf{y}_1\mathbf{y}_2\cdots\mathbf{y}_s$ and an eye subpath by $\bar{z} = \mathbf{z}_1\mathbf{z}_2\cdots\mathbf{z}_t$ where s and t are the number of vertices in each subpath. And we denote the complete path generated by connecting the vertices $\mathbf{y}_s$ and $\mathbf{z}_t$ by $\bar{x}_{s,t} = \bar{y}\bar{z} = \mathbf{y}_1\mathbf{y}_2\cdots\mathbf{y}_s\mathbf{z}_t\cdots\mathbf{z}_2\mathbf{z}_1$. We note that the strategy can be uniquely indexed by the number of vertices used for each subpath, so we denote the sampling strategy to generate the path by a tuple (s,t).

We note that $s = 0$ or $t = 0$ means the path is constructed by direct hit to light or camera. For instance when $s = 0$, the light subpath is empty ($\bar{y} = \emptyset$) and the connected path is defined by $\bar{x}_{0,t} = \bar{z}$.

Each subpath is generated by combination of local sampling strategies, where a new vertex is generated by a conditional density with respect to the previous vertex except for the initial vertex. This process is denoted by

$$\mathbf{v}_1 \sim p(\cdot), \quad \mathbf{v}_i \sim p(\cdot \mid \mathbf{v}_{i-1}),$$

where $\mathbf{v}$ is either $\mathbf{z}$ or $\mathbf{y}$, and $p(\cdot)$ is a PDF with respect to area measure. Note that multiple vertices can be samples in the same time (joint importance sampling) but for simplicity we assume the vertices are sampled by the conditional densities only depends on the previous vertex.

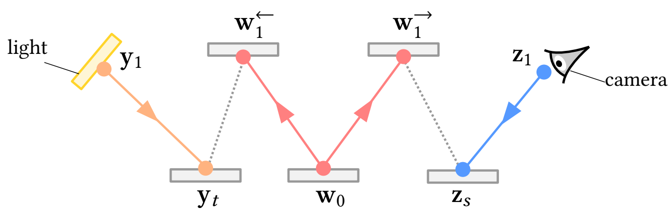

Figure 1: Path construction by intermediate subpath. A path is constructed by connecting subpaths with two connections, vertices between light (orange) and intermediate (red) subpaths, and vertices between eye (blue) and intermediate (red) subpaths. Arrows show the directions of the traced rays.

The PDF for constructing the path $\bar{x}$ by the strategy (s, t) is therefore,

$$
\begin{aligned}
p_{s,t}(\bar{x}) &= p(\bar{y})\, p(\bar{z}) \\
p(\bar{y}) &= \begin{cases} 1 & s = 0 \\ p(\mathbf{y}_1) \prod_{i=2}^{s} p(\mathbf{y}_i \mid \mathbf{y}_{i-1}) & \text{otherwise} \end{cases} \\
p(\bar{z}) &= \begin{cases} 1 & t = 0 \\ p(\mathbf{z}_1) \prod_{i=2}^{t} p(\mathbf{z}_i \mid \mathbf{z}_{i-1}) & \text{otherwise.} \end{cases}
\end{aligned}
\tag{2}
$$

A path $\bar{x}$ can be generated by arbitrary strategies (s, t). BDPT utilizes multiple importance sampling to combine the estimate from the multiple strategies. The multi-sample estimate of Eq. 1 is written as

$$
\hat{I}_{\mathrm{BDPT}} = \sum_{s,t} w_{s,t}\left(\bar{x}_{s,t}\right) \frac{f\left(\bar{x}_{s,t}\right)}{p_{s,t}\left(\bar{x}_{s,t}\right)},
$$

where $\bar{x}_{s,t} \sim p_{s,t}(\cdot)$ and $w_{s,t}\left(\bar{x}_{s,t}\right)$ being a MIS weight which satisfies the conditions in the original paper [6]. The most notable weighting scheme is *balance heuristic.*

3 Path Sampling with Intermediate Subpath

Overview. We will develop a new path construction technique based on the subpath generated from the arbitrary scene surface, called *intermediate subpath.* A complete path is constructed by two connections, one by connecting vertices between eye and intermediate subpaths, and the other by connecting vertices between light and intermediate subpaths (Fig. 1). Followed by the formulation, we will introduce portal-based bidirectional path sampling as a special case in Sec. 4.

Generating Intermediate Subpath. An intermediate subpath is generated by first sampling a point on the surface $\mathbf{w}_0 \in \mathcal{M}$, followed by tracing a sequence of points on the scene surface in *two directions*. The vertices samples according to the directions are categorized by the possible connection to the vertices either in light or eye subpaths. We denote the vertices connected to light subpath by the superscript $\leftarrow$ and the vertices connected by the eye subpath by $\rightarrow$. We denote the sequence of vertices in each direction by $\bar{w}^{\leftarrow} := \mathbf{w}_1^{\leftarrow}\mathbf{w}_2^{\leftarrow}\cdots\mathbf{w}_{s'}^{\leftarrow}$ and $\bar{w}^{\rightarrow} := \mathbf{w}_1^{\rightarrow}\mathbf{w}_2^{\rightarrow}\cdots\mathbf{w}_{t'}^{\rightarrow}$, where s' and t' are number of extended vertices in each direction. Concatenating the vertices, an intermediate subpath $\bar{w}$ can be represented by

$$\bar{w} = \mathbf{w}_{s'}^{\leftarrow}\cdots\mathbf{w}_1^{\leftarrow}\mathbf{w}_0\mathbf{w}_1^{\rightarrow}\cdots\mathbf{w}_{t'}^{\rightarrow}.$$

Similar to the generation of light and eye subpaths, the vertices except $\mathbf{w}_0$ are sampled according to the local sampling strategy given the previous vertex. This process is summarized by

$$\mathbf{w}_0 \sim p\left(\cdots\right), \quad \mathbf{w}_i^{\leftarrow} \sim p\left(\cdot \mid \mathbf{w}_{i-1}^{\leftarrow}\right), \quad \mathbf{w}_{i'}^{\rightarrow} \sim p\left(\cdot \mid \mathbf{w}_{i'-1}^{\rightarrow}\right),$$

where $i = 1,\ldots,s'$, $i' = 1,\ldots,t'$, and $\mathbf{w}_0 \equiv \mathbf{w}_0^{\leftarrow} \equiv \mathbf{w}_0^{\rightarrow}$. Therefore, the joint PDF for the intermediate path can be written as

$$\begin{aligned} p\left(\bar{w}\right) &= p\left(\mathbf{w}_0\right) p\left(\mathbf{w}_1^{\leftarrow}\cdots\mathbf{w}_{s'}^{\leftarrow}\right) p\left(\mathbf{w}_1^{\rightarrow}\cdots\mathbf{w}_{t'}^{\rightarrow}\right) \\ &= p\left(\mathbf{w}_0\right) \cdot \left[\prod_{i=1}^{s'} p\left(\mathbf{w}_i^{\leftarrow} \mid \mathbf{w}_{i-1}^{\leftarrow}\right)\right] \cdot \left[\prod_{i=1}^{t'} p\left(\mathbf{w}_i^{\rightarrow} \mid \mathbf{w}_{i-1}^{\rightarrow}\right)\right]. \end{aligned}$$

Connecting Subpaths. Similar to BDPT, a complete path is constructed by connecting vertices in subpaths, but with two vertex connections. We use the same notations for light/eye subpaths as in Sec. 2. The first connection is the vertices between light and intermediate subpaths. We denote the connecting vertices by $\mathbf{y}_s$ and $\mathbf{w}_{s'}^{\leftarrow}$. Similarly, the second connection is done between the vertices in eye and intermediate subpaths, denoted by $\mathbf{z}_t$ and $\mathbf{w}_{t'}^{\rightarrow}$. Therefore, the connected path can be represented by

$$\bar{x} = \bar{y}\bar{w}\bar{z} = \mathbf{y}_1\cdots\mathbf{y}_s\mathbf{w}_{s'}^{\leftarrow}\cdots\mathbf{w}_1^{\leftarrow}\mathbf{w}_0\mathbf{w}_1^{\rightarrow}\cdots\mathbf{w}_{t'}^{\rightarrow}\mathbf{z}_t\cdots\mathbf{z}_1, \tag{3}$$

which generate a path with $s + s' + t' + t + 1$ vertices. The path sampling strategy involving the intermediate subpath can be indexed by 4-tuple (s, s', t', t).

We note that the connection strategy can be naturally defined for the cases when either of s, s', t', t is zero, similar to the connection in BDPT. For instance, when $s = 0$

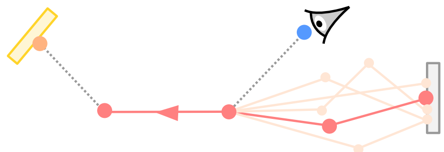

Figure 2: Empirical sampling strategy reusing a part of intermediate path (red). In this case, the computation of the path PDF is not feasible because the part of the intermediate subpath might be taken from the arbitrary paths (light orange). We circumvent this problem by considering the sampling strategy so that the intermediate subpath always contains $\mathbf{w}_0$.

and $s' > 0$, the path is constructed by the direct hit to the light by the intermediate subpath, that is, $\bar{x} = \bar{w}\bar{z}$.

We note that the path sampled by the strategy with intermediate path can also be sample by the strategies by BDPT described in Sec. 2, which implies we can combine both strategies via MIS. To combine both cases, we augment the index with the number of connections $c \in \{1, 2\}$, resulting 5-tuple index $\mathbf{j} = (c, s, s', t', t) \in \mathcal{J}$ where $\mathcal{J}$ is an indexed family of the valid strategies. Then the path connection strategy can be defined for both cases:

$$\bar{x} = \begin{cases} \bar{y}\bar{z} & c = 1 \\ \bar{y}\bar{w}\bar{z} & c = 2. \end{cases}$$

Therefore the PDF for constructing a path $\bar{x}$ by the strategy $\mathbf{j}$ can be written as

$$p_{\mathrm{j}}(\bar{x}) = \begin{cases} p(\bar{y})\, p(\bar{z}) & c = 1 \\ p(\bar{y})\, p(\bar{w})\, p(\bar{z}) & c = 2. \end{cases}$$

Combining Strategies. We can directly apply MIS for the all possible strategies $\mathbf{j}$. The multi-sample estimate can be written as

$$\hat{I}_{\mathrm{BDPT-IM}} = \sum_{\mathrm{j} \in \mathcal{J}} w_{\mathrm{j}}(\bar{x}_{\mathrm{j}}) \frac{f(\bar{x}_{\mathrm{j}})}{p_{\mathrm{j}}(\bar{x}_{\mathrm{j}})}, \tag{4}$$

where $\bar{x}_{\mathrm{j}} \sim p_{\mathrm{j}}(\cdot)$.

Discussion. Our path sampling strategy involving in intermediate subpath uses two pre-defined categories of vertices to be connected either to the vertices in light or eye subpaths. In other words, the intermediate subpath is always selected so that it contains the vertex $\mathbf{w}_0$. This design choice is intentional, since otherwise the sampling strategy cannot be uniquely identified.

For instance, we consider an empirical sampling strategy reusing a part of intermediate subpath (Fig. 2), by connecting the vertices $\mathbf{y}_s$ and $\mathbf{w}^{\leftarrow}_{s_1}$, and the vertices $\mathbf{y}_s$ and $\mathbf{w}^{\leftarrow}_{s_2}$ where $s_1 > s_2 > 0$. In this case, the local vertex PDF $p\left(\mathbf{w}^{\leftarrow}_{s_2}\right)$ is in fact a marginalized PDF, because the vertex can be samples from arbitrary paths with the number of vertices s_2 from $\mathbf{w}_0$. This strategy is thus not feasible to be estimated.

We also note that one of the parts of intermediate subpath in each direction can be sampled from the conditional distribution of the other part, while in the above formulation we assumed the two distributions are independent. This is analogous to the fact that the light subpath generation can actually be dependent on the already-sampled eye subpath (e.g., next event estimation). We will use this fact in the next section to design the path sampling technique using a portal.

4 Portal-based Bidirectional Path Sampling

Overview. In this section, we will develop a bidirectional path sampling technique based on *portal.* A portal is a virtual object in the scene that helps to improve the efficiency of sampling, typically involving the situation where the blocking geometries are the cause of inefficiency. The portal is mainly used in the context of path tracing, where the information of portals are used to construct the local directional sampling distribution [1]. We will extend the idea, by redefining the concept in the context of bidirectional path sampling. Our formulation is based on the path sampling with intermediate subpath described in Sec. 3.

The idea is to consider a portal as a part of scene surface $\mathcal{M}$ with *pass-through* material, where the incoming ray doesn't change its direction after the interaction on the portal. This idea makes it possible to handle the portal in the context bidirectional path sampling with intermediate subpath, by constructing a position sampling distribution on the portal.

Representation of Portals. We consider a portal as a part of scene surface $\mathcal{M}$. We denote a set of points on the portal by $\mathcal{M}_p \subset \mathcal{M}$. We can use any representation of the surface, such as triangle mesh or parametric surface, as long as we can construct the position sampling distribution on the surface $\mathcal{M}_p$. This representation naturally describes how we handle the multiple portals.

A point on the portal is in a vacuum thus it involves in no scattering event. This scattering event can be described by assigning the *pass-through* material on the portal surfaces, which doesn't change the incoming ray direction. A pass-through material

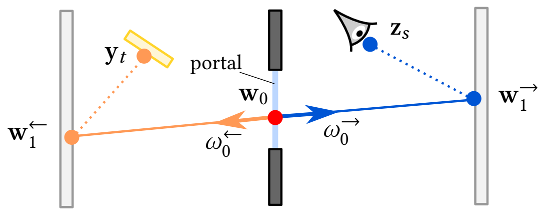

Figure 3: Generation of intermediate subpath from a portal. We consider a portal as scene surface with pass-through material. A vertex $\mathbf{w}_0$ is sampled from the surface on the portal. A part of subpath $\bar{w}^{\rightarrow}$ is initially generated by tracing rays from $\mathbf{w}_0$. The initial direction of the opposite part of the subpath $\bar{w}^{\leftarrow}$ starts from the direction $\omega_0^{\leftarrow}$, which is aligned to the initial direction $\omega_0^{\rightarrow}$ of a part of subpath $\bar{w}^{\rightarrow}$.

can be modeled by the BSDF:

$$f_s(\mathbf{x}, \omega_o, \omega_i) = \delta_\sigma(H(\omega_0, \omega_i)), \tag{5}$$

where $\int_\Omega \delta_\sigma(H(\omega_o, \omega))\, d\sigma^\perp(\omega) = 1$ and $H(\omega, \omega') = \omega + \omega'$ is the unnormalized half vector. Note that the idea behind this scattering is conceptually analogous to null scattering discussed in the rendering of heterogeneous volume [4]. We note that importance sampling of the BSDF corresponds to the deterministic operation to change the incoming direction: $\omega_o = -\omega_i$.

Intermediate Subpath From Portal. Since we considered the portal as a part of the scene surface, we can consider the subpath sampled from the portal as an intermediate subpath. Here we will describe the sampling process (Fig. 3).

First we sample a point on the portal $\mathbf{w}_0 \sim p_A(\cdot)$ according to the sampling distribution constructed on the portal surface $\mathcal{M}_P$. Next we sample a direction $\omega_0^{\rightarrow} \sim p_{\sigma^\perp}(\cdot \mid \mathbf{w}_0)$ from the point $\mathbf{w}_0$, then finds the intersected surface $\mathbf{w}_1^{\rightarrow}$ in the direction of the ray $(\mathbf{w}_0, \omega_0^{\rightarrow})$. The following process is similar to the sampling of light/eye subpaths and we have a part of the intermediate subpath $\bar{w}^{\rightarrow}$. Note that you can arbitrary choose the initial directional sampling distribution with PDF $p_{\sigma^\perp}(\omega_0 \mid \mathbf{w}_0)$, e.g., uniform distribution on an unit sphere around $\mathbf{w}_0$.

The initial direction for the second part of the subpath $\omega_0^{\leftarrow}$ are sampled from the delta distribution: $\omega_0^{\leftarrow} \sim \delta_\sigma\left(H\left(\cdot, \omega_0^{\rightarrow}\right)\right)$, that is, $\omega_0^{\leftarrow} = -\omega_0^{\rightarrow}$. The following process is similar to the sampling of light/eye subpaths and we have $\bar{w}^{\leftarrow}$.

Therefore, the PDF of the strategy (s', t') to generate the intermediate subpath

$\bar{w} = \bar{w}^{\leftarrow}\mathbf{w}_0\bar{w}^{\rightarrow}$ can be written as

$$\begin{aligned}
p_{s',t'}(\bar{w}) &= p(\mathbf{w}_0)\, p(\bar{w}^{\leftarrow})\, p(\bar{w}^{\rightarrow}) \\
&= p(\mathbf{w}_0) \cdot \left[\delta_\sigma \left(H\left(\omega_0^{\leftarrow}, \omega_0^{\rightarrow}\right)\right) \frac{G\left(\mathbf{w}_0, \mathbf{w}_1^{\leftarrow}\right)}{\left|\mathbf{N}_{\mathbf{w}_0} \cdot \omega_0^{\leftarrow}\right|} \prod_{i=2}^{s'} p\left(\mathbf{w}_i^{\leftarrow} \mid \mathbf{w}_{i-1}^{\leftarrow}\right) \right] \\
&\qquad \cdot \left[p_{\sigma^{\perp}}\left(\omega_0 \mid \mathbf{w}_0\right) G\left(\mathbf{w}_0, \mathbf{w}_1^{\rightarrow}\right) \prod_{i=1}^{t'} p\left(\mathbf{w}_i^{\rightarrow} \mid \mathbf{w}_{i-1}^{\rightarrow}\right) \right],
\end{aligned}$$

where the terms $G\left(\mathbf{w}_0, \mathbf{w}_1^{\leftarrow}\right) / \left|\mathbf{N}_{\mathbf{w}_0} \cdot \omega_0^{\leftarrow}\right|$ and $G\left(\mathbf{w}_0, \mathbf{w}_1^{\rightarrow}\right)$ are necessary to convert the measure of the PDFs to area measure.

Since $\mathbf{w}_1^{\leftarrow}$ is sampled from delta distribution, the PDF contains delta function. This term will be canceled out when we evaluate the path contribution $f(\bar{x}) / p(\bar{x})$ since the numerator contain the same delta function (Eq. 5).

Combining Strategies. We can apply MIS to the paths constructed from the portal (Eq. 4). We note that MIS weight computation needs to care about the possibility that a path can contain multiple points on the portal, even if the path is constructed without intermediate path. Also we need to be careful that the interaction involving in delta function cannot be sampled with other directional sampling strategies. Thus we need to exclude the strategy $\mathbf{j}'$ from the computation of MIS weight if the path $\bar{x}$ is sampled by the strategy $\mathbf{j} \neq \mathbf{j}'$ cannot be sampled by the strategy $\mathbf{j}'$. This means in MIS computation the position of the vertex $\mathbf{x} \in \mathcal{M}_P$ in a path can be fixed, since the path cannot be sampled if the position of the vertices on portal is different.

Known Issue. Since we modeled a portal as virtual scene surface with pass-through material, the aforementioned strategy based on intermediate subpath cannot sample the sequence of consecutive vertices $\mathbf{v}_i\mathbf{v}_{i+1}\mathbf{v}_{i+2}$ where $\mathbf{v}_i, \mathbf{v}_{i+2} \in \mathcal{M}_P$ and $\mathbf{v}_{i+1} \notin \mathcal{M}_P$. This is because we cannot connect vertices $\mathbf{v}_i\mathbf{v}_{i+1}$ or $\mathbf{v}_{i+1}\mathbf{v}_{i+2}$ since one of two vertices contains delta component. This issue has the same nature to well-known problem that specular-diffuse-specular vertices cannot be sampled by vertex connection. Presumably, this issue can be fixed by defining extended connection though portal vertices employing the similar idea to manifold next event estimation [2].

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