Enforce and Inform: the Power of Collective Action *

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Abstract

This paper introduces a model of informal collective action. Each citizen decides if to take action—join a protest, sign a petition, speak out in favor of reform in a citizen advisory committee. The citizens' actions can signal private information to a policy-maker who decides over a one-dimensional policy. The policy-maker is allowed to be biased towards high or low policies. There exist equilibria with welfare-efficient policy-choices, independent of the bias, if and only the citizens have "skin in the game". That is, if collective action can be directly consequential for policy choice, e.g., by enforcing a minimal level of reform. Our result bears relevance for the design of participatory democratic mechanisms, such as citizen advisory committees, and provides an informational argument for the delegate model of representation of Edmund Burke.

This paper proposes and analyzes a model of informal collective action of citizens. Applications include protests, petitions, and citizen advisory committees. Previous academic and public debate has discussed two key functions of informal collective action. This paper offers the first integrated model in which both are present.

The first function of collective action is that it may contrain the set of feasible choices of a policy-maker. To develop concepts most clearly, this constraining

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effect of collective action is exogenous: if a *critical mass* of citizens becomes active, policy-making will be constrained. Such contraints may arise due to media coverage and the implied political attention. Similarly, policy makers might be forced into action when some collective protest becomes persistent in social networks.[1] Network and sociological theory suggests that such persistence realizes whenever the level of activity passes some minimum level so that it sustains itself (see, e.g., Schelling, 1978; Granovetter, 1978, and the broader literature on networks in economics and statistics).

The idea that collective action has direct exogenous effects on policy-making is prevalent in political science. It is a key assumption in the literature on regime change (Weingast, 1997; De Mesquita, 2010; Little *et al.*, 2015) as well as that on private politics (Baron, 2003; Baron and Diermeier, 2007).[2] This idea has also garnered substantial attention in public debates and shapes agendas of activist movements as a fundament for strategy and marketing. For example, *Extinction rebellion* is upholding the idea of a 3.5% rule (Chenoweth *et al.*, 2011), i.e., the notion that no government can withstand a challenge of 3.5% of its population without accommodating the movement.[3]

The second function of collective action is an informational one. Citizen activity may signal relevant information to policy-makers. This informational or signaling function of collective action has been analyzed in a literature following Lohmann (1993, 1994).[4]

In the model, a binary state captures if the costs of reform outweigh its benefits. The citizens and a policy-maker share a common prior about the state but disagree about the cost-benefit ratio (not its sign) in some of them. Each citizen holds a piece of private information about the necessity of reform in the form of a conditionally i.i.d. signal about the state and decides if to "take action" ($a = 1$) or not ($a = 0$). The citizens' actions may be informative for policy choice since the policy-maker observes them before deciding on a reform

[1]For example, in 2013-2014, opposition leaders posted calls for protests on social media after the Ukrainian president Viktor Yanukovych failed to sign an agreement with the EU. The protests then grew rapidly and spread widely, eventually culminating in the ouster of Yanukovych.

[2]For example, in the literature on regime change, an incumbent authority has to make costly concessions when protest is larger (Little *et al.*, 2015, see, e.g.).

[3]See `https://extinctionrebellion.uk/act-now/campaigns/project-3-5/`.

[4]See Banerjee and Somanathan (2001); Battaglini and Benabou (2003); Battaglini (2017); Ekmekci *et al.* (2022).

level $x \in [0, x_h]$. Further, the citizens' actions may enforce *some* level of reform: if sufficiently many citizens take action, the policy-maker's choice is constrained in that he cannot stick with the status quo $x = 0$ but has to choose at least $x = x_l > 0$.

As a main result, we show that there *always* exists an equilibrium in which welfare-efficient reform levels are chosen in each state if $x_l > 0$. In contrast, if collective action could not constrain the feasible choices of the policy-maker, $x_l = 0$, the citizen actions would be pure cheap talk. Then, only inefficient babbling equilibria exist, for natural combinations of parameters (Battaglini, 2017). For example, this happens when the policy-maker is sufficiently biased against reform because he holds a more conservative cost estimate. Our result highlights the efficiency of collective action as soon as the citizens' have the ability to enforce *some* policy change. The citizens have "skin in the game", and we show that this makes them take into account their private information, meaning that responsive equilibria exist. Our result suggests that the motives that drive collective action are a meaningful determinant of its consequences.

A caveat is that the citizens face a coordination problem. The welfare-efficient equilibrium is not the only responsive equilibrium. There exists another, inefficient equilibrium when the policy-maker is sufficiently biased *against* reform. In this equilibrium, the policy-maker's bias creates a counter-bias in the citizen's behaviour. The outcomes are systematically biased *towards* reform: regardless of the state, the citizens coordinate on taking high levels of collective action, making it likely that the policy-maker will be constrained to enforce some reform.

Our results also have a normative interpretation. They may inform about good design of partipatory democratic mechanisms such as citizen advisory committees, deliberative polls, formal petitions etc. Our main result suggests that such mechanisms should be equipped with some formal "bite", meaning that policy-makers commit to some direct consequences to outcomes of these mechanisms. This is in line with the recent report by OECD (2020) on "innovative citizen participation and new democratic institutions" which identifies such commitment as one of two key desiderata.[5] Full commitment in the sense of promising exact policies is oftentimes not realistic in real political situations. To this end, our results suggest that partial commitment to a limited range of possible policies may be sufficient in view of the efficiency of outcomes.

[5] The other desideratum is "representativeness".

Finally, the results contribute to the classic question, dating back to the founding fathers, of whether political representation should be organized in the form of trustees or delegates (Burke, 1774; Mill, 1947 [1861]). Our results provide an argument in favor of delegate representation: leaving the citizens with some decision power ("skin in the game") strenghtens the incentives for responsive and therefore informative citizen behaviour. There is very few formal work on the trustee-delegate trade-off. Notable exceptions are Kartik *et al.* (2017); Fox and Shotts (2009). Fox and Shotts (2009) shows that delegate representation can emerge even in situations when it is not welfare-efficient. Kartik *et al.* (2017) show that giving some intermediate level of discretion to the elected politician in a Downsian model benefits the citizens since the politician can adapt the policy to his information. Our setting is more close to the literature on the Condorcet jury theorem (Condorcet, 1785) and applies to situations in which the policy-maker's choice may benefit from the citizens' dispersed private information.[6]

The paper is structured as follows: Section 1 presents the model and some preliminary analysis. Section 2 presents and discusses the main result. Section 3 presents a formal derivation of the main result. Section 4 presents another interpretation of the model. Section 5 contains concluding remarks.

1 Model

There is a body of $N = 2n + 1$ citizens. Each citizen (he) $i = 1, \ldots, N$ decides if to take action ($a = 1$) or not ($a = 0$). A policy-maker (she) observes the citizens' actions and subsequently choose a policy x. The policy choice can range from the status quo, $x = 0$, to a maximal level of reform, $x = x_h$ if $\frac{\sum_{i=1,\ldots,N} a_i}{N} \leq t^{en}$, for some $t^{\mathrm{en}} > 0$. The policy choice is constrained to $x \in [x_l, x_h]$ if the aggregate level of action passes the threshold t, that is, when $\frac{\sum_{i=1,\ldots,N} a_i}{N} > t^{en}$. This captures that a sufficient level of collective action enforces some policy change. The number of citizens needed to take action is k^{en}, defined as the smallest integer strictly larger than t^{en}.

There are two states of the world $\omega \in \{\gamma, \beta\}$. The prior probability of γ is $\Pr(\gamma) \in (0, 1)$. The marginal benefit of reform is state-dependent. It is 1 in the

[6]See Feddersen and Pesendorfer (1997) for the most general version of the Condorcet jury theorem.

"good" state γ and 0 in the "bad" state β. Marginal cost of the citizens are $0 < c < 1$. The politician may have different cost $0 < c_p < 1$. A citizen's utility from $x \in [0, x_h]$ in ω is

$$u(\omega, x) = \begin{cases} x(1-c) & \text{if } \omega = \gamma, \\ x(-c) & \text{if } \omega = \beta. \end{cases} \tag{1}$$

The policy maker's utility is

$$u_p(\omega, x) = \begin{cases} x(1-c_p) & \text{if } \omega = \gamma, \\ x(-c_p) & \text{if } \omega = \beta. \end{cases} \tag{2}$$

A citizen with belief p regarding the likelihood of γ has the following expected utility from a policy x:

$$x(p-c). \tag{3}$$

Hence, he prefers $x = x_h$ over any other policy, including the status quo $x = 0$ if his belief exceeds his threshold of doubt,

$$p \geq c. \tag{4}$$

The policy-maker prefers higher policies if $p \geq c_p$. If the belief p falls below the *threshold of doubt* c, a citizen prefers the status quo over all other policies, and the policy-maker's preference is correspondingly.

Each citizen privately observes a binary signal $s \in \{g, b\}$ drawn independently from a commonly known distribution with $0 < \frac{\Pr(g|\gamma)}{\Pr(g|\beta)} < \frac{\Pr(b|\gamma)}{\Pr(b|\beta)} < \infty$. Signal g is indicative of γ and signal b is indicative of β. A citizen's strategy is a pair $\sigma = (\sigma(g), \sigma(b))$ where $\sigma(s)$ is the probability of taking action after s. A strategy of the policy-maker is a feasible mapping $\psi : \{1, \ldots, N\} \to \Delta([0, x_h])$ where $\psi(k)$ is the distribution over policies x when k citizens take action. We analyse the (Bayes-Nash) equilibria of the game of the citizens and the policy-maker that are symmetric with respect to the citizens. Slightly abusing standard notation, we call a strategy *responsive* if $\sigma(g) > \sigma(b)$ (whereas standard notation would just require $\sigma(g) \neq \sigma(b)$). For any responsive strategy, the expected collective action in γ is larger than in β, i.e.,

$$\mathrm{E}(\sigma(s)|\gamma) > \mathrm{E}(\sigma(s)|\beta), \tag{5}$$

where $\mathrm{E}(\sigma(s)|\omega) = \sum_{s\in\{g,b,\}} \Pr(s|\omega)\sigma(s)$. This way, taking action is a signal in favour of γ.

1.1 Policy Maker's Best Reponse

Upon observing that k' citizens take action, the policy maker makes an inference about the relative likelihood of the states,

$$\frac{\Pr(\gamma|k';\sigma;n)}{\Pr(\beta|k';\sigma;n)} = \frac{\Pr(\gamma)}{\Pr(\beta)}\frac{\binom{N}{k'}}{\binom{N}{k'}}\left(\frac{\mathrm{E}(\sigma(s)|\gamma)}{\mathrm{E}(\sigma(s)|\beta)}\right)^{k'}\left(\frac{1-\mathrm{E}(\sigma(s)|\gamma)}{1-\mathrm{E}(\sigma(s)|\beta)}\right)^{N-k'}, \tag{6}$$

given $\mathrm{E}(\sigma(s)|\beta) \in (0,1)$. Fix σ with $\sigma(g) > \sigma(b)$. If $\Pr(\gamma|k';\sigma;n) < c_p$ for all $1 \le k' \le N$, the policy-maker prefers to keep the status quo regardless of the observed actions. Then, we set $k = N$. If $\Pr(\gamma|k';\sigma;n) > c_p$ for all $1 \le k' \le N$, the policy maker prefers x_h over any other policy, including the status quo, regardless of the observed actions. Then, we set $k = 0$. In all other cases, there is a unique $0 < k < N$ such that

$$\Pr(\gamma|k;\sigma;n) < c_p \le \Pr(\gamma|k+1;\sigma;n). \tag{7}$$

Then, the policy maker prefers to keep the status when at most k citizens take action. Otherwise, she prefers x_h over any other policy, including the status quo. When $c_p = \Pr(\gamma|k+1;\sigma;n)$, the policy maker‘s best response to k^*+1 can involve mixing over different policies. Without loss of generality, we restrict to best responses in which she mixes over $x = x_h$ and the lowest feasible policy. We identify the pure strategy in which she chooses $x = x_h$ if she observes more than k but more than t citizens take action and the lowest feasible policy otherwise with k. Given our restriction, any best response is a mixture of cutoffs k and $k+1$ for some $0 \le k \le N$. In the following, we only consider such strategies and identify them with their random cutoff $\tilde{k}$. A policy-maker strategy is *responsive* if $\mathrm{supp}(\tilde{k}) \cap \{1,\ldots,N-1\} \ne \emptyset$ and a strategy profile is *responsive* if the strategies of all agents are responsive. Note that for any strategy sequence $(\sigma_n)_{n\in\mathbb{N}}$ with

$\sigma = \lim_{n\to\infty} \sigma_n$ and $\sigma(g) > \sigma(b)$, the policy-maker's best response is responsive for n large enough.

1.2 Citizens' Best Response

Citizens use their choice both to constrain the policy-maker's choice set (*enforcement motive*) as well as to transmit information to the authority to influence the policy maker's choice (*signaling motive*).

Given a responsive citizen strategy σ and a responsive strategy $\tilde{k}$ of the policy-maker, there may be two events in which a citizen's vote is pivotal for the policy outcome. For any realized strategy of the policy-maker, $\text{piv}^{\text{en}} = \emptyset$ if $k \leq k^{\text{en}}$. Otherwise, we label by piv^{en} the event in which when t of the other citizens take action and $2n - t$ do not. In this event, the policy-maker is constrained in her choice only if the citizen takes action. Moreover, this constraint is binding given $k > k^{\text{en}}$ and the policy-maker chooses $x = x_l$ if the citizen takes action but $x = 0$ otherwise. We set $\text{piv}^{\text{sg}} = \emptyset$ if $k \in \{0, 2n\}$. Otherwise, we label by piv^{sg} the event in which the realized strategy of the policy-maker has a cutoff k and k of the other citizens take action and $2n - k$ do not. In this event, the policy-maker chooses $x = x_h$ only if the citizen takes action. Otherwise, she chooses $x = 0$ if $k \leq k^{\text{en}}$ and $x = x_l$ if $k > t$. Note that $\text{piv}^{\text{sg}} \cap \text{piv}^{\text{en}} = \emptyset$.

Given a realized cutoff $k^{\text{en}} < k$, the likelihood of piv^{en} in ω is

$$\Pr\left(\text{piv}^{\text{en}} | \omega; k, \sigma, n\right) = \binom{2n}{k^{\text{en}}} \mathrm{E}(\sigma(s)|\omega)^{k^{\text{en}}} (1 - \mathrm{E}(\sigma(s)|\omega))^{2n - k^{\text{en}}}. \tag{8}$$

For any cutoff $k^{\text{en}} < k$, the likelihood ratio of being pivotal for the policy maker's choice set is

$$\frac{\Pr\left(\text{piv}^{\text{en}} | \gamma; \sigma, k, n\right)}{\Pr\left(\text{piv}^{\text{en}} | \beta; \sigma, k, n\right)} = \frac{\mathrm{E}(\sigma(s)|\gamma)^t (1 - \mathrm{E}(\sigma(s)|\gamma))}{\mathrm{E}(\sigma(s)|\beta)^n (1 - \mathrm{E}(\sigma(s)|\beta))}^{2n-t}. \tag{9}$$

The likelihood ratio of the states conditional on being pivotal for the policy maker's choice set and having observed $s \in \{g, b\}$ is

$$\frac{\Pr(\gamma | \text{piv}^{\text{en}}, s; \sigma, k, n)}{\Pr(\beta | \text{piv}^{\text{en}}, s; \sigma, k, n)} = \frac{\Pr(\gamma)}{\Pr(\beta)} \frac{\Pr(s|\gamma)}{\Pr(s|\beta)} \frac{\Pr\left(\text{piv}^{\text{en}} | k, \gamma; \sigma, n\right)}{\Pr\left(\text{piv}^{\text{en}} | k, \beta; \sigma, n\right)} \tag{10}$$

where we used the conditional independence of the citizens' signals. Given a cutoff $0 < k < N$ of the policy-maker, the likelihood ratio of being pivotal for the policy maker's choice is

$$\frac{\Pr\left(\mathrm{piv}^{\mathrm{sg}}|\gamma;\sigma,k,n\right)}{\Pr\left(\mathrm{piv}^{\mathrm{sg}}|\beta;\sigma,k,n\right)}=\frac{\binom{2n}{k}\mathrm{E}(\sigma(s)|\gamma)^{k}(1-\mathrm{E}(\sigma(s)|\gamma))^{2n-k}}{\binom{2n}{k}\mathrm{E}(\sigma(s)|\beta)^{k}(1-\mathrm{E}(\sigma(s)|\beta))^{2n-k}}. \tag{11}$$

The likelihood ratio of the states conditional on being pivotal for the policy maker's choice and having observed $s \in \{g, b\}$ is

$$\frac{\Pr(\gamma|\mathrm{piv}^{\mathrm{sg}}.s;\sigma,k,n)}{\Pr(\beta|\mathrm{piv}^{\mathrm{sg}},s;\sigma,k,n)}=\frac{\Pr(\gamma)}{\Pr(\beta)}\frac{\Pr(s|\gamma)}{\Pr(s|\beta)}\frac{\Pr\left(\mathrm{piv}^{\mathrm{sg}}|\gamma;\sigma,k,n\right)}{\Pr\left(\mathrm{piv}^{\mathrm{sg}}|\beta;\sigma,k,n\right)} \tag{12}$$

Finally, we characterise the citizen's best response. Consider any realization k of the cutoff of the policy-maker's strategy. Recall that when $k \leq k^{\mathrm{en}}$, if more than k citizens take action, the policy-maker chooses x_h and otherwise the status quo is kept. In this case, given k, the expected utility of a citizen who received $s \in \{g, b\}$ from any reform exceeds that of the status quo if

$$x_h\Big[\Pr(\gamma|\mathrm{piv}^{\mathrm{sg}};\sigma,k,n)-c_v\Big]\geq 0, \tag{13}$$

where we used (2). Observe that only the signaling motive is present. Recall that, when $k > k^{\mathrm{en}}$, if at most k^{en} citizens take action, the policy-maker chooses $x = 0$, if more than k^{en} citizens take action, depending on the vote tally, the policy-maker chooses x_h or x_l. In this case, given k, the expected utility of a citizen who received $s \in \{g, b\}$ from any reform exceeds that of the status quo if

$$\begin{aligned}&\Pr(\mathrm{piv}^{\mathrm{en}}|s;\sigma,k,n)x_l\Big[\Pr(\gamma|\mathrm{piv}^{\mathrm{en}},s;\sigma,k,n)-c_v\Big]\\&+\Pr(\mathrm{piv}^{\mathrm{sg}}|s;\sigma,k,n)(x_h-x_l)\Big[\Pr(\gamma|\mathrm{piv}^{\mathrm{sg}},s;\sigma,k,n)-c_v\Big]\geq 0;\end{aligned} \tag{14}$$

Summing over all realizations of the policy-maker's cutoff it is optimal for a citizen who received $s \in \{g, b\}$ to take action if

$$\begin{aligned}&\sum_{k\leq t}\Pr(\tilde{k}=k)\Pr\left(\mathrm{piv}^{\mathrm{sg}}|\gamma;\sigma,k,n\right)x_h\Big[\Pr(\gamma|\mathrm{piv}^{\mathrm{sg}};\sigma,k,n)-c_v\Big]\\+&\sum_{k>t}\Pr(\tilde{k}=k)\Pr(\mathrm{piv}^{\mathrm{en}}|s;\sigma,k,n)x_l\Big[\Pr(\gamma|\mathrm{piv}^{\mathrm{en}},s;\sigma,k,n)-c_v\Big]\\+&\Pr(\mathrm{piv}^{\mathrm{sg}}|s;\sigma,k,n)(x_h-x_l)\Big[\Pr(\gamma|\mathrm{piv}^{\mathrm{sg}},s;\sigma,k,n)-c_v\Big]\geq 0.\end{aligned}$$

2 Results

We characterise all responsive equilibria.[7] Section 2 reports and discusses our results and their intuition, and Section 3 contains the formal analysis. We call an equilibrium sequence $(\sigma_n)_{n\to\infty}$ *efficient* if the policy that maximizes social welfare is elected with probability converging to 1 as $n \to \infty$. Note that the efficient policy is $x = x_h$ in γ and $x = 0$ in β since the policy maker and the citizens have aligned interests under full information.

Theorem 1 *There exists an efficient equilibrium sequence.*

This result holds, regardless of the bias of the policy-maker. Even when it is arbitrarily hard to convince the policy-maker of the necessity of reform, the bias will not hinder efficient outcomes.

The presence of the enforcement motive is essential. If collective action would not have the potential to enforce some policy change by constraining the policy-maker's choice set (the case $k^{\text{en}} > N$), it would be pure cheap talk. Then, citizen behaviour is solely driven by the signaling motive. Battaglini (2017) has shown that, in this cheap talk model, no responsive equilibria exist when the policy-maker's bias is sufficiently large,

$$\frac{c_p}{1-c_p}\frac{\Pr(a|\beta)}{\Pr(a|\gamma)}\frac{\Pr(b|\gamma)}{\Pr(b|\beta)} \geq \frac{c_v}{1-c_v}, \tag{15}$$

That is, all equilibria are inefficient, given (15). The logic behind this result is as follows: Suppose there were an equilibrium in which, when a citizen takes action ($a = 1$), this transmits information about the state. When considering the event in which the transmitted information convinces the policy-maker to the stronger reform x_h, the citizen understands that the policy-maker must hold sufficient information in favor of reform (from observing the other citizen actions), given her large bias. But then, since the citizen is more easily convinced of the need of reform, conditional on this event, the citizen prefers the stronger reform regardless of her private information and best responds by taking action after all signals.

[7] Note that, if $\Pr(\gamma) < c_p$ there always exist trivial equilibria in which the policy-maker uses a strategy that is not responsive and the citizens choose a (potentially) responsive strategy σ so that $\Pr(\gamma|k;\sigma) < c_p$ for all $k \in \{0,\ldots,N\}$. Similarly, there always exist trivial equilibria in which the citizen strategies are responsive and the policy-maker's strategy is not responsive when $\Pr(\gamma) > c_p$.

This cheap talk logic fails when the citizen's actions can enforce some policy change directly. This possibility of enforcement gives the citizens "skin in the game" and will imply that, in equilibrium, citizens sometimes do not take action after private information indicating high cost of reforms. In other words, responsive equilibria exist. Particularly suprising may be that, for this logic, *some* "skin in the game" is sufficient when the body of citizens is large.

A caveat is that the welfare-efficient equilibrium is not the only responsive equilibrium. There exists another responsive but inefficient equilibrium when the policy-maker's bias is large, that is, when (15) holds.

Theorem 2 *Let* (15) *hold.*

1. *There exists an equilibrium sequence* $(\sigma_n^*, \tilde{k}_n)_{n\in\mathbb{N}}$ *in which the policy-maker's choices is constrained in all states and the policy-maker chooses the highest feasible policy in* γ *and the lowest feasible policy in* β*, as* $n \to \infty$*,*

$$\lim_{n\to\infty} \Pr(x_h|\gamma; \sigma_n^*, \tilde{k}_n n) = 1, \tag{16}$$

$$\lim_{n\to\infty} \Pr(x_l|\beta; \sigma_n^*, \tilde{k}_n n) = 1. \tag{17}$$

2. *Any sequence of responsive equilibria is either efficient or the policy-maker is constrained in all states with probability going to* 1 *as* $n \to \infty$ *(as in 1.).*

In the inefficient equilibrium, the citizen's will constrain the policy-maker's choice set and enforce some reform in *all* states, even when any type of reform is Pareto-dominated by the status quo ex-post. Theorem 2 shows that the citizens face a coordination problem when taking collective action. The reason for this coordination problem derives from a *complementarity*: when many citizens join the collective action so that, with a high likelihood, some reform is enforced no matter which state it is, the enforcement motive becomes less important for citizen behaviour and the signaling motive relatively more important. Following the cheap talk-logic laid out in the previous paragraph, the signaling motive creates incentives to take action and sustains high levels of collective action under the best response.

In Section 3, we present the equilibrium analysis and the proofs of Theorem 1 and Theorem 2.

3 Reduced Form Equilibrium Characterization

Unlike in related classical models of collective choice, in our setting, there are *multiple* pivotal events. This poses a significant technical challenge.[8]

We solve this by leveraging insights from the statistical theory of large deviations (Cramér, 1938; Sanov, 1958). The key result is a compact reduced form characterization of *all* sequences of responsive citzen strategies that are part of an equilbrium sequence in terms of the Kullback-leibler distance

$$H(t,q) = \left[q \log\left(\frac{q}{t}\right) + (1-q)\log\left(\frac{1-q}{1-t}\right) \right] \tag{18}$$

between the expected collective action $q = \mathrm{E}(a|\omega; \sigma_n)$ in the two states and the thresholds $t = t^{en}$ and $t = t^{sg} := \lim_{n\to\infty} t_n^{\mathrm{sg}}$, where $t_n^{\mathrm{sg}} = \frac{k_n}{2n+1}$ and k_n any realization of the cut-off of the policy-maker's strategy. Note that the definition of t^{sg} does not depend on the choice of k_n since the policy-maker mixes only over "adjacent" cutoffs k_n and $k_n + 1$ for some $0 \le k_n \le N-1$, in equilibrium. Thus, t^{sg} is a function of $(\sigma_n)_{n\in\mathbb{N}}$.

Proposition 1 *Let* (15) *hold. For a sequence of responsive strategies* $(\sigma_n)_{n\in\mathbb{N}}$*, there is* $\bar{n} \in \mathbb{N}$ *so that* σ_n *is part of an equilibrium* $(\sigma_n, \tilde{k}_n)_{n\in\mathbb{N}}$ *for* $n \ge \bar{n}$ *if and only if* $\lim_{n\to\infty} \mathrm{E}(\sigma_n(s)|\gamma) > t^{\mathrm{en}}$ *and*

$$\lim_{n\to\infty} \min_{\omega\in\{\alpha,\beta\}} H(t^{\mathrm{sg}}, \mathrm{E}(\sigma_n(s)|\omega) = \lim_{n\to\infty} \min_{\omega\in\{\alpha,\beta\}} H(t^{\mathrm{en}}, \mathrm{E}(\sigma_n(s)|\omega). \tag{19}$$

This characterization makes the analysis tractable. In particular, it allows us to characterize *all* equilibria.

Section 3.1 - Section 3.3 provide the main insights that lead to the reduced form characterization of Proposition 1. Based on this, Section 3.4 shows that (19) and $\lim_{n\to\infty} \mathrm{E}(\sigma_n(s)|\gamma) > t^{\mathrm{en}}$ are necessary conditions. Section 3.5 shows how the necessary conditions of Proposition 1 imply that only the two types of equilibrium sequences as in Theorem 1 and 2 are possible, given (15). Section 3.6 presents a fixed-point argument that establishes the sufficiency of the conditions.

[8]The difficulty of dealing with multiple, distinct pivotal events also arises in models of majority elections with signaling motives (e.g., Razin, 2003), with multiple issues (Ahn and Oliveros, 2012), or due to the possibility of a recount of the votes (Damiano *et al.*, 2015). However, we believe that our paper is the first to analyse *all* equilibria and provide a compact and tractable equilibrium characterization in a setting with multiple pivotal events.

3.1 The Citizen's Inference: Sanov's Theorem

The presence of multiple pivotal events changes the citizen's inference relative to classical models of collective choice. First, citizens infer whether they are pivotal for the choice set of the policy-maker ($\mathrm{piv}^{\mathrm{en}}$) or the policy-maker's choice ($\mathrm{piv}^{\mathrm{sg}}$). Second, they update their beliefs about the state. We show that for the citizen's inference the "distance" between $t \in \{t^{\mathrm{en}}, t^{\mathrm{sg}}\}$ and expected collective action $q = \mathrm{E}(\omega(s)|\omega)$ in the two states is key. The smaller the distance, the more likely a citizen is pivotal.

This intuition can be made more precise. The relevance of a pivotal event may be fathomed by considering how many actions have to be flipped on average for the pivotal event to arise. Since actions are binary, this amounts to a measure of "bits". Indeed, the proof of the following Lemma 1 shows that likelihood ratio of the pivotal events can be expressed in terms of the Kullback-Leibler distances $2nH(t, \mathrm{E}(\sigma(s)|\omega)$ between the distribution of the actions of the other $2n$ citizens in ω to the counterfactual action distribution that would arise when the others would vote take action with a likelihood of t instead of $\mathrm{E}(\sigma(s)|\omega)$, making the pivotal event related to t most likely.[9]

Lemma 1 *Take any sequence of responsive strategies $(\sigma_n)_{n\in\mathbb{N}}$ with $\sigma = \lim_{n\to\infty}\sigma_n$ and any sequence $(\tilde{k})_{n\in\mathbb{N}}$ of responsive best responses of the policy-maker.*

1. *For the inference about the pivotal events: if $H(t^{\mathrm{en}}, \mathrm{E}(\sigma(s)|\omega) > (<) H(t^{\mathrm{sg}}, \mathrm{E}(\sigma(s)|\omega)$ and $t^{en} < t^{sg} < 1$, then,*

$$\lim_{n\to\infty} \frac{\Pr(\mathrm{piv}^{\mathrm{en}}|\omega; \sigma_n, \tilde{k}_n, n)}{\Pr(\mathrm{piv}^{\mathrm{sg}}|\omega; \sigma_n, \tilde{k}_n, n)} = 0 \ \ (\infty). \tag{20}$$

   for $\omega \in \{\alpha, \beta\}$.

2. *For the inference about the state: if $H(t^{\mathrm{mot}}, \mathrm{E}(\sigma(s)|\alpha) > (<) H(t^{\mathrm{mot}}, \mathrm{E}(\sigma(s)|\beta)$, then,*

$$\lim_{n\to\infty} \frac{\Pr(\alpha|\mathrm{piv}^{\mathrm{mot}}; \sigma, \tilde{k}_n, n)}{\Pr(\beta|\mathrm{piv}^{\mathrm{mot}}; \sigma, \tilde{k}_n, n)} = 0 \ \ (\infty). \tag{21}$$

[9] The elegant idea of using a counterfactual distribution is due to Cramér (1938). The more general insight that "large deviations"—as the pivotal events in our setting—can be described by Kullback-Leibler distances is due to Sanov (1958).

for mot $\in \{\mathrm{en}, \mathrm{sg}\}$.

Proof. Consider the sequences of the binomials $X_n = \mathcal{B}(2n, t_n^{sg})$ and $Y_n = \mathcal{B}(2n, t_n^{en})$. Consider any realization $k_n < 2n+1$ from the support of the policy-maker's strategy $\tilde{k}_n$. Then,

$$\begin{aligned} \frac{\Pr(\mathrm{piv}^{\mathrm{sg}}|\alpha;\sigma_n,k_n,n)}{\Pr(X_n=(2n+1)t_n^{sg})} &= (\frac{\mathrm{E}(\sigma_n(s)|\gamma)}{t_n^{sg}})^{(2n+1)t_n^{sg}}(\frac{1-\mathrm{E}(\sigma_n(s)|\gamma)}{1-t_n^{sg}})^{(2n+1)(\frac{2n}{2n+1}-t_n^{sg})} \\ &\approx e^{-(2n+1)H(t_n^{sg},\mathrm{E}(\sigma_n(s)|\gamma)} \end{aligned} \tag{22}$$

where the factorials $\binom{2n}{(2n+1)t_n^{sg}}$ cancel out on the first line. For the second line, we used that $\frac{2n}{2n+1} \approx 1$.[10] In the same way,

$$\frac{\Pr(\mathrm{piv}^{\mathrm{en}}|\gamma;\sigma_n,k_n,n)}{\Pr(Y_n=(2n+1)t_n^{en})} \approx e^{-(2n+1)H(t_n^{en},\mathrm{E}(\sigma_n(s)|\gamma)} \tag{23}$$

We prove the first item. Combining (22) and (23),

$$\begin{aligned} &\frac{\Pr(\mathrm{piv}^{\mathrm{sg}}|\gamma;\sigma_n,k_n,n)}{\Pr(\mathrm{piv}^{\mathrm{en}}|\gamma;\sigma_n,k_n',n)} \\ \approx\ & e^{-(2n+1)\left[H(t_n^{\mathrm{sg}},\mathrm{E}(\sigma_n(s)|\gamma)-H(t_n^{\mathrm{en}},\mathrm{E}(\sigma_n(s)|\gamma)\right]}\frac{\Pr(Y_n=(2n+1)t_n^{en})}{\Pr(X_n=(2n+1)t_n^{\mathrm{sg}})}. \end{aligned} \tag{24}$$

An application of the local central limit theorem for triangular arrays of binomials (see, e.g., Davis and McDonald, 1995), $\Pr(X_n = (2n+1)t_n^{sg}) \approx \phi(0)\left[t_n^{sg}(1-t_n^{sg})\right]^{-1}$ and $\Pr(Y_n = (2n+1)t^{en}) \approx \phi(0)\left[t^{en}(1-t^{en}\right]^{-1}$ where ϕ is the density of the standard normal. The claim of the first item follows since we assumed $0 < t^{en} < t^{sg} = \lim_{n\to\infty} t_n^{sg} < 1$.

We prove the second item. Using (22) and (23) for both $\omega = \gamma$ and $\omega = \beta$

$$\frac{\Pr(\mathrm{piv}^{\mathrm{mot}}|\gamma;\sigma_n,k_n,n)}{\Pr(\mathrm{piv}^{\mathrm{en}}|\beta;\sigma_n,k_n',n)} \approx e^{-(2n+1)\left[H(t_n^{\mathrm{mot}},\mathrm{E}(\sigma_n(s)|\gamma)-H(t_n^{\mathrm{mot}},\mathrm{E}(\sigma_n(s)|\beta)\right]},$$

which implies the second item. ∎

[10]For two sequences x_n, y_n, we write $x_n \approx y_n$ if $\frac{x_n}{y_n} \to 1$ as $n \to \infty$.

3.2 The Policy Maker's Cutoff Rule

Lemma 2 *For any sequence of responsive strategies* $(\sigma_n)_{n\in\mathbb{N}}$ *with* $\sigma = \lim_{n\to\infty}\sigma_n$ *and any sequence of responsive best responses of the policy-maker,*

$$H(t^{\text{sg}}, \text{E}(\sigma(s)|\gamma)) = H(t^{\text{sg}}, \text{E}(\sigma(s)|\beta)). \tag{25}$$

Proof. Since signals are boundedly informative, the action of a single citizen is boundedly informative. Formally, $\frac{\Pr(b|\gamma)}{\Pr(b|\beta)} \le \frac{q(\gamma;\sigma)}{q(\beta;\sigma)} \le \frac{\Pr(g|\beta)}{\Pr(g|\gamma)}$ for any candidate strategy σ. As a consequence, the inference after seeing k_n supporters (where observing k_n+1 supporters would tip the policy maker's choice) leads to a belief with bounded distance to the policy maker's critical belief $c_p \in (0,1)$. Thus, This implies that the citizen's belief conditional on piv^{sg} is bounded uniformly for all n,

$$\lim_{n\to\infty}\frac{\Pr(\gamma|\text{piv}^{\text{sg}}, s;\sigma, n)}{\Pr(\beta|\text{piv}^{\text{sg}}, s;\sigma, n)} \in (0,\infty), \tag{26}$$

which implies (25), given the second item of Lemma 1. ■

3.3 No Motive Dominates

Both the signaling and the enforcement motive stay relevant for the citizens' decision-making when the population grows large.

Lemma 3 *Let* (15) *hold. For any sequence of responsive equilibria* $(\sigma_n, \tilde{k}_n)_{n\in\mathbb{N}}$,

$$0 < \lim_{n\to\infty}\frac{\Pr(\text{piv}^{\text{en}}|\sigma_n, \tilde{k}_n, n)}{\Pr(\text{piv}^{\text{sg}}|\sigma_n, \tilde{k}_n, n)} < \infty \tag{27}$$

Proof. For any responsive equilibrium sequence $(\sigma_n, \tilde{k}_n)_{n\to\infty}$, let $d_n = \frac{\Pr(\text{piv}^{\text{en}}|\sigma_n,\tilde{k}_n,n)}{\Pr(\text{piv}^{\text{sg}}|\sigma_n,\tilde{k}_n,n)}$. First, suppose that $\lim_{n\to\infty} d_n = 0$. Then, actions are almost pure "cheap talk" when n is large. The following replicates the logic why in pure cheap talk settings efficient equilibria cannot exist (see Battaglini, 2017).

Since the action of a given citizen is a garbling of her private signal, $\frac{\text{E}(\sigma_n(s)|\gamma)}{\text{E}(\sigma_n(s)|\beta)} \le \frac{\Pr(a|\gamma)}{\Pr(a|\beta)}$. Since conditional on piv^{sg}, the authority's decision depends on the action of the given citizen, $\lim_{n\to\infty}\frac{\Pr(\gamma|\text{piv}^{sg};\sigma_n,k_n,n)}{\Pr(\beta|\text{piv}^{sg};\sigma_n,k_n,n)} > \frac{c_p}{1-c_p}\frac{\Pr(a|\beta)}{\Pr(a|\gamma)}$ for all sequences of realizations $(k_n)_{n\in\mathbb{N}}$ with $0 < k_n < N$ and $k_n \in \operatorname{supp}(\tilde{k}_n)$. Thus,

$$\lim_{n\to\infty} \frac{\Pr(\gamma|\text{piv}^{sg}, b; \sigma_n, k_n, n)}{\Pr(\gamma|\text{piv}^{sg}, b; \sigma_n, k_n, n)} > \frac{c_p}{1-c_p} \frac{\Pr(a|\beta)}{\Pr(a|\gamma)} \frac{\Pr(b|\gamma)}{\Pr(b|\beta)} > c_v. \tag{28}$$

where we used the large bias assumption (15) for the second inequality. When $d_n \to 0$, (14) simplifies and is satisfied (with strict equality) for $s \in \{g, b\}$ and n large enough if (28) holds. In other words, for large n, the unique best response is to take action after both signals. This contradicts with the reponsiveness of the equilibria.

Second, suppose that $\lim_{n\to\infty} d_n = \infty$. We argue that the inference from the pivotal event piv^{rf} stays bounded as $n \to \infty$,

$$\lim_{n\to\infty} \frac{\Pr(\gamma|\text{piv}^{\text{en}}; \sigma_n, k_n, n)}{\Pr(\beta|\text{piv}^{\text{en}}; \sigma_n, k_n, n)} \in (0, \infty), \tag{29}$$

for all sequences of realizations $(k_n)_{n\in\mathbb{N}}$ with $0 < k_n < N$ and $k_n \in \operatorname{supp}(\tilde{k}_n)$. Given $d_n \to \infty$, (14) simplifies and we see that otherwise, for n large enough, the citizens have the same unique best response after any signal $s \in \{g, b\}$ This contradicts with the responsiveness of the equilibria.

We claim that

$$0 < \lim_{n\to\infty} \mathrm{E}(\sigma_n(s)|\beta) < t^{\text{en}} < \lim_{n\to\infty} \mathrm{E}(\sigma_n(s)|\gamma) < 1; \tag{30}$$

First, we argue that $\lim_{n\to\infty} \mathrm{E}(\sigma_n(s)|\gamma) > t^{\text{en}}$. Otherwise, Lemma 2 would imply $t^{\text{sg}} < t$. But this would imply that the constraint on the choice set of the policy-maker is not binding for n large enough, so that $\text{piv}^{en} = \emptyset$. This contradicts with $d_n \to \infty$. Second, suppose that $\lim_{n\to\infty} \mathrm{E}(\sigma_n(s)|\beta) \geq t^{\text{en}}$. Then, $\lim_{n\to\infty} \frac{\Pr(\gamma|\text{piv}^{sg};\sigma_n,k_n,n)}{\Pr(\gamma|\text{piv}^{sg};\sigma_n,k_n,n)} = 0$, given Lemma 1. This contradicts with (29).

Now, Lemma 2 implies

$$\mathrm{E}(\sigma_n(s)|\beta) \leq t_n^{\text{en}} \leq \mathrm{E}(\sigma_n(s)|\gamma). \tag{31}$$

for n large enough. Since the probability mass function of the binomial with mean $\mathrm{E}(\sigma_n(s)|\omega)$ is weakly increasing below the mean and weakly decreasing above the mean, (26) and (30) - (31) together imply $\lim_{n\to\infty} d_n \in (0, \infty)$, which contradicts the initial assumption $d_n \to \infty$. ■

3.4 Proof of Proposition 1 (only if-part)

We explain how the auxiliary results of Lemma 1 - 3 imply that the conditions (19) and $\mathrm{E}(\sigma(s)|\gamma) > t^{\mathrm{en}}$ of Proposition 1 are necessary conditions for any responsive equilibrium sequence. The sufficiency of the conditions 1 is established by the fixed-point arguments in Section 3.6.

Lemma 2 and Lemma 3 imply that no limit equilibrium σ can have $\mathrm{E}(\sigma(s)|\gamma) \leq t^{\mathrm{en}}$ since otherwise $t^{\mathrm{sg}} < t^{\mathrm{en}}$ and the threshold t^{en} would be inconsequential, which would imply that the signaling motive would dominate, $d_n \to 0$. However, this contradicts Lemma 3. This together with Lemma 1 and Lemma 3 implies that any responsive equilibrium must satisfy (19) or $t^{\mathrm{sg}} = 1$. Suppose that the latter is the case. Using (24) and that $\Pr(Y_n = (2n+1)t^{en}) \approx \phi(0)\Big[t^{en}(1-t^{en}\Big]^{-1}$ for ϕ the density of the standard normal,

$$\lim_{n\to\infty} \frac{\Pr(\mathrm{piv}^{\mathrm{sg}}|\gamma;\sigma_n,k_n,n)}{\Pr(\mathrm{piv}^{\mathrm{en}}|\gamma;\sigma_n,k'_n,n)} \tag{32}$$

$$\geq \lim_{n\to\infty} e^{-(2n+1)\Big[H(t_n^{\mathrm{sg}},\mathrm{E}(a|\gamma;\sigma_n)-H(t_n^{\mathrm{en}},\mathrm{E}(a|\gamma;\sigma_n))\Big]}\phi(0)(t^{\mathrm{en}}(1-t^{\mathrm{en}}))^{-1} \tag{33}$$

Now, note that $t^{\mathrm{sg}} = 1$ implies $\mathrm{E}(\sigma(s)|\omega) \to 1$ for $\omega \in \{\gamma, \beta\}$, given Lemma 2. Plugging in,

$$\lim_{n\to\infty} \frac{\Pr(\mathrm{piv}^{\mathrm{sg}}|\gamma;\sigma_n,k_n,n)}{\Pr(\mathrm{piv}^{\mathrm{en}}|\gamma;\sigma_n,k'_n,n)} \geq \lim_{n\to\infty} e^{(2n+1)H(t_n^{\mathrm{en}},1)}(t^{\mathrm{en}}(1-t^{\mathrm{en}}))^{-1} = \infty. \tag{34}$$

This implies that the signaling motive dominates, $d_n \to \infty$, which cannot be, given Lemma 3.

3.5 Illustration of the Equilibria

We graphically illustrate how the necessary conditions of Proposition 1 imply that only the two types of equilibrium sequences as in Theorem 1 and 2 are possible when the policy-maker is sufficiently biased, (15). For this, first, we derive a compact one-dimensional representation of responsive equilibria.

Lemma 4 *Any responsive strategy with $\sigma(g) > \sigma(b)$ that is part of an equilibrium falls in one of the following two categories:*

1. *Citizens take action with probability* 0 *after a b-signal,* $\sigma(b) = 0$ *and mix*

after a g-signal.

2. *Citizens take action with probability* 1 *after a g-signal,* $\sigma(g) = 1$ *and mix after a b-signal.*

Proof. First, given (10), (12), and $\frac{\Pr(g|\gamma)}{\Pr(g|\beta)} < \frac{\Pr(b|\gamma)}{\Pr(b|\beta)}$, citizens with a g-signal believe the state γ to be more likely than citizens with a b-signal, i.e.

$$\Pr(\gamma|\mathrm{piv}^{\mathrm{mot}}, g; \sigma, n) > \Pr(\gamma|\mathrm{piv}^{\mathrm{mot}}, b; \sigma, n) \tag{35}$$

for mot $\in \{\mathrm{en}, \mathrm{sg}\}$. Second, $\sigma(g) > \sigma(b)$ implies that take action is a signal indicative of γ, and hence may tip the policy-maker only towards higher policies. Therefore, (14) and (35) together yield the results. ■

We consider all strategies that fulfill the necessary condition of Lemma 4. We call them *candidate strategies* and parametrize them along their expected collective action $\mathrm{E}(\sigma(s)|\beta)$,

$$\mathrm{E}(\sigma(s)|\beta) = \Pr(g|\beta)\sigma(g) + \Pr(b|\beta)\sigma(b).$$

For the candidate strategies of the first class ($\sigma(g) = 1$) it must hold that $\mathrm{E}(\sigma(s)|\beta) \geq \Pr(g|\beta)$ and for the candidate strategies of the second class ($\sigma(b) = 0$) it must hold that $\mathrm{E}(\sigma(s)|\beta) \leq \Pr(g|\beta)$. We can therefore parametrize the whole set of candidate strategies by their vote share $\mathrm{E}(\sigma(s)|\beta)$: let for any q be σ^q the strategy that results in an expected collective action of q, i.e.

$$\mathrm{E}(\mathrm{E}(\sigma^q(s)|\beta) = q.$$

We have

$$(\sigma^q(g), \sigma^q(b)) = \begin{cases} (0, \frac{q}{\Pr(g|\beta)}), & \text{for} \quad q \leq \Pr(g|\beta), \\ (\frac{q-\Pr(g|\beta)}{\Pr(b|\beta)}, 1), & \text{else.} \end{cases}$$

Figure 1 illustrates the limit candidate strategies σ^q satisfying the necessary conditions of Proposition 1. The figure "plots" the condition (19) for all limit candidate strategies satisfying the second condition $\mathrm{E}(a|\gamma; \sigma^q) > t^{\mathrm{en}}$. The plot shows that there only two candidate strategies satisfy both conditions. They are given by $\hat{q} = \mathrm{E}(\sigma^{\hat{q}}(s)|\beta) < \mathrm{t}^{en}$ and $\bar{q} = \mathrm{E}(\sigma^{\bar{q}}(s)|\beta) > \mathrm{t}^{en}$. Lemma 2 implies that

$\mathrm{E}(\sigma^q(s)|\beta) < t^{\mathrm{sg}} < \mathrm{E}(\sigma^q(s)|\gamma)$ for $q \in \{\hat{q}, \bar{q}\}$. Thus, an application of the law of large numbers implies that the policy x_h is chosen in γ and the policy $x = 0$ is chosen in β in any equilibrium sequence converging to $\sigma^{\hat{q}}$. The policy x_h is chosen in γ and the policy $x = x_l > 0$ is chosen in β in any equilibrium sequence converging to $\sigma^{\bar{q}}$. This establishes the second item of Theorem 2.

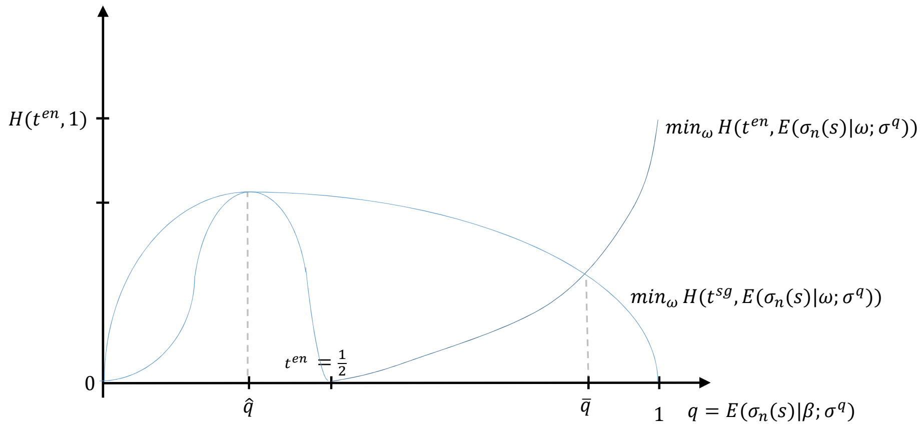

Figure 1: The figure illustrates the limit candidate strategies σ^q for which no motive dominates, $\lim_{n\to\infty} d_n \in (0, \infty)$, fixing $\mathrm{t}^{en} = \frac{1}{2}$. There are two such candidate strategies, given by $\hat{q} = \mathrm{E}(\sigma^{\hat{q}}(s)|\beta) < \mathrm{t}^{en}$ and $\bar{q} = \mathrm{E}(\sigma^{\bar{q}}(s)|\gamma) < \mathrm{t}^{en}$.

Section 3.6 provides the proof of Theorem 1 and the proof of the first item of Theorem 2.

3.6 Fixed Point Arguments

3.6.1 Equilibrium as a One-Dimensional Fixed Point

Take the set of candidate strategies σ^q, parametrized by $q = \mathrm{E}(\sigma^q(s)|\beta) \in (0, 1)$. A responsive citizen strategy that is part of an equilibrium is a strategy σ^{q^*} satisfying $\sigma^{q^*} \in \mathrm{BR}(\sigma^{q^*})$, where BR is the best response correspondence when the other $2n$ citizens choose σ^{q^*} and the policy-maker chooses a best response given σ^{q^*}.[11] Alternatively, equilibrium strategies of the citizens can be characterized as a number q^* satisfying

$$q^* \in \mathrm{E}(\mathrm{BR}(\sigma^{q^*})(s)|\gamma). \tag{36}$$

[11]We do not fix the best response of the policy-maker. The correspondence takes into account all best responses of the citizens to any best response of the policy-maker.

where we slightly abuse notation by applying the expectation operator to a set of random variables. Any q^* satisfying (36) together with a best-response $\tilde{k}^*$ of the policy-maker so that $q^* \in \mathrm{E}(\mathrm{BR}(\sigma^{q^*})(s)|\gamma)$ constitutes an equilibrium.

Recall the two possible limit candidate strategies $\sigma^{\bar{q}}$ and $\sigma^{\hat{q}}$ with $\bar{q} < \frac{1}{2}$ and $\hat{q} > \frac{1}{2}$. We prove Theorem 1 and the first item of Theorem 2 by constructing an efficient and an inefficient equilibrium sequence. The citizen strategy of the efficient equilibrium sequence will converge to $\sigma^{\bar{q}}$ and the citizen strategy of the inefficient equilibrium sequence will converge to $\sigma^{\hat{q}}$. This way, we also establish the sufficiency of the conditions of Proposition 1.

3.6.2 Proof of Theorem 1: Efficient Equilibrium Sequence

Fix $q < \bar{q}$ for which $\mathrm{E}(\sigma^q(s)|\gamma) > t^{\mathrm{en}}$. Note that $H(\mathrm{E}(\sigma^q(s)|\gamma), t^{\mathrm{en}}) < H(q, t^{\mathrm{en}})$ and $H(\mathrm{E}(\sigma^q(s)|\gamma), t^{\mathrm{en}}) < \min_{\omega\in\{\alpha,\beta\}} \lim_{n\to\infty} H(\mathrm{E}(\sigma^q(s)|\omega), t^{sg}_n)$ (compare to Figure 1). Lemma 1 implies $\Pr(\gamma|\mathrm{piv}^{\mathrm{en}}; \sigma^q, \tilde{k}_n, n) \to 1$ for any best response $\tilde{k}_n$ of the policy-maker and that the enforcement motive dominates. Thus, (14) simplifies, and we see that the best response of the citizens is to take action after both signals when n is large enough. Hence, $\mathrm{E}(\mathrm{BR}(\sigma^q)(s)|\gamma) = 1$. Analogously, fixing $q = \frac{1}{2}$, Lemma 1 and (14) imply that the best response of the citizens satisfies $\mathrm{E}(\mathrm{BR}(\sigma^q)(s)|\gamma) = 1$ for n large enough. Finally, since the best response correspondence is upper-hemicontinuous, an application of Kakukani's fixed point theorem yields the existence of a sequence $q^*_n \to \bar{q}$ satisfying (36). The corresponding sequence of strategies $\sigma^{q^*_n}$ is responsive and satisfies $\lim_{n\to\infty} \mathrm{E}(\sigma^q(s)|\beta) < t^{\mathrm{en}} < \lim_{n\to\infty} \mathrm{E}(\sigma^q(s)|\gamma)$. Lemma 2 implies $t^{sg} < \lim_{n\to\infty} \mathrm{E}(\sigma^q(s)|\gamma)$. Thus, it follows from the law of large numbers that $x = x_h$ is chosen in γ and $x = 0$ chosen in β with probability converging to 1 as $n \to \infty$. We conclude that $\sigma^{q^*_n}$ and the corresponding sequence of best responses of the policy-maker constitute an efficient equilibrium sequence.

3.6.3 Proof of Theorem 2, item 1: Inefficient Equilibrium Sequence

Fix $q > \hat{q}$. Note that $\lim_{n\to\infty} \min_{\omega\in\{\alpha,\beta\}} H(\mathrm{E}(\sigma^q(s)|\omega); t^{\mathrm{sg}}_n) < \lim_{n\to\infty} \min_{\omega\in\{\alpha,\beta\}} H(\mathrm{E}(\sigma^q(s)|\omega); t^{\mathrm{en}})$ (compare to Figure 1). Lemma 1 implies $\lim_{n\to\infty} \frac{\Pr(\mathrm{piv}^{\mathrm{en}}|\sigma_n, \tilde{k}_n, n)}{\Pr(\mathrm{piv}^{\mathrm{sg}}|\sigma_n, \tilde{k}, n)} \to \infty$ for any best response $\tilde{k}_n$ of the policy-maker. Then, (28) implies that the best response of the citizens is to take action after both signals when n is large enough, so that $\mathrm{E}(\mathrm{BR}(\sigma^q)(s)) = 1$. Since we showed in the previous paragraph that $\mathrm{E}(\mathrm{BR}(\sigma^q)|\gamma) = 0$ for $q = t^{\mathrm{en}}$ and n large enough, another application of Kakukani's fixed point theorem yields the existence of a sequence $q_n^* \to \hat{q}$ satisfying (36). The corresponding sequence of strategies $\sigma^{q_n^*}$ is responsive and satisfies $t^{\mathrm{en}} < \lim_{n\to\infty} \mathrm{E}(\sigma^{q^*}(s)|\beta) < \lim_{n\to\infty} \mathrm{E}(\sigma^{q^*}(s)|\gamma)$. Lemma 2 implies $t^{sg} < \lim_{n\to\infty} \mathrm{E}(\sigma^q(s)|\gamma)$. Thus, it follows from the law of large numbers that $x = x_h$ is chosen in γ and $x = x_l$ in β with probability converging to 1 as $n \to \infty$. We conclude that $\sigma^{q_n^*}$ and the corresponding sequence of best responses of the policy-maker constitute an inefficient equilibrium sequence.

4 Another Interpretation of the Model

Our model has an alternative interpretation as a model of a majority election with *implementation uncertainty*, as follows: One can show that, when the politician has a large bias, (15), then, in all responsive equilibrium sequences, $t^{\mathrm{en}} < t_n^{\mathrm{sg}}$ when n is large. Thus, when less then a share t^{en} of citizens take action, the outcome is the status quo.[12]

As a consequence, both the inefficient and the efficient equilibrium sequence would also arise in the alternative model in which we fix the outcome to be the status quo if the share t^{en} of citizens taking action is less than a threshold t^{en}. This amounts to a model of a standard majority election between the status quo $x = 0$ and a reform $x > 0$, in which the implementation level x of the reform is subject to the discretionary choice of a political authority—e.g., a parliament or a government—following majority support for reform.

In many election settings, including those of direct democracy, such *implementation uncertainty* arises quite naturally. Think of the Brexit referenda in which details about the form of separation of the UK from Europe (e.g., "soft" Brexit versus "hard" Brexit) were uncertain at the time of the referendum. An-

[12]This observation is immediate for the inefficient equilibrium sequence, since there $t^{\mathrm{en}} < \lim_{n\to\infty} t_n^{\mathrm{sg}}$. Take the efficient equilibrium sequence and assume that the opposite would hold, i.e., for all n sufficiently large, the maximal realization of the policy-maker's cutoff is so that $k^{\mathrm{en}} \geq k_n^{\mathrm{sg}}$. Thus, the minimal realization satisfies $k_n^{\mathrm{sg}} \geq k^{\mathrm{en}} - 1$ Using the ordering $\mathrm{E}(\sigma_n(s)|\beta) < t^{\mathrm{en}}, t^{\mathrm{sg}} < \mathrm{E}(\sigma_n(s)|\gamma)$, and applying Theorem 2 of Arratia and Gordon (1989) yields $\lim_{n\to\infty} \Pr(\gamma|\mathrm{piv}^{\mathrm{sg}}; \sigma_n, \tilde{k}_n, n) \geq \lim_{n\to\infty} \Pr(\gamma|\mathrm{piv}^{\mathrm{en}}, b; \sigma_n, \tilde{k}_n, n)$. However, given (15), $\Pr(\gamma|\mathrm{piv}^{\mathrm{en}}, b; \sigma_n, \tilde{k}_n, n) > c_v$, see (28). Given (14), this implies that, after both signals $s \in \{g, b\}$, the unique best response is to take action, when n is large enough. This contradicts with the efficiency of the equilibrium sequence.

other recent example is the 2020 national plebiscite in Chile in which a supermajority of voters called for the writing of a new constitution. However, specifics of the new constitution were open at the time of the referendum. In particular, the members of the constitutional convention would be selected in a follow-up election in 2020 (which was postponed twice to take place eventually in May 2021, making a case in point).

In view of this alternative model interpretation, our paper contributes to the literature on the Condorcet jury theorem. This literature has shown that elections effectively aggregate exogenous information that is dispersed among many voters, so that outcomes in all equilibria are"as if" there is no uncertainty about the state (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1997).[13] Most related in this literature is Razin (2003) who studies a Condorcet model in which citizens have a signaling motive. A particular focus is on situations in which in all citizens prefer more right policies in a "right state" relative to a "left state", but the election candidates' ideal policies are extreme, so that there is no overlap with the citizens' ideal policies ("polarization"). He shows that all equilibria that satisfy some symmetry condition are inefficient. If the correct candidate would be elected in both states, he perfectly infers the state. Thus, in the right state, the rightist candidate chooses an even more right policy, whereas citizens would prefer him to choose more moderately. In fact, there is a feasibility problem, and no citizen strategy can imply efficient outcomes. The feasibility problem arises since the politician and the citizens have misaligned interest under full information. By way of contrast, in our setting, citizens and policy-maker have aligned interest under full information, and there exists an equilibrium sequence that aggregates information. We report a failure of information aggregation that is due to mis-coordination of voters (Theorem 1).If there is no implementation uncertainty ($x_l = x_h$) in our setting, a version of the Condorcet jury theorem holds (Feddersen and Pesendorfer, 1998, see). Thus, our results imply that introducing just minimal implementation uncertainty can upset information aggregation.

We believe that the topic of political implementation uncertainty is underexplored, but highly relevant. Future papers may investigate the possible dimensions of implementation uncertainty. For example, in this paper, implementation uncertainty is tied to one decision-maker. More realistically, it arises from an

[13]See also Myerson (1998), Wit (1998), and Duggan and Martinelli (2001).

after-game that may capture parliamentary debate or supranational bargaining over reform implementation.

5 Conclusion

This paper has introduced and analyzed a model of informal collective action in which citizen behaviour can both inform as well as enforce policy choices. Our main result highlights that the ability to enforce or constrain the policy range of a policy-maker gives the citizens “skin in the game” and enables informative citizen behaviour in equilibrium. As a consequence, welfare-efficient equilibrium sequences exist, even when the policy-maker has a large preference bias.

Our results contribute to very classic but also recent discussions about “good” design of democratic mechanisms. It provides an informational argument for the delegate model of representation of Edmund Burke. It qualifies the central desideratum of “commitment” to policy responses of OECD (2020) for the design of “new democratic institutions”.

Finally, we have discussed another interpretation of our model as one of a majority election with implementation uncertainty that may arise due to discretionary choices of policy-makers and how this interpretation opens spaces for future work (Section 4).

Appendix

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