Voter Attention and Distributive Politics [*]

Carl Heese [†]

Carl Heese [†]

Click here for latest version

November 7, 2019

Abstract

I study the role of heterogeneous attention among voters for distributive politics. Groups in the population pay systematically different attention to politics and acquire different levels of information. I study the effects of heterogenous attention when a reform may benefit one group at the expense of others. In the benchmark, when the information of voters is exogenous, a median voter theorem holds and a welfare-enhancing reform is not be adopted if it is not preferred by a majority. When information is endogenous, attention shifts election outcomes into a direction that is welfare-improving. Even when a welfare-enhancing reform is not preferred by a majority ex-post, under certain conditions, there are equilibria where the reform is adopted. The key driver of the results is that voters who are more severely affected by a proposed reform will pay more attention, consistent with empirical studies ("issue publics hypothesis", Converse (1964)). This information advantage translates into voting power precluding the majority from exerting its dominance.

[†]University of Bonn, Department of Economics, heese@uni-bonn.de.

1 Introduction

A rich literature in distributive politics seeks to understand if and when elections select policies that maximize social welfare.[1] I introduce a novel aspect into this discussion, namely, endogenous attention to politics, which is guided by the empirical observation that voters that care more about a political issue, will acquire more information about it[2]

I propose a model that is able to describe the relation between the information level of different voter groups with conflicting interests and their voting power. The main result shows that equilibria can be described by comparing a certain power index of the groups of voters with conflicting interests (Theorem 1 and Observation 2). This characterization will imply that, for a large class of settings, there are equilibria where the first-best (utilitarian) outcome is elected, even if the outcome is not preferred by a majority of the voters ex-post (Theorem 2). In such situations, a minority of the voters is more severely affected by the reform, but will be better informed about it, and is able to translate the information advantage into voting power, thereby shifting the election outcome towards efficiency.

Examples of reforms with uncertain distributive consequence are numerous: a trade reform opens new markets for exporting firms but threatens the prospects in other sectors, a public health policy reform makes certain treatments more accessible to citizens, while it implies price increases for a range of pharmaceuticals needed by other citizens, and a new tax reform reduces the tax burden for some citizens, but elevates it for others. In all these examples, most voters are ex ante uninformed about the consequences of the reform, e.g. which sectors gain from a trade reform, or what the complex implications of a tax reform are. However, they hold have private information about their exposure to the proposed reform, that is, about the magnitude of their preference intensities:

---

[1]See e.g. Fernandez & Rodrik (1991), Alesina & Rodrik (1994) and Persson & Tabellini (1994). Similar to this paper, Bhattacharya (2013a) and Ali *et al.* (2018) transport the informational approach to elections (Austen-Smith & Banks (1996), Feddersen & Pesendorfer (1997)) to the literature on distributive politics.

[2]This is known as the "issue publics hypothesis" (Converse (1964)). See e.g. Carmines & Stimson (1980), Miller & Krosnick (2000), and Iyengar *et al.* (2008).

more affluent citizens have more money at stake, and medically dependent citizens are more strongly affected by changes in public health policy.

I consider a modified version of the canonical setting by Feddersen & Pesendorfer (1997). Relative to Feddersen & Pesendorfer (1997), the voters information about the policies is endogenous *and* I allow that the voters have conflicting interests (distributive politics). Both modifications have been studied before, but not together: Bhattacharya (2013b) has studied a variant with conflicting interests and Martinelli (2006) has studied a variant with endogenous information.[3] The novelty is that I study both aspects together and this will be critical to be able to describe the relation of preference intensities, the information level and the electoral power of groups with conflicting interests.

There are two possible policies, a reform and a status quo. Voters' preferences over policies are heterogeneous and depend on an unknown, binary state in a general way (some voters may prefer the reform only in the first state, some prefer the reform only in the second state, and some "partisans" may prefer one of the policies independently of the state). The preferences are each voter's private information. In addition, all voters can receive information about the state in the form of a noisy signal and each voter freely chooses the precision of his private signal. More precise information is more costly. Upon receiving their signals, all citizens vote simultaneously. The election determines the outcome by a simple majority rule.

Feddersen & Pesendorfer (1997) have shown that, when voters receive conditionally i.i.d. signals of some exogenous quality and preferences are "monotone",[4] in all equilibria of large elections, the outcome preferred by the median voter is elected. In many situations, where voters have conflicting interests, this is not the first-best outcome: for example, when 51% of the citizens benefit from a reform marginally, but at the same time 49% of the citizens are impaired by it severely. Martinelli (2006) has shown that the median voter theorem also holds in a setting where information is endogenous, but only

---

[3]I discuss these contributions in more detail momentarily

[4]The preference distribution of the voters is "monotone" if a higher belief in the first state entails that more voters prefer the reform.

if relevant political information can be acquired at cost that are "not too high". Formally, what matters for his result is how fast cost go to zero when a voter chooses an arbitrarily uninformative signal. The critical condition is that the first three derivatives of the cost function are zero at the precision of the uninformative signal. Bhattacharya (2013a) has shown that Feddersen & Pesendorfer (1997)s result generalizes to settings with conflicting interests.[5] Importantly, his model does not allow to study the role of the intensity of preferences since the result is invariant to scaling the intensities of specific groups of voters.

The main analysis in this paper describes the political power of the heterogenous voters. All equilibria can be represented by an index rule (Lemma 2). The *power index* of a group of voters sharing common interests is a relative measure of its electoral power and is, in particular, proportional to the size of the group and increasing in the welfare that is at stake for the group. When relevant political information can be acquired at cost that are "not too high", there are equilibria where competitive information acquisition shapes the election and where the election yields the outcome preferred by the group with the higher power index. I relate this index representation to social welfare, under a certain independence condition on the preference distribution that implies that the preferences are "monotone.[6] Roughly speaking, I show that, when information cost are in a certain intermediate range, there is an equilibrium that leads to first-best outcomes; to put it differently, there is an intermediate range of cost functions where the information cost screen the voters intensities appropriately. Besides the condition as in Martinelli (2006), another condition is necessary for first-best outcomes, namely, that cost should not go to zero too fast, where the strictness of the condition will depend globally on the distribution of the voters' preferences.

---

[5]Bhattacharya (2013a) also shows that the result breaks down when preferences are non-monotone.

[6]When the preferences are non-monotone, there may be many equilibria, similar to the results in Bhattacharya (2013b). I characterize all of them; they are all "index equilibria, but generally, the power index of the group, depends on what Bhattacharya (2013b) calls the *induced prior belief* of the given equilibrium.

In the second part, I deepen the equilibrium analysis. By doing so, I point out determinants of efficiency and the political power of voter groups that are novel to the existing literature. First, I analyze the robustness of the equilibrium with first-best outcomes when varying the voters cost of acquiring relevant information about the policy consequences. The more similar is the welfare at stake, that is the aggregate intensities, of the citizens who benefit relative to those who are harmed by a reform, the smaller the intermediate range of cost functions for which there is an equilibrium with first-best outcomes (Theorem 3). Intuitively, more similar intensities make it more difficult for the information cost to screen voter intensities appropriately. Second, I show that the electoral power of a voter group is decreasing in the polarization of the preference intensities within the group (Theorem 4).

In the last part of the paper, I study the situations where the voters differ not only in their exposure to the proposed reform, but also in their ability to access and interpret political information. Formally, the voters are subject to different cost functions. First, I provide an equivalence result: the extended setting with heterogenous cost is equivalent to a setting with homogenous cost: intuitively, the cost of information and preference intensities are strategically equivalent such that differences in cost translate into differences in intensities. My previous efficiency results therefore extend when the cost types are independent of the preference intensities (Theorem 7). Second, I vary the richness of the information choice of the voters in the extended setting and show that coarsening the voters information choice may have positive welfare effects. I compare the situation where the voters have only access to a binary set of signals to the previous setting where voters have access to a rich set of signals and can freely choose their signal precision without constraints. In the coarse setting, there are always equilibria where the welfare-maximizing outcome is elected in all states (Theorem 8), unlike in the setting with rich information choice where this depends on how similar the welfare at stake is for the citizens who benefit relative to those who are harmed by a reform. This is particularly surprising, since a richer choice set for the voters should intuitively facilitate an appropriate screening of preference intensities in equilibrium, and not pre-

vent it.

The paper contributes to the literature on information aggregation in large elections. Condorcet's Jury Theorem (1785) says that if voters have common interests but information is dispersed in the electorate, majority rule results in socially optimal outcomes. Information aggregates in the sense that electoral outcomes correspond to the choices of a fully informed welfare-maximizing social planner. Austen-Smith & Banks (1996), Feddersen & Pesendorfer (1998) have established a "modern" version of the Condorcet Jury Theorem in a setting where voters are strategic. However, as argued, elections might fail to elect socially optimal outcomes when voters have conflicting interests, for example when 51% of the citizens benefit from a reform marginally, but at the same time 49% of the citizens are impaired by it severely. This paper points at an empirical observation that has been mostly overlooked, namely, that the dispersion of the voters' information is endogenous, and I show how this can imply socially optimal outcomes, independent of the distribution of the voters' preference intensities.

The paper contributes to the literature on elections with costly information acquisition by studying a general setup that, in particular, allows the voters to have conflicting interests like in distributive politics. The previous literature has studied situations where preference types might be heterogenous, but where all voters share a common interest (e.g. as in Martinelli (2006) and Oliveros (2013a)). Martinelli (2006)) and Oliveros (2013a) are concerned with the possibility of information aggregation in large elections when information is costly and have shown that information aggregation is possible in setups with symmetric preference distributions.[7] For the common interest case, I generalize the results of the literature by providing an analysis for general continuous preference distributions and by characterizing all equilibria of the voting game. In particular, I show that information aggregation is generally possible under the conditions on the cost function provided in Martinelli (2006).[8]

---

[7]Further more distantly related papers are Triossi (2013) and Martinelli (2007) who study heterogenous cost in common interest setups and Oliveros (2013b) who studies the relation of abstention and information cost.

[8]The settings in Martinelli (2006) and in Oliveros (2013b) are special cases of my model.

The paper is related to work on elections with voting cost and vote-buying. Krishna & Morgan (2011) and Krishna & Morgan (2015) have shown that elections yield first-best outcomes when voting is voluntary and costly. These results are analogous to my findings for the setting with a coarse, binary information choice. The main model with a rich information choice is more closely related to the literature on vote buying. Lalley & Weyl (2018) have shown that equilibrium outcomes of a large electorate are first-best when each voter can buy any number of votes at a total price quadratic in the number bought. Similarly, in this model, the analysis will show that when the cost of information is arbitrarily close to 'cubic' as a function of the precision of the signal, there are equilibria with first-best outcomes for almost all preference distributions. Eguia & Xefteris (2018) show that vote-buying mechanisms with general price functions implement the set of Bergsonian welfare functions.[9] Similarly, my results show that majority elections with information cost implement the Bergsonian welfare functions that satisfy a fairness principle known as the Pigou-Dalton principle.[10]

The rest of the paper is organized as follows. Section 1.1 provides a simple example that illustrates the main result. Section 2 presents the model and the preliminary analysis. Section 3.3 discusses the power (index) of voter groups. Section 3 shows the existence of equilibria with first-best outcomes and discusses their robustness and the role of preference polarization. Section 5 characterizes all other equilibria and, in particular, shows existence of another equilibrium that converges to "voting according to the prior belief" about the unknown state. In this equilibrium, the outcome that is preferred by the majority of the voters given the prior belief, is elected (Theorem 5). Section 6 studies heterogenous cost and a setting where the information choice of the voters is coarse.

---

[9]A Bergsonian welfare function selects the outcome that maximizes a certain weighted sum of the agents' utilities, see Burk (1936).

[10]See Moulin (2004) for a definition of the Pigou-Dalton principle.

1.1 Two-Type example

The following extreme setting shows how information cost can screen the preference intensities appropriately, and illustrates thereby how intensities translate into voting power and how first-best outcomes can be elected, even when a majority of the voters does not prefer the outcome ex-post. There are $2n+1$ voters. With probability $1 > \lambda > \frac{1}{2}$, a voter is *aligned* and prefers the reform A over the status quo B only in α and B over A in β. Otherwise, a voter is *contrarian* and prefers A only in β.[11] A contrarian voter gets utility of 1 when her preferred policy is elected and an aligned voter gets a small utility of $\epsilon > 0$ when her preferred policy is elected. Voters have a binary choice, to get a private, perfect signal about the state at a given cost $c > 0$ or an uninformative signal at no cost. The common prior about the state is uniform, i.e., $\Pr(\alpha) = \frac{1}{2}$. To maximize welfare, the election should implement what the contrarians want if they care much more, i.e., when ϵ is sufficiently small $(\epsilon(1 - \lambda) < \lambda)$.

Consider three scenarios: zero, intermediate and high cost. When cost are zero, i.e. $c = 0$, all voters get perfectly informed about the state and the outcome preferred by the median voter is elected in each state. When cost are very high, e.g. $c > 1$, nobody gets informed and the policy elected is independent of the state. When cost come from a certain intermediate range, they screen types such that only types with high intensities get informed. In fact, for any n, there is a range of such intermediate cost c and a *symmetric* equilibrium given c where the aligned vote for each policy with the same probability, i.e. $50 - 50$, and the contrarians vote for their preferred outcome in each state. Thus, in each state, the outcome preferred by the contrarians, which represent a minority in expectation, receives a strict majority of the votes in expectation. Therefore, when the electorate grows large, the contrarian-preferred outcome is elected and this maximizes welfare.[12] To see why such an equilibrium exists,

---

[11] The terminology to label the voter types carries no economic meaning whatsoever, but only relates to the notation. Aligned voters prefer the outcome that is "aligned" with the state.

[12] I do not discuss the welfare effects of the cost for the example since it turns out that the aggregate cost are arbitrarily small in the equilibria of the main result when the electorate

note that without the private signal, a citizen is indifferent between voting for either of the policies. This is, because the event, in which the citizen's vote affects the outcome, is equally likely in each state in the symmetric candidate equilibrium.[13] The citizen only benefits from being informed about the state in the situations when her vote affects the outcome of the election. Thus, the benefit is proportional to the likelihood of this pivotal event piv and when the cost are intermediate, i.e. when $\epsilon \Pr(\text{piv}) < c < \Pr(\text{piv})$, only the contrarians would like to buy the signal. Note that the closer the utility at stake for the aligned and the contrarians, the smaller the range of intermediate cost that screen the types.

2 Model

There are $2n + 1$ voters (or citizens), two policies A and B, and two states of the world $\omega \in \{\alpha, \beta\} = \Omega$. The prior probability of α is $\Pr(\alpha) \in (0, 1)$.

Voters have heterogeneous and state-dependent preferences. A voter's preference is described by a type $t = (t_\alpha, t_\beta)$, where $t_\omega \in [-1, 1]$ is the utility of A in ω. The utility of B is normalized to zero, so that t_ω is the difference of the utilities of A and B in ω. The types are identically distributed across voters according to a cumulative distribution function $H : [-1, 1]^2 \to [0, 1]$ that has a continuous density h. A voter's type is her private information. Each voter observes a binary signal $s \in \{a, b\}$. The observed signal is the private information of the voter as well. The joint distribution of the type and the signal of a voter is independent of the distribution of the signals and the types of the other voters.

The voting game is as follows. First, nature draws the state and the profile of types $\mathbf{t}$ according to H. Second, after observing her type, each voter chooses a precision $x(t) \in [\frac{1}{2}, 1]$ of her signal, that is $\frac{1}{2} + x(t) = \Pr(a|\alpha) = \Pr(b|\beta)$. Then, private signals realize. After observing her private signal, each voter

---

is large.

[13]In one state the reform wins with a margin of $\frac{1}{2}(1 - \lambda)$ and in the other state the reform loses with a margin $\frac{1}{2}(1 - \lambda)$ in expectation, implying that $\Pr(\alpha|\text{piv}) = \frac{1}{2}$.

simultaneously submits a vote for A or B. Finally, the submitted votes are counted and the majority outcome is chosen.

There is a strictly increasing, convex and twice continuously differentiably *cost function* $c : [0, \frac{1}{2}] \to \mathbb{R}_+$ and when choosing precision x, the voter bears a cost $c(x)$ where $c(0) = 0$. There is a $d > 1$ such that $\lim_{x \to \infty} \frac{c'(x)}{x^{d-1}} \in \mathbb{R}$.[14] A strategy $\sigma = (x, \mu)$ of a voter consists of a function $x : [-1, 1]^2 \to [0, \frac{1}{2}]$ mapping types to signal precisions and a function $\mu : [-1, 1]^2 \times \{a, b\} \to [0, 1]$ where $\mu(t, s)$ is the probability that a voter of type t with signal s votes for A. I consider only non-degenerate strategies.[15] I analyze the Bayes-Nash equilibria of the Bayesian game of voters in symmetric strategies, henceforth called *equilibria*.

The next section analyzes the voting behavior of the types and derives a useful representation of the types that I will use from then on.

2.1   Preliminaries

In this section, I analyze the best response of the voters (Sections 2.1.2, 2.1.4, 2.1.5, and 2.1.7), introduce key objects of the analysis (Sections 2.1.1 and 2.1.3) and provide a useful representation of the voter types that I will use from then on (Section 2.1.6).

2.1.1   Aggregate Preferences

A central object of the analysis is the *aggregate preference function*

$$\Phi(p) := \Pr_H(\{t : p \cdot t_\alpha + (1-p) \cdot t_\beta \geq 0\}), \tag{1}$$

which maps a belief $p \in [0, 1]$ to the probability that a random type t prefers A under p. Figure 1 illustrates Φ: The dashed (blue) line corresponds to the

---

[14]It will be a direct insight from the preliminary results in the next section that without the condition $d > 1$, no voter acquires any information in equilibrium when n is sufficiently large; see in particular (18).

[15]A strategy σ is *degenerate* if $\mu(t, s) = 1$ for all (t, s) or if $\mu(t, s) = 0$ for all (s, t). When all voters follow the same degenerate strategy and there are at least three voters, if one voter deviates to any other strategy, the outcome is the same. So, the degenerate strategies with $x = 0$ are trivial equilibria.
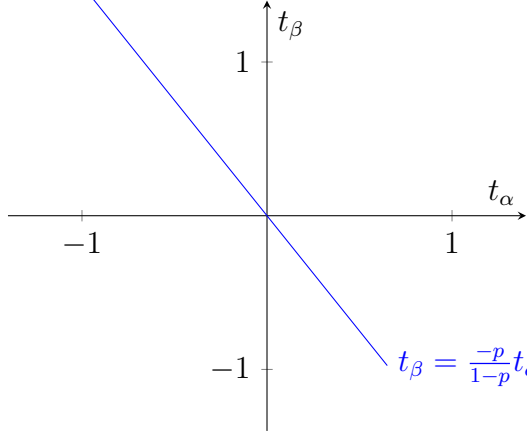
Figure 1: For any given belief $p = \Pr(\alpha) \in (0,1)$, the plane of types t with threshold of doubt $y(t) = p$ is $t_\beta = \frac{-p}{1-p}t_\alpha$.

set of types $t = (t_\alpha, t_\beta)$ that are indifferent between policy A and policy B when holding the belief p. Voters having types to the north-east prefer A given p. The indifference set has a slope of $\frac{-p}{1-p}$ and a change in p corresponds to a rotation of it. Given that H has a continuous density, Φ is continuously differentiable in p.

Voters having types t in the north-east quadrant prefer A for all beliefs and voters having types t in the south-west quadrant always prefer B (*partisans*). Voters having types t in the south-east quadrant prefer A in state α and B in β (*aligned voters*) and voters having types t in the north-west quadrant prefer B in state α and A in β (*contrarian voters*). I assume that

$$\Phi(0) < \frac{1}{2}, \text{ and } \Phi(1) > \frac{1}{2} \tag{2}$$

such that the median-voter preferred outcome is A in α and B in β. In particular, this excludes the cases with a majority of partisans for one policy in expectation. Henceforth, I call distributions H that have a continuous density and satisfy (2) simply *preference distributions*. In the following, the set of aligned types is denoted $L = \{t : t_\alpha > 0, t_\beta < 0\}$ and the set of contrarian

11

types is denoted $C = \{t : t_\alpha < 0, t_\beta > 0\}$.

2.1.2 Pivotal Voting and the Voter's Threshold of Doubt

How do the voters decide which alternative to vote for? To simplify the exposition, in the following, I only consider strategies σ where partisans use the (weakly) dominant strategy to vote for their preferred policy in the following.[16] This section shows that the aligned and contrarian voters follow a cutoff rule when deciding between A and B: what matters is if the posterior conditional on the information from the private signal and conditional on the event where the citizens vote decides the election exceeds a type-dependent *threshold of doubt*,

$$y(t) = \frac{-t_\beta}{t_\alpha - t_\beta}. \tag{3}$$

From the viewpoint of a given voter and given any strategy $\sigma' = (x', \mu')$ used by the other voters, the pivotal event, piv, is the event in which the realized types and signals of the other $2n$ voters are such that exactly n of them vote for A and n for B. For this event, if she votes A, the outcome is A; if she votes B, the outcome is B. In any other event, the outcome is independent of her vote. Thus, a strategy is optimal if and only if it is optimal conditional on the pivotal event. Take any two strategies $\sigma' = (x', \mu')$ and $\sigma = (x, \mu)$. For any type t of the given voter, and given the precision choice $x(t)$, let $\Pr(\alpha|s, \text{piv}; \sigma', n)$ be the posterior probability of α conditional on *being pivotal* and conditional on s when the other voters use σ'.

Lemma 1 *Take any strategy σ'. The function μ is part of a best response $\sigma = (x, \mu)$ to σ' if and only if for all aligned types $t \in L$, given the precision*

---

[16]In fact, for any non-degenerate strategy, I show that the likelihood of the pivotal event is non-zero such that voting for the preferred policy while not acquiring any information is the unique strict best respose.

$x(t)$,

$$\Pr(\alpha|s, \mathrm{piv}; \sigma', n) > y(t) \Rightarrow \mu(t, s) = 1, \qquad (4)$$

$$\Pr(\alpha|s, \mathrm{piv}; \sigma', n) < y(t) \Rightarrow \mu(t, s) = 0; \qquad (5)$$

and for all contrarian types $t \in C$, *given the precision* $x(t)$,

$$\Pr(\alpha|s, \mathrm{piv}; \sigma', \sigma, n) > y(t) \Rightarrow \mu(t, s) = 0, \qquad (6)$$

$$\Pr(\alpha|s, \mathrm{piv}; \sigma', \sigma, n) < y(t) \Rightarrow \mu(t, s) = 1, \qquad (7)$$

Proof. Given some x, the function μ is part of a best response if and only if for all $t = (t_\alpha, t_\beta)$ and for the signal precision $x(t)$,

$$\Pr(\alpha|s, \mathrm{piv}; \sigma', n) \cdot t_\alpha + (1 - \Pr(\alpha|s, \mathrm{piv}; \sigma', n)) \cdot t_\beta > 0 \Rightarrow \mu(s, t) = 1, \quad (8)$$

$$\Pr(\alpha|s, \mathrm{piv}; \sigma', n) \cdot t_\alpha + (1 - \Pr(\alpha|s, \mathrm{piv}; \sigma', n)) \cdot t_\beta < 0 \Rightarrow \mu(s, t) = 0, \quad (9)$$

that is, a voter supports A if the expected value of A conditional on being pivotal and s is strictly positive and supports B otherwise. Rearranging yields (4)- (7). ∎

Note that indifference holds only for a set of types that has zero measure since the type distribution H is continuous. For all other types, the best response is pure. It follows that there is no loss of generality to consider pure strategies with $\mu(t, s) \in \{0, 1\}$ for all (s, t).[17]

2.1.3 Vote Shares and the Likelihood of the Pivotal Event

Take any strategy $\sigma = (x, \mu)$ of the voters. The probability that a voter of random type votes for A in state $\omega \in \{\alpha, \beta\}$ is denoted $q(\omega; \sigma)$. A simple calculation shows that

$$q(\alpha; \sigma) = \int_{t \in [-1,1]^2} (\frac{1}{2} + x(t))\mu(t, a) + (\frac{1}{2} - x(t))\mu(t, b)dHt \qquad (10)$$

---

[17]When writing "*for all*," I ignore zero measure sets here and in the following.

and

$$q\left(\beta;\sigma\right) \;=\; \int_{t\in[-1,1]^2} (\frac{1}{2} - x(t))\mu(t,a) + (\frac{1}{2} + x(t))\mu(t,b)dHt. \qquad (11)$$

I refer to $q\left(\omega;\sigma\right)$ also as the *(expected) vote share* of A in ω. Since, conditional on the state, the type and the signal of a voter is independent of the types and the signals of the other voters, the probability of a tie in the vote count is

$$\Pr\left(\mathrm{piv}|\omega;\sigma,n\right) = \binom{2n}{n} \left(q\left(\omega;\sigma\right)\right)^n \left(1 - q\left(\omega;\sigma\right)\right)^n. \qquad (12)$$

I use a Stirling approximation of the binomial coefficient and (12) to obtain[18] [19]

$$\Pr\left(\mathrm{piv}|\omega;\sigma,n\right) \approx 4^n(n\pi)^{-\frac{1}{2}}\left[q(\omega;\sigma)(1 - q(\omega;\sigma))\right]^n. \qquad (13)$$

Note that, given (13), the likelihood of the pivotal event is non-zero for any (non-degenerate) strategy, but converges to zero since the function $q(1 - q)$ takes the maximum $\frac{1}{4}$ on $[0,1]$.

2.1.4 Choices of Informed Voters

Take any strategy σ' and let $\sigma = (x,\mu)$ be a best response to σ'. Since signal a is indicative of α and b of β, voters with a signal a believe state α to be more likely than voters with a signal b. In fact, given any $x > 0$, the psoteriors are ordered as

$$\Pr\left(\alpha|a,\mathrm{piv};\sigma,n\right) > \Pr\left(\alpha|b,\mathrm{piv};\sigma,n\right). \qquad (14)$$

---

[18]The notation $x_n \approx y_n$ describes that two sequences $(x_n)_{n\in\mathbb{N}}$ and $(y_n)_{n\in\mathbb{N}}$ are *asymptotically equivalent* in the following sense: $\lim_{n\to\infty} \frac{x_n}{y_n} = 1$.

[19]Stirling's formula yields $(2n)! \approx (2\pi)^{\frac{1}{2}} 2^{2n+\frac{1}{2}} n^{2n+\frac{1}{2}} e^{-2n}$ and $(n!)^2 \approx (2\pi) n^{2n+1} e^{-2n}$. Consequently, $\binom{2n}{n} \approx (2\pi)^{-\frac{1}{2}} 2^{2n+\frac{1}{2}} n^{-\frac{1}{2}} = 4^n(n\pi)^{-\frac{1}{2}}$.

14

To see why, note that the posterior likelihood ratio of the states conditional on a signal $s \in \{a, b\}$ and the event that the voter is pivotal is

$$\frac{\Pr(\alpha|s, \text{piv}; \sigma', n)}{\Pr(\beta|s, \text{piv}; \sigma', n)} = \frac{\Pr(\alpha)}{\Pr(\beta)} \frac{\Pr(\text{piv}|\alpha; \sigma', n)}{\Pr(\text{piv}|\beta; \sigma', n)} \frac{\Pr(s|\alpha; \sigma)}{\Pr(s|\beta; \sigma)}, \quad (15)$$

if $\Pr(\text{piv}|\beta; \sigma', n) > 0$, where I used the conditional independence of the types and signals of the other voters from the signal of the given voter. The order of the likelihood ratios follows from $\Pr(a|\alpha; \sigma) = \Pr(b|\beta; \sigma) = \frac{1}{2} + x$.

Suppose that a voter acquires information, i.e. $x(t) > 0$, and suppose that she votes for the same policy after both signals, i.e. $\mu(t, a) = \mu(t, b)$. Then, she would be strictly better off by not paying for the information and simply voting for this policy. Therefore, (4)-(7), and (14) together imply that aligned types that acquire information vote A only after a, and contrarian types that acquire information vote A only after b, that is

$$\forall t \in L : x(t) > 0 \Rightarrow \mu(t, a) = 1 \text{ and } \mu(t, b) = 0, \quad (16)$$

$$\forall t \in C : x(t) > 0 \Rightarrow \mu(t, a) = 0 \text{ and } \mu(t, b) = 1. \quad (17)$$

Let us turn to the question how much information voters acquire when they choose to acquire any at all. For any type t, a first-order condition pins down the equilibrium precision of the types that get informed. I derive the first-order condition in the Appendix; it equates marginal cost and marginal benefit,

$$c'(x) = \Pr(\text{piv}; \sigma', n) E_\omega(|t_\omega| | \text{piv}; \sigma', n). \quad (18)$$

Recall that a voter only benefits from better information in the pivotal event when her vote affects the outcome of the election. It turns out that, therefore, the marginal benefit of a higher signal precision is proportional to the likelihood of the pivotal event and the expected welfare at stake.[20] Using the implicit

---

[20]For example, the expected utility of an aligned type who chooses precision $x > 0$ is given by the expected utility, K, from all the events when her vote does not affect the outcome, by the cost, and by the expected utility from the pivotal event. The expected

function, I show that there is a unique solution to (18), denoted $x^*(t; \sigma', n)$, when n is large.

Lemma 2 *Take any strategy σ' and let the precision choice x be part of a best response. Then, when n is large enough,*

$$x(t) > 0 \Rightarrow x(t) = x^*(t; \sigma', n),$$

where $x^*(t; \sigma', n)$ is given by (18) and continuously differentiable in t.

2.1.5 Limit Vote Shares

I note that the posterior conditional on the pivotal event pins down the limit of the vote share in both states, as $n \to \infty$. To see why, recall that for any strategy sequence, the likelihood of the pivotal event converges to zero. Therefore, the first-order condition (18) implies that the precision of all types converges to zero uniformly. This implies $\lim_{n \to \infty} \Pr(\alpha|\text{piv}, s) - \Pr(\alpha|\text{piv}; \sigma_n^*) = 0$, given the equilibrium precision $x(t)$ of any type. Hence, given (4)-(6) the limit vote share of A is given by the share of voters preferring A given the belief $\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n)$, meaning that $\lim_{n \to \infty} q(\omega; \sigma_n^*) = \Phi(\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n^*))$. Using that Φ is continuous, I conclude:

Lemma 3 *Take any sequence of strategies $(\sigma_n)_{n \in \mathbb{N}}$. For all n, let $\hat{\sigma}_n$ be any best response to σ_n. For any $\omega \in \{\alpha, \beta\}$,*

$$\lim_{n \to \infty} q(\omega; \hat{\sigma}_n) = \lim_{n \to \infty} \Phi(\Pr(\alpha|\text{piv}; \sigma_n)). \tag{19}$$

The next section provides a useful representation of the voter types that I will use from then on.

---

utility from the pivotal event is

$$\Pr(\text{piv})\Big[t_\alpha \Pr(\alpha|\text{piv}; \sigma)(\frac{1}{2} + x) + t_\beta(\Pr(\alpha|\text{piv}; \sigma)(\frac{1}{2} - x)\Big].$$

To see why, note that when the citizen is pivotal and votes B, she receives utility of zero. When she is pivotal and votes A, she receives utility of t_ω, depending on the state. Given (4), it is optimal to vote A only after a; and she receives a in α with probability $\frac{1}{2} + x$ and in β with probability $\frac{1}{2} - x$. Taking derivatives, yields the first-order condition (18).

2.1.6   Voter-Type Representation

Voter types differ in their *(total) intensity*

$$k(t) = |t_\alpha| + |t_\beta|. \tag{20}$$

In fact, the total intensity and the threshold of doubt $y(t) = \frac{-t_\beta}{t_\alpha - t_\beta}$ relate one-to-one to the type if we restrict to either the aligned or the contrarian types. Precisely, for all aligned types $t \in L$,

$$t_\alpha = k(t)(1 - y(t)) \quad \text{and} \quad t_\beta = -k(t)y(t), \tag{21}$$

and for all contrarian types $t \in C$,

$$t_\alpha = -k(t)(1 - y(t)) \quad \text{and} \quad t_\beta = k(t)y(t). \tag{22}$$

From now on, I will sometimes slightly abuse the previous notation and use the notation $t = (y, k)$ for types, typically specifying if the type is aligned of contrarian, i.e. if $g = L$ or $g = L$. Note that $|t_\omega|$ and therefore the solution $x^*(t; \sigma, n)$ to the first-order condition (18) only depends on $y(t)$ and $k(t)$ and not on the group, aligned or contrarians, that the type belongs to.

2.1.7   Information Acquisition Regions

When deciding if to acquire information, voters trade off the cost of information with the benefits. The *critical types* t with $y(t) = \Pr(\alpha|\text{piv}; \sigma', n)$ are indifferent between A and B without further information, given (4) - (7). On the other hand, voters with extreme treshold of doubts $y(t) \approx 0$ or $y(t) \approx 1$ have strong preferences for one alternative without further information. Intuitively, these extreme types value information less than the critical types. The following result verifies this intuition and shows that only types with intermediate threshold of doubts acquire information, including the critical types. Second, intuitively, types with higher (total) intensities have more incentives to acquire information. The next result also shows that the interval around
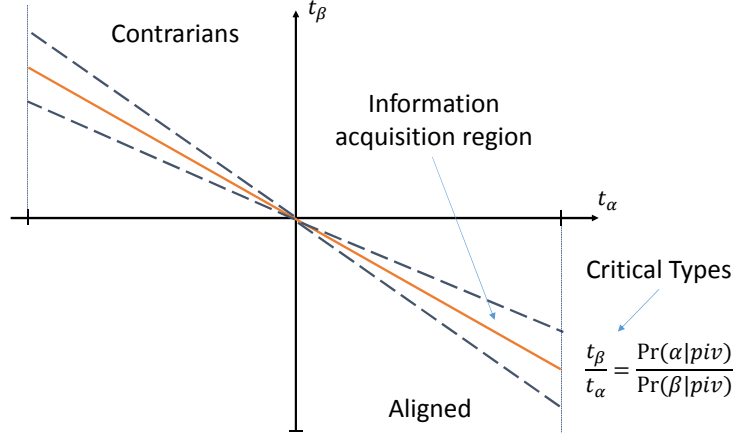
17

Figure 2: Information Acquisition regions of group g with boundaries the graphs of $\phi_g^-(k)$ and $\phi_g^+(k)$ (dashed lines).

the critical threshold $y(t) = \Pr(\alpha|\text{piv}; \sigma', n)$ of the types that get informed only depends on the (total) intensity k.

Lemma 4 *Let σ' be a strategy with $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma', n) \in (0,1)$. When n is large enough, for any $k \in (0,2)$ and any $g \in \{L, C\}$ there are $\phi_g^- < \Pr(\alpha|\text{piv}; \sigma, n) < \phi_g^+$ for such that for any best response $\sigma = (x, \mu)$ to σ' and any type $(y, k) \in g$,*

$$x(y,k) > 0 \iff y \in [\phi_g^-(k), \phi_g^+(k)], \tag{23}$$

Figure 2 illustrates the functions ϕ_g^- and ϕ_g^+ which limit the regions of types that acquire information under the best response. A later example illustrates the result (see Section 3.4). The proof is provided in the Appendix.

3   Voting Power and Welfare

In this section, I provide the main analysis, which describes the relation between the preference intensities, the information level of voter groups and

their voting power. For this, I consider a sequence of elections along which the electorate's size $2n + 1$ grows. For each n and a strategy σ_n, I calculate the probability that a policy $z \in \{A, B\}$ wins the support of the majority of the voters in state ω, denoted $\Pr(z|\omega; \sigma_n, n)$. I will be interested in the limits of $\Pr(z|\omega; \sigma_n^*, n)$ for equilibrium sequences $(\sigma_n^*)_{n \in \mathbb{N}}$. I am particularly interested in equilibrium sequences where citizens vote in an informed manner such that the election outcomes differ across the states,

$$\lim_{n \to \infty} \Pr(A|\alpha; \sigma_n, n) \neq \lim_{n \to \infty} \Pr(A|\beta; \sigma_n, n), \tag{24}$$

which I call *informative*. What will matter for the outcomes in informative equilibrium sequences is which voter group has the larger *power index*, defined as follows. For any $p \in [0, 1]$, any $g \in \{L, C\}$, let

$$W(g, p) = \Pr(g) f(p|t \in g) \mathrm{E}(k(t)^{\frac{2}{d-1}} | t \in g, y(t) = p), \tag{25}$$

where f is the density of the (conditional) distribution of the threshold of doubt. For any sequence of strategies $(\sigma_n)_{n \in \mathbb{N}}$ and any $g \in \{C, L\}$,

$$W(g, \hat{p}) \tag{26}$$

is the *power index of group g*, where $\hat{p} = \lim_{n \to \infty} \Pr(\alpha|\mathrm{piv}; \sigma_n, n)$. The power index is proportional to the likelihood of the critical types and the mean of the weighted intensities of the critical types as $n \to \infty$, and will be a relative measure of the electoral power of the voter group. Throughout, I will impose the following genericity condition on the preference distribution H,

$$\Phi(\Pr(\alpha)) \neq \frac{1}{2}, \tag{27}$$
$$W(L, p) \neq W(C, p) \text{ for any } p \in \Phi^{-1}(\frac{1}{2})$$

3.1 Result: Index Equilibrium Sequence

The main result shows: when information of low precision $x \approx 0$ is sufficiently cheap, which will be captured by a condition on the elasticity at zero, $\lim_{x \to 0} \frac{c'(x)x}{c(x)}$,[21] there is an informative equilibrium sequence where the outcome preferred by the group with the larger power index is elected with probability converging to 1 as $n \to \infty$.[22]

Theorem 1 *Let $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$. Take any preference distribution H satisfying (27). There is an informative equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$ satisfying*

$$\lim_{n \to \infty} \Pr(A|\alpha; \sigma_n^*, n) = \lim_{n \to \infty} \Pr(B|\beta; \sigma_n^*, n) = \begin{cases} 0 \ \ if \ W(L, \hat{p}) < W(C, \hat{p}), \\ 1 \ \ if \ W(L, \hat{p}) > W(C, \hat{p}). \end{cases} \quad (28)$$

where $\hat{p} = \lim_{n \to \infty} \Pr(\alpha|\mathrm{piv}; \sigma_n^*, n)$.

Section 3.2 discusses the welfare implications of Theorem 1. In Section 3.3, I explain the two central observations for the proof of Theorem 1. These observations describe the electoral power of the different types of voters and, imply another weaker result for *all* equilibria (Corollary 1). In Section 3.4, I give a detailed sketch of proof for Theorem 1 for a special case. Then, in Section 3.5, I sketch how the results for the special case carry over to the general case when the electorate is large, $n \to \infty$. A thorough proof of Theorem 1 is in Section B of the Appendix.

3.2 First-Best Outcomes

In this section, I study the welfare properties of the election. I describe the welfare effect of the disparate information acquisition of voters. For this note that policy A is welfare-maximizing in state ω if $\mathrm{E}_H(t_\omega) \geq 0$. The main result of this section is that, for a large class of settings, elections have equilibria that yield outcomes maximizing social welfare in each state, unless information cost

---

[21]The same condition appears in Martinelli (2006)'s model.
[22]The characterization of all equilibrium sequences in Section 5 shows the result of Lemma 1, in fact, extends to all informative equilibrium sequences.

are too extreme, that is, too low or too high.

I consider the following settings: I consider preference distributions for which the conditional distribution of the threshold of doubt, $F(\cdot|g)$ is independent of the voter group, i.e. for all $g \in \{L, C\}$,

$$F(\cdot|g) = F. \tag{29}$$

and the conditional distribution $J(\cdot|g)$ of the total intensities of types $t \in g$ is independent from G , that is, for all $g \in \{C, L\}$ and all $y \in [0, 1]$

$$J(\cdot|g, y(t) = y) = J(\cdot|g). \tag{30}$$

Clearly, information cost cannot screen the intensities of the partisans since all the partisans stay uninformed and simply vote their preferred policy. Therefore, I consider the settings where screening of the intensities of the partisans is obsolete: the welfare at stake for the A-partisans is, in expectation, of the same magnitude as the welfare at stake for the B-partisans, i.e. for all $\omega \in \{\alpha, \beta\}$,

$$\mathrm{E}_H(|t_\omega||\omega, \{t : t_\alpha > 0, t_\beta > 0\}) = \mathrm{E}_H(|t_\omega||\omega, \{t : t_\alpha < 0, t_\beta < 0\}). \tag{31}$$

One specific case of (31) is the situaton without partisans.

First-Best (Utilitarian) Outcomes Given the independence assumptions (29) - (31), for each state ω, the outcome preferred by the voter group, aligned or contrarians, with the larger likelihood weighted mean of the total intensities is the welfare-maximizing outcome; that is, the welfare-maximizing outcome is A in α and B in β if

$$\mathrm{Pr}_H(t \in L)\mathrm{E}_H(k(t)|t \in L) > \mathrm{Pr}_H(t \in C)\mathrm{E}_H(k(t)|t \in C) \tag{32}$$

and otherwise B in α and A in β. To see why, consider, for example, state α. By definition, A is the welfare-maximizing outcome if $E_H(t_\alpha) > 0$. Given (31) and given (21) and (22), this is equivalent to $\mathrm{Pr}_H(t \in L)\mathrm{E}(k(t)(1 - y(t))|t \in$

21

$L) - \Pr_H(t \in C)\mathrm{E}(k(t)(1 - y(t))|t \in C) > 0$. Given (29) and (30), this is equivalent to (32).

Theorem 2 *Take any preference distribution H satisfying (27) and (29)-(31). There is $\bar{d} > 3$ such that for any cost function, for which $\lim_{x \to 0} \frac{c'(x)x}{c(x)} \in (3, \bar{d})$, there is an equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$ for which the welfare-maximizing outcome is elected with probability converging to 1 as $n \to \infty$.*

Proof. First, note that Lemma 7 in the Appendix shows that $\lim_{x \to 0} \frac{c'(x)x}{x} = d$. Now, Theorem 1 implies Theorem 2 as follows. Given the independence assumptions (29) and (30),

$$
\begin{aligned}
W(L, \cdot) &> W(C, \cdot) \\
\Leftrightarrow \Pr(L)\mathrm{E}(k(t)^{\frac{2}{d-1}}|t \in L) &> \Pr(C)\mathrm{E}(k(t)^{\frac{2}{d-1}}|t \in C).
\end{aligned}
\tag{33}
$$

Comparing (32) and (33), for each preference distribution H satisfying the conditions of the lemma, there is an open set of elasticities $d > 3$ close to $d = 3$, and an equilibrium sequences for any d from this set such that the welfare-maximizing outcome is elected in each state. ∎

3.3 The Critical Observations

3.3.1 The Uninformed

The first observation is the following: if we would just consider the uninformed votes, the outcome of the election were random. Let

$$
\Pr(A|\omega; \sigma_n^*; \tilde{\pi}_n)
\tag{34}
$$

be the likelihood of outcome A in the game $\tilde{\pi}_n$ of $2n+1$ voters where a random citizen votes A with the same probability as a random uninformed voter given σ_n^*, i.e. with probability $\int_t \frac{1}{2}\Big[\mu(a, t) + \mu(b, t)\Big]dH(t|x(t) = 0)$.

Observation 1 *Take any preference distribution H satisfying (27). For any informative equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$, in each state $\omega \in \{\alpha, \beta\}$, the out-*

22

come implied by the vote share of the uninformed voters is random, i.e. $\Pr(A|\omega; \sigma_n^*; \tilde{\pi}_n) \in (0, 1)$.

The proof is provided in Section 5, where I finish the characterization of all equilibria. The observation is stated here already to provide guidance and intuition for the coming analysis, but not needed for the coming proofs.

What is key is that for an equilibrium sequence to be informative, some voters must acquire a relevant amount of information. For the voters to be willing to do so, the election must be sufficiently close to being tied such that the likelihood of being the pivotal voter is relatively large from the perspective of a single voter. In fact, it turns out that for any informative equilibrium sequence the *induced prior* converges to a belief p^*,

$$\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n^*) = p^*, \tag{35}$$

given which a randomly drawn type would vote exactly $50 - 50$, i.e. $\Phi(p^*) = \frac{1}{2}$.[23] Suppose for a second that $\Pr(\alpha|\text{piv}; \sigma_n^*) = p^*$. Then, without further information about the state, a random type prefers A with a likelihood of 50%, which suggests the result of Observation 1.[24] I prove (35) in Section B.1.1) of the Appendix.

3.3.2 The Partially Informed: Competition in Information Acquisition

Suppose that there is an informative equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$. Given Observation 1, the election is indeterminate when considering only the votes of the uninformed. Intuitively, the election will therefore be decided by the voters that get partially informed. The partially informed citizens vote for their prefered policy with a likelihood of $\frac{1}{2} + x(t)$, and therefore, shift the

---

[23]The term induced prior was introduced by Bhattacharya (2013b).

[24]There are two subtleties to the proof of Observation 1: first, one has to show that the convergence to p^* is sufficiently fast. Second, one has to show there are approximately as many citizens that get informed and would have voted A without further information and citizens that get informed and would have voted B without further information. Otherwise, an asymmetry might shift the conditional vote share of the uninformed below or above $\frac{1}{2}$.

vote share towards their prefered policy in each state. On the aggregate, the difference in the expected vote share for policy A across the states is

$$q(\alpha; \sigma_n^*) - q(\beta; \sigma_n^*) \quad = \quad 2\Big[\int_{t \in L} x(t)dH(t) - \int_{t \in C} x(t)dH(t)\Big]. \qquad (36)$$

where $\int_{t \in g} x(t)dH(t) = \Pr_H(g)\mathrm{E}_H(x(t)|t \in g)$ is the average precision acquired by a type of the group $g \in \{L, C\}$, weighted by the likelihood of a random type belonging to the group. We see that the order of the likelihood-weighted average precision determines the order of the vote shares in the states and therefore, in which state A is more likely to be elected.

The second critical observation for Theorem 1 is that there is a simple rule describing which group has the higher likelihood-weighted average precision: it is the group with the higher power index.

Observation 2 *Take any sequence of strategies* $(\sigma_n)_{n \in \mathbb{N}}$. *For all* n, *let* $\hat{\sigma}_n$ *be any best response to* σ_n. *If* $\hat{p} = \lim_{n \to \infty} \Pr(\alpha|\mathrm{piv}; \sigma_n, n) \in (0, 1)$ *the sequence of best responses satisfies*

$$\lim_{n \to \infty} \frac{\int_{t \in L} x(t)dH(t)}{\int_{t \in C} x(t)dH(t)} = \frac{W(L, \hat{p})}{W(C, \hat{p})}. \qquad (37)$$

I provide a sketch of the proof for in Section 3.5. The proof is in Section B.1 of the Appendix.

Intuition for Theorem 1. Note that the vote share of the informed is given by

$$\frac{1}{2} + \int_{t \in L} x(t)dH(t) - \int_{t \in C} x(t)dH(t). \qquad (38)$$

Hence, Observation 2 implies that the vote share of the informed voters is larger than 50% in α and smaller than 50% in β if and only if the power index of the aligned types is larger.[25] Given Observation 1, the uninformed votes split almost $50 - 50$ such that both observations together suggest that

25

any informative equilibrium sequence implements the outcome preferred by the group with the larger power index, when there is sufficiently much information acquisition overall, i.e. when information is sufficiently "cheap". Intuitively, this will be the case when the condition that is known from the literature (Martinelli (2006)) holds, i.e. when $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$, which is the one given in Theorem 1. Concluding, I have provided intuition for Theorem 1 except for the existence of informative equilibrium sequences in the first place.

3.4 Sketch of Proof: Uniform Types

3.4.1 Setting

Each voter is of an aligned type with probability $\lambda > \frac{1}{2}$ and of a contrarian type with probability $(1-\lambda)$ independently. The cost are polynomial $c(x) = x^d$ and the total intensity is the same for all aligned and contrarian voters respectively, but might depend on the voter group, that is $k(t) = k_L$ for all $t \in L$ and $k(t) = k_C$ for all $t \in C$. The prior belief about the state is uniform and the conditional distribution of the threshold of doubt is the same for the aligned and contrarian types respectively, denoted F, and uniform.

3.4.2 Symmetric Candidate Equilibrium

I claim that there is a *symmetric equilibrium*, meaning that the expected margin of victory is the same in both states, i.e. $q(\alpha; \sigma_n^*) - \frac{1}{2} = \frac{1}{2} - q(\beta; \sigma_n^*)$. This implies that the election is equally close to being tied in both states, and voters do not learn anything from the pivotal event, i.e.

$$\Pr(\alpha | \text{piv}) = \frac{1}{2}, \tag{39}$$

given (12). The expected utility of an aligned type who chooses precision $x > 0$ is given by the expected utility, K, from all the events when her vote does not affect the outcome, by the cost $c(x)$, and by the expected utility from

the pivotal event. The expected utility from the pivotal event is

$$\Pr(\text{piv})\left[t_\alpha \Pr(\alpha|\text{piv};\sigma)(\frac{1}{2}+x)+t_\beta(\Pr(\alpha|\text{piv};\sigma)(\frac{1}{2}-x)\right]. \tag{40}$$

To see why, note that when the citizen is pivotal and votes B, she receives utility of zero. When she is pivotal and votes A, she receives utility of t_ω, depending on the state. Given (4), she only votes A after a; and she receives a in α with probability $\frac{1}{2}+x$ and in β with probability $\frac{1}{2}-x$.

3.4.3 Equilibrium Conditions

Take any aligned type. Using that $\Pr(\alpha|\text{piv})=\frac{1}{2}$ in any symmetric candidate equilibrium, and using $t_\alpha = k(1-y)$ and $t_\beta = -ky$ (see (21)), the first-order condition, which equates marginal cost and marginal benefits, is

$$\Pr(\text{piv})\frac{k_L}{2}=c'(x). \tag{41}$$

The first-order condition pins down the precision $x_L^* > 0$ chosen by the aligned types that get partially informed; compare to Lemma 2 and (18). Which are these types? Using (21) and (40), the expected utility from an informed choice, with precision $x_L^* > 0$, is lower than the expected utility from an uninformed vote for B if $\Pr(\text{piv})\frac{1}{2}\left[(\frac{1}{2}+x)k(1-y)-(\frac{1}{2}-x)k_g y\right]-c(x) \geq 0$. This is equivalent to $\Pr(\text{piv})\frac{k}{2}\left[\frac{1}{2}(1-y)-\frac{1}{2}y\right]+\Pr(\text{piv})\frac{k}{2}x\left[(1-y)+y\right]-c(x)<0$. Using the first-order condition, this is equivalent to $c'(x_L^*)(\frac{1}{2}-y)+c'(x_L^*)x_L^*-c(x_L^*)<0$. Dividing by $c(x_L^*)$ yields $x^*(1-\frac{c(x_L^*)}{x_L^* c'(x_L^*)})<(y-\frac{1}{2})$. So, all types with

$$y > \frac{1}{2} + x_L^{**} \tag{42}$$

prefer B where $x_L^{**} = x_L^*(1-\frac{c(x_L^*)}{x_L^* c'(x_L^*)})$. Analogously, all types with

$$y < \frac{1}{2} - x_L^{**} \tag{43}$$

prefer A. A similar analysis shows that there is a precision $x_C > 0$ chosen by all the contrarian types that get partially informed. This precision is given by the first-order condition

$$\Pr(\text{piv}) \frac{k_C}{2} = c'(x), \tag{44}$$

and pins down the contrarian types that get partially informed. These are the types with a threshold of doubt

$$\frac{1}{2} - x_C^{**} \leq y \leq \frac{1}{2} + x_C^{**}, \tag{45}$$

where $x_C^{**} = x_C(1 - \frac{c(x_C)}{x_C c'(x_C)})$.

3.4.4 Fixed Point Argument

Using the symmetry of the signals, and that the distribution of the threshold of doubt y is uniform, (42), (43) and (45) together imply that the best response σ to any symmetric candidate equilibrium is symmetric, i.e. the vote shares satisfy $q(\alpha; \sigma) - \frac{1}{2} = \frac{1}{2} - q(\beta; \sigma)$. An application of Kakutani's fixed point theorem implies that there is a symmetric equilibrium. In this equilibrium, only the types $y \in [\phi_g^-(k), \phi_g^+(k)]$ get partially informed with $\phi_g^+(k) = \frac{1}{2} - x_g^{**}$ and $\phi_g^-(k) = \frac{1}{2} + x_g^{**}$.

3.4.5 On the Critical Observations

Observation 1. In the symmetric equilibrium of the example, $\Pr(\alpha|\text{piv}; \sigma_n^*) = \frac{1}{2}$ for all n, and $\Phi(\frac{1}{2}) = \frac{1}{2}$. This means, that without further information about the state, the election would be exactly tied in expectation and outcomes would be completely random. The vote of the uninformed is, split exactly $50 - 50$ in expectation: those aligned types critical of policy A, with threshold of doubt $y > \frac{1}{2} + x_L^{**}$, vote B, and those critical of policy B, with threshold of doubt $y < \frac{1}{2} - x_L^{**}$, vote A. Analogously, for the contrarian types. This implies Observation 1 for the symmetric equilibrium of the special case with uniform

27

types.

Observation 2. The types that get partially informed are those for which $y(t) \in [\frac{1}{2} - x_g^*(1 - \frac{c(x_g^*)}{x_g^* c'(x_g^*)}), \frac{1}{2} + x_g^*(1 - \frac{c(x_g^*)}{x_g^* c'(x_g^*)})]$. I conclude,

$$
\begin{aligned}
\lim_{n \to \infty} \frac{\int_{t \in L} x(t) dH(t)}{\int_{t \in C} x(t) dH(t)} &= \lim_{n \to \infty} \frac{\Pr(L)}{\Pr(C)} \frac{(x_L^*)^2 (1 - \frac{c(x_L^*)}{x_L^* c'(x_L^*)})}{(x_C)^2 (1 - \frac{c(x^*)}{x_g^* c'(x^*)})} \\
&= \frac{\Pr(L)}{\Pr(C)} \frac{(x_L^*)^2}{(x_C)^2},
\end{aligned}
\tag{46}
$$

where, for the last line, I used $\frac{c(x)}{x c'(x)} = \frac{1}{d}$, given that $c(x) = x^d$. The first-order condition (41) pins down the ratio of the precision of aligned and contrarians, $\frac{x_L^*}{x_C^*} = (\frac{k_L}{k_C})^{\frac{1}{d-1}}$. Finally, the definition of the power indices $W(g)$, i.e. (26), together with (46) implies (37) for the symmetric equilibrium of the special case with uniform types.

Given the symmetry of the equilibrium, the margin of victory in each state is $| \int_{t \in L} x(t) dH(t) - \int_{t \in C} x(t) dH(t)$. When information is sufficiently "cheap", outcomes of the election will be determinate in each state. Intuitively, this will be the case when the condition that is known from the literature (Martinelli (2006)) holds, i.e. when $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$, which is the condition in Theorem 1.

3.5 Sketch of Proof: General Case

In this section, I sketch the proof of Observation 2 for the general case. The analysis follows along similar lines as in the special case with uniform types. However, one has to show that the order of several types of asymmetric behaviour is sufficiently small such that the results from the example carry over asymptotically as $n \to \infty$. Before I do this, a preliminary remark: note that the boundaries $\phi_g^+(k)$ and $\phi_g^-(k)$ of the information acquisition regions are

implicitly determined by

$$\frac{1}{2} + x^{**}(\phi_g^-(k), k) = \frac{\Pr(\beta|\text{piv})\phi_g^-(k)}{\Pr(\alpha|\text{piv})(1 - \phi_g^-(k)) + \Pr(\beta|\text{piv})\phi_g^-(k)}, \quad (47)$$

$$\frac{1}{2} - x^{**}(\phi_g^+(k), k) = \frac{\Pr(\beta|\text{piv})\phi_g^+(k)}{\Pr(\alpha|\text{piv})(1 - \phi_g^+(k)) + \Pr(\beta|\text{piv})\phi_g^+(k)}, \quad (48)$$

where $x^{**}(y, k; \sigma, n) = x^*(y, k; \sigma, n)(1 - \frac{c(x^*(y,k;\sigma,n))}{x^*(y,k;\sigma,n)c'(x^*(y,k;\sigma,n))})$ is defined as in the uniform types case and $x^*(y, k; \sigma, n)$ is the solution to the first-order condition (18); see for example the proof of Lemma 4 in Section A of the Appendix. Note that the conditions (47) and (48) reduce to the conditions (42), (43) and (45) from the uniform types example when $\Pr(\alpha|\text{piv}; \sigma_n^*) = \frac{1}{2}$.

Now, first, even when fixing the total intensity, different types choose different precisions in general. I show that this heterogeneity is sufficiently small and the information precision of all types that get partially informed is well approximated by the precision of the critical types, which satisfy $y = \Pr(\alpha|\text{piv}; \sigma_n, n)$. Using a Taylor approximation, for all k,

$$x(y, k) > 0 \Rightarrow \frac{x(y, k)}{x(\Pr(\alpha|\text{piv}; \sigma_n, n), k)} \approx 1. \quad (49)$$

The basic intuition is that only types arbitrarily close to the critical types acquire information when n is growing large. The second insight comes from a close inspection of the first-order condition (18). Changes in the marginal benefit on the right hand side due to changes in the threshold of doubt y are of an order of the marginal cost $c'(x(y, k))$. These changes relate to infinitely smaller changes in the equilibrium precision on the left hand side since the second derivative $c''(x(y, k))$ of the cost function is much larger than the first derivative $c'(x(y, k))$ when the precision is small.[26]

Second, the information acquisition region might be asymmetric around the critical types with $y = \Pr(\alpha|\text{piv}; \sigma_n, n)$. I show that this asymmetry is of a

---

[26]Consider the example of polynomial cost $c(x) = x^d$. Then $\lim_{x \to 0} \frac{c''(x)}{c'(x)} = \lim_{x \to \infty} \frac{d-1}{x} = \infty$.

sufficiently small order. Similar to the uniform types example,

$$\frac{x(\hat{y},k)(1 - \frac{c(x(\hat{y},k))}{c'(x(\hat{y},k))x(\hat{y},k)})}{\left[\phi_g^-(k) - \Pr(\alpha|\text{piv};\sigma_n,n)\right]} \approx \frac{x(\hat{y},k)(1 - \frac{c(x(\hat{y},k))}{c'(x(\hat{y},k))x(\hat{y},k)})}{\left[\Pr(\alpha|\text{piv};\sigma_n,n) - \phi_g^+(k)\right]} \approx K, \qquad (50)$$

where K is a constant that only depends on $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n,n) = \hat{p}$. To show this, I consider the linear (Taylor) approximations at $\hat{y}_n = \Pr(\alpha|\text{piv};\sigma_n^*)$ of the left hand side, $\chi(y)$, of the equations (47) and (48) that define the boundaries $\phi_g^+(k)$ and $\phi_g^-(k)$, . Since the left hand side takes the value $\frac{1}{2}$ at $\hat{y}_n$, this yields $\chi'(\hat{y}_n)\left[\phi_g^+(k) - \hat{y}_n)\right] \approx x^{**}(\phi_g^+(k))$ and $\chi'(\hat{y}_n)\left[\hat{y}_n - \phi_g^-(k)\right] \approx x^{**}(\phi_g^+(k))$.[27] The intuition here is again that only types arbitrarily close to the critical types acquire information when n is large such that the boundaries are arbitrarily close to $\hat{y}$ and the linear approximation is asymptotically precise. Finally, (50) follows from (49) and the Taylor approximations of (47) and (48).

Third, when the distribution of the thresholds is not uniform, the linear (Taylor) approximation of the distribution around the critical types is asymptotically precise to describe the mass of types that acquire information,

$$\frac{f(\hat{y},k|k'=k,g)\left[\phi^g(k) - \psi^g(k)\right]}{F(\psi^g(k)|k'=k,g) - F(\phi^g(k)|k'=k,g)} \approx 1. \qquad (51)$$

where $F(-|k'=k,g)$ is the conditional distribution of the thresholds and $f(-|k'=k,g)$ its density.

---

[27]The approximation errors are of an order $\left[\phi_g^-(k) - \hat{y}_n\right]^2$ and $\left[\phi_g^+(k) - \hat{y}_n\right]^2$ respectively. Since only types arbitrarily close to the critical types acquire information as $n \to \infty$, it holds $\phi_g^-(k) \overset{n\to\infty}{\to} \phi_g^+(k)$ for all $k > 0$. Hence, the error terms are negligible.

Finally, given (49), (51), and (50),

$$\int_{t \in g} x(t) dH(t)$$

$$\approx \int_k f(\hat{y}_n, k | k' = k, g) 2K x(\hat{y}_n, k)^2 (1 - \frac{c(x(\hat{y}_n, k))}{c'(x(\hat{y}_n, k)) x(\hat{y}_n, k)}) dJ(k|g)$$

$$= \Pr(g) f(\hat{y}_n | g) \mathrm{E}_k (x(y, k)^2 (1 - \frac{c(x(y, k))}{c'(x(y, k)) x(y, k)}) | y = \hat{y}_n, g). \qquad (52)$$

where $J(\cdot|g)$ is the conditional distribution of the total intensity of the types of group g. Thus,

$$\lim_{n \to \infty} \frac{\int_{t \in g} x(t) dH(t)}{\Pr(g) f(\hat{p}|g) \mathrm{E}_k \left[x(y, k)^2 | y = \hat{p}, g \right] \frac{2K(d-1)}{d}} = 1 \qquad (53)$$

where I used the notation $\hat{p} = \lim_{n \to \infty} \Pr(\alpha | \mathrm{piv}, \sigma_n^*) = p^*$ and, second, that $(1 - \frac{c(x(y,k))}{c'(x(y,k)) x(y,k)}) \to \frac{d-1}{d}$ as $n \to \infty$, which follows from Lemma 7 in the Appendix and since $x(y, k) \to 0$ for all (y, k). The first-order condition (18) together with $\lim_{x \to 0} \frac{c'(x)}{x^{d-1}} \in \mathbb{R}$ implies $x(\hat{p}, k) \approx k^{\frac{1}{d-1}} x(\hat{p}, 1)$ for all $k > 0$. Thus, it follows from (53) that the ratio of the power indices $W(g)$ defined by (26) is asymptotically equivalent to the ratio of the likelihood-weighted average precision $\int_{t \in g} x(t) dH(t)$, that is (37) holds.

Given Observation 2, whenever the contrarians have a larger power index, policy A is more likely to be elected in β than in α. Clearly, this implies that the median-voter preferred outcome is less likely to be elected in one of the states, since the median voter prefers A only in α.

Corollary 1 *Let $W(L, p) < W(C, p)$ for any $p \in [0, 1]$ with $\Phi(p) = \frac{1}{2}$. For any equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$, there is a state $\omega \in \{\alpha, \beta\}$ where the median-voter preferred outcome is less likely to be elected as $n \to \infty$, i.e.*

$$\lim_{n \to \infty} \Pr(A|\alpha; \sigma_n^*, n) \leq \frac{1}{2} \quad or \quad \lim_{n \to \infty} \Pr(B|\beta; \sigma_n^*, n) \leq \frac{1}{2}. \qquad (54)$$

Proof. In the Appendix, Lemma 8 (a corollary of the proof of (35) shows that

any equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$ satisfies $\lim_{n \to \infty} \Pr(\alpha | \text{piv}; \sigma_n^*, n) \in \{\Pr(\alpha), \Phi^{-1}(\frac{1}{2})\}$. Since $\Phi(0) \neq \frac{1}{2}$, $\Phi(1) \neq \frac{1}{2}$, and $\Pr(\alpha) \in (0, 1)$ by assumption, the corollary follows from Lemma 2. ∎

4 Determinants of Elections

4.1 The Degree of Political Conflict

In this section, consider the settings witout partisans,

$$\Pr(L) = 1 - \Pr(C) > 0, \tag{55}$$

mainly for the simplicity of exposition. Theorem 2 shows that information cost can screen intensities perfectly and this implies that the contrarians, which represent a minority in expectation, might win the election. Conversely, without information cost, all voters get perfectly informed and elections choose the median-voter preferred outcome in each state.

When the welfare at stake for both groups is very similar, screening becomes more difficult. Intuitively, this benefits the group of the aligned voters, which is larger in expectation. For any preference distribution H satisfying (29) - (57) and $\Pr(C) > 0$, the *degree of conflict* is

$$C(H) = \left[\left| \frac{\Pr(L)}{\Pr(C)} \frac{\mathrm{E}(k(t) | t \in L)}{\mathrm{E}(k(t) | t \in C)} - 1 \right| \right]^{-1}. \tag{56}$$

The degree of conflict is larger, when the welfare at stake for the voter groups, aligned and contrarians, is more similar in expectation, thereby capturing how contested the election is.

The next result verifies the intuition and shows that the election being more contested is typically in the interest of the group that is larger in expectation. For this result, I focus on the informative equilibrium sequence of Lemma 1[28] and preference distributions such that all voters of the same group g share the

---

[28]The later characterization of all equilibrium sequences in Section 5 shows that, generically, there are no other informative equilibrium sequences when $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$.

same (total) intensity $k_g > 0$, that is, for all $g \in \{C, L\}$ there is a $k_g > 0$ such that

$$t \in g \Rightarrow k(t) = k_g. \tag{57}$$

I show that, when the degree of conflict is arbitrarily large, the outcome preferred by the contrarians, which are a minority in expectation, is not elected, except for an arbitrarily small set of cost functions.

Theorem 3 *There is a function $\epsilon : \mathbb{R}^{>0} \to \mathbb{R}^{>0}$ with $\lim_{\kappa \to \infty} \epsilon(\kappa) = 0$ such that the following holds. Take $\kappa > 0$ and any preference distribution H with $C(H) > \kappa$, and satisfying (29), (55) (57), and $W(L,p) \neq W(C,p)$ for any $p \in [0,1]$ with $\Phi(p) = \frac{1}{2}$. Then, for a measure $1 - \epsilon(\kappa)$ of cost functions with $\lim_{x \to \infty} \frac{c'(x)x}{c(x)}$, there is an equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$ such that*

$$\lim_{n \to \infty} \Pr(A|\alpha; \sigma_n^*) = \lim_{n \to \infty} \Pr(B|\beta; \sigma_n^*) = 1, \tag{58}$$

Proof. Fix the elastictiy $d = \lim_{x \to \infty} \frac{c'(x)x}{c(x)} > 3$ and the preference distribution. Take any equilibrium sequence $(\sigma_n^*)_{n \in \ltimes}$ and let $\hat{p} = \lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n)$. Suppose that $\frac{W(L,\hat{p})}{W(C,\hat{p})} < 1$. Given (29) and (57),

$$\frac{W(L,\hat{p})}{W(C,\hat{p})} < 1 \Leftrightarrow \left[\frac{k_L}{k_C}\right]^{\frac{2}{d-1}} < \frac{\Pr(C)}{\Pr(L)} \tag{59}$$

Given the definition (56), $\kappa \to \infty$ implies $\left[\frac{k_L}{k_C}\right] \to \frac{\Pr(C)}{\Pr(L)}$. Since $0 < \frac{\Pr(C)}{\Pr(L)} < 1$, for any κ large enough, the ratio $\left[\frac{k_L}{k_C}\right]^{\frac{2}{d-1}}$ is strictly increasing in d and the derivative has a positive lower bound.[29] Thus, for κ arbitrarily large, $\frac{W(L,\hat{p})}{W(C,\hat{p})} < 1$ can only hold for elasticities d arbitrarily close to $d = 3$. Consequently, Lemma 1 implies the theorem. ∎

---

[29] Calculation shows $\frac{\partial a^{\frac{2}{d-1}}}{\partial d} = -\frac{2a^{\frac{2}{d-1}} log(a)}{(d-1)^2}$ which is strictly bounded above zero when $0 < a < 1$.

4.2 Polarization of Preferences

This section argues that groups of voters that share common interests are less likely to win an election when the preference intensities vary more strongly across the voters of the group.

For any preference distribution H and $g \in \{L, C\}$, let F_H^g be the conditional distribution of the threshold of doubt $y(t)$ of the types $t \in g$, and let J_H^g be the conditional distribution of the (total) intensities $k(t)$ of the types $t \in g$. A distribution H is a *g-intensity spread of H* if

$$H(-|t \in g') \quad = \quad H'(-|t \in g') \text{ for } g' \neq g \in \{C, L\}, \tag{60}$$

$$F_H^g \quad = \quad F_{H'}^g, \tag{61}$$

$$J_H^g \quad <_{\text{mps}} \quad J_{H'}^g, \tag{62}$$

where (62) means that $J_{H'}^g$ is a mean-preserving spread of J_H^g.

The next result shows that a voter group acquires less information relative to other groups when the preference intensitities are more dispersed within the group. In other words, when the intensities are more polarized, this aggravates the free-rider problem of the group relative to the free-rider problem of the other groups. Given Lemma 2, the ratio $\frac{W_H(L,\hat{p})}{W_H(C,\hat{p})}$ of the power indices of the voter groups pins down the ratio of the equilibrium amount of information acquired by the voter groups, where for clarity I added the preference distribution as a subscript.

Lemma 5 *Let* $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$. *Let* $g \in \{C, L\}$. *Take preference distributions* H, H' *satisfying* (29) - (31). *Take any equilibrium sequence* $(\sigma_n^*)_{n \in \mathbb{N}}$ *given* H *and any equilibrium sequence* $(\hat{\sigma}_n)_{n \in \mathbb{N}}$ *given* H' *such that* $\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n) = \lim_{n \to \infty} \Pr(\alpha|\text{piv}; \hat{\sigma}_n, n)$. *If* H' *is a g-intensity spread of* H, *for* $g' \neq g \in \{C, L\}$,

$$\frac{W_{H'}(g, \hat{p})}{W_{H'}(g', \hat{p})} < \frac{W_H(g, \hat{p})}{W_H(g', \hat{p})}. \tag{63}$$

The proof is in Section B of the Appendix.

I use the previous results of Lemma 2 and Lemma 1 that characterize equilibrium outcomes depending on the order of the power indices and show that when the intensities within a voter group are sufficiently dispersed, there is no equilibrium sequence where the outcome preferred by the group is more likely to be elected in both states. Second, there is an equilibrium sequence where the outcome that is preferred by the group is *never* elected as $n \to \infty$.

Theorem 4 *Let* $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$. *Let* $g = L$ ($g = C$). *Take any preference distribution* H *satisfying* (29) - (31). *There is a g-intensity spread* H' *of* H *such that:*

1. *For all equilibrium sequences* $(\sigma_n^*)_{n \in \mathbb{N}}$ *given* H',

$$\lim_{n \to \infty} \Pr(A|\alpha; \sigma_n^*, n) \leq (\geq)\frac{1}{2} \quad or \quad \lim_{n \to \infty} \Pr(B|\beta; \sigma_n^*, n) \leq (\geq)\frac{1}{2}.$$

2. *There is an equilibrium sequence* $(\sigma_n^*)_{n \in \mathbb{N}}$ *given* H' *such that*

$$\lim_{n \to \infty} \Pr(A|\alpha; \sigma_n^*, n) = \lim_{n \to \infty} \Pr(B|\beta; \sigma_n^*, n) = 0(1).$$

The proof is in Section B of the Appendix.

5 All Equilibria

5.1 Tyranny of the Uninformed Majority

The two-type example in the introduction illustrated how information acquisition can be complementary in election settings and how this complementarity entails a multiplicity of equilibria.

In this section, I show that, generically, there is an equilibrium sequence where the outcomes are as if all voters would have no access to further information about the state when the electorate is large. The theorem implies that when signals of low precision are sufficiently cheap, i.e. when $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$,

generically, there are multiple types of equilibrium sequences: an informative equilibrium sequence (see Lemma 1) and an equilibrium sequence that is not informative.

Theorem 5 *Let $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$. There exists an equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$ for which*

$$\lim_{n \to \infty} \Pr(A|\alpha; \sigma_n^*, n) = \lim_{n \to \infty} \Pr(A|\alpha; \sigma_n^*, n) = \begin{cases} 1 & \text{if } \Phi(\Pr(\alpha)) > \frac{1}{2}, \\ 0 & \text{if } \Phi(\Pr(\alpha)) < \frac{1}{2}, \end{cases} \tag{64}$$

Proof. Recall that equilibrium can be alternatively characterized in terms of the vector of the expected vote shares of outcome A in state α and β, i.e. (121). Let

$$\mathrm{B}_\delta = \{\mathbf{q} = (q(\alpha), q(\beta)) : |\mathbf{q} - (\Phi(\Pr(\alpha)), \Phi(\Pr(\alpha)))| < \delta. \tag{65}$$

Given $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$, there is $\delta > 0$ small enough such that the vote shares in B_δ are all larger than $\frac{1}{2}$ or all smaller than $\frac{1}{2}$. Hence, it follows from (13) that the likelihood of the pivotal event is exponentially small, given any such $\mathbf{q}$. The verbatim argument of the proof of (35) implies (91), i.e.

$$\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \mathbf{q}, n) = \Pr(\alpha). \tag{66}$$

The intuition is that the precision of each voter is exponentially small since the value of information is proportional to the likelihood of the pivotal event. Consequently, the difference of the vote shares in the two states is exponentially small, which implies that that the pivotal event contains no information about the relative probability of α and β as the electorate grows large. Consequently, voting according to the prior belief is optimal as the electorate grows large. Now, take any $\mathbf{q} \in \mathrm{B}_\delta$. Lemma 3 together with (66) implies that the vote shares of $\sigma^{\mathbf{q}}$ are again in $\mathbf{B}_\delta$ when n is large enough, i.e. $\mathbf{q}(\sigma^{\mathbf{q}}) \in \mathrm{B}_\delta$. An application of Kakutani's fixed point theorem shows that there is a sequence of equilibrium vote shares $(\mathbf{q}_n^*)_{n \in \mathbb{N}}$, i.e. vote shares satisfying (121), and for

36

all $\omega \in \{\alpha, \beta\}$,

$$\lim_{n\to\infty} q_n^*(\omega) = \Phi(\Pr(\alpha)). \tag{67}$$

The theorem follows from the weak law of large numbers, given $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$.
∎

I state a corollary of the proof of Theorem 5 that will be useful for the characterization of all equilibrium sequences in Section 5.2.

Corollary 2 *Take any equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$. *Either* $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n) = \Pr(\alpha)$ *or* $\Phi(\hat{p}) = \frac{1}{2}$ *for* $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n)$.

5.2 Characterization Result

In this section, I finish the characterization of equilibrium sequences.

Theorem 6 *Take any preference distribution H satisfying $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$ and $W(L, p) \neq W(C, p)$ for all $p \in \Phi^{-1}(\frac{1}{2})$.*

1. *If $\lim_{x\to 0} \frac{c'(x)x}{c(x)} < 3$, all equilibrium sequences satisfy (64).*

2. *If $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$, there are three types of equilibrium sequences. There is an equilibrium sequence satisfying (28). There is an equilibrium satisfying (64) , and there is an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfying*

$$\lim_{n\to\infty} \Pr(A|\alpha; \sigma_n^*, n) = \lim_{n\to\infty} \Pr(B|\beta; \sigma_n^*, n)$$

$$= \begin{cases} 0 & \text{if } \text{sgn}(W(L, \hat{p}) - W(C, \hat{p})) \neq \text{sgn}(\Pr(\alpha)) - \hat{p} \\ 1 & \text{if } \text{sgn}(W(L, \hat{p}) - W(C, \hat{p})) = \text{sgn}(\Pr(\alpha)) - \hat{p}. \end{cases} \tag{68}$$

where $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n)$. Any equilibrium sequence satisfies either (28), (68) or (64).

Let $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$. Then, Theorem 1 establishes the existence of an equilibrium sequence satisfying (28). Theorem 5 establishes the existence of an equilibrium sequence satisfying (64). I prove another theorem in the Appendix, Theorem 9, that establishes the existence of an equilibrium sequence satisfying (68). The remaining parts of the proof of Theorem 6 are in Section D of the Appendix.

Using the characterization of equilibrium sequences, I prove Observation 1 in Section D of the Appendix.

6 Extensions

6.1 Heterogenous Cost

In this section, I show that cost of information and preference intensities are strategically equivalent such that the previous results extend to a setting where the information cost of the voters are heterogenous. I show, in particular, that there are equilibria with first-best outcomes when the heterogeneity of the cost is independent of the ordinal preference type of the voters.

Let the information cost of the voters depend on a private type γ. For a given cost function c, a voter of *effort type* γ pays $c(\gamma, x) = \gamma c(x)$ for a signal of precision x. The effort type γ is distributed independently and identically across voters according to some distribution G, with density g, the *effort type distribution*, and independently of the the signals of the voters and the preference types of the other voters. The support of G is bounded below by a strictly positive constant.

The model with heterogenous information cost is equivalent to a model with homogenous cost and a suitable distribution of preferences since the best response of an aligned or contrarian voter with effort type γ, total intensity k and threshold of doubt y is the same as that of the voter with effort type $\gamma' = 1$, total intensity $\frac{k}{\gamma}$ and threshold of doubt y, given the characterization of the best response, (4)-(7), (18), (47) and (48).

Consider the settings with heterogenous cost where the effort type is inde-

pendent of the voter's own preference type, that is $G \perp\!\!\!\perp H$, and where the distribution H of the preference types satisfies (29) - (31). Then, the corresponding preference distribution with homogenous cost types satisfies (29) - (31) also. As a consequence, Theorem 2 extends to settings with heterogenous information cost. With the appropriate extension of the definitions to the heterogenous cost model:

Theorem 7 *Take any preference distribution H that satisfies (29)- (31) and $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$, and for which $W(L,p) \neq W(C,p)$ for any $p \in [0,1]$ with $\Phi(p) = \frac{1}{2}$. Take any effort type distribution G that is independent of H. There is an open set of cost functions $c(\cdot)$ and an equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$ given c for which the welfare-maximizing outcome is elected with probability converging to 1 as $n \to \infty$.*

I conjecture that theorem 7 extends to settings where the distribution of effort types has a support with lower end $\gamma = 0$, and more generally, that the equilibrium representation through power indices (as in Lemma 2) extends, given effort types that are independent of the ordinal preference types: for any fixed γ, certainly, the ratio of the power indices captures the ratio of the information acquired. But then, this remains true, when integrating over γ.

6.2   Rich vs Coarse Choice of Information Quality

In this section, I illustrate that the 'richness' of the informational choice set of the voters matters. For this, I compare the previous results of the model where voters choose the quality of their information from a continuous set to a setting where the informational choice of the citizens is coarse. Interestingly, in the setting with a coarse information choice, there are *always* equilibrium sequences where, in each state, the welfare-maximizing outcome is elected, whereas in the continuous setting this depends on the degree of conflict κ, that is how similar the welfare at stake is for voters that gain from policy A and those that are harmed by it; see Section 4.1. This finding suggests that the richness of the informational choice present in modern societies might sometimes be harmful to social welfare. This is particularly surprising, since a richer

choice set for the voters should intuitively facilitate the screening of intensities.

Binary Precision Setting. A random voter is of an aligned type with probability $\lambda = \Pr(L) > \frac{1}{2}$, and of a contrarian type with probability $1 - \lambda = \Pr(C)$. All types share a common threshold of doubt $y(t) = \frac{1}{2}$. Each voter can choose an uninformative signal at no cost or a binary, symmetric signal of precision $\frac{1}{2} + x$ with $x > 0$ at a cost $c \geq 0$, $\Pr(a|\alpha) = \Pr(b|\beta) = \frac{1}{2} + x$. The cost c are drawn independently and identically across voters from the uniform distribution and are the private information of the voter. The cost of a voter is independent of the (preference) types and the signals of the voters. The common prior of the voters is uniform.

Now, take any symmetric strategy σ_n, meaning that $|q(\alpha; \sigma_n) - \frac{1}{2}| = |q(\beta; \sigma_n) - \frac{1}{2}|$. Given (12), this implies that $\Pr(\alpha|\text{piv}; \sigma_n, n) = \frac{1}{2}$. The expected utility from the pivotal event of an aligned voter who chooses the informative signal is

$$\Pr(\text{piv})\left[k_L \Pr(\alpha|\text{piv}; \sigma)(\frac{1}{2} + x) - k_L(\Pr(\alpha|\text{piv}; \sigma)(\frac{1}{2} - x)\right] - c \quad (69)$$

$$= \Pr(\text{piv})\frac{k_L}{2}x - c. \quad (70)$$

To see why, note that when the citizen is pivotal and votes B, she receives utility of zero. When she is pivotal and votes A, she receives utility of k_L or $-k_L$, depending on the state. Note that Lemma 16 extends to this setting. Given Lemma 4, she only votes A after a; and she receives a in α with probability $\frac{1}{2} + x$ and in β with probability $\frac{1}{2} - x$.

Thus, it is optimal for an aligned voter to choose the informative signal if and only if $c \leq \Pr(\text{piv})\frac{k}{2}x$. Given that the cost are uniformly distributed, the likelihood-weighted mean precision of the aligned types is

$$\int_{t \in L} x(t)dH(t) = \int_{t \in L} \frac{k(t)}{2}\Pr(\text{piv})xdH(t)$$
$$= \Pr(\text{piv})x\lambda E_H(k(t)|t \in L). \quad (71)$$

Similarly, the likelihood-weighted mean precision of the contrarian types is

$$\int_{t \in C} x(t) dH(t) \;=\; \Pr(\text{piv}) x (1 - \lambda) \mathrm{E}_H(k(t)|t \in C). \tag{72}$$

When the types that choose the uninformative signal vote $50 - 50$ for policy A and B, the vote shares of the best response are symmetric to $\frac{1}{2}$. An application of Kakutani's fixed point theorem implies that, for all n, there is a symmetric equilibrium σ_n^*. In this equilibrium, the vote shares are ordered as follows,

$$q(\alpha; \sigma_n^*) \;>\; q(\beta; \sigma_n^*) \tag{73}$$

$$\Leftrightarrow \lambda \mathrm{E}_H(k(t)|t \in L) \;>\; (1 - \lambda) \mathrm{E}_H(k(t)|t \in C). \tag{74}$$

Note that the preference distribution satisfies (30) and (29) such that the welfare-maximizing outcome is given by (32). Thus, (73) implies that the welfare-maximizing policy is more likely to be elected in each state. The next result shows that the welfare-maximizing outcome is elected with probability converging to 1 in each state as the electorate grows large.

Theorem 8 *Consider the setting with a binary choice of the signal precision. For any preference distribution H with $(1 - \lambda)\mathrm{E}_H(k(t)|t \in C) \neq \lambda \mathrm{E}_H(k(t)|t \in L)$, there is an equilibrium sequence $(\sigma_n)_{n \in \mathbb{N}}$, for which the welfare-maximizing outcome is elected with probability converging to 1.*

Proof. Consider a sequence $(\sigma_n)_{n \in \mathbb{N}}$ of symmetric equilibria, as I just constructed. Clearly, Lemma 14 is valid in this setting such that it remains to show that $\delta_\omega \in \{\infty, -\infty\}$ for all $\omega \in \{\alpha, \beta\}$. By symmetry of the equilibria, $\delta_\alpha = -\delta_\beta$. Suppose that $\delta_\alpha \in \mathbb{R}$. Given (71) and (72),

$$(q(\alpha) - \frac{1}{2}) s(\alpha; \sigma_n^*)^{-1}$$

$$= \frac{1}{2} \Pr(\text{piv}) x \Big(\Big[\mathrm{E}_H(k(t)|t \in L) - (1 - \lambda) \mathrm{E}_H(k(t)|t \in C) \Big] \Big)$$

Multiplication of both sides with the inverse of the standard deviation $s(\alpha; \sigma_n^*)^{-1} =$

$\frac{q(\omega;\sigma_n^*)(1-q(\omega;\sigma_n^*))}{2n+1}$ and taking limits $n \to \infty$ and using Lemma 12 yields

$$\delta_\alpha = \lim_{n\to\infty} s(\alpha;\sigma_n^*)^{-1}\phi(\delta_\alpha)x\Big(\Big[\mathrm{E}_H(k(t)|t \in L) - (1-\lambda)\mathrm{E}_H(k(t)|t \in C)\Big] \quad (75)$$

The right hand side diverges, and, by assumption, the left hand side does not diverge, $\delta_\alpha \in \mathbb{R}$. This yields a contradiction. Hence, $\delta_\alpha = -\delta_\beta \in \{\infty, -\infty\}$, and the theorem follows from Lemma 14 and (73). ∎

6.3 Aggregate Cost

I show that the sum of the voters' cost converges to zero in all equilibrium sequences when $\lim_{x\to 0} \frac{c'(x)x}{c(x)} \neq 3$. This shows that the equilibrium sequences with first-best outcomes imply first-best results, even when taking into account the cost of voters.

Lemma 6 *Let $\lim_{x\to 0} \frac{c'(x)x}{c(x)} \neq 3$. Take any equilibrium sequence $(\sigma_n)_{n\in\mathbb{N}}$ and let x_i be the realisation of the precision of voter $i \in \{1, \ldots, 2n+1\}$. Then,*

$$\lim_{n\to\infty}\Big[\sum_{i=1,\ldots,2n+1} c(x_i)\Big] = 0. \quad (76)$$

The proof is provided in Section E of the Appendix.

Appendices

A Information Acquisition

A.1 Proof of Lemma 2

The benefits of information acquisition depend on how often the voters' ballot decides the election, that is the likelihood of the pivotal event. Consider an aligned type. The expected utility of an aligned type who chooses precision $x > 0$ is given by the expected utility, K, from all the events when her vote

does not affect the outcome, by the cost, and by the expected utility from the pivotal event. The expected utility from the pivotal event is

$$\Pr(\text{piv})\Big[t_\alpha \Pr(\alpha|\text{piv};\sigma)(\frac{1}{2}+x) + t_\beta(\Pr(\alpha|\text{piv};\sigma)(\frac{1}{2}-x)\Big]. \qquad (77)$$

To see why, note that when the citizen is pivotal and votes B, she receives utility of zero. When she is pivotal and votes A, she receives utility of t_ω, depending on the state. Given (4), it is optimal to vote A only after a; and she receives a in α with probability $\frac{1}{2}+x$ and in β with probability $\frac{1}{2}-x$. Equating marginal cost and marginal benefits gives

$$c'(x) = \Pr(\text{piv}|\sigma')\Big[\Pr(\alpha|\text{piv};\sigma')t_\alpha - \Pr(\beta|\text{piv};\sigma')t_\beta\Big]. \qquad (78)$$

I claim that there is a unique solution $x^*(t;\sigma',n)$ to (78) when n is large enough. First, the marginal cost are strictly increasing since c is strictly convex. So, any solution to (78) is unique. Since $\lim_{x\to 0}\frac{c'(x)}{x^{d-1}} \in \mathbb{R}$ for some $d>1$ and since c is continuously differentiable, marginal cost at zero are zero, i.e. $c'(0)=0$. Given $c'(0)=0$, $c'(1)>0$ and since $0 \le \Pr(\text{piv}|\sigma',n) < c'(1)$ for any n large enough, given (13), it follows from the intermediate value theorem that there is a solution to (78). It follows from the implicit function theorem that $x^*(t;\sigma',n)$ is continuously differentiable. The argument for the contrarian types is analogous. This finishes the proof of the lemma.

A.2   Proof of Lemma 4.

Lemma 7 $\lim_{x\to 0}\frac{xc'(x)}{c(x)} = d.$

Proof. Recall that $\lim_{n\to\infty}\frac{c'(x)}{x^{d-1}} \in \mathbb{R}$. Since $\lim_{x\to 0}c(x) = c(0) = 0$, it follows from l' Hospital's rule that $\lim_{x\to 0}\frac{c(x)}{x^d} = \lim_{x\to 0}\frac{1}{d}\frac{c'(x)}{x^{d-1}}$. Therefore, $\lim_{n\to\infty}\frac{c(x)}{c'(x)x} = \lim_{x\to 0}\frac{c(x)}{x^d}\frac{x^d}{c'(x)x} = \frac{1}{d}.$ ∎

Step 1 *There is $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$: take any strategy σ' and any type t. It is a best response for a type t to acquire information $x(t) > 0$ if and only if*

$$\frac{1}{2} + x^{**}(t) \geq \frac{\Pr(\beta|\text{piv};\sigma',n)\phi_g^-(k)}{\Pr(\alpha|\text{piv};\sigma',n)(1-\phi_g^-(k)) + \Pr(\beta|\text{piv};\sigma',n)\phi_g^-(k)}, \quad (79)$$

$$\frac{1}{2} - x^{**}(t) \leq \frac{\Pr(\beta|\text{piv};\sigma',n)\phi_g^+(k)}{\Pr(\alpha|\text{piv};\sigma',n)(1-\phi_g^+(k)) + \Pr(\beta|\text{piv};\sigma',n)\phi_g^+(k)}, \quad (80)$$

where $x^{**}(t;\sigma',n) = x^*(t;\sigma',n)(1 - \frac{c(x^*(t;\sigma',n))}{x^*(t;\sigma',n)c'(x^*(t;\sigma',n))})$ and $x^*(t;\sigma',n)$ is the unique solution to the first-order condition (18).

Consider an aligned type. Recall Lemma 2 which says that if an aligned type acquires information, then $x(t) = x^*(t;\sigma',n)$. Recall Lemma 16 which says that if an aligned type acquires information, the type votes A after a and B after b. So, the comparison of the expected utility from voting A or B without further information and the expected utility from choosing $x = x^*(t;\sigma',n)$ and voting A after a and B after b minus the cost $c(x^*(t;\sigma',n))$, pins down the aligned types that acquire information.

Using (77), an aligned type prefers choosing precision $x = x^*(t;\sigma',n)$ over voting A without further information if

$$\Pr(\text{piv}|\sigma',n)\left[\Pr(\alpha|\text{piv};\sigma',n)(\frac{1}{2}+x)t_\alpha + \Pr(\beta|\text{piv};\sigma',n)(\frac{1}{2}-x)t_\beta\right] - c(x)$$

$$\geq \Pr(\text{piv}|\sigma',n)\left[\Pr(\alpha|\text{piv};x,\sigma',n)t_\alpha + \Pr(\beta|\text{piv};\sigma',n)t_\beta\right]. \quad (81)$$

I rewrite (81) as

$$\Pr(\text{piv}|\sigma',n)\left[(\frac{1}{2}+x)\left[\Pr(\alpha|\text{piv};\sigma',n)t_\alpha - \Pr(\beta|\text{piv};\sigma',n)t_\beta\right] + \Pr(\beta|\text{piv};\sigma',n)t_\beta\right] - c(x)$$

$$\geq \Pr(\text{piv}|\sigma',n)\left[\Pr(\alpha|\text{piv};\sigma',n)t_\alpha - \Pr(\beta|\text{piv};\sigma',n)t_\beta + 2\Pr(\beta|\text{piv};\sigma',n)t_\beta\right] \quad (82)$$

44

Let $x = x^*(t; \sigma', n)$ in the following. Plugging (78) into (82) gives

$$(\frac{1}{2} + x)c'(x) - c(x) + \Pr(\text{piv}|\sigma', n)\Pr(\beta|\text{piv}; \sigma')t_\beta$$
$$\geq c'(x) + 2\Pr(\text{piv}|\sigma', n)\Pr(\beta|\text{piv}; \sigma', n)t_\beta. \qquad (83)$$

I divide by $c'(x)$ rearrange, and use (78) again,

$$(\frac{1}{2} + x) - \frac{c(x)}{c'(x)} \geq 1 + \frac{\Pr(\beta|\text{piv}; \sigma', n)(t_\beta)}{\Pr(\alpha|\text{piv}; \sigma', n)t_\alpha + \Pr(\beta|\text{piv}; \sigma', n)(-t_\beta)}. \qquad (84)$$

I use (21) and (22),

$$(\frac{1}{2} + x) - \frac{c(x)}{c'(x)} \geq 1 + \frac{-\Pr(\beta|\text{piv}; \sigma', n)y(t)}{\Pr(\alpha|\text{piv}; \sigma', n)(1 - y(t)) + \Pr(\beta|\text{piv}; \sigma', n)y(t)}. \qquad (85)$$

Rearranging gives (80). In the same way one shows that an aligned type prefers choosing precision $x = x^*(t; \sigma', n)$ over voting B without further information only if (79) holds. The argument for the contrarian types is analogous. This finishes the proof of the first step.

Fix a group of voter types, aligned or contrarians. Recall that any type of the group is uniquely determined by the pair $(y(t), k(t))$, see (21) and (22).

Step 2 *For any n large enough: for any $g \in \{C, L\}$ and for any $k > 0$,*

$$\frac{1}{2} + x^{**}(y, k) - \chi(y), \quad and \qquad (86)$$

$$\frac{1}{2} - x^{**}(y, k) - \chi(y). \qquad (87)$$

are strictly increasing in $y \in (0, 1)$, where $\chi(y) = \frac{\Pr(\beta|\text{piv};\sigma',n)y}{\Pr(\beta|\text{piv};\sigma',n)y + \Pr(\alpha|\text{piv};\sigma',n)(1-y)}$. For any $\epsilon > 0$, there is $\delta > 0$ such that the derivatives of (86) and (87) are bounded below by δ.

Consider the derivatives of the summands individually: recall that $x^*(y, k; \sigma', n)$ is implicitly defined by the first-order condition 18. First, it follows from im-

plicit differentiation[30] that the derivative of $x^*(y, k; \sigma', n)$ with respect to y converges to zero uniformly as $n \to \infty$. Intuitively, changes in the marginal benefit on the right hand side of (18) due to changes in the threshold of doubt y are of an order of the marginal cost $c'(x(y, k))$. These changes relate to infinitely smaller changes in the equilibrium precision $x^*(y, k; \sigma', n)$ on the left hand side since the derivative of the marginal cost, $c''(x(y, k))$, is much larger than the marginal cost $c'(x(y, k))$ when the precision is small. Lemma 7 together with the definition of $x^{**}(y, k; \sigma', n)$ implies that the derivative of $x^{**}(y, k; \sigma', n)$ with respect to y converges to zero uniformly as $n \to \infty$. Second, note that χ is continuously differentiable in y; moreover, for any $\epsilon > 0$, there is $\delta > 0$ such that $\chi'(y) > \delta$ for any $y \in (\epsilon, 1 - \epsilon)$ and any n.[31] I conclude that for n large enough, $\frac{1}{2} + x^{**}(y, k) - \chi(y)$ and $\frac{1}{2} - x^{**}(y, k) - \chi(y)$ are strictly increasing.

Step 3 *For any n large enough, there are $\phi_g^-(k), \phi_g^+(k)$ with $\phi_g^-(k) < \hat{y} < \phi_g^+(k)$ such that (47) and (48) hold, and*

$$\chi(y) \leq \frac{1}{2} + x^{**}(y, k) \quad \Leftrightarrow \quad y \leq \phi_g^+(k), \tag{88}$$

$$\chi(y) \geq \frac{1}{2} - x^{**}(y, k) \quad \Leftrightarrow \quad y \geq \phi_g^-(k). \tag{89}$$

Note that $\chi(\hat{y}) = \frac{1}{2}$ and $x^{**}(\hat{y}, k) > 0$ for $\hat{y} = \Pr(\alpha|\text{piv}; \sigma', n)$. Thus, $\chi(\hat{y}) \leq \frac{1}{2} + x^{**}(\hat{y}, k)$. The claim of the step follows from Step 2 and since $\lim_{n \to \infty} x^{**}(\hat{y}, k) = 0$.

Finally, Lemma 4 follows from Step 1 and Step 3. Note that it follows from the implicit function theorem that the functions $\phi_g^-(k)$ and $\phi_g^+(k)$ are

---

[30]For any continuously differentiable $F : [0, 1] \times [0, \frac{1}{2}] \to \mathbb{R}$ and an (implicit) function $g : [0, 1] \to [0, \frac{1}{2}]$ with $F(a, g(a)) = 0$, $g'(a) = \frac{\frac{\partial F(x,y)}{\partial x}|_{(x,y)=(a,g(a))}}{\frac{\partial F(x,y)}{\partial y}|_{(x,y)=(a,g(a))}}$, which is an implication of the chain rule of differentiation.

[31]For any $p \in (0, 1)$, $\frac{\partial}{\partial y}\left(\frac{py}{py+(1-p)(1-y)}\right) = \frac{(1-p)p}{(p(2y-1)-y+1)^2}$. Thus, for any $\epsilon > 0$, there is $\delta > 0$ such that for all $p \in (\epsilon, 1 - \epsilon)$, $\frac{\partial}{\partial y}\left(\frac{py}{py+(1-p)(1-y)}\right) > \delta$. The assumption $\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma', n) \in (0, 1)$ implies that, moreover, there is $\delta > 0$ such that $\chi'(y)$ is uniformly bounded below by a positive constant for any n.

continuously differentiable in k.

B Proof of Theorem 1

This section proves Lemma 1. Section B.1 proves (35) and Observation 2. Section B.2 and Section B.3 prepare a fixed point argument. Section B.2 analyzes the inference of the voters from the pivotal event. Section B.3 analyzes which cost functions allow for sufficient information acquisition by the voters, as needed for election outcomes to be *informative*, $\lim_{n\to\infty} \Pr(A|\alpha; \sigma_n, n) \neq \lim_{n\to\infty} \Pr(B|\alpha; \sigma_n, n)$ and determinate in both states, $\lim_{n\to\infty} \Pr(B|\omega; \sigma_n, n) \in \{0, 1\}$.

B.1 Critical Observations

B.1.1 Proof of (35)

Suppose that $\lim_{n\to\infty} \Phi(\Pr(\alpha|\text{piv}; \sigma_n^*)) \neq \frac{1}{2}$. Then, the pivotal likelihood is exponentially small when n is large, given (13). Thus, the first-order condition (18) implies that the precision $x(t)$ of each voter is exponentially small. It follows from (36) that the difference in the vote shares is exponentially small,

$$q(\alpha; \sigma_n^*) - q(\beta; \sigma_n^*) = \lim_{n\to\infty} 2\Big[\int_{t\in L} x(t)dH(t) - \int_{t\in C} x(t)dH(t) \Big] < z^n c_3 \quad (90)$$

for some constant $c_3 \neq 0$ and n large enough. Consider the voter's inference about the relative likelihood of α and β. Intuitively, given (90), the pivotal event contains no information about the relative probability of α and β as the electorate grows large. I claim that (90) implies

$$\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n) = \Pr(\alpha). \quad (91)$$

To see why, consider the likelihood ratio

$$\frac{\Pr(\text{piv}|\alpha; \sigma_n^*, n)}{\Pr(\text{piv}|\beta; \sigma_n^*, n)} = \Big[\frac{q(\alpha; \sigma_n^*)(1 - q(\alpha; \sigma_n^*))}{q(\beta; \sigma_n^*)(1 - q(\beta; \sigma_n^*))} \Big]^n. \quad (92)$$

The inequality (90) states that the difference in vote shares is exponentially small such that $(1 - \frac{1}{n^2})^n \leq \frac{\Pr(\alpha|\text{piv};\sigma_n^*,n)}{\Pr(\beta|\text{piv};\sigma_n^*,n)} \leq (1 + \frac{1}{n^2})^n$ for all n large enough. Then, the description $\lim_{n\to\infty}(1 - \frac{x}{n})^n = e^x$ of the e-function implies that the likelihood ratio converges to 1 and in turn (91).

If $\Phi(\Pr(\alpha)) = \frac{1}{2}$, (35) follows from (91) since Φ is continuous. Suppose that $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$. Note that (91) implies that $\lim_{n\to\infty} q(\omega;\sigma_n^*) = \Phi(\Pr(\alpha))$ for $\omega \in \{\alpha,\beta\}$, given Lemma 3 and since Φ is continuous. Then, the weak law of large numbers implies $\lim_{n\to\infty} \Pr(A|\alpha;\sigma_n^*,n) = \lim_{n\to\infty} \Pr(B|\beta;\sigma_n^*,n) \in \{0,1\}$. This contradicts with the equilibrium sequence being informative; see the definition (24). I conclude, that any informative equilibrium sequence must satisfy (35). This finishes the proof of (35).

I record the following corollary of the proof of (35).

Lemma 8 *For any equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$, $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) \in (0,1)$.

Proof. Suppose, for example that $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) = 0$. Then, Lemma 3 implies that $\lim_{n\to\infty} q(\omega;\sigma_n^*) = \Phi(0)$ for any $\omega \in \{\alpha,\beta\}$. Since $\Phi(0) \neq \frac{1}{2}$, the proof of (35) implies (91) which is a contradiction to $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) = 0$ since $\Pr(\alpha) > 0$. The assumption $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) = 1$ leads to a contradiction in the same way. This finishes the proof of Lemma 8. ∎

B.1.2   Proof of Observation 2

I show a more general lemma, which implies Lemma 2.

Lemma 9 *Take any sequence of strategies* $(\sigma_n)_{n\in\mathbb{N}}$. *For all n, let $\hat{\sigma}_n$ be any best response to σ_n. If $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n,n) \in (0,1)$, for any $g \in \{L,C\}$, the sequence of best responses satisfies*

$$\int_{t\in g} x(t)dH(t) \approx c_4 \left[\Pr(\text{piv}|\sigma_n^*,n) \right]^{\frac{2}{d-1}} W(g,\hat{p})$$

for some constant $c_4 > 0$ that does not depend on g.

Proof. For the simplicity of the exposition, in the following, we consider cost functions $c(x) = \gamma x^d$ for some $d > 1$. More generally, recall that any cost function considered in this paper satisfies $\lim_{x \to 0} \frac{c'(x)}{x^{d-1}}$ for some $d > 1$ such that l'Hospital's rule implies that $\lim \frac{c(x)}{x^d} \in \mathbb{R}$. The proof for a general cost function is verbatim except that one has to replace the equality $c(x) = \gamma x^d$ with the approximation $c(x) \approx \gamma x^d$.

Notation. Recall that, when fixing the voter group $g \in \{L, C\}$, we can view the conditional distribution $H(-|t \in g)$ of the preference types $t \in g$ as a distribution of the threshold of doubt $y(t)$ and the total intensity $k(t)$ of the types $t \in g$, see (21) and (22). The marginal distribution $F(-|t \in g)$ of the threshold of doubt of the types $t \in g$ has the density $f^g(y) = \int_{k \in [0,2]} h^g(y,k) dk$ where h^g is the density of $H(-|t \in g)$. The marginal distribution J of the total intensity of the voter types $t \in g$ has the density $j^g(k) = \int_{y \in [0,1]} h^g(y,k) dy$. Similarly, let the densities of the conditional distributions $F(y'|k = k', t \in g) = \int_{y \le y'} \frac{h^g(y,k)}{j^g(k')}$ and $J(k'|y = y', t \in g) = \int_{k \le k'} \frac{h^g(y,k)}{f^g(y')}$ be $f^g(y|k = k')$ and $j^g(k|y = y')$, for any $k' \in [0,2]$ and any $y' \in [0,1]$. In the following, I consider $x(t)$, $x^*(t; \sigma_n, n)$ and $x^{**}(t; \sigma_n, n)$ as functions of (y,k). Recall from 2 that $x^*(t; \sigma_n, n)$ and $x^{**}(t; \sigma_n, n)$ only depend on the treshold of doubt and the total intensity, and are independent of the group that the type t belongs to.

Take any sequence of strategies $(\sigma_n)_{n \in \mathbb{N}}$ with $\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n, n) \in (0, 1)$. The first step shows that the region $[\phi_g^-(k), \phi_g^+(k)]$ of types that acquire information, i.e. for which $x(y, k) > 0$, is sufficiently symmetric around the critical types with $\hat{y}_n = \Pr(\alpha|\text{piv}; \sigma_n)$. This first step implies (50), as in the sketch of proof in the main text.

Step 1 *For all $k \in (0, 2)$,*

$$\lim_{n \to \infty} \frac{\phi_g^+(k) - \hat{y}_n}{x^{**}(\hat{y}, k; \sigma_n, n)} = -\lim_{n \to \infty} \frac{\phi_g^-(k) - \hat{y}_n}{x^{**}(\hat{y}_n, k; \sigma_n, n)} = \lim_{n \to \infty} \frac{1}{\chi'(\hat{y}_n)}. \tag{93}$$

where $\hat{y}_n = \Pr(\alpha|\text{piv}; \sigma_n)$.

Let $\chi(y) = \frac{\Pr(\beta|\text{piv};\sigma_n,n)y}{\Pr(\beta|\text{piv};\sigma',n)y+\Pr(\alpha|\text{piv};\sigma_n,n)(1-y)}$, which is the left hand side of (47) and (48). Recall that $\chi(\hat{y}_n) = \frac{1}{2}$. A Taylor expansion of χ at $\hat{y}_n = \Pr(\alpha|\text{piv};\sigma_n)$ implies

$$\phi_g^-(k) - \hat{y}_n = -\frac{1}{\chi'(\epsilon_{1,n})}x^{**}(\phi_g^-(k),k) \tag{94}$$

for some $\epsilon_{1,k,n} \in [\phi_g^-(k),\hat{y}_n]$. I plug in the definition of $x^{**}(y,k)$,

$$\phi_g^-(k) - \hat{y}_n = -\frac{1}{\chi'(\epsilon_{1,k,n})}x^*(\phi_g^-(k),k;\sigma_n,n)\left[1 - \frac{x^*(\phi_g^-(k),k;\sigma_n,n)c'(x^*(\phi_g^-(k),k;\sigma_n,n))}{c(x^*(\phi_g^-(k),k;\sigma_n,n))}\right].(95)$$

A Taylor expansion of $x^*(y,k;\sigma_n,n)$ at $\hat{y}_n = \Pr(\alpha|\text{piv};\sigma_n,n)$ yields

$$x^*(\phi_g^-(k),k;\sigma_n,n) = x^*(\hat{y}_n,k;\sigma_n,n) + (x^*(\epsilon_{2,k,n},k;\sigma_n,n))'\left[\phi_g^-(k) - \hat{y}_n\right]. \tag{96}$$

for some $\epsilon_{2,n} \in [\phi_g^-(k),\hat{y}_n)]$. As in the proof of Lemma 9, implicit differentiation shows that the derivative of the function $x^*(y,k;\sigma_n,n)$ with respect to y converges to zero uniformly. Therefore, (95), (96) and together imply

$$\lim_{n\to\infty}\frac{\phi_g^-(k)-\hat{y}_n}{x^{**}(\hat{y},k;\sigma_n,n)} = -\lim_{n\to\infty}\frac{1}{\chi'(\epsilon_{1,n})} = -\lim_{n\to\infty}\frac{1}{\chi'(\hat{y}_n)}. \tag{97}$$

where the last equality follows from the continuity of χ' and since $\epsilon_{1,k,n} \to \hat{y}_n$ as $n \to \infty$. The analogous argument for $\phi_g^+(k)$ shows that

$$\lim_{n\to\infty}\frac{\phi_g^+(k)-\hat{y}_n}{x^{**}(\hat{y}_n,k;\sigma_n,n)} = \lim_{n\to\infty}\frac{1}{\chi'(\hat{y}_n)}, \tag{98}$$

which finishes the proof of (93).

The next step shows (49), as in the sketch of the proof in the main text.

Step 2 *For all $k \in (0,2)$,*

$$x(y,k) > 0 \Rightarrow \frac{x(y,k)}{x(y^*,k)} \approx 1. \tag{99}$$

where the convergence is uniform across (y, k).

Take any (y, k) such that $x(y, k) > 0$. Lemma 4 implies that $y \in [\phi_g^-(k), \phi_g^+(k)]$. A Taylor expansion of $x^*(y, k; \sigma_n, n)$ at $\hat{y}_n = \Pr(\alpha | \text{piv}; \sigma_n, n)$ implies

$$x(y, k) = x^*(\hat{y}_n, k; \sigma_n, n) + (x^*(\epsilon_{y,k,n}, k; \sigma_n, n))'(y - \hat{y}_n) \tag{100}$$

for some $\epsilon_{y,k,n} \in [y, \hat{y}_n]$. Since $\lim_{n \to \infty} x^*(\hat{y}, k; \sigma_n, n) = 0$, Lemma 7 implies that

$$\lim_{n \to \infty} \frac{x^*(\hat{y}_n, k; \sigma_n, n)}{x^{**}(\hat{y}_n, k; \sigma_n, n)} = \frac{d}{d-1}. \tag{101}$$

Hence, Step 1 implies $\lim_{n \to \infty} \frac{x^*(\hat{y}_n, k; \sigma_n, n)}{\phi_g^+(k) - \hat{y}_n} \in \mathbb{R}$, and, thus $\lim_{n \to \infty} \frac{x^*(\hat{y}_n, k; \sigma_n, n)}{y - \hat{y}_n} \in \mathbb{R}$ Recall that the derivative of $x^*(y, k; \sigma_n, n)$ with respect to y converges to zero uniformly. Therefore, I conclude that (100) yields (99).

The next step shows (51), as in the sketch of the proof in the main text.

Step 3 *For all $k' \in (0, 2)$ and any $g \in \{L, C\}$,*

$$\frac{f^g(\hat{y}_n, k; \sigma_n, n | k = k') \left[\phi_g^-(k') - \phi_g^+(k') \right]}{F(\phi_g^+(k) | k(t) = k, t \in g) - F(\phi_g^-(k) | k = k', t \in g)} \approx 1. \tag{102}$$

A Taylor expansion of $F(- | k = k', t \in g)$ at $\hat{y} = \Pr(\alpha | \text{piv}; \sigma_n, n)$ implies

$$F(\phi(k) | k = k', t \in g) = F(\hat{y} | k = k', t \in g) + f^g(\epsilon_{3,k',n} | k = k') \left[\phi_g^-(k') - \hat{y}_n \right] \tag{103}$$

$$F(\psi(k) | k = k', t \in g) = F(\hat{y} | k = k', t \in g) + f^g(\epsilon_{4,k',n} | k = k') \left[\phi_g^+(k') - \hat{y}_n \right] \tag{104}$$

for some $\epsilon_{3,k,n} \in [\phi_g^-(k'), \hat{y}_n]$ and some $\epsilon_{4,k,n} \in [\hat{y}_n, \phi_g^+(k')]$. Since $\lim_{n \to \infty} \phi_g^-(k') - \phi_g^-(k) = 0$, the continuity of $f^g(- | k = k')$ implies

$$\lim_{n \to \infty} \frac{f^g(\epsilon_{3,k',n} | k = k')}{f^g(\hat{y}_n | k = k')} = 1, \tag{105}$$

$$\lim_{n \to \infty} \frac{f^g(\epsilon_{4,k',n} | k = k')}{f^g(\hat{y}_n | k = k')} = 1. \tag{106}$$

51

Finally, (105) and (106) together with (103) and (104) imply (102).

The last two steps finish the proof of Lemma 2.

Step 4 *For any $g \in \{C, L\}$ and any $k' \in [0, 2]$,*

$$\lim_{n\to\infty} \frac{\int_y x(y, k') dF(y|k = k', t \in g)}{x^*(\hat{y}_n, k'; \sigma_n, n)^2} = \lim_{n\to\infty} \frac{2 f^g(\hat{y}_n|k = k')}{\chi'(\hat{y}_n)} \frac{d-1}{d}. \quad (107)$$

I use the first three steps,

$$\lim_{n\to\infty} \int_y x(y, k') dF(y|k = k', t \in g)$$

$$\approx \left[F(\psi^g(k')|k = k') - F(\phi^g(k')|k = k') \right] x^*(\hat{y}_n, k'; \sigma_n, n)$$

$$\approx f^g(\hat{y}_n, k'|k = k') \left[\phi^g(k') - \psi^g(k') \right] x^*(\hat{y}_n, k'; \sigma_n, n)$$

$$\approx 2\frac{f^g(\hat{y}_n, k'|k = k')}{\chi'(\hat{y}_n)} x^{**}(\hat{y}_n, k'; \sigma_n, n) x^*(\hat{y}_n, k'; \sigma_n, n)$$

$$\approx 2\frac{f^g(\hat{y}_n, k'|k = k')}{\chi'(\hat{y}_n)} x^*(\hat{y}_n, k'; \sigma_n, n)^2 \frac{d-1}{d},$$

where I used Step 2 for the second line, Step 3 for the third line, Step 1 for the fourth line and (101) for the last line.

Step 5 *For all $g \in \{L, C\}$,*

$$\int_{t \in g} x(t) dH(t) \approx \frac{2(d-1)}{d} \frac{x^*(\hat{p}, 1; \sigma_n, n)^2}{\chi'(\hat{p})} \Pr(g) f^g(\hat{p}) \mathrm{E}(k(t)^{\frac{2}{d-1}}|t \in g, y(t) = \hat{p}).$$

for $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\mathrm{piv}; \sigma_n, n)$.

First, I change coordinates,

$$\int_{t \in g} x(t) dG(t) = \Pr(g) \int_{k' \in [0,2]} \int_{y \in [0,1]} x(y, k') f^g(y, k') dy dk' \quad (108)$$

and, then, apply (107) to obtain

$$\int_{t\in g} x(t)dG(t)$$

$$\approx \ \Pr(g)\int_{k'\in[0,2]} j^g(k')x^*(\hat{y}_n,k')^2\frac{f^g(\hat{y}_n|k=k')}{\chi'(\hat{y}_n)}\frac{2(d-1)}{d}dk'. \quad (109)$$

It follows from the first-order condition (18) and since $c(x) = \gamma x^d$ that for all $k' \in [0,2]$,

$$x^*(\hat{y}_n,k';\sigma_n,n) = (k')^{\frac{1}{d-1}}x^*(\hat{y}_n,1;\sigma_n,n). \quad (110)$$

Combining (109) and (110),

$$\int_{t\in g} x(t)dG(t) \ \approx \ \Pr(g)\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2\int_{k'\in[0,2]} (k')^{\frac{2}{d-1}}j^g(k')f^g(\hat{y}_n|k=k')dk'.$$

I rewrite,

$$\int_{t\in g} x(t)dG(t) \ \approx \ \Pr(g)\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2 f^g(\hat{y}_n)\int_{k'\in[0,2]} (k')^{\frac{2}{d-1}}\frac{h^g(\hat{y}_n,k')}{f^g(\hat{y}_n)}dk'$$

$$= \ \Pr(g)\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2 f^g(\hat{y}_n)\int_{k'\in[0,2]} (k')^{\frac{2}{d-1}}j^g(k'|y=\hat{y}_n)dk'$$

$$= \ \Pr(g)\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2 f^g(\hat{y}_n)\mathrm{E}_H((k')^{\frac{2}{d-1}}|y=\hat{y}_n)$$

$$\approx \ c_{4,n}\Pr(g)f^g(\hat{p})\mathrm{E}_H((k')^{\frac{2}{d-1}}|y=\hat{p}) \quad (111)$$

where $c_{4,n} = \frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2$. I used (35) and the continuity of f^g, and $f^g(k'|y=y')$ in y' for the statement on the last line. The continuity of χ' and $x^*(\cdot,1;\sigma_n,n)$ in y implies $\lim_{n\to\infty}\frac{c_4}{c_{4,n}} = 1$ for $c_4 = \frac{2(d-1)}{d}\frac{x^*(\hat{p},1;\sigma_n,n)^2}{\chi'(\hat{p})} > 0$. This finishes the proof of Step 4. ∎

B.2 Preliminaries: Inference in Large Elections

I record the intuitive fact that voters update toward the substate in which the vote share is closer to $1/2$, that is, in which the election is closer to being tied
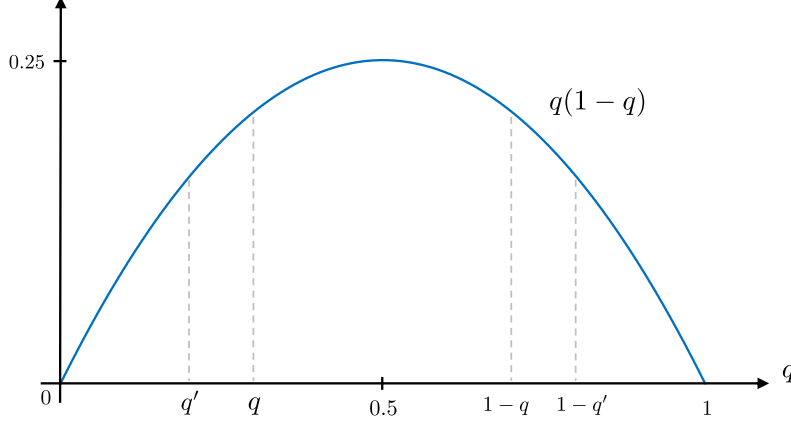
Figure 3: The function $q(1 - q)$ for $q \in [0, 1]$. If $|q - \frac{1}{2}| < |q' - \frac{1}{2}|$, then $q(1 - q) > q'(1 - q')$.

in expectation.

Lemma 10 *Take any strategy σ for which $\Pr(\mathrm{piv}|\beta; \sigma) \in (0, 1)$. If*

$$\left| q(\alpha; \sigma) - \frac{1}{2} \right| < (\leq) \left| q(\beta; \sigma) - \frac{1}{2} \right|, \tag{112}$$

then

$$\frac{\Pr(\mathrm{piv}|\alpha; \sigma, n)}{\Pr(\mathrm{piv}|\beta; \sigma, n)} > (\geq)1. \tag{113}$$

Proof. The function $q(1 - q)$ has an inverse u-shape on $[0, 1]$ and is symmetric around its peak at $q = \frac{1}{2}$, as is illustrated in Figure 3. So, $|q - \frac{1}{2}| < (\leq) |q' - \frac{1}{2}|$ implies that $q(1 - q) > (\geq)q'(1 - q')$. Thus, it follows from (12) that (112) implies (113). ∎

I show that Lemma 10 extends in an extreme form as the electorate grows large ($n \to \infty$): the event that the election is tied is infinitely more likely in the state in which the election is closer to being tied in expectation. In fact, the likelihood ratio of the pivotal event diverges exponentially fast.

54

Lemma 11 *Consider any sequence of strategies* $(\sigma_n)_{n \in \mathbb{N}}$. *If,*

$$\lim_{n \to \infty} \left| q(\alpha; \sigma_n) - \frac{1}{2} \right| < (>) \lim_{n \to \infty} \left| q(\beta; \sigma_n) - \frac{1}{2} \right|, \tag{114}$$

then, for any $\kappa \geq 0$,

$$\lim_{n \to \infty} \frac{\Pr(\text{piv}|\alpha; \sigma_n, n)}{\Pr(\text{piv}|\beta; \sigma_n, n)} n^{-\kappa} = \infty(0). \tag{115}$$

Proof. Let

$$k_n = \frac{q(\alpha; \sigma_n)(1 - q(\alpha; \sigma_n))}{q(\beta; \sigma_n)(1 - q(\beta; \sigma_n))}.$$

From (12), the left-hand side of (115) is $\frac{(k_n)^n}{n^\kappa}$. The function $q(1-q)$ has an inverse u-shape on $[0, 1]$ and is symmetric around its peak at $q = \frac{1}{2}$, as is illustrated in Figure 3. So, (114) implies that $\lim_{n \to \infty} k_n > 1$. So, $\lim_{n \to \infty} (k_n)^n = \infty$. Moreover, $(k_n)^n$ diverges exponentially fast and, hence, dominates the denominator n^κ, which is polynomial. ∎

B.3 Preliminaries: Value and Cost of Information

Value of Information. The marginal value of information is proportional to the likelihood of the pivotal event, given (18). How large is this likelihood? Since the vote share in a state is the empirical mean of $2n + 1$ i.i.d. Bernoulli variables which take the value 1 with probability $q(\omega; \sigma_n)$, an application of the local central limit[32] tells how likely events close to the expected vote share are. In particular, the next result shows that when the expected vote share in a state is sufficiently close to the majority threshold $\frac{1}{2}$, the pivotal likelihood is proportional to the standard deviation of the vote share, which is of order $n^{-\frac{1}{2}}$.[33] For any sequence of strategies $(\sigma_n)_{n \in \mathbb{N}}$ and any n, let $s(\omega; \sigma_n)$ be the

---

[32]See Gnedenko (1948), and McDonald (1980) for the local limit theorem for triangular arrays of integer-valued variables.

[33]The number of A-votes is distributed according to a binomial distribution with parameters n and $q_{\omega,n}$. Hence, its variance is $nq(\omega; \sigma_n)(1 - q(\omega; \sigma_n))$, and the standard deviation $(nq(\omega; \sigma_n)(1 - q(\omega; \sigma_n)))^{\frac{1}{2}}$. Consequently, the standard deviation of the vote share is distributed according to $\frac{1}{n} \mathcal{B}(n, q(\omega; \sigma_n))$, so its standard deviation is $\left(\frac{q(\omega; \sigma_n)(1 - q(\omega; \sigma_n))}{n} \right)^{\frac{1}{2}}$.

standard deviation of the vote share. Let

$$\delta_\omega = \lim_{n \to \infty} \frac{1}{s(\omega; \sigma_n)} \left[q(\omega; \sigma_n) - \frac{1}{2} \right] \tag{116}$$

be the normalized distance of the expected vote share to the majority threshold as $n \to \infty$.

Lemma 12 *For any sequence of strategies $(\sigma_n)_{n \in \mathbb{N}}$ and for all $\omega \in \{\alpha, \beta\}$,*

$$\lim_{n \to \infty} \Pr(\text{piv}|\omega; \sigma_n) s(\omega; \sigma_n)^{-1} = \phi(\delta_\omega), \tag{117}$$

where ϕ the probability density function of the standard normal distribution.

The proof is omitted since the result directly follows from the local central limit theorem.

Figure 4 provides intuition for Lemma 14.

Cost of Information. It depends on the cost of information, how the value of information translates into information acquisition and election outcomes. In what follows, I describe a condition on the cost function that allows for informative and determinate outcomes under the best response to any strategies with vote shares sufficiently close to $\frac{1}{2}$. Conversely, I give a condition when this is not the case; then, intutively, determinate and informative outcomes are not possible under the best response to *any* strategy since the value of information will just be lower if the election is less close to being tied. It turns out that the key statistic is the elasticity of the cost function for low levels of precision $x \approx 0$. To see why, in a first step, I show the following result.

Lemma 13 *Take a sequence of strategies $(\sigma_n)_{n \in \mathbb{N}}$ with $\delta_\alpha \in \mathbb{R}$ and suppose that $W(L, \hat{p}) \neq W(C, \hat{p})$ for $\hat{p} = \lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n, n)$. Let $\hat{\sigma}_n$ be a best reponse to σ_n.*
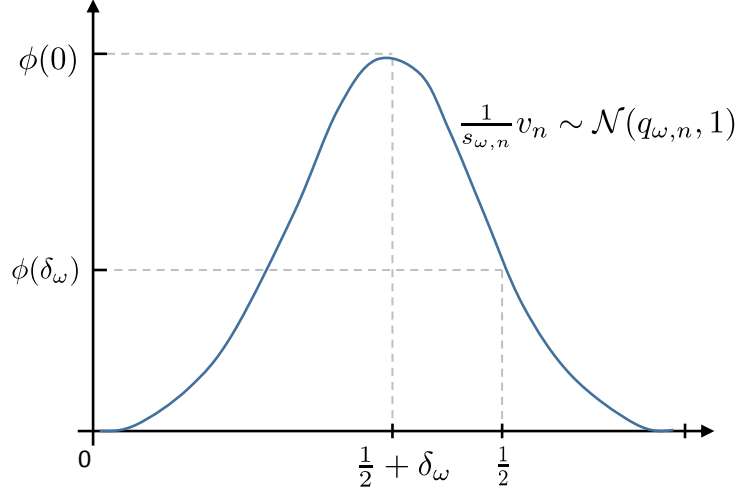
Figure 4: Conditional on ω, the realized vote share v_n follows a binomial distribution with parameters $q(\omega; \sigma_n^*)$ and $2n + 1$. It follows from the Central Limit Theorem for triangular arrays that the distribution of the normalized vote share converges a normal distribution.

1. If $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$, then

$$\lim_{n \to \infty} \frac{q(\alpha; \hat{\sigma}_n) - q(\beta; \hat{\sigma}_n)}{s(\alpha; \sigma_n)} = \infty. \tag{118}$$

2. If $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$, then

$$\lim_{n \to \infty} \frac{q(\alpha; \hat{\sigma}_n) - q(\beta; \hat{\sigma}_n)}{s(\alpha; \sigma_n)} = 0. \tag{119}$$

The proof is provided in Section B of the Appendix.

How do the information acquisition and expected vote shares translate into election outcomes? Take a strategy $(\sigma_n)_{n \in \mathbb{N}}$ with $\delta_\alpha \in \mathbb{R}$ and let $d > 3$. Given Lemma 13, it might be that the expected vote shares are arbitrarily many standard deviations away from the majority threshold under the best response, i.e. $\delta_\omega((\hat{\sigma}_n)_{n \in \mathbb{N}}) \in \{\infty, -\infty\}$ for all $\omega \in \{\alpha, \beta\}$. The following result

57

characterizes the distribution of the election outcomes in state ω as $n \to \infty$ for any given sequence of strategies as a function of $\delta_\omega \in \mathbb{R} \cup \{\infty, -\infty\}$. The result implies that outcomes are determinate in ω as $n \to \infty$ when $\delta_\omega \in \{\infty, -\infty\}$.

Lemma 14 *Consider any sequence of strategies $(\sigma_n)_{n \in \mathbb{N}}$ and any state $\omega \in \{\alpha, \beta\}$. The probability that A gets elected in ω converges to*

$$\lim_{n \to \infty} \Pr(A | \omega_i; \sigma_n) = \Phi(\delta_\omega),$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal distribution.

The proof relies on an application of the central limit theorem and is provided in the Appendix. Figure 4 illustrates Lemma 14.

In the next section, I prove Lemma 1, using the previous lemmas 13 and 14. Thereby, I show the existence of informative equilibrium sequences with both determinate and different outcomes in the states, given $\lim_{n \to \infty} \frac{c'(x)x}{c(x)} > 3$.

B.3.1 Proof of Lemma 13

Recall that $d = \lim_{x \to 0} \frac{c'(x)x}{c(x)}$ (see Lemma 9). What will matter is if the mean square of the precision of the critical types, i.e. $\mathrm{E}_k \Big[x(y, k)^2 | y = \hat{p} \Big]$, is of an order larger or smaller than the pivotal likelihood. Consider the first-order condition, (18),

$$
\begin{aligned}
c'(x^*(t; \sigma_n, n)) &= \Pr(\mathrm{piv} | \sigma_n, n) \mathrm{E}_\omega(|t_\omega| | \mathrm{piv}; \sigma_n, n). \\
\Rightarrow x^*(t; \sigma_n, n)^2 &\approx \left[\Pr(\mathrm{piv} | \sigma_n, n) \frac{\mathrm{E}_\omega(|t_\omega| | \mathrm{piv}; \sigma_n, n)}{d} \right]^{\frac{2}{d-1}}. \quad (120)
\end{aligned}
$$

We see that the critical elasticity is $d = 3$. For $d > 3$, the squared precision is of an order larger than the likelihood of the pivotal event, $\lim_{n \to \infty} \frac{x^*(t; \sigma_n, n)^2}{\Pr(\mathrm{piv} | \sigma_n, n)} = \infty$. Let $d > 3$ in the following. Then, given (53) and the observation just made, the likelihood-weighted average precision of a voter group is of an order larger than the likelihood of the pivotal event, $\lim_{n \to \infty} \frac{\int_{t \in g} x(t) dH(t)}{\Pr(\mathrm{piv} | \sigma_n, n)} = \infty$. Take any sequence of strategies $(\sigma_n)_{n \in \mathbb{N}}$ with $\delta_\alpha \in \mathbb{R}$. Then, the likelihood of the pivotal

58

event in α is of the order of the standard deviation of the vote share, $s(\alpha; \sigma_n)$, given Lemma 12. I conclude that the likelihood-weighted average precision of each voter group is as large as arbitrarily many standard deviations of the vote share in α, as $n \to \infty$, i.e. $\lim_{n \to \infty} \frac{\int_{t \in g} x(t) dH(t)}{s(\alpha; \sigma_n)} = \infty$. Hence, the first item of the lemma follows from (36), Lemma 2 and the assumption that $W(L, \hat{p}) \neq W(C, \hat{p})$. The proof of the second item is analogous.

B.3.2 Proof of Lemma 14

Proof. Let $q_n = q(\omega, \sigma_n)$. By using the normal approximation[34]

$$\mathcal{B}(2n+1, q_n) \simeq \mathcal{N}((2n+1)q_n, (2n+1)q_n(1-q_n)),$$

we see that the probability that A wins the election in ω converges to

$$\Phi\left(\frac{\frac{1}{2}(2n+1) - (2n+1) \cdot q_n}{((2n+1)q_n(1-q_n))^{\frac{1}{2}}}\right).$$

Taking limits $n \to \infty$, gives

$$
\begin{aligned}
&\lim_{n \to \infty} \Phi\left(\frac{\frac{1}{2}(2n+1) - (2n+1) \cdot q_n}{(2n+1)q_n(1-q_n))^{\frac{1}{2}}}\right) \\
=\ &\lim_{n \to \infty} \Phi\left(\frac{(2n+1)\frac{1}{2} - (2n+1)(\frac{1}{2} + (q_n - \frac{1}{2}))}{((2n+1)^{\frac{1}{2}}(q_n(1-q_n))^{\frac{1}{2}}}\right) \\
=\ &\lim_{n \to \infty} \Phi\left((q_n - \frac{1}{2})\left[\frac{(2n+1)}{q_n(1-q_n)}\right]^{\frac{1}{2}}\right) \\
=\ &\Phi(\delta_\omega),
\end{aligned}
$$

---

[34]For this normal approximation, we cannot rely on the standard central limit theorem, because q_n varies with n. Recall that for any undominated strategy, types t with $t_\alpha > 0, t_\beta > 0$ vote A and types t with $t_\alpha < 0, t_\beta < 0$ vote B. Hence, since the type distribution has a strictly positive density, there exists $\epsilon > 0$ such that $\epsilon < q_n < 1 - \epsilon$ for all $n \in \mathbb{N}$. As a consequence, we can apply the Lindeberg-Feller central limit theorem (see Billingsley (2008), Theorem 27.2). To see why, one checks that a sufficient condition for the the Lindeberg condition is that $(2n+1)q_n(1-q_n) \to \infty$ as $n \to \infty$ since this implies that for n sufficiently large the indicator function in the condition takes the value zero.

where the equalities on the last two lines hold both when $\delta_\omega \in \{\infty, -\infty\}$ and when $\delta_\omega \in \mathbb{R}$. For the equality on the last line, I used that the standard deviation of the vote share is given by $s(\omega; \sigma_n) = \left(\frac{q_n(1-q_n)^{\frac{1}{2}}}{2n+1}\right)$. ∎

B.4 Fixed Point Argument

First, I provide a useful compact representation of equilibrium.

Equilibrium Vote Shares. It follows from the analysis of the best response in Section 2 that, for n large enough, an equilibrium is a (non-degenerate) strategy $\sigma = (x, \mu)$ that satisfies (4)-(7), with $\sigma' = \sigma$ and (18) for all types t with $x(t) > 0$.

I claim that equilibrium can be alternatively characterized in terms of the vector of the expected vote shares of outcome A in state α and β, i.e.

$$\mathbf{q}(\sigma) = (q(\alpha; \sigma), q(\beta; \sigma)). \tag{121}$$

Note that for any σ and any $\omega \in \{\alpha, \beta\}$, the vote share $q(\omega; \sigma)$ pins down the likelihood of the pivotal event conditional on ω, given (12). Given (4)-(7) and (18), the vector of the pivotal likelihoods is a sufficient statistic for the best response, and therefore $\mathbf{q}(\sigma)$ as well. Given some vector of expected vote shares $\mathbf{q} = (q(\alpha), q(\beta)) \in (0, 1)$, let $\sigma^{\mathbf{q}}$ be the best response to $\mathbf{q}$. So, σ^* is an equilibrium if and only if $\sigma^* = \sigma^{\mathbf{q}(\sigma^*)}$. Conversely, an equilibrium can be described by a vector of vote shares $\mathbf{q}^*$ that is a fixed point of $\mathbf{q}(\sigma^\cdot)$, that is[35]

$$\mathbf{q}^* = \mathbf{q}\left(\sigma^{\mathbf{q}^*}\right). \tag{122}$$

In the following, I use the notation $\Pr(\alpha|\text{piv}; \mathbf{q})$ to denote the posterior consistent with (12) and the vote shares $\mathbf{q}$, and also similar notation like $s(\omega; \mathbf{q})$, analogous to the previous notation.

---

[35]The ability to write an equilibrium as a finite-dimensional fixed point via (122) is a significant advantage. A similar reduction to finite dimensional equilibrium beliefs has been useful in other settings as well; see Bhattacharya (2013b), Ahn & Oliveros (2012) and Heese & Lauermann (2017).

Proof of Lemma 1. First, I simplify the problem of finding an informative equilibrium sequence further to a problem of finding fixed points of one-dimensional functions.

For this, I define a constrained best response $\hat{q}(\omega, \sigma^{\mathbf{q}})$ as follows. Let $\hat{p} \in [0, 1]$ be the minimal belief for which $\Phi(\hat{p}) = \frac{1}{2}$ and $\Phi'(p^*) > 0$.[36] Consider the case when $\Pr(\alpha) < \hat{p}$ and $W(L, \hat{p}) < W(C, \hat{p})$. Take $\delta > 0$ small enough such that Φ is strictly increasing on $[\hat{p} - \delta, \hat{p} + \delta]$ and $\Pr(\alpha) < \hat{p} - \delta$. Now, if $\Pr(\alpha | \text{piv}; \mathbf{q}) \in [\hat{p} - \delta, \hat{p} + \delta]$, then let $\hat{q}(\omega, \sigma^{\mathbf{q}}) = q(\omega, \sigma^{\mathbf{q}})$. If $\Pr(\alpha | \text{piv}; \mathbf{q}) < \hat{p} - \delta$, then let $\hat{q}(\omega, \sigma^{\mathbf{q}})$ be the vote share of the best response to any strategy σ' with induced prior $\Pr(\alpha | \text{piv}; \sigma', n) = \hat{p} - \delta$ and for which the likelihood of the pivotal event is $\Pr(\text{piv} | \mathbf{q}, n)$. Conversely, if $\Pr(\alpha | \text{piv}; \mathbf{q}) > \hat{p} + \delta$, then let $\hat{q}(\omega, \sigma^{\mathbf{q}})$ be the vote share of the best response to any strategy σ' with induced prior $\Pr(\alpha | \text{piv}; \sigma', n) = \Phi(\hat{p} + \delta)$ and for which the likelihood of the pivotal event is $\Pr(\text{piv} | \mathbf{q}, n)$. The constrained best response is a 'truncation' of $q(\omega; \sigma^{\mathbf{q}})$ and therefore continuous in $\mathbf{q}$.

Step 1 *For any $\epsilon > 0$ small enough, any $\frac{1}{2} - \frac{\epsilon}{2} \leq q(\alpha) \leq \frac{1}{2}$, and any n large enough, there is $q_n^*(\beta) \geq \frac{1}{2}$ such that*

$$q(\alpha) = \hat{q}(\alpha; \sigma^{(q(\alpha), q_n^*(\beta))}) \tag{123}$$
$$= q(\alpha; \sigma^{(q(\alpha), q_n^*(\beta))}). \tag{124}$$

and $q^*(\beta)$ is continuous in $q(\alpha)$.

Take any $q(\alpha) \in [\frac{1}{2}, \frac{1}{2} - \frac{\epsilon}{2}]$. Let $\mathbf{q} = (q(\alpha), q(\beta))$ in the following.

Step 1.1 *If $q(\beta) = \frac{1}{2} + \epsilon$, then, for ϵ small enough and n large enough,*

$$\hat{q}(\alpha; \sigma^{\mathbf{q}}) > q(\alpha). \tag{125}$$

The election is more close to being tied in α, and, by Lemma 11, voters become convinced that the state is α, that is $\lim_{n \to \infty} \Pr(\alpha | \text{piv}; \mathbf{q}, n) = 1$. It follows

---

[36]Recall that $\Phi(0) < \frac{1}{2}$ and $\Phi(1) > \frac{1}{2}$ by assumption and that Φ is continuously differentiable and not constant on any open interval. This implies the existence of $\hat{p}$.

from the definition of $\hat{q}(\omega; \sigma^{\mathbf{q}})$ and Lemma 3 that $\lim_{n\to\infty} \hat{q}(\alpha; \sigma^{\mathbf{q}}) = \Phi(\hat{p} + \delta)$. Finally, (125) follows when ϵ is small enough, since $\Phi(\hat{p} + \delta) > \frac{1}{2}$.

Step 1.2 *If $q(\beta) = \frac{1}{2}$, then for ϵ small enough and any n,*

$$\hat{q}(\alpha; \sigma^{\mathbf{q}}) < q(\alpha). \tag{126}$$

The election is more close to being tied in β, and, by Lemma 10, voters update towards β, that is $\Pr(\alpha|\text{piv}; \mathbf{q}, n) \leq \Pr(\alpha)$. Since $\Pr(\alpha) < \hat{p} - \delta$, it follows from the definition of $\hat{q}(\alpha; \sigma^{\mathbf{q}})$ and Lemma 3 that $\lim_{n\to\infty} \hat{q}(\alpha; \sigma^{\mathbf{q}}) = \Phi(\hat{p} - \delta)$. Finally, (126) follows for $\epsilon > 0$ small enough since $\Phi(\hat{p} - \delta) < \frac{1}{2}$.

Since $\hat{q}(\alpha; \sigma^{\mathbf{q}})$ is continuous in $q(\beta)$, it follows from Step 1.1, Step 1.2, and the intermediate value theorem that, for n large enough, there is $q^*(\beta)$ such that (123) holds. It follows from the implicit function theorem that $q^*(\beta)$ is continuous. Now, suppose that $\Pr(\alpha|\text{piv}; \sigma^{(q(\alpha), q_n^*(\beta))}, n) \notin [\hat{p} - \delta, \hat{p} + \delta]$. Then, Lemma 3 together with the definition of $\hat{q}(\alpha; \sigma^{\mathbf{q}})$ implies that $\lim_{n\to\infty} \hat{q}(\alpha; \sigma^{\mathbf{q}}) \in \{\Phi(\hat{p} - \delta), \Phi(\hat{p} + \delta)\}$. This contradicts with (123) since $\Phi(\hat{p} - \delta) < q(\alpha) < \Phi(\hat{p} + \delta)$ for $\epsilon > 0$ small enough. Hence, $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma^{(q(\alpha), q^*(\beta))}, n) \in [\hat{p} - \delta, \hat{p} + \delta]$ and therefore, given the definition of the 'truncation' $\hat{q}$, (123) implies (124).

In what follows, I show that, for any n large enough, there is a vote share $q_n^*(\alpha)$ such that $\mathbf{q}_n^* = (q_n^*(\alpha), q^*(\beta))$ is a fixed point of $\mathbf{q}(\sigma^{\cdot})$, thereby constructing equilibria $\sigma^{\mathbf{q}_n^*}$ of the voting game. Given (123), it is sufficient to show the following.

Step 2 *For any n large enough, there is $q_n^*(\alpha)$ such that*

$$q(\beta) = q(\beta; \sigma^{(q(\alpha), q^*(\beta))}). \tag{127}$$

I consider sequences of vote shares $(\mathbf{q}_n)_{n\in\mathbb{N}}$ with $\mathbf{q}_n = (q(\alpha)_n, q^*(\beta)_n)$.

Step 2.1 *If* $\lim_{n\to\infty}(\frac{1}{2} - q(\alpha)_n)s(\alpha; \mathbf{q}_n) \in \mathbb{R}^{\geq 0}$, *then, for* n *large enough,*

$$q^*(\beta)_n < q(\beta; \sigma^{\mathbf{q}_n}). \tag{128}$$

Recall that $\lim_{x\to 0} \frac{c'(x)x}{c(x)} = d$ (see Lemma 9). Now, (124) together with the assumption $\lim_{n\to\infty}(\frac{1}{2} - q(\alpha)_n)s(\alpha; \mathbf{q}_n) \in \mathbb{R}^{\geq 0}$ implies

$$\delta(\alpha)((\sigma^{\mathbf{q}_n})_{n\in\mathbb{N}}) = \lim_{n\to\infty} (\frac{1}{2} - q(\alpha; \sigma^{\mathbf{q}_n}))s(\alpha; \mathbf{q}_n) \in \mathbb{R}^{\geq 0}. \tag{129}$$

Then, Lemma 3 implies $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \mathbf{q}_n) \in \Phi^{-1}(\frac{1}{2})$. Given the definition of the 'truncated' best response $\hat{\sigma}$,

$$\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \mathbf{q}_n) = \hat{p}. \tag{130}$$

Since I consider the case where $W(L, \hat{p}) < W(C, \hat{p})$, it follows from Lemma 2 and (36) that $q(\alpha; \sigma^{\mathbf{q}_n}) < q(\beta; \sigma^{\mathbf{q}_n})$ for n large enough. Given (129), Lemma 13 implies that

$$\lim_{n\to\infty} \left[q(\beta; \sigma^{\mathbf{q}_n}) - \frac{1}{2}\right]|s(\alpha; \mathbf{q}_n) = \infty. \tag{131}$$

Given (130) and since $\hat{p} \in (0, 1)$, the inference from the pivotal event is bounded. Therefore, Lemma 12 implies that

$$(q^*(\beta) - \frac{1}{2})s(\beta; \mathbf{q}_n) \in \mathbb{R} \tag{132}$$

since otherwise $\lim_{n\to\infty} \frac{\Pr(\alpha|\text{piv}; \mathbf{q})}{\Pr(\alpha|\text{piv}; \mathbf{q})} = \frac{\Phi(\delta_\alpha((\sigma^{\mathbf{q}_n})_{n\in\mathbb{N}}))}{\Phi(\delta_\beta((\sigma^{\mathbf{q}_n})_{n\in\mathbb{N}}))} = 0$. Note that $\lim_{n\to\infty} \frac{s(\alpha; \mathbf{q}_n)}{s(\beta; \mathbf{q}_n)} = 1$.[37] Therefore, (131) and (132) imply (128) for n large enough.

Step 2.2 *If* $q(\alpha) = \frac{1}{2} - \frac{\epsilon}{2}$, *then, for* n *large enough,*

$$q_n^*(\beta) > q(\beta; \sigma^{\mathbf{q}_n}). \tag{133}$$

---

[37]The standard deviation of the vote share in ω is $(\frac{q(\omega)(1-q(\omega)_n)}{n})^{\frac{1}{2}}$. Note that $\lim_{n\to\infty} q(\alpha)_n - q(\beta)_n = 0$, which implies that the ratio of the standard deviations converges to 1 as $n \to \infty$.

Lemma 3 together with (123) implies that $\lim_{n\to\infty} q(\beta; \sigma^{\mathbf{q}}) = \frac{1}{2} - \epsilon$. Since $q^*(\beta) \geq \frac{1}{2}$, clearly, (133) holds for n large enough.

Finally, since $q(\beta; \sigma^{\mathbf{q}})$ is continuous in $q(\alpha)$, an application of the intermediate value theorem shows that, for all n large enough, there is $q^*(\alpha) < \frac{1}{2}$ for which (127) holds. This finishes the proof of Step (2.2). The corresponding strategies $\sigma_n^* = \sigma^{(q_n^*(\alpha), q^*(\beta))}$ form an equilibrium sequence .

It remains to show that the election chooses the outcome preferred by the contrarians in each state with probability converging to 1, given the equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$. Since $q(\alpha; \sigma_n^*) \leq \frac{1}{2} \leq q(\beta; \sigma_n^*)$, Lemma 12 implies that it remains to show that $|\delta_\omega| = \infty$ for all states ω.

Suppose that $\delta_\omega \in \mathbb{R}$ for some $\omega \in \{\alpha, \beta\}$. I claim that this implies that $\delta_\omega \in \mathbb{R}$ for all states. Otherwise, Lemma 14 implies that the inference from the pivotal event is not bounded, that is $\lim_{n\to\infty} \frac{\Pr(\alpha|\text{piv}; \sigma_n^*)}{\Pr(\alpha|\text{piv}; \sigma_n^*)} = \frac{\Phi(\delta_\alpha)}{\Phi(\delta_\beta)} \in \{0, 1\}$. But then $\lim_{n\to\infty} q(\omega; \sigma_n^*) \in \{\Phi(0), \Phi(1)\}$ and $\delta_\omega = \infty$ for all states since $\Phi(0) < \frac{1}{2}$ and $\Phi(1) > \frac{1}{2}$. Second, when $\delta_\alpha \in \mathbb{R}$, Lemma 13 implies that there is a state ω for which $\delta_\omega = \infty$. However, this contradicts with the observation that $\delta_\omega \in \mathbb{R}$ for all states. Consequently, it must be that $|\delta_\omega| = \infty$ for all states ω. This finishes the proof of Lemma 1 for the case when $\Pr(\alpha) < \hat{p}$ and $W(L, \hat{p}) < W(C, \hat{p})$. The proof of the other cases is analogous.

Note that in the same way that I constructed an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ with $q(\alpha; \sigma_n^*) \leq \frac{1}{2} \leq q(\beta; \sigma_n^*)$ in the case when $\hat{p} > \Pr(\alpha)$ and $W(L, \hat{p}) < W(C, \hat{p})$, I can construct an equilibrium sequence with $q(\alpha; \sigma_n^*) \leq q(\beta; \sigma_n^*) \leq \frac{1}{2}$ for n large enough, in this case; essentially since the likelihood of the pivotal event as a function of the vote shares $\mathbf{q}$ is symmetric around $\frac{1}{2}$. The proof is verbatim and only the relevant inequality signs need to be flipped. Conversely, if $\hat{p} > \Pr(\alpha)$ and $W(L, \hat{p}) > W(C, \hat{p})$, I can construct an equilibrium sequence with $\frac{1}{2} \leq q(\alpha; \sigma_n^*) \leq q(\beta; \sigma_n^*)$ for n large enough. By symmetry of the other cases, I conclude:

Theorem 9 *Let* $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$. *Take any preference distribution H for which $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$ and $W(L, p) \neq W(C, p)$ for any $p \in [0, 1]$ with $\Phi(p) = \frac{1}{2}$.*

There is an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfying (68).

C   Determinants of Elections

C.1   Proof of Lemma 5

Proof. Recall that $\lim_{x\to 0} \frac{c'(x)x}{c(x)} = d$ (Lemma 7). The basic intuition is that that power index $W(g,\hat{p})$ of the group g is proportional to $\mathrm{E}_H(k^{\frac{2}{d-1}})$. Since $k^{\frac{2}{d-1}}|^{t\in g}$ is strictly concave, given $d > 3$, an application of Jensen's inequality shows that for any g-intensity spread H' of H and $g \neq g' \in \{L, C\}$,

$$\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g) < \mathrm{E}_H(k^{\frac{2}{d-1}}|t \in g), \tag{134}$$

$$\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g') = \mathrm{E}_H(k^{\frac{2}{d-1}}|t \in g'). \tag{135}$$

First, note that by the definition of a g-intensity spread, $\mathrm{Pr}_{H'}(L) = \mathrm{Pr}_H(L)$ and $\mathrm{Pr}_{H'}(C) = \mathrm{Pr}_H(C)$; see (60)-(61). Second, $\hat{p} = \lim_{n\to\infty} \mathrm{Pr}(\alpha|\mathrm{piv}; \sigma_n^*, n) = \lim_{n\to\infty} \mathrm{Pr}(\alpha|\mathrm{piv}; \hat{\sigma}_n, n)$ and (61) together imply $f_H^g(\lim_{n\to\infty} \mathrm{Pr}(\alpha|\mathrm{piv}; \sigma_n^*, n)) = f_{H'}^g(\lim_{n\to\infty} \mathrm{Pr}(\alpha|\mathrm{piv}; \hat{\sigma}_n, n))$. Third, since H and H' satisfy (30), $\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g) = \mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g, y(t) = \hat{p})$ and $\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g') = \mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g', y(t) = \hat{p})$. These observations together with (134) and (135) imply (63), given the definition (26) of the power indices. ∎

C.2   Proof of Lemma 4

Consider the group of the aligned voters. Given Lemma 2 and Lemma 1, it remains to show that for any H there is a L-intensity spread such that $\frac{W_{H'}(L,\hat{p})}{W_{H'}(C,\hat{p})} < 1$ across all equilibrium sequences. Note that $W_H(C\hat{p}) = W_{H'}(C,\hat{p})$ for any L-intensity spread H' of H. I show that for any H and for any $\epsilon > 0$, there is an L-intensity spread H' of H such that for all equilibrium sequences $(\sigma_n^*)_{n\in\mathbb{N}}$ given H',

$$W_{H'}(L, \hat{p}) < \epsilon. \tag{136}$$

Note that the density $f^g(y)$ of the threshold of doubt is continuous and therefore bounded. Given (29) and (30), it is therefore sufficient to show that for any $\epsilon > 0$, there is a a g-intensity spread with

$$\mathrm{E}_{H'}(k^{\frac{2}{d-1}} | t \in g) < \epsilon. \tag{137}$$

For this, consider g-intensity spreads $H'(\kappa)$ of H such that the conditional distributions $J^g_{H'(\kappa)}$ of the intensities k of the types of group g are concentrated in neighbourhoods around 0 and $K > 0$, that is $\mathrm{Pr}_{J^g_{H'(\kappa)}}(\kappa \leq k \leq \kappa + \delta) + \mathrm{Pr}_{J^g_{H'(\kappa)}}(0 \leq k \leq \epsilon) \geq 1 - \delta$. Since the mean of the intensities is the same under the g-intensity spread, the iterated law of expectation implies $\lim_{\delta \to 0} \mathrm{Pr}_{J^g_{H'(\kappa)}}(\kappa \leq k \leq \kappa + \delta)\kappa = E_{J^g_H}(k)$. Hence,

$$\lim_{\delta \to 0} \mathrm{E}_{J^g_{H'(\kappa)}}(k^{\frac{2}{d-1}}) = \frac{E_{J^g_H}(k)}{\kappa} \kappa^{\frac{2}{d-1}} \overset{\kappa \to \infty}{\to} 0, \tag{138}$$

where I used that $d = \lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$ and hence $\frac{2}{d-1} < 1$. Finally, (138) shows that $W_{H'(\kappa)}(L, \hat{p}) < \epsilon$ when κ is large enough and δ small enough. This finishes the proof of the theorem.

D   Characterization of All Equilibria

D.1   Proof of Theorem 6

Case $\lim_{x \to 0} \frac{c'(x)x}{c(x)} > 3$.
I show that any equilibrium sequence satisfies either (28), (68) or (64).
Take any equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$. It follows from Corollary 2 that $\hat{p} = \lim_{n \to \infty} \mathrm{Pr}(\alpha | \mathrm{piv}; \sigma_n^*) \in \Phi^{-1}(\frac{1}{2}) \cup \{\mathrm{Pr}(\alpha)\}$. Given Lemma 2, the order of the vote shares is given by the order of $W(L, \hat{p})$ and $W(C, \hat{p})$. Consider the case when $W(L, \hat{p}) < W(C, \hat{p})$ such that there are the following two cases:

Case 1 *For any n large enough, $\frac{1}{2} \leq q(\alpha; \sigma_n^*) \leq q(\beta; \sigma_n^*)$.*

Suppose that $\delta(\alpha) \in \mathbb{R}$. Then, Lemma 13 implies that $\delta(\alpha) = \infty$, a contradiction. Hence $\delta(\alpha) = \delta(\beta) = \infty$. Lemma 14 implies that the equilibrium

66

sequence satisfies (9).

Case 2 *For any n large enough, $q(\alpha; \sigma_n^*) \leq \frac{1}{2} \leq q(\beta; \sigma_n^*)$.*

Suppose that $\delta(\alpha) \in \mathbb{R}$. This implies $\delta(\beta) \in \mathbb{R}$. To see why, note that otherwise Lemma 12 implies that $\lim_{n\to\infty} \frac{\Pr(\alpha|\text{piv};\sigma_n^*,n)}{\Pr(\alpha|\text{piv};\sigma_n^*,n)} = \frac{\Phi(\delta_\alpha)}{\Phi(\delta_\beta)} = 1$. Then, Lemma 3 implies that $\lim_{n\to\infty} q(\alpha; \sigma_n^*) = \Phi(1)$. This yields a contradiction to $q(\alpha; \sigma_n^*) \leq \frac{1}{2}$ since $\Phi(1) > \frac{1}{2}$ by assumption. On the other hand, Lemma 13 implies that $\delta(\alpha) \in \{\infty, -\infty\}$ or $\delta(\beta) \in \{\infty, -\infty\}$. Clearly, this contradicts the earlier observation that $\delta(\omega) \in \mathbb{R}$ for any $\omega \in \{\alpha, \beta\}$. In the same way, the assumption $\delta(\beta) \in \mathbb{R}$ leads to a contradiction. Consequently, $-\delta(\alpha) = \delta(\beta) = \infty$ and Lemma 14 implies that the equilibrium sequence satisfies (28).

This finishes the proof for the case when $W(L, \hat{p}) < W(C, \hat{p})$. The proof for the case when $W(L, \hat{p}) > W(C, \hat{p})$ is analogous.

Case $\lim_{x\to 0} \frac{c'(x)x}{c(x)} < 3$.

Take any equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$. First, recall Corollary 2, which implies that $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*) \in \Phi^{-1}(\frac{1}{2}) \cup \{\Pr(\alpha)\}$. If $\hat{p} = \Pr(\alpha)$, Lemma 3 together with $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$ and the law of large numbers imply that $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfies (64). Let $\hat{p} \in \Phi^{-1}(\frac{1}{2})$ in the following.

Step 1 *Let $\delta_{\omega,n} = (q(\omega; \sigma_n^*) - \frac{1}{2})s(\omega; \sigma_n^*)^{-1}$ for $\omega \in \{\alpha, \beta\}$. Then,*

$$\lim_{n\to\infty} \delta_{\alpha,n} - \delta_{\beta,n} = 0. \tag{139}$$

Note that $s(\omega; \sigma_n^*) = \frac{q(\omega;\sigma_n^*)(1-q(\omega;\sigma_n^*))}{2n+1}$ such that $\hat{p} \in \Phi^{-1}(\frac{1}{2})$ together with Lemma 3 implies $\lim_{n\to\infty} \frac{s(\alpha;\sigma_n^*)}{s(\beta;\sigma_n^*)} = 1$. Note that Lemma 13 provides an upper bound for the limit of the difference of the vote shares measured in standard deviations, since it considers the case when the vote share in one or both of the states is arbitrarily close to $\frac{1}{2}$. Consequently, Lemma 13 implies (139).

Step 2 $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*) = \Pr(\alpha)$.

Consider the ratio of the likelihoods of the pivotal event in the two states,

$$
\frac{\Pr(\text{piv}|\alpha; \sigma_n^*, n)}{\Pr(\text{piv}|\beta; \sigma_n^*, n)}
$$

$$
= \left[\frac{q(\alpha; \sigma_n^*)(1 - q(\alpha; \sigma_n^*))}{q(\beta; \sigma_n^*)(1 - q(\beta; \sigma_n^*))} \right]^n.
$$

$$
= \left[1 - \frac{(q(\alpha; \sigma_n^*) - \frac{1}{2})^2 - (q(\beta; \sigma_n^*) - \frac{1}{2})^2}{(q(\beta; \sigma_n^*) - \frac{1}{2})^2 - \frac{1}{4}} \right]^n
$$

$$
= \left[1 - \frac{1}{2n+1} \left(\frac{q(\alpha; \sigma_n^*)(1 - q(\alpha; \sigma_n^*))}{(q(\beta; \sigma_n^*) - \frac{1}{2})^2 - \frac{1}{4}} \delta_{\alpha,n}^2 - \frac{q(\beta; \sigma_n^*)(1 - q(\beta; \sigma_n^*))}{(q(\beta; \sigma_n^*) - \frac{1}{2})^2 - \frac{1}{4}} \delta_{\beta,n}^2 \right) \right]^n.
$$

Let

$$
x_n = \left(\frac{q(\alpha; \sigma_n^*)(1 - q(\alpha; \sigma_n^*))}{(q(\beta; \sigma_n^*) - \frac{1}{2})^2 - \frac{1}{4}} \delta_{\alpha,n}^2 - \frac{q(\beta; \sigma_n^*)(1 - q(\beta; \sigma_n^*))}{(q(\beta; \sigma_n^*) - \frac{1}{2})^2 - \frac{1}{4}} \delta_{\beta,n}^2 \right. \tag{140}
$$

The likelihood ratio simplifies to

$$
\frac{\Pr(\text{piv}|\alpha; \sigma_n^*, n)}{\Pr(\text{piv}|\beta; \sigma_n^*, n)} = \left[1 - \frac{1}{2n+1} x_n \right]^n. \tag{141}
$$

I rewrite,

$$
\frac{\Pr(\text{piv}|\alpha; \sigma_n^*, n)}{\Pr(\text{piv}|\beta; \sigma_n^*, n)} = \left(\left[1 - \frac{1}{2n+1} x_n \right]^n - e^{-\frac{1}{2}x_n} \right) + e^{-\frac{1}{2}x_n} \tag{142}
$$

and analyse the two summands separately in the following. First, note that Lemma 3 together with $\hat{p} \in \Phi^{-1}(\frac{1}{2})$ implies $\lim_{n\to\infty} q(\omega; \sigma_n^*) = \frac{1}{2}$ for $\omega \in \{\alpha, \beta\}$. Thus, Step 1 and (140) together imply

$$
\lim_{n\to\infty} x_n = 0. \tag{143}
$$

This yields

$$
\lim_{n\to\infty} e^{-\frac{1}{2}x_n} = 1, \tag{144}
$$

Second, using the Lemmas 4.3 and 4.3 in Durrett (1991) [p.94], for all $y \in \mathbb{R}$,

$$|(1 - \frac{y}{(2n+1)})^n - e^{-y}| \leq \frac{y^2}{(2n+1)^3} \tag{145}$$

Finally, it follows from (142) - (145) that

$$\lim_{n \to \infty} \frac{\Pr(\text{piv}|\alpha; \sigma_n^*, n)}{\Pr(\text{piv}|\beta; \sigma_n^*, n)} = 1, \tag{146}$$

which was to be shown.

Now, the result of Step 2 is a contradiction to $\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n^*) \in \Phi^{-1}(\frac{1}{2})$ since $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$ by assumption. Consequently, for all equilibrium sequences $\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n^*) = \Pr(\alpha)$. It follows from the law of large numbers that all equilibrium sequences satisfy (64).

D.2 Proof of Observation 1

Take any informative equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$. Let $\hat{q}(\omega; \sigma_n^*)$ be the likelihood that a randomly drawn uninformed type votes A in ω; $\hat{q}(\omega; \sigma_n^*)$ is the *vote share of the uninformed*. Let $\tilde{q}(\omega; \sigma_n^*)$ be the likelihood that a randomly drawn uninformed type votes A in ω; $\tilde{q}(\omega; \sigma_n^*)$ is the *vote share of the informed*. Now, first, Theorem 6 implies that $(\sigma_n^*)_{n \in \mathbb{N}}$ satisfies (28). Therefore, $q(\alpha; \sigma_n^*) < \frac{1}{2} < q(\beta; \sigma_n^*)$ for all n large enough or $q(\beta; \sigma_n^* < \frac{1}{2} < q(\alpha; \sigma_n^*)$ for all n large enough. Second, since the precision of the signals of the voters is symmetric across the states, Claim 16 implies that the vote shares in the two states α and β of the informed voters are symmetric to $\frac{1}{2}$. Third, suppose that $\lim_{n \to \infty} |\hat{q}(\omega; \sigma_n^*) - \frac{1}{2})|n^{-\frac{1}{2}} = \infty$. Then, Lemma 14 implies $\lim_{n \to \infty} \Pr(\alpha|\text{piv}; \sigma_n^*) \in \{0, 1\}$, given the symmetry of the vote share of the informed. This contradicts with (35). Finally, Observation 1 follows from Lemma 14.

E Extensions

E.1 Proof of Lemma 6

Given Corollary 2, it is sufficient to consider two cases.

Case 3 $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*) = \Pr(\alpha)$.

Since $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$, (13) implies that the likelihood of the pivotal event is exponentially small,

$$\Pr(\text{piv}|\alpha; \sigma_n^*) < z^n c_1 \tag{147}$$

for some constant $c_1 \in \mathbb{R}$, some $0 < z < 1$ and for n large enough. This translates into the information acquisition of the voters. For all t,

$$x^*(t) \approx \Pr(\text{piv}|\sigma_n^*)^{\frac{1}{d-1}} \left[\Pr(\alpha|\text{piv}; \sigma_n^* t_\alpha + \Pr(\beta|\text{piv}; \sigma_n^*) t_\beta \right] < z^n c_2 \tag{148}$$

for some constant $c_2 \in \mathbb{R}$, where I used the first-order condition (78) and that $\lim_{x\to\infty} \frac{c'(x)}{x^{d-1}} \in \mathbb{R}$ for the approximation. The approximation (148) implies that the sum of the voters cost converges to zero since cost are approximately polynomial when $x \approx 0$. This finishes the proof of Case 3.

Case 1 $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*) \in \Phi^{-1}(\frac{1}{2})$.

Fix a group $g \in \{\ell, s\}$. Any type $t \in g$ can be uniquely described by the total intensity and the threshold of doubt; see Section 2.1.7. It follows from (35) that

$$\lim_{n\to\infty} \hat{p} = \frac{\Pr(\text{piv}|\alpha; \sigma_n^*)}{\Pr(\text{piv}|\beta; \sigma_n^*)} = \frac{\Pr(\beta)}{\Pr(\alpha)} \frac{\hat{p}}{1 - \hat{p}}. \tag{149}$$

For the ease of exposition, let $\alpha = \arg\max \left(|q(\alpha; \sigma_n^*) - \frac{1}{2}|, |q(\alpha; \sigma_n^*) - \frac{1}{2}| \right)$. Multiplication of the first-order condition (78) by $n^{\frac{1}{2}}$ and rearranging yields

$$n^{\frac{1}{2}} c'(x^*(p^*, 1)) = n^{\frac{1}{2}} \Pr(\alpha) \Pr(\text{piv}|\alpha; \sigma_n^*) \left[t_\alpha - t_\beta \frac{\Pr(\text{piv}|\alpha; \sigma_n^*)}{\Pr(\text{piv}|\beta; \sigma_n^*)} \right]. \tag{150}$$

Note that

$$
\begin{aligned}
\forall q \in (0,1): 4^n(q(1-q))^n &= 4^n\Big[(\tfrac{1}{2} - (\tfrac{1}{2} - q))(\tfrac{1}{2} + (\tfrac{1}{2} - q)\Big]^n \\
&= 4^n(\tfrac{1}{4} - (\tfrac{1}{2} - q)^2)^n \\
&= (1 - 4\frac{(n^{\frac{1}{2}}(\tfrac{1}{2} - q))^2}{n})^n.
\end{aligned}
\tag{151}
$$

Combining (13) with (150) and (151) gives

$$
n^{\frac{1}{2}} c'(x^*(p^*, 1)) \approx c_{11}(1 - 4\frac{(n^{\frac{1}{2}}(\tfrac{1}{2} - q(\alpha; \sigma_n^*)))^2}{n})^n
$$

for some constant $c_{10} > 0$. Multiplication of both sides with $\delta_n = n^{\frac{1}{2}}|q(\alpha; \sigma_n^*) - \tfrac{1}{2}|$ yields

$$
\delta_n n^{\frac{1}{2}} c'(x^*(p^*, 1)) \approx c_{10} \delta_n e^{-4\delta_n} + \delta_n \Big[(1 - 4\frac{\delta_n^2}{n})^n - e^{-4\delta_n}\Big].
\tag{152}
$$

for some constant $c_{10} > 0$. Using Lemmas 4.3 and 4.3 in Durrett (1991),

$$
(1 - 4\frac{\delta_n^2}{n})^n - e^{-4\delta_n} \leq \frac{\delta_n^4}{n^3}.
\tag{153}
$$

Therefore, $\lim_{n \to \infty} \delta_n \Big[(1 - 4\frac{\delta_n^2}{n})^n - e^{4\delta_n}\Big] = 0$. It follows from Theorem 6 that $\lim_{n \to \infty} \delta_n = \infty$ which implies $\lim_{n \to \infty} \delta_n e^{-4\delta_n} = 0$. I conclude that

$$
\lim_{n \to \infty} \delta_n n^{\frac{1}{2}} c'(x^*(p^*, 1)) = 0.
\tag{154}
$$

Recall (36) which states that

$$
q(\alpha; \sigma_n^*) - q(\beta; \sigma_n^*) = 2\Big[\int_{t \in \ell} c(x(t)) dH(t) - \int_{t \in s} c(x(t)) dH(t)\Big].
\tag{155}
$$

I combine (108) and (155) and $W(\ell, \hat{p}) \neq W(s\hat{p})$ to obtain

$$
\begin{aligned}
x^*(p^*, 1)^2 &\leq c_{11}\Big[|q(\alpha; \sigma_n^*) - \frac{1}{2}| + |q(\beta; \sigma_n^*) - \frac{1}{2}|\Big] \\
&\leq 2c_{11}\frac{\delta_n}{n^{-\frac{1}{2}}}
\end{aligned}
\tag{156}
$$

for n large enough and some constants c_{11}, where I used the triangle equality on the first line. Hence, (154) implies

$$
\lim_{n \to \infty} n x^*(p^*, 1)^2 c'(x^*(p^*, 1)) = 0
\tag{157}
$$

Using Lemma 7,

$$
\lim_{n \to \infty} \frac{x^2 c'(x)}{x c(x)} = d.
\tag{158}
$$

Recall that $x^*(p^*, 1)$ converges to zero as $n \to \infty$, such that combining (157) and (158) gives

$$
\lim_{n \to \infty} n x^*(p^*, 1) c(x^*(p^*, 1)) = 0.
\tag{159}
$$

I claim that any equilibrium sequence $(\sigma_n^*)_{n \in \mathbb{N}}$ satisfies

$$
\int_{t \in g} c(x(t)) dH(t) \approx \frac{c(x^*(p^*, 1)) x^*(p^*, 1)}{h'(p^*)} \Pr(g) f(p^*|t \in g) \mathrm{E}(k(t)^{\frac{2}{d-1}}|t \in g, y(t) = p^*)
\tag{160}
$$

The proof of (160) follows from previous arguments, analogous to the ones in Step 5 of the Proof of Observation 2. The proof is verbatim with the required changes in notation. Finally, the lemma follows from (159) and (160).

References

Ahn, David S, & Oliveros, Santiago. 2012. Combinatorial voting. *Economet-rica*, **80**(1), 89–141.

Alesina, Alberto, & Rodrik, Dani. 1994. Distributive politics and economic

growth. *The quarterly journal of economics*, **109**(2), 465–490.

Ali, S Nageeb, Mihm, Maximilian, & Siga, Lucas. 2018. *Adverse Selection in Distributive Politics*. Tech. rept. Working paper, Penn State University.

Austen-Smith, David, & Banks, Jeffrey S. 1996. Information aggregation, rationality, and the Condorcet jury theorem. *American political science review*, **90**(1), 34–45.

Bhattacharya, Sourav. 2013a. Preference monotonicity and information aggregation in elections. *Econometrica*, **81**(3), 1229–1247.

Bhattacharya, Sourav. 2013b. Preference monotonicity and information aggregation in elections. *Econometrica*, **81**(3), 1229–1247.

Billingsley, Patrick. 2008. *Probability and measure*. John Wiley & Sons.

Burk, Abram. 1936. Real Income, Expenditure Proportionality, and Frisch's" New Methods of Measuring Marginal Utility". *The Review of Economic Studies*, **4**(1), 33–52.

Carmines, Edward G, & Stimson, James A. 1980. The two faces of issue voting. *American Political Science Review*, **74**(1), 78–91.

Converse, Philip E. 1964. The nature of belief systems in mass publics (1964). *Critical review*, **18**(1-3), 1–74.

Durrett, R. 1991. Probability: Theory and Examples, Wadsworth & Brooks/Cole, Pacific Grove. *MR1068527*.

Eguia, Jon X, & Xefteris, Dimitrios. 2018. Implementation by vote-buying mechanisms.

Feddersen, Timothy, & Pesendorfer, Wolfgang. 1997. Voting behavior and information aggregation in elections with private information. *Econometrica: Journal of the Econometric Society*, 1029–1058.

Feddersen, Timothy, & Pesendorfer, Wolfgang. 1998. Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *American Political science review*, **92**(1), 23–35.

Fernandez, Raquel, & Rodrik, Dani. 1991. Resistance to reform: Status quo bias in the presence of individual-specific uncertainty. *The American economic review*, 1146–1155.

Gnedenko, Boris Vladimirovich. 1948. On a local limit theorem of the theory of probability. *Uspekhi Matematicheskikh Nauk*, **3**(3), 187–194.

Heese, Carl, & Lauermann, Stephan. 2017. *Persuasion and Information Aggregation in Elections*. Tech. rept. Working Paper.

Iyengar, Shanto, Hahn, Kyu S, Krosnick, Jon A, & Walker, John. 2008. Selective exposure to campaign communication: The role of anticipated agreement and issue public membership. *The Journal of Politics*, **70**(1), 186–200.

Krishna, Vijay, & Morgan, John. 2011. Overcoming ideological bias in elections. *Journal of Political Economy*, **119**(2), 183–211.

Krishna, Vijay, & Morgan, John. 2015. Majority rule and utilitarian welfare. *American Economic Journal: Microeconomics*, **7**(4), 339–375.

Lalley, Steven, & Weyl, E Glen. 2018. Nash equilibria for quadratic voting. *Available at SSRN 2488763*.

Martinelli, César. 2006. Would rational voters acquire costly information? *Journal of Economic Theory*, **129**(1), 225–251.

Martinelli, César. 2007. Rational ignorance and voting behavior. *International Journal of Game Theory*, **35**(3), 315–335.

McDonald, David R. 1980. On local limit theorem for integer-valued random variables. *Theory of Probability & Its Applications*, **24**(3), 613–619.

Miller, Joanne M, & Krosnick, Jon A. 2000. News media impact on the ingredients of presidential evaluations: Politically knowledgeable citizens are guided by a trusted source. *American Journal of Political Science*, 301–315.

Moulin, Hervé. 2004. *Fair division and collective welfare*. MIT press.

Oliveros, Santiago. 2013a. Abstention, ideology and information acquisition. *Journal of Economic Theory*, **148**(3), 871–902.

Oliveros, Santiago. 2013b. Aggregation of endogenous information in large elections.

Persson, Torsten, & Tabellini, Guido. 1994. Is inequality harmful for growth? *The American economic review*, 600–621.

Triossi, Matteo. 2013. Costly information acquisition. Is it better to toss a coin? *Games and Economic Behavior*, **82**, 169–191.