Crypto-Portfolio Optimization

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Summary

How to scatter a capital amongst diverse assets ?

Specific problem

Target portfolio :

- **Long positions only**
- **Low risk** (volatility $< 25\,\%$)
- **Diversification** $> 70\,\%$
- **Transaction fees** $< 0{,}55\,\%$

Available assets :

- 20 Crypto currencies
- Treasury bond (risk free, $5\,\%$)

Portfolio theory

Portfolio theory

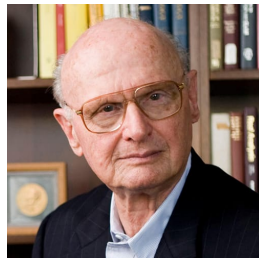**Markowitz Portfolio Theory (1950s)**

$$\min_{w} \ w^\top \Sigma w \quad \text{s.t. } w^\top m = r, \ w^\top 1 = 1$$

- w : weights

- Σ : covariance matrix

- m : returns

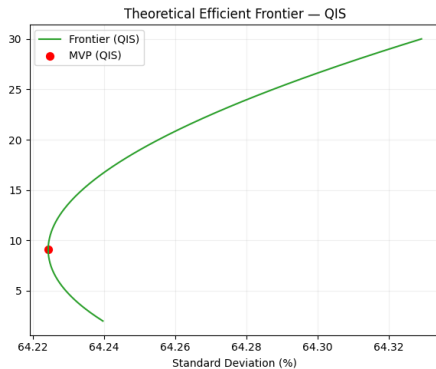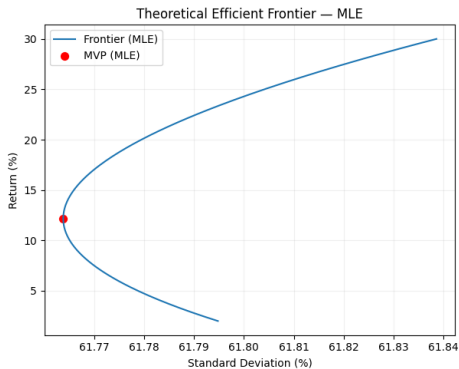- **Solution :** $w^* = \lambda \Sigma^{-1} 1_N + \gamma \Sigma^{-1} m$

Limitations :

- Σ and m unstable $\rightarrow$ assume constant over a short enough period

- Doesn't consider the specific constraints

- Correlations estimation

Harry Markowitz
Nobel 1990

Portfolio theory

Our solutions

Weights optimization : Markowitz boosted

Objective : Optimize a long-only portfolio by balancing return and risk, with an explicit diversification constraint.

Mathematical formulation :

$$\min_{w} \ w^{\top}\Sigma w \ - \ \gamma \cdot w^{\top}\mu \ \text{ subject to : } \sum_{i=1}^{n} w_i = 1, \ w_i \geq 0, \ D(w) \geq d_{\min}$$

Diversification measure $D(w)$:

$$D(w) = \frac{1}{n \cdot H(w)}, \quad H(w) = \frac{\sum_{i=1}^{n} w_i^2}{\left(\sum_{i=1}^{n} w_i\right)^2}$$

Interpretation :

- Σ : Covariance matrix estimated from rolling windows
- μ : Expected returns
- γ : Risk–return trade-off coefficient
- $D(w)$: Diversification index in $[0, 1]$

Covariance estimation - Analytic methods

MLE :

- Maximum Likelihood Estimator
- Classic and basic method
- Conclusive results for these specific data

QIS :

- State-of-the-art numerical method
- However, suffers from many heavy hypotheses
- Unconclusive for these specific data

Parameters seeking

- **Gamma**
  Higher Gamma :
  higher return, higher vol

- **Window size**
  Number of past days used to
  estimate the covariance matrix

- **Horizon**
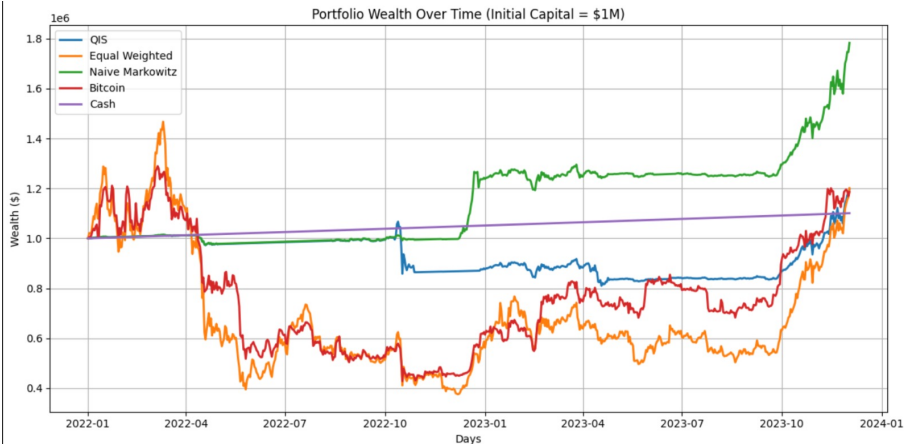 number of future days
  before rebalancing

- **Result**
  Gamma = 0.1
  Window size = 100 days
  Horizon = 20 days

Indicators	Value
Sharpe Ratio	1.41
Annualized Volatility	0.253
Annualized Return	0.409

Table 1 – Result after training

Results

Backtest - Wealth

Portfolio Wealth Over Time (Initial Capital = $1M)

Backtest - Volatility

Rolling Annualized Volatility (100-day window)

Rolling Sharpe Ratio (100-day window)
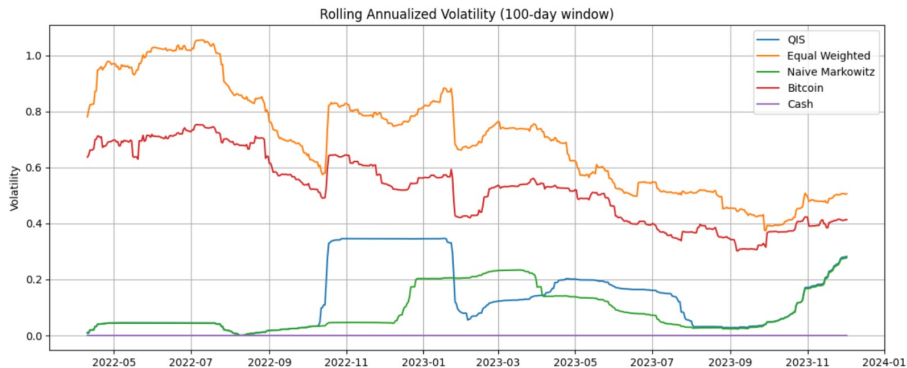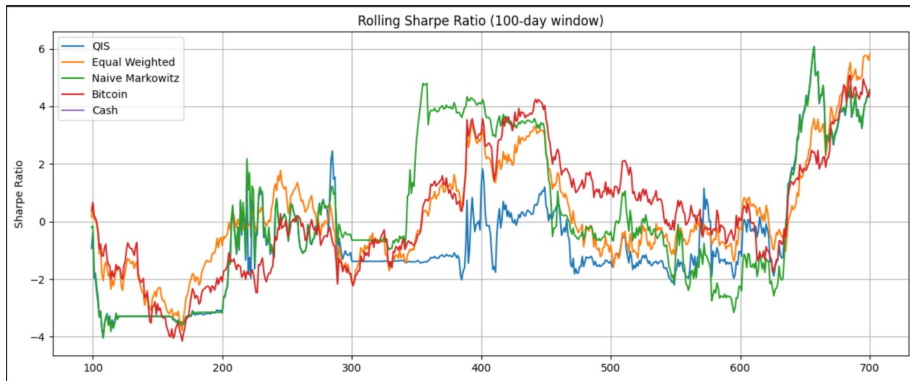
Backtest - Equally Weighted

Backtest - Markowitz Boosted
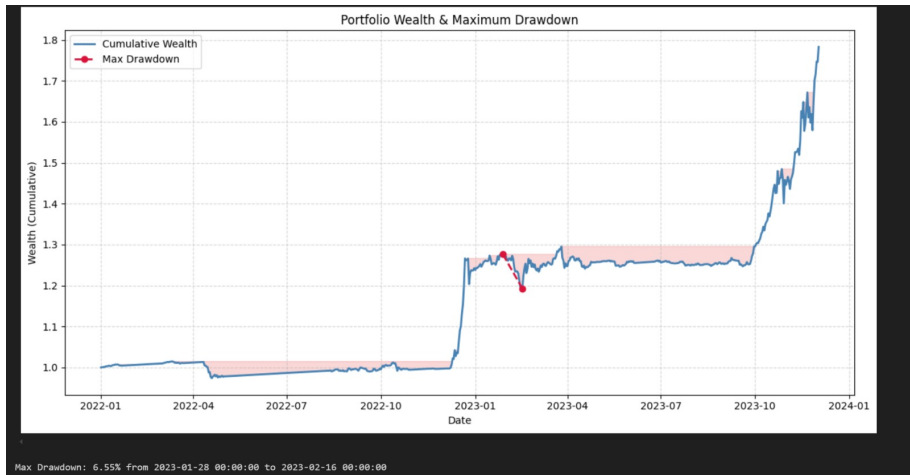
|	Sharpe Ratio	Annualized Volatility	Annualized Return	wealth
Performance for QIS	0.329446	0.187985	0.103113	1.198034e+06
Performance for equal_weighted	0.429469	0.723191	0.104551	1.200795e+06
Performance for Sample Covariance	1.812623	0.144294	0.407655	1.782921e+06
Performance for bitcoin	0.345456	0.548695	0.096577	1.185480e+06

Total transaction fees paid : $4,094.80
This represents 0.41% of the initial capital ($1,000,000.00)
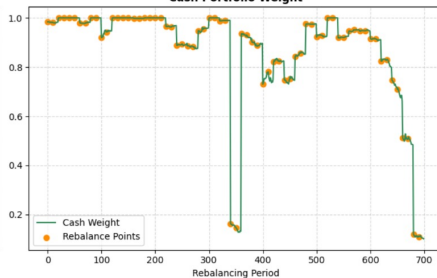
Backtest - Diversification

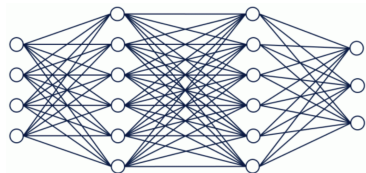Asset Allocation Over Time for Markowitz boost (Separate Scales)
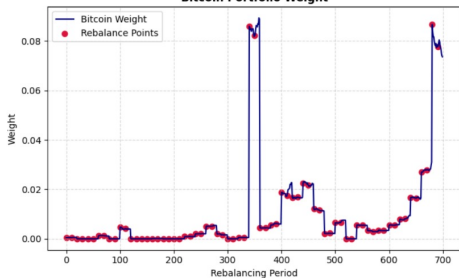
Another method : Neural Network

Machine Learning

- The covariance matrix's evolution dynamics are complex
- Neural networks are able to capture such dynamics

$\rightarrow$ **Estimation/prediction of Σ via a relevantly built neural network**

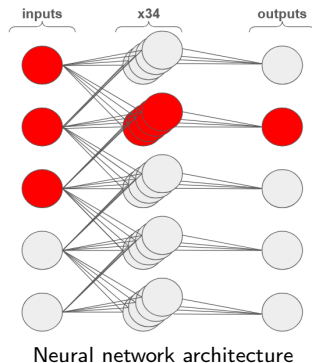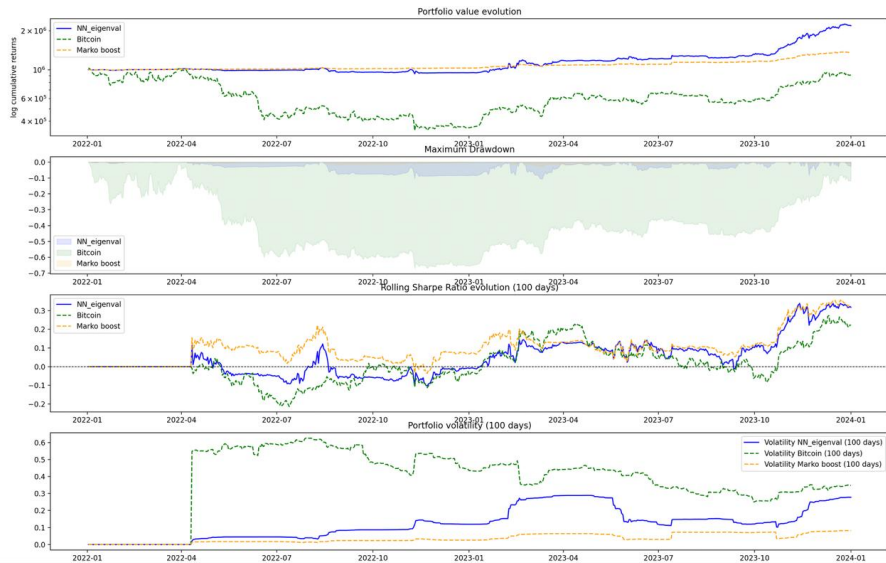**What architecture ?**

- **Input** : past eigenvalues
  batch size : 32

- **Convolution layer**
  Activation function : leaky relu
  kernel size : 3

- **Dense layer**
  Activation function : softplus

$$\hat{\lambda}_i = \log(1 + e^{x_i})$$
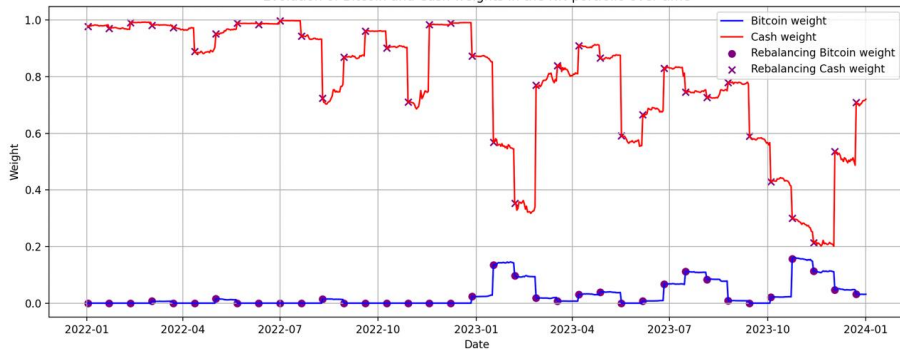
- **Output** : predicted eigenvalues

Neural network architecture

Backtest - Neural Network

Backtest - Neural Network

Evolution of Bitcoin and Cash weights in the NN portfolio over time

Conclusion

Performance Summary

Volatility
$< \mathbf{25\%}$ Achieved : 14%

Cost Fee
$< \mathbf{0.55\%}$ Achieved : 0.41%

Diversification
$> \mathbf{70\%}$ Achieved

Positioning
Long Only   Achieved

> All constraints were fully met.
>
> Our portfolio achieved a
> **Sharpe ratio of 1.8**,
> despite a predominantly bearish
> cryptocurrency market.

Thank you for your attention