Voter Attention and Distributive Politics *

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Abstract

This paper studies the role of heterogeneous attention among voters for distributive politics. Groups in the population pay systematically different attention to politics and acquire different levels of information. This paper studies the effects of heterogeneous attention when a reform may benefit one group at the expense of others. In the benchmark, when the information of voters is exogenous, a median voter theorem holds, and a welfare-enhancing reform is not adopted if it is not preferred by a majority. When information is endogenous, attention shifts election outcomes into a direction that is welfare-improving. Even when a welfare-enhancing reform is not preferred by a majority ex-post, under certain conditions, there are equilibria where the reform is adopted. The key driver of the results is that voters who are more severely affected by a proposed reform will pay more attention, consistent with empirical studies ("issue publics hypothesis," Converse (1964)). This information advantage translates into voting power, precluding the majority from exerting its dominance.

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1 Introduction

A rich literature in distributive politics seeks to understand if, and when, elections select policies that maximize social welfare.[1] This paper introduces a novel aspect into this discussion; namely, endogenous attention to politics, which is guided by the empirical observation that voters that care more about a political issue will acquire more information about it.[2] For instance, older people care more about healthcare issues, and citizens with children care more about education issues (Iyengar *et al.* , 2008).

I propose a model that can describe the relationship between the information levels of different voter groups with conflicting interests and their voting power. The main result shows that equilibria can be described by comparing a certain power index of the groups of voters with conflicting interests (Theorem 1 and Observation 2). This characterization will imply that, for a large class of settings, there are equilibria where the first-best (utilitarian) outcome is elected, even if this is not the preferred outcome of the majority of the voters *ex-post* (Theorem 2). In such equilibria, a minority of the voters is more severely affected by the reform, but will be better informed about it, and can translate the information advantage into voting power, thereby shifting the election outcome towards efficiency.

Examples of reforms with uncertain distributive consequences are numerous: a trade reform opens new markets for exporting firms but threatens the prospects in other sectors; a public health policy reform makes certain treatments more accessible to some citizens, while implying price increases for a range of pharmaceuticals needed by others; and a new education reform benefits some children but affects others negatively. In all these examples, most voters are *ex-ante* uninformed about the consequences of the reform, e. g. which sec-

[1] See e.g. Fernandez & Rodrik (1991), Alesina & Rodrik (1994) and Persson & Tabellini (1994). Similar to this paper, Bhattacharya (2013a) and Ali *et al.* (2018) transport the informational approach to elections (Austen-Smith & Banks (1996), Feddersen & Pesendorfer (1997)) to the literature on distributive politics.

[2] This is known as the "issue publics hypothesis" (Converse, 1964). See e.g. Krosnick (1990) and Henderson (2014), and Carpini & Keeter (1996) for an overview about the American public's factual knowledge about politics.

tors gain from a trade reform, or if their child benefits from education reform. However, they hold private information about their exposure to the proposed reform, that is, about the magnitude of their preference intensities: more affluent citizens have more personal wealth at stake, changes in education policy are more relevant to citizens with children, and citizens dependent on medication are more profoundly affected by changes in public health policy.

This paper considers a modified version of the canonical setting by Feddersen & Pesendorfer (1997). Relative to Feddersen & Pesendorfer (1997), the voters' information about the policies is endogenous *and* the setting allows that the voters have conflicting interests (distributive politics). Both modifications have been studied before, but not together: Bhattacharya (2013b) has studied a variant with conflicting interests and Martinelli (2006) has studied a variant with endogenous information.[3] The novelty is that I study both aspects together and this will be critical to be able to describe the relationship of preference intensities, the information level and the electoral power of groups with conflicting interests.

There are two possible policies: a reform and a status quo. Voters' preferences over policies are heterogeneous and depend on an unknown, binary state in a general way (some voters may prefer the reform only in the first state, others may prefer the reform only in the second state, while some "partisans" may prefer one of the policies independently of the state). The preferences are each voter's private information. Besides, all voters can receive information about the state in the form of a noisy signal, and each voter freely chooses the precision of her private signal. More precise information is more costly. Upon receiving their signals, all citizens vote simultaneously. The election determines the outcome by simple majority rule.

Feddersen & Pesendorfer (1997) have shown that when voters receive conditionally i.i.d. signals of some exogenous quality and their preferences are "monotone" in all equilibria of large elections the outcome preferred by the

[3] I will discuss these papers in more detail momentarily.

median voter is elected state-by-state.[4] In many situations where voters have conflicting interests this is not the first-best outcome: for example, when 51% of the citizens marginally benefit from a reform, while the other 49% are severely impacted by it. Martinelli (2006) has shown that the median voter theorem also holds in a common interest setting where information is endogenous, but only if voters can acquire relevant political information can at a cost that is "not too high." Formally, what matters for the result is how fast cost goes to zero when a voter chooses an arbitrarily uninformative signal. The critical condition is that the first three derivatives of the cost function are zero at the precision of the uninformative signal. Bhattacharya (2013a) has shown that Feddersen & Pesendorfer (1997)'s result generalizes to settings with conflicting interests.[5] Importantly, his model does not allow to study the role of the intensity of preferences since the result is invariant to scaling the intensities of specific groups of voters.

The principal analysis in this paper describes the political power of the heterogeneous voters. All equilibria can be represented by an index rule (Lemma 2). The *power index* of a group of voters sharing common interests is a relative measure of its electoral power. In particular, it is proportional to the size of the group and is increasing in the welfare that is at stake for the group. When voters can acquire relevant political information at a cost that is "not too high," there are equilibria where competitive information acquisition shapes the election and where the election yields the outcome preferred by the group with the higher power index. I relate this index representation to social welfare, under a certain independence condition on the preference distribution that implies that the preferences are "monotone:[6] In general terms, I show that when the cost of information is in a certain intermediate range, there

[4] The preference distribution of the voters is "monotone" if a higher belief in the first state entails that more voters prefer the reform.

[5] Bhattacharya (2013a) also shows that the result breaks down when preferences are non-monotone.

[6] When the preferences are non-monotone, there may be many equilibria, similar to the results in Bhattacharya (2013b). I characterize all of them; they are all "index equilibria, but generally, the power index of the group, depends on what Bhattacharya (2013b) calls the *induced prior belief* of the given equilibrium.

is an equilibrium that leads to first-best outcomes. In other words, there is an intermediate range of cost functions where the information cost screen the voters intensities appropriately.[7] As well as the condition in Martinelli (2006), generally, another condition is necessary for first-best outcomes: namely, that cost should not go to zero too fast, where the strictness of the condition will depend globally on the distribution of the voters' preferences.

In the second part, I deepen the equilibrium analysis. By doing so, I point out determinants of efficiency and the political power of voter groups that are novel to the existing literature. First, I analyze the robustness of the equilibrium with first-best outcomes by varying the voters' cost of acquiring relevant information about the policy consequences. The more similar the welfare at stake–that is, the aggregate intensities, of the citizens who benefit relative to those who are harmed by reform– the smaller the intermediate range of cost functions for which there is an equilibrium with first-best outcomes (Theorem 3). Intuitively, more similar intensities make it more difficult for the information cost to be able to screen voter intensities appropriately. Second, I show that the electoral power of a voter group is decreasing with the polarization of the preference intensities within the group (Theorem 4).

In the last part of the paper, I study the situations where the voters do not only differ in their exposure to the proposed reform in terms of preference intensities but also in their ability to access and interpret political information. Formally, the voters are subject to different cost functions. First, I provide an equivalence result: the extended setting with a heterogeneous cost is equivalent to a setting with a homogeneous cost. Intuitively, the cost of information and preference intensities are strategically equivalent such that differences in cost translate into differences in intensities. The previous efficiency results, therefore, extend when the cost types are independent of the preference intensities (Theorem 7). Second, I vary the richness of the information choice of the voters in the extended setting with heterogeneous cost, and show that

[7] I also show that aggregate cost of the voters converge to 0 as the electorate grows large such that the equilibrium sequences with first-best outcomes imply first-best results, even when taking into account the cost of voters, see Lemma 9.

coarsening the voters' information choices may have positive welfare effects. I compare the situation where the voters only have access to a binary set of signals to the previous setting where voters have access to a rich set of signals and can freely choose their signal precision without constraints. In the coarse setting, there are always equilibria where the welfare-maximizing outcome is elected in all states (Theorem 8), unlike in the setting with the rich information choice where this depends on how similar the welfare at stake is for the citizens who benefit from a particular reform relative to those who are harmed by it. This is particularly surprising since a richer choice set for the voters should intuitively facilitate, rather than prevent, an appropriate screening of preference intensities in equilibrium.

This paper contributes to the literature on information aggregation in large elections. Condorcet's Jury Theorem (1785) states that if voters have common interests, but the information is dispersed throughout the electorate, then majority rule results in socially optimal outcomes. Information aggregates in the sense that electoral outcomes correspond to the choices of a fully informed welfare-maximizing social planner. Austen-Smith & Banks (1996), Feddersen & Pesendorfer (1998) have established a "modern" version of Condorcet's Jury Theorem in a setting where citizens vote strategically. However, as has been argued, elections might fail to elect socially optimal outcomes when voters have conflicting interests, fsuch as when 51% of citizens marginally benefit from a reform, while the other 49% are severely impacted by it. This paper points at an empirical observation that has been mostly overlooked in this context: namely, that the dispersion of the voters' information is endogenous. I show how this can lead to socially optimal outcomes, independent of the distribution of the voters' preference intensities.

This paper also contributes to the literature on elections with costly information acquisition by studying a general setup that, in particular, allows the voters to have conflicting interests, like in distributive politics. The previous literature has studied situations where preference types might be heterogeneous, but where all voters share a common interest (e. g., as in Martinelli (2006) and Oliveros (2013a)). Martinelli (2006) and Oliveros (2013a) are concerned with the possibility of information aggregation in large elections when information is costly and have shown that information aggregation is possible in setups with symmetric preference distributions.[8] For the common interest case, I generalize the results of the literature by providing an analysis for general continuous preference distributions and by characterizing all the equilibria of the voting game. In particular, the main result of the paper will imply that, in common interest settings, information aggregation is generally possible under the conditions on the cost function provided in Martinelli (2006).[9]

This paper is related to work on elections with voting cost and vote-buying. Krishna & Morgan (2011, 2015) have shown that elections yield first-best outcomes when voting is voluntary and costly. These results are analogous to the findings for the setting with a coarse, binary information choice. The main model with a rich information choice is more closely related to the literature on vote-buying. Lalley & Weyl (2018) have shown that equilibrium outcomes of a large electorate are first-best when each voter can buy any number of votes at a total price that is quadratic in the number bought. Similarly, in this model, the analysis shows that when the cost of information is arbitrarily close to "cubic" as a function of the precision of the signal, there are equilibria with first-best outcomes for almost all preference distributions. Eguia & Xefteris (2018) show that vote-buying mechanisms with general price functions implement the set of Bergsonian welfare functions.[10] Similarly, my results show that majority elections with information costs implement the Bergsonian welfare functions that satisfy a fairness principle known as the Pigou-Dalton principle (see Section 7.3).[11]

This paper is related to papers studying the interaction of limited attention

[8]Further more distantly related papers include those by Triossi (2013) and Martinelli (2007) who study heterogeneous cost in common interest setups, and Oliveros (2013b) who studies the relationship of abstention and information cost.

[9]The settings in Martinelli (2006) and in Oliveros (2013b) are special cases of my model. Note that when voters have a common interest, the full-information equivalent outcome is utilitarian.

[10]A Bergsonian welfare function selects the outcome that maximizes a certain weighted sum of the agents' utilities (see Burk (1936)).

[11]See Moulin (2004) for a definition of the Pigou-Dalton principle.

of voters and the policy choices of political platforms. Matějka & Tabellini (2017) study this question in a probabilistic voting model and Hu & Li (2018) study it using a spatial model with non-strategic voters.

A Two-Type example

The following extreme setting shows how information cost can screen the preference intensities appropriately. Thereby, it illustrates how intensities translate into voting power and how first-best outcomes can be elected, even when a majority of the voters do not prefer the outcome *ex-post*. There are $2n+1$ voters. With probability $1 > \lambda > \frac{1}{2}$, a voter is *aligned* and prefers the reform A over the status quo B only in α and B over A in β. Otherwise, a voter is *contrarian* and prefers A in β and B in α.[12] An aligned voter gets a small utility of $\epsilon > 0$ when her preferred policy is adopted, while a contrarian voter gets a utility of 1 when her preferred policy is adopted. Voters have a binary choice: either to get a private, perfect signal about the state at a given cost $c > 0$ or an uninformative signal at no cost. The common prior about the state is uniform, i.e., $\Pr(\alpha) = \frac{1}{2}$. In order to maximize welfare, the election should implement what the contrarians want if they care much more, i.e., when ϵ is sufficiently small such that $\epsilon(1-\lambda) < \lambda$.

Consider three scenarios: zero, intermediate and high cost. When cost is zero, i.e. $c = 0$, all voters become perfectly informed about the state and the outcome preferred by the median voter is elected in each state. When the cost is very high, e. g. $c > 1$, nobody gets informed, and the policy elected is independent of the state. When the cost comes from a certain intermediate range, they screen types such that only types with high intensities get informed. In fact, for any n, there is a range of such intermediate cost c and a *symmetric* equilibrium given c where the aligned vote for each policy with the same probability, i.e., $50-50$, and the contrarians vote for their preferred outcome in each state. Thus, in each state, the outcome preferred by the contrarians,

[12] The terminology used to label the voter types carries no economic meaning whatsoever but only relates to the notation. Aligned voters prefer the outcome that is "aligned" with the state.

which represent a minority in expectation, receives a strict majority of the votes in expectation. Therefore, when the electorate grows large, thethe outcome preferred by the contrarians is elected, which maximizes welfare.[13] To see why such an equilibrium exists, note that without the private signal, a citizen is indifferent between voting for either of the policies. This is because the event in which the citizen's vote affects the outcome is equally likely in each state in the symmetric candidate equilibrium.[14] The citizen only benefits from being informed about the state in the situations when her vote affects the outcome of the election. Thus, the benefit of being informed is proportional to the likelihood of this pivotal event, piv, and when the cost is intermediate, i.e., when $\epsilon \Pr(\text{piv}) < c < \Pr(\text{piv})$, only the contrarians would like to buy the signal. Note that the closer the utility at stake for the aligned relative to the contrarians, the smaller is the range of intermediate costs that screen the types.

The rest of the paper is organized as follows. Section 2 presents the model and the preliminary analysis. Section 3 discusses the power (index) of voter groups and shows the existence of equilibria with first-best outcomes. Section 4 describes the range of the cost functions for which they exist, and discusses the role of preference polarization. Section 5 characterizes all other equilibria and, in particular, shows the existence of another equilibrium that converges to "voting according to the prior belief" about the unknown state. In this equilibrium, the outcome that is preferred by the majority of the voters given the prior belief is elected (Theorem 5). Section 6 studies an extended setting with heterogeneous cost functions and a setting where the information choice of the voters is coarse. Section 7 contains further remarks.

[13]I do not discuss the welfare effects of the cost for the example since it turns out that the aggregate cost is arbitrarily small in the equilibria of the main result when the electorate is large, $n \to \infty$ (see Section 7.2).

[14]In one state the reform wins with a margin of $\frac{1}{2}(1-\lambda)$ and in the other state the reform loses with a margin $\frac{1}{2}(1-\lambda)$ in expectation, implying that $\Pr(\alpha|\text{piv}) = \frac{1}{2}$, where piv is the event where a given citizen's vote affects the outcome of the election.

2 Model

There are $2n+1$ voters (or citizens), two policies A and B, and two states of the world $\omega \in \{\alpha, \beta\} = \Omega$. The prior probability of α is $\Pr(\alpha) \in (0,1)$.

Voters have heterogeneous and state-dependent preferences. A voter's preference is described by a type $t = (t_\alpha, t_\beta)$, where $t_\omega \in [-1,1]$ is the utility of A in ω. The utility of B is normalized to zero, so that t_ω is the difference between the utilities of A and B in ω. The types are identically distributed across voters according to a cumulative distribution function $H : [-1,1]^2 \to [0,1]$ that has a continuous density h. A voter's type is her private information. Each voter observes a binary signal $s \in \{a, b\}$. The observed signal is also the private information of the voter. The joint distribution of the type and the signal of a voter is independent of the distribution of the signals and the types of the other voters.

The voting game is as follows. First, nature draws the state and the profile of types $\mathbf{t}$ according to H. Second, after observing her type, each voter chooses a precision $x(t) \in [0, \frac{1}{2}]$ of her signal, that is $\frac{1}{2} + x(t) = \Pr(a|\alpha) = \Pr(b|\beta)$. Then, private signals realize. After observing her private signal, each voter simultaneously submits a vote for A or B. Finally, the submitted votes are counted and the majority outcome is chosen.

There is a strictly increasing, convex, and twice continuously differentiable *cost function* $c : [0, \frac{1}{2}] \to \mathbb{R}_+$ and when choosing precision x, the voter bears a cost $c(x)$ where $c(0) = 0$. There is a $d > 1$ such that[15]

$$\lim_{x \to \infty} \frac{c'(x)}{x^{d-1}} \in \mathbb{R}. \tag{1}$$

A strategy $\sigma = (x, \mu)$ of a voter consists of a function $x : [-1,1]^2 \to [0, \frac{1}{2}]$ mapping types to signal precisions and a function $\mu : [-1,1]^2 \times \{a,b\} \to [0,1]$ mapping types and signals to probabilities to vote A, i.e., $\mu(t,s)$ is the probability that a voter of type t with signal s votes for A. I consider only non-

[15]It will be a direct insight from the preliminary results in the next section that without the condition $d > 1$, no voter acquires any information in equilibrium when n is sufficiently large; see (19).

degenerate strategies.[16] I analyze the Bayes-Nash equilibria of the Bayesian game of voters in symmetric strategies, called *equilibria* henceforth.

2.1 Preliminaries

In this section, I analyze the best response of the voters (Sections 2.1.2, 2.1.4, 2.1.5, and 2.1.7), introduce key objects of the analysis (Sections 2.1.1 and 2.1.3) and provide a useful representation of the voter types that I will use sometimes from then on (Section 2.1.6).

2.1.1 Aggregate Preferences

A central object of the analysis is the *aggregate preference function*

$$\Phi(p) = \Pr_H(\{t : p \cdot t_\alpha + (1-p) \cdot t_\beta \geq 0\}), \tag{2}$$

which maps a belief $p \in [0,1]$ about the state to the probability that a random type t prefers A under p. Figure 1 illustrates Φ: the (colored) line corresponds to the set of types $t = (t_\alpha, t_\beta)$ that are indifferent between policy A and policy B when holding the belief p. Voters having types to the north-east prefer A given p (shaded area); these types have mass $\Phi(p)$. The indifference set has a slope of $\frac{-p}{1-p}$ and an increase in p corresponds to a left rotation of it. Given that H has a continuous density, Φ is continuously differentiable in p.

Voters having types t in the north-east quadrant prefer A for all beliefs and voters having types t in the south-west quadrant always prefer B (*partisans*). Voters having types t in the south-east quadrant prefer A in state α and B in β (*aligned voters*), and voters having types t in the north-west quadrant prefer

[16] A strategy σ is *degenerate* if $\mu(t,s) = 1$ for all (t,s) or if $\mu(t,s) = 0$ for all (s,t). When all voters follow the same degenerate strategy and there are at least three voters, if one voter deviates to any other strategy, then the outcome is the same. Therefore, the degenerate strategies with $x = 0$ are trivial equilibria.

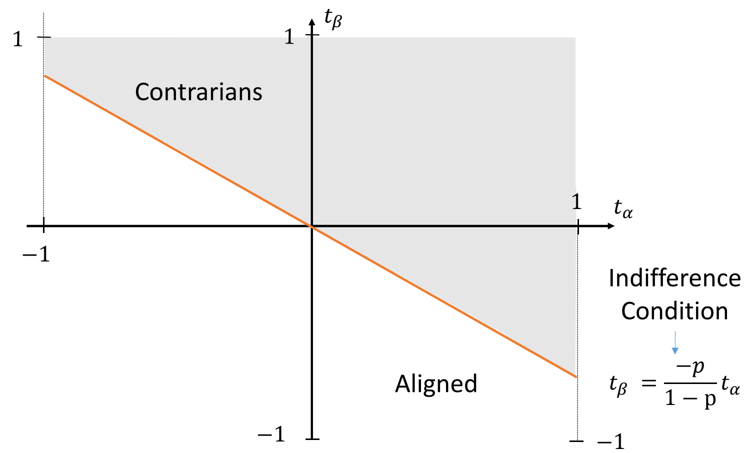


Figure 1: For any given belief $p = \Pr(\alpha) \in (0, 1)$, the set of types t with a threshold of doubt $y(t) = p$ is given by $t_\beta = \frac{-p}{1-p} t_\alpha$.Voter types north-east of the indifference line (shaded area) prefer A given p.

B in state α and A in β (*contrarian voters*). I assume that

$$\Phi(0) < \frac{1}{2}, \text{ and } \Phi(1) > \frac{1}{2} \tag{3}$$

such that the median-voter preferred outcome is A in α and B in β. In particular, this excludes the cases when there is a majority of partisans for one policy in expectation. I also make the genericity assumption that Φ is not constant on any open interval.[17] Henceforth, I will call distributions H that have a continuous density and satisfy (3) simply *preference distributions*. The set of the aligned types is denoted $L = \{t : t_\alpha > 0, t_\beta < 0\}$ and the set of the contrarian types is denoted $C = \{t : t_\alpha < 0, t_\beta > 0\}$ and $g \in \{L, C\}$ is the generic symbol for a voter group, aligned or contrarians.

2.1.2 Pivotal Voting and the Voter's Threshold of Doubt

How do the voters decide which alternative to vote for? To simplify the exposition, in the rest of the paper, I only consider strategies σ where the partisans

[17] This assumption is known from the literature, see Bhattacharya (2013b).

use the (weakly) dominant strategy to vote for their preferred policy.[18] This section shows that the aligned and contrarian voters follow a cutoff rule when deciding between A and B: what matters is if the posterior conditional on the information from the private signal and conditional on the event where the citizen's vote decides the election outcome exceeds a type-dependent *threshold of doubt*,

$$y(t) = \frac{-t_\beta}{t_\alpha - t_\beta}. \tag{4}$$

From the viewpoint of a given voter and given any strategy σ' used by the other voters, the pivotal event, piv, is the event in which the realized types and signals of the other $2n$ voters are such that exactly n of them vote for A and n for B. For this event, if she votes A, the outcome is A; if she votes B, the outcome is B. In any other event, the outcome is independent of her vote. Thus, a strategy is optimal if and only if it is optimal conditional on the pivotal event. Take any two strategies $\sigma' = (x', \mu')$ and $\sigma = (x, \mu)$. For any type t of the given voter, and given the precision choice $x(t)$, let $\Pr(\alpha|s, \text{piv}; \sigma', n)$ be the posterior probability of α conditional on *being pivotal* and conditional on the private signal s when the other voters use σ'.

Lemma 1 *Take any strategy σ'. The function μ is part of a best response $\sigma = (x, \mu)$ to σ' if and only if for all aligned types $t \in L$, given the precision $x(t)$,*

$$\Pr(\alpha|s, \text{piv}; \sigma', n) > y(t) \Rightarrow \mu(t, s) = 1, \tag{5}$$

$$\Pr(\alpha|s, \text{piv}; \sigma', n) < y(t) \Rightarrow \mu(t, s) = 0; \tag{6}$$

and for all contrarian types $t \in C$, given the precision $x(t)$,

$$\Pr(\alpha|s, \text{piv}; \sigma', \sigma, n) > y(t) \Rightarrow \mu(t, s) = 0, \tag{7}$$

$$\Pr(\alpha|s, \text{piv}; \sigma', \sigma, n) < y(t) \Rightarrow \mu(t, s) = 1, \tag{8}$$

[18] In fact, for any non-degenerate strategy, I show that the likelihood of the pivotal event is non-zero (see Section 2.1.3) such that voting for the preferred policy while not acquiring any information is the unique strict best response for all partisans.

Proof. Given some x, the function μ is part of a best response if and only if for all $t = (t_\alpha, t_\beta)$ and for the signal precision $x(t)$,

$$\Pr(\alpha|s, \mathrm{piv}; \sigma', n) \cdot t_\alpha + (1 - \Pr(\alpha|s, \mathrm{piv}; \sigma', n)) \cdot t_\beta > 0 \Rightarrow \mu(s,t) = 1, \quad (9)$$

$$\Pr(\alpha|s, \mathrm{piv}; \sigma', n) \cdot t_\alpha + (1 - \Pr(\alpha|s, \mathrm{piv}; \sigma', n)) \cdot t_\beta < 0 \Rightarrow \mu(s,t) = 0, \quad (10)$$

that is, a voter supports A if the expected value of A conditional on being pivotal and s is strictly positive and otherwise supports B. Rearranging yields (5)- (8). ■

Note that indifference holds only for a set of types that has a zero measure since the type distribution H is continuous. For all other types, the best response is pure. It follows that there is no loss of generality to consider pure strategies with $\mu(t, s) \in \{0, 1\}$ for all (s, t).[19]

2.1.3 Vote Shares and the Likelihood of the Pivotal Event

Take any strategy $\sigma = (x, \mu)$ of the voters. The probability that a voter of random type votes for A in state $\omega \in \{\alpha, \beta\}$ is denoted $q(\omega; \sigma)$. A simple calculation shows that

$$q(\alpha; \sigma) = \int_{t \in [-1,1]^2} (\frac{1}{2} + x(t))\mu(t, a) + (\frac{1}{2} - x(t))\mu(t, b) dHt, \quad (11)$$

and

$$q(\beta; \sigma) = \int_{t \in [-1,1]^2} (\frac{1}{2} - x(t))\mu(t, a) + (\frac{1}{2} + x(t))\mu(t, b) dHt. \quad (12)$$

I also refer to $q(\omega; \sigma)$ as the *(expected) vote share* of A in ω. As it is, conditional on the state, the type and the signal of a voter is independent of the types and the signals of the other voters. Therefore, the probability of a tie in the

[19] When writing "*for all*," I ignore zero measure sets here and in the following.

vote count is

$$\Pr\left(\text{piv}|\omega;\sigma,n\right)=\binom{2n}{n}\left(q\left(\omega;\sigma\right)\right)^{n}\left(1-q\left(\omega;\sigma\right)\right)^{n}. \tag{13}$$

I use a Stirling approximation of the binomial coefficient and (13) to obtain[20] [21]

$$\Pr\left(\text{piv}|\omega;\sigma,n\right)\approx 4^{n}(n\pi)^{-\frac{1}{2}}\Big[q(\omega;\sigma)(1-q(\omega;\sigma)\Big]^{n}. \tag{14}$$

Note that, given (14), the likelihood of the pivotal event is non-zero for any (non-degenerate) strategy, but converges to zero as the electorate size grows large since the function $q(1-q)$ takes the maximum $\frac{1}{4}$ on $[0,1]$.

2.1.4 Choices of Informed Voters

Take any strategy σ' and let $\sigma=(x,\mu)$ be a best response to σ'. Since signal a is indicative of α and b of β, voters with a signal a believe state α to be more likely than voters with a signal b. In fact, given any $x>0$, the posteriors are ordered as

$$\Pr\left(\alpha|a,\text{piv};\sigma,n\right)>\Pr\left(\alpha|b,\text{piv};\sigma,n\right). \tag{15}$$

To see why, note that the posterior likelihood ratio of the states conditional on a signal $s\in\{a,b\}$ with precision $x(t)$ and the event that the voter is pivotal is

$$\frac{\Pr\left(\alpha|s,\text{piv};\sigma',n\right)}{\Pr\left(\beta|s,\text{piv};\sigma',n\right)}=\frac{\Pr\left(\alpha\right)}{\Pr\left(\beta\right)}\frac{\Pr\left(\text{piv}|\alpha;\sigma',n\right)}{\Pr\left(\text{piv}|\beta;\sigma',n\right)}\frac{\Pr(s|\alpha;\sigma)}{\Pr(s|\beta;\sigma)}, \tag{16}$$

if $\Pr\left(\text{piv}|\beta;\sigma',n\right)>0$, where I used the conditional independence of the types and signals of the other voters from the signal of the given voter. Then, the order of the likelihood ratios in (16) follows from $\Pr(a|\alpha;\sigma)=\Pr(b|\beta;\sigma)=\frac{1}{2}+x$.

[20] The notation $x_n\approx y_n$ describes that two sequences $(x_n)_{n\in\mathbb{N}}$ and $(y_n)_{n\in\mathbb{N}}$ are *asymptotically equivalent* in the following sense: $\lim_{n\to\infty}\frac{x_n}{y_n}=1$.

[21] Stirling's formula yields $(2n)!\approx(2\pi)^{\frac{1}{2}}2^{2n+\frac{1}{2}}n^{2n+\frac{1}{2}}e^{-2n}$ and $(n!)^2\approx(2\pi)n^{2n+1}e^{-2n}$. Consequently, $\binom{2n}{n}\approx(2\pi)^{-\frac{1}{2}}2^{2n+\frac{1}{2}}n^{-\frac{1}{2}}=4^n(n\pi)^{-\frac{1}{2}}$.

Suppose that a voter acquires information, i.e. $x(t) > 0$, and suppose that she votes for the same policy after both signals, i.e. $\mu(t, a) = \mu(t, b)$. Then, she would be strictly better off by not paying for the information and simply voting for this policy. Therefore, (5)-(8), and (15) together imply that aligned types that acquire information vote A only after a, and contrarian types that acquire information vote A only after b. That is

$$\forall t \in L : x(t) > 0 \Rightarrow \mu(t, a) = 1 \text{ and } \mu(t, b) = 0, \tag{17}$$

$$\forall t \in C : x(t) > 0 \Rightarrow \mu(t, a) = 0 \text{ and } \mu(t, b) = 1. \tag{18}$$

Let us turn to the question of how much information voters acquire, if they choose to acquire any at all. For any type t with $x(t) > 0$, a first-order condition pins down the equilibrium precision. I derive the first-order condition in the Appendix; it equates marginal cost and marginal benefit,

$$c'(x) = \Pr(\text{piv}; \sigma', n)\Big[\Pr(\alpha|\text{piv}; \sigma', n)|t_\alpha| + \Pr(\beta|\text{piv}; \sigma', n)|t_\beta|. \tag{19}$$

To get an intuition for (19), recall that a voter only benefits from better information in the event piv when her vote affects the outcome of the election. It turns out that, therefore, the marginal benefit of a higher signal precision is proportional to the likelihood of the pivotal event and the expected welfare at stake.[22] Using the implicit function, I show that, when n is large, there is a unique solution to (19), denoted $x^*(t; \sigma', n)$.

Lemma 2 *Take any strategy σ' and let the precision choice x be part of a best*

[22]For example, the expected utility of an aligned type who chooses precision $x > 0$ is given by the expected utility, K, from all the events when her vote does not affect the outcome, by the cost, and by the expected utility from the pivotal event. The expected utility from the pivotal event is

$$\Pr(\text{piv})\Big[t_\alpha \Pr(\alpha|\text{piv}; \sigma)(\frac{1}{2} + x) + t_\beta(\Pr(\alpha|\text{piv}; \sigma)(\frac{1}{2} - x)\Big].$$

To see why, note that when the citizen is pivotal and votes B, she receives a utility of zero. When she is pivotal and votes A, she receives a utility of t_ω, depending on the state. Given (17), it is optimal to vote A only after a; and she receives a in α with probability $\frac{1}{2} + x$ and in β with probability $\frac{1}{2} - x$. Taking derivatives, yields the first-order condition (19).

response. Then, when n is large enough, for all t,

$$x(t) > 0 \Rightarrow x(t) = x^*(t; \sigma', n),$$

where $x^*(t; \sigma', n)$ is given by (19) and continuously differentiable in t.

2.1.5 Limit Vote Shares

I note that the posterior conditional on the pivotal event pins down the limit of the vote share in both states, as $n \to \infty$. To see why, recall that for any strategy sequence, the likelihood of the pivotal event converges to zero. Therefore, the first-order condition (19) implies that the precision of all types converges to zero uniformly. For any equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$, this implies $\lim_{n\to\infty} \Pr(\alpha|\text{piv}, s; \sigma_n^*, n) - \Pr(\alpha|\text{piv}; \sigma_n^*, n) = 0$, given the equilibrium precision $x(t)$ of any type. Therefore, given (5)-(7), the limit vote share of A is equal to the mass of voters prefering A given the belief $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n)$, meaning that $\lim_{n\to\infty} q(\omega; \sigma_n^*) = \Phi(\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*))$. Using that Φ is continuous, I conclude:

Lemma 3 *Take any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$. For all n, let $\hat{\sigma}_n$ be any best response to σ_n. For any $\omega \in \{\alpha, \beta\}$,*

$$\lim_{n\to\infty} q(\omega; \hat{\sigma}_n) = \lim_{n\to\infty} \Phi(\Pr(\alpha|\text{piv}; \sigma_n)). \tag{20}$$

Even though, the limit of the vote share of policy A is the same in both states, note that, for any finite n, the vote shares will typically differ. The next section provides a useful representation of the voter types that I will use sometimes from then on.

2.1.6 Voter-Type Representation

Voter types differ in their *(total) intensity*

$$k(t) = |t_\alpha| + |t_\beta|. \tag{21}$$

In fact, the total intensity and the threshold of doubt $y(t) = \frac{-t_\beta}{t_\alpha - t_\beta}$ relate one-to-one to the type (t_α, t_β) if we restrict to either the aligned or the contrarian types. To be precise, for all aligned types $t \in L$,

$$t_\alpha = k(t)(1 - y(t)) \quad \text{and} \quad t_\beta = -k(t)y(t), \tag{22}$$

and for all contrarian types $t \in C$,

$$t_\alpha = -k(t)(1 - y(t)) \quad \text{and} \quad t_\beta = k(t)y(t). \tag{23}$$

From now on, I will occasionally slightly abuse the previous notation and use the notation $t = (y, k)$ for types, typically specifying if the type is aligned or contrarian, i.e. if $g = L$ or $g = C$. Note that $|t_\omega|$ and therefore the solution $x^*(t; \sigma, n)$ to the first-order condition (19) only depend on $y(t)$ and $k(t)$ and not on the group, i.e., aligned or contrarians, that the type belongs to.

2.1.7 Information Acquisition Regions

When deciding whether or not to acquire information, voters trade off the cost of information against the benefits. The *critical types* t with $y(t) = \Pr(\alpha|\text{piv}; \sigma', n)$ are indifferent between A and B without further information, given (5) - (8). On the other hand, voters with an extreme threshold of doubt $y(t) \approx 0$ or $y(t) \approx 1$ have strong preferences for one alternative without further information. Intuitively, these extreme types value information less than the critical types. First, the following result verifies this intuition and shows that only types with an intermediate threshold of doubt acquire information, including the critical types. Second, intuitively, types with higher (total) intensities have more incentives to acquire information. The following result also shows that the interval around the critical threshold $y = \Pr(\alpha|\text{piv}; \sigma', n)$ consisting of the types that get informed only depends on the (total) intensity k.

Lemma 4 *Let σ' be a strategy with $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma', n) \in (0, 1)$. When n is large enough, for any $k \in (0, 2)$ and any $g \in \{L, C\}$ there are $\phi_g^-(k) <$*

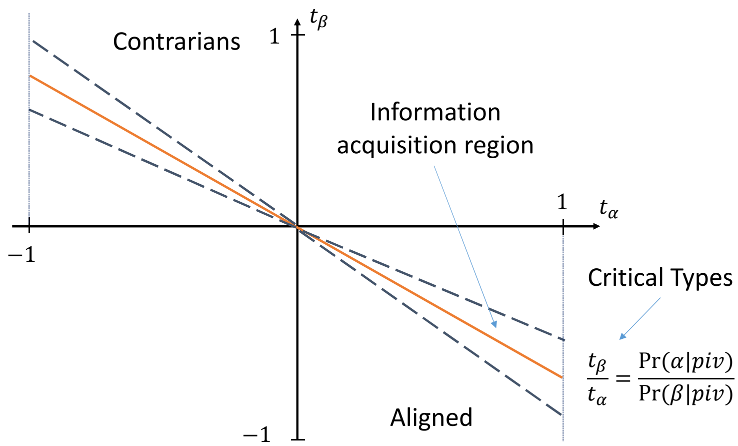


Figure 2: Information Acquisition regions of group g with boundaries the graphs of $\phi_g^-(k)$ and $\phi_g^+(k)$ (dashed lines).

$\Pr(\alpha|\text{piv};\sigma',n) < \phi_g^+(k)$ *for such that for any best response* $\sigma = (x,\mu)$ *to* σ' *and any type* $(y,k) \in g$,

$$x(y,k) > 0 \quad \Leftrightarrow \quad y \in [\phi_g^-(k), \phi_g^+(k)], \tag{24}$$

Figure 2 illustrates the functions ϕ_g^- and ϕ_g^+, which limit the regions of types that acquire information under the best response. A later example illustrates the result (see Section 3.4). The proof is provided in the Appendix.

3 Voting Power and Welfare

In this section, I provide the main analysis, whichh describes the relationship between the preference intensities, the information level of the different voter groups and their voting power. For this, I consider a sequence of elections along which the electorate's size $2n+1$ grows. For each n and a strategy σ_n, I calculate the probability that a policy $z \in \{A, B\}$ wins the support of the majority of the voters in state ω, denoted $\Pr(z|\omega;\sigma_n,n)$. I will be interested in the limits of $\Pr(z|\omega;\sigma_n^*,n)$ for equilibrium sequences $(\sigma_n^*)_{n\in\mathbb{N}}$. I will be particularly interested in equilibrium sequences where citizens vote in an informed manner such that the election outcomes differ across the states,

$$\lim_{n\to\infty}\Pr(A|\alpha;\sigma_n,n)\neq\lim_{n\to\infty}\Pr(A|\beta;\sigma_n,n), \tag{25}$$

which I call *informative*. What will matter for the outcomes in informative equilibrium sequences is which voter group has the larger *power index*, defined as follows. For any $p\in[0,1]$, any $g\in\{L,C\}$, let

$$W(g,p)=\Pr(g)f(p|t\in g)\mathrm{E}(k(t)^{\frac{2}{d-1}}|t\in g,y(t)=p), \tag{26}$$

where f is the density of the (conditional) distribution of the threshold of doubt $y(t)$ of the types $t\in g$, and $d=\lim_{x\to 0}\frac{c'(x)x}{c(x)}$ is the limit elasticity of the cost function.[23] For any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$ and any $g\in\{C,L\}$,

$$W(g,\hat{p}) \tag{27}$$

is the *power index of group g*, where $\hat{p}=\lim_{n\to\infty}\Pr(\alpha|\mathrm{piv};\sigma_n,n)$. The power index is proportional to the likelihood of the critical types and the mean of the weighted intensities of the critical types as $n\to\infty$, and will serve as a relative measure of the electoral power of the voter group. Throughout the analysis, I will impose the following genericity condition on the preference distribution H,

$$\begin{aligned}\Phi(\Pr(\alpha))&\neq\frac{1}{2},\\ W(L,p)\neq W(C,p)\ &\text{ for any }\ p\in\Phi^{-1}(\frac{1}{2}),\\ \mathrm{E}(k(t)^{\frac{2}{d-1}}|t\in g,y(t)=p)>0\ &\text{ for any }\ g\in\{L,C\},p\in[0,1],\\ f(\cdot|t\in g)>0\ &\text{ for any }\ g\in\{L,C\}.\end{aligned} \tag{28}$$

[23] A proof of this statement is given right after the main result, see Section 3.1.

3.1 Result: Index Equilibrium Sequence

The main result shows that when information of low precision $x \approx 0$ is sufficiently cheap, which will be captured by a condition on the elasticity at zero, $\lim_{x\to 0} \frac{c'(x)x}{c(x)}$,[24] there is an informative equilibrium sequence where the outcome preferred by the group with the larger power index is elected with probability converging to 1 as $n \to \infty$.[25]

Theorem 1 *Let* $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$. *Take any preference distribution* H *satisfying (28). There is an informative equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$ *satisfying*

$$\lim_{n\to\infty} \Pr(A|\alpha; \sigma_n^*, n) = \lim_{n\to\infty} \Pr(B|\beta; \sigma_n^*, n) = \begin{cases} 0 & \textit{if } W(L, \hat{p}) < W(C, \hat{p}), \\ 1 & \textit{if } W(L, \hat{p}) > W(C, \hat{p}). \end{cases} \tag{29}$$

and $W(L, \hat{p}) \neq W(C, \hat{p})$ *where* $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n)$.

Section 3.2 discusses the welfare implications of Theorem 1. In Section 3.3, I explain the three central observations for why the power index of the voter groups, aligned and contrarians, determines election outcomes. In Section 3.4, I give a detailed sketch of proof for Theorem 1 for a special case. Then, in Section 3.5, I sketch how the results for the special case carry over to the general case when the electorate is large, $n \to \infty$. In the same section, I also sketch the formal fixed point argument for the existence of informative equilibrium sequences. A thorough proof of Theorem 1 is in Section B of the Appendix.

The following lemma shows that the condition on the cost function in Theorem 1 can be written as a condition on the parameter d.

Lemma 5 $\lim_{x\to 0} \frac{xc'(x)}{c(x)} = d$.

Proof. Recall that $\lim_{n\to\infty} \frac{c'(x)}{x^{d-1}} \in \mathbb{R}$. Since $\lim_{x\to 0} c(x) = c(0) = 0$, it follows from l'Hospital's rule that $\lim_{x\to 0} \frac{c(x)}{x^d} = \lim_{x\to 0} \frac{1}{d} \frac{c'(x)}{x^{d-1}}$. Therefore, $\lim_{n\to\infty} \frac{c(x)}{c'(x)x} = \lim_{x\to 0} \frac{c(x)}{x^d} \frac{x^d}{c'(x)x} = \frac{1}{d}$. ■

[24] The same condition appears in Martinelli (2006)'s model.

[25] The characterization of all equilibrium sequences in Section 5 shows that the result of Theorem 1, in fact, extends to all informative equilibrium sequences.

3.2 First-Best Outcomes

In this section, I study the welfare properties of the election. I describe the welfare effect of the disparate information acquisition of voters. Policy A is welfare-maximizing in state ω if $\mathrm{E}_H(t_\omega) \geq 0$. The main result descibed in this section is that, for a large class of settings, elections have equilibria that yield outcomes maximizing utilitarian welfare in each state, unless the information cost is too extreme, i.e., too low or too high.

I consider the following settings: I consider preference distributions for which the conditional distribution of the threshold of doubt, $F(\cdot|t \in g)$ is independent of the voter group, i.e. for all $g \in \{L, C\}$,

$$F(\cdot|t \in g) = F. \tag{30}$$

The conditional distribution $J(\cdot|t \in g)$ of the total intensities of types $t \in g$ is independent from H, that is, for all $g \in \{C, L\}$ and all $y \in [0, 1]$

$$J(\cdot|t \in g, y(t) = y) = J(\cdot|t \in g). \tag{31}$$

Clearly, the information cost cannot screen the intensities of the partisan voters since all the partisans stay uninformed and simply vote for their preferred policy. Therefore, I consider the settings where the screening of the intensities of the partisans is obsolete. The welfare at stake for the A-partisans is, in expectation, of the same magnitude as the welfare at stake for the B-partisans, i.e., for all $\omega \in \{\alpha, \beta\}$,

$$\begin{aligned} & \Pr(\{t : t_\alpha > 0, t_\beta > 0\})\mathrm{E}_H(|t_\omega||\omega, \{t : t_\alpha > 0, t_\beta > 0\}) \\ = \;& \Pr(\{t : t_\alpha < 0, t_\beta <\})\mathrm{E}_H(|t_\omega||\omega, \{t : t_\alpha < 0, t_\beta < 0\}). \end{aligned} \tag{32}$$

One specific case of (32) is the situaton without partisans.

First-Best (Utilitarian) Outcomes. Given the independence assumptions (30) - (32), for each state ω, the outcome preferred by the voter group, aligned or contrarians, with the larger likelihood weighted mean of the total intensities is the welfare-maximizing outcome, that is, the welfare-maximizing outcome is A in α and B in β if

$$\Pr_H(t \in L)\mathrm{E}_H(k(t)|t \in L) > \Pr_H(t \in C)\mathrm{E}_H(k(t)|t \in C) \tag{33}$$

and otherwise B in α and A in β. To see why, consider, for example, state α. By definition, A is the welfare-maximizing outcome if $E_H(t_\alpha) > 0$. Given (32) and given (22) and (23), $\mathrm{E}_H(t_\alpha)$ is equivalent to $\Pr_H(t \in L)\mathrm{E}(k(t)(1-y(t))|t \in L) - \Pr_H(t \in C)\mathrm{E}(k(t)(1-y(t))|t \in C) > 0$.[26] Given (30) and (31), this is equivalent to (33).

Theorem 2 *Take any preference distribution H satisfying (28) and (30)-(32). There is $\bar{d} > 3$ such that for any cost function for which $\lim_{x\to 0}\frac{c'(x)x}{c(x)} \in (3, \bar{d})$, there is an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ for which the welfare-maximizing outcome is elected with probability converging to 1 as $n \to \infty$.*

Proof. First, note that Lemma 5 in the Appendix shows that $\lim_{x\to 0}\frac{c'(x)x}{x} = d$. Now, Theorem 1 implies Theorem 2 as follows. Given the independence assumptions (30) and (31),

$$\begin{aligned} W(L,\cdot) &> W(C,\cdot) \\ \Leftrightarrow \Pr(L)\mathrm{E}(k(t)^{\frac{2}{d-1}}|t \in L) &> \Pr(C)\mathrm{E}(k(t)^{\frac{2}{d-1}}|t \in C). \end{aligned} \tag{34}$$

Comparing (33) and (34), for each preference distribution H satisfying the conditions of the lemma, there is an open set of elasticities $d > 3$ close to $d = 3$, and an equilibrium sequences for any d from this set such that the welfare-maximizing outcome is elected in each state. ■

Full-Information Equivalence with Common Interests. When all non-partisan voters share a common interest, i.e. when $\Pr_H(t \in C) = 0$, Theorem 1 implies that whenever information of low precision $x \approx 0$ is sufficiently cheap,

[26] $E_H(t_\alpha) = \Pr_H(t \in L)\mathrm{E}(t_\alpha|t \in L) + \Pr_H(t \in C)\mathrm{E}(t_\alpha|t \in C) + \Pr_H(t \notin (L \cup C))\mathrm{E}(t_\alpha|t \notin (L \cup C))$.

there is an equilibrium of the large election where the outcomes are equivalent to the outcome with publicly known states ("full-information equivalence").

Corollary 1 *Let* $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$. *Take any preference distribution* H *satisfying (28) and* $\Pr(t \in L) = 1$. *There is an equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$ *satisfying*

$$\lim_{n\to\infty} \Pr(A|\alpha; \sigma_n^*, n) = \lim_{n\to\infty} \Pr(B|\beta; \sigma_n^*, n) = 1. \tag{35}$$

Proof. Note that $\Pr(t \in C) = 0$ implies $W(L, \cdot) = 0$. Thus, the corollary is a special case of Theorem 1. ■

3.3 The Critical Observations

This section presents three observations about informative equilibrium sequences that are central to the proof of Theorem 1. The first observation is that, in any informative equilibrium sequence, the outcome of the election would be random if we considered only the votes of the uninformed voters; in particular, the vote shares of the uninformed for the two policies are almost $50-50$. The second observation implies that if we considered only the votes of the citizens that get partially informed, i.e., $x(t) > 0$, then, the vote share of policy A is larger in α than in β only if the aligned types have the higher power index, and vice versa. The first two observations together suggest that, in each state, the outcome preferred by the group with the higher power index is more likely to be elected for n large. The third observation describes which cost functions permit equilibrium sequences with informative and determinate outcomes, i.e. $\lim_{n\to\infty} \Pr(A|\alpha; \sigma_n^*, n) = \lim_{n\to\infty} \Pr(B|\alpha; \sigma_n^*, n) \in \{0, 1\}$. Finally, in Section 3.3.4, I explain how the three observations together suggest the result of Theorem 1.

3.3.1 The Uninformed

The first observation is the following: if we would just consider the uninformed votes, the outcome of the election would be random. Let

$$\Pr(A|\omega; \sigma_n^*; \tilde{\pi}_n) \tag{36}$$

be the likelihood of outcome A in the game $\tilde{\pi}_n$ of $2n+1$ voters where a random citizen votes A with the same probability as a random uninformed voter given σ_n^*, i.e. with probability $\int_t \frac{1}{2}\Big[\mu(a,t) + \mu(b,t)\Big] dH(t|x(t) = 0)$.

Observation 1 *Take any preference distribution H satisfying (28). For any informative equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$, in each state $\omega \in \{\alpha, \beta\}$, the outcome implied by the vote share of the uninformed voters is random, i.e.*

$$\Pr(A|\omega; \sigma_n^*; \tilde{\pi}_n) \in (0, 1).$$

The proof is provided in Section 5, where I finish the characterization of all the equilibria. The observation is stated here already to provide guidance and intuition for the coming analysis, but it is not needed for the upcoming proofs. What is key is that for an equilibrium sequence to be informative, some voters must acquire a relevant amount of information. For the voters to be willing to do so, the election must be sufficiently close to being a tie such that the likelihood of being the pivotal voter is relatively large from the perspective of a single voter. In fact, it turns out that for any informative equilibrium sequence the *induced prior* converges to a belief p^*,

$$\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*) = p^*, \tag{37}$$

given which a randomly drawn type would vote exactly $50-50$, i.e. $\Phi(p^*) = \frac{1}{2}$.[27] Suppose for a second that $\Pr(\alpha|\text{piv}; \sigma_n^*) = p^*$. Then, without further information about the state, a random type prefers A with a likelihood of

[27]The term *induced prior* was introduced by Bhattacharya (2013b).

50%, which suggests the result of Observation 1.[28] I prove (37) in Section B.1.1) of the Appendix.

3.3.2 The Partially Informed: Competition in Information Acquisition

Suppose that there is an informative equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$. Given Observation 1, the election is indeterminate when considering only the votes of the uninformed. Intuitively, the election will, therefore, be decided by the voters that get partially informed. The partially informed citizens vote for their preferred policy with a likelihood of $\frac{1}{2}+x(t)$, and therefore, shift the vote share towards their preferred policy in each state. On the aggregate, the difference in the expected vote share for policy A across the states is

$$q(\alpha;\sigma_n^*) - q(\beta;\sigma_n^*) \;\;=\;\; 2\Big[\int_{t\in L} x(t)dH(t) - \int_{t\in C} x(t)dH(t)\Big]. \qquad (38)$$

where $\int_{t\in g} x(t)dH(t) = \Pr_H(t\in g)\mathrm{E}_H(x(t)|t\in g)$ is the average precision acquired by a type of the group $g\in\{L,C\}$, weighted by the likelihood of a random type belonging to the group. Given (38), we see that the order of the likelihood-weighted average precision determines the order of the vote shares in the states, and therefore in which state A is more likely to be elected.

The second critical observation for Theorem 1 is that there is a simple rule describing which group has the higher likelihood-weighted average precision: it is the group with the higher power index.

Observation 2 *Take any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$. For all n, let $\hat{\sigma}_n$ be any best response to σ_n. If $\hat{p}=\lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma_n,n)\in(0,1)$ the sequence*

[28] There are two subtleties to the proof of Observation 1: first, one has to show that the convergence to p^* is sufficiently fast. Second, one has to show that there are approximately as many citizens that get informed and would have voted A without further information as citizens that get informed and would have voted B without further information. Otherwise, a relevant asymmetry might shift the conditional vote share of the uninformed to either significantly below or above $\frac{1}{2}$.

of best responses satisfies

$$\lim_{n\to\infty} \frac{\int_{t\in L} x(t)dH(t)}{\int_{t\in C} x(t)dH(t)} = \frac{W(L,\hat{p})}{W(C,\hat{p})}. \tag{39}$$

Note that (39) implies that, for all n large enough, the order of the power indices pins down the order of the vote shares in the two states,

$$q(\alpha;\hat{\sigma_n}) > q(\alpha;\hat{\sigma_n}) \Leftrightarrow W(L,\hat{p}) > W(C,\hat{p}). \tag{40}$$

I provide a sketch of the proof in Section 3.5. The proof can be found in Section B.1 of the Appendix.

3.3.3 Value and Cost of Information

The third observation describes which cost functions permit sufficient information acquisition, meaning that there is a strategy sequence $(\sigma_n)_{n\in\mathbb{N}}$ for which the best response $(\hat{\sigma_n})_{n\in\mathbb{N}}$ yields informative and determinate outcomes, i.e. $\lim_{n\to\infty}\Pr(A|\alpha;\hat{\sigma}_n,n) = \Pr(B|\beta;\hat{\sigma}_n,n) \in \{0,1\}$. What matters is how fast cost go to zero when a voter acquires a signal of arbitrarily low precision. The critical condition is that the elasticity of the cost function close to zero exceeds three, i.e.

$$\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3.$$

The third observation is that, given (41), for any sequence of strategies with vote shares $q(\alpha;\sigma_n) \le \frac{1}{2} \le q(\beta;\sigma_n)$ or $q(\alpha;\sigma_n) \ge \frac{1}{2} \ge q(\beta;\sigma_n)$ *sufficiently close* to $\frac{1}{2}$, the outcomes under the sequence of best responses are both informative and determinate, as $n\to\infty$. Conversely, if $\lim_{x\to 0}\frac{c'(x)x}{c(x)} < 3$, then the outcomes under the sequence of best responses are uniformly random as $n\to\infty$.

For any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$ and any n, let $s(\omega;\sigma_n)$ be the standard deviation of the vote share. Let

$$\delta_\omega = \lim_{n\to\infty} \frac{1}{s(\omega;\sigma_n)}\Big[q(\omega;\sigma_n) - \frac{1}{2}\Big] \tag{41}$$

be the normalized distance of the expected vote share to the majority threshold as $n \to \infty$.

Observation 3 *Take any preference distribution H satisfying (28). Take a sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$ with $q(\alpha;\sigma_n) \leq \frac{1}{2} \leq q(\beta;\sigma_n)$ or $q(\alpha;\sigma_n) \geq \frac{1}{2} \geq q(\beta;\sigma_n)$ for all n and with $\delta_\omega \in \mathbb{R}$ for some $\omega \in \{\alpha,\beta\}$. Let $\hat{\sigma_n}$ be a best reponse to σ_n.*

1. *If $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$, then*

$$\lim_{n\to\infty} \Pr(A|\alpha;\hat{\sigma}_n,n) = \Pr(B|\beta;\hat{\sigma}_n,n) \in \{0,1\}. \tag{42}$$

2. *If $\lim_{x\to 0} \frac{c'(x)x}{c(x)} < 3$, then*

$$\lim_{n\to\infty} \Pr(A|\alpha;\hat{\sigma}_n,n) = \Pr(B|\beta;\hat{\sigma}_n,n) = \frac{1}{2}. \tag{43}$$

Note that, under weak additional smoothness conditions on c, (41) implies that the first three derivatives at zero are zero,

$$c'(0) = c''(0) = c'''(0) = 0 \tag{44}$$

which is the condition that appears in Martinelli (2006),[29] who shows that the condition (44) is necessary and sufficient for full-information equivalent outcomes in a setting with uniform types and common interest (only aligned

[29]Suppose that c is four times continuously differentiable, and $d > 3$, but for example, that $c'(0) = c''(0) = 0$ and $c'''(0) > 0$. Applying l'Hospital's rule multiple times yields $\lim_{x\to 0} \frac{c(x)}{x\cdot c'(x)} = \lim_{x\to 0} \frac{c'(x)}{x\cdot c''(x)+c'(x)} = \lim_{x\to 0} \frac{c''(x)}{x\cdot c'''(x)+c''(x)+c''(x)} = \lim_{x\to 0} \frac{c'''(x)}{x\cdot c''''(x)+3c'''(x)} = \frac{1}{3}$ since $c'''(0) > 0$. However, this constradicts with Lemma 5 which shows that $\lim_{x\to 0} \frac{c(x)}{x\cdot c'(x)} = \frac{1}{d}$.

types).

3.3.4 Intuition for Theorem 1

Note that the *vote share of the informed voters* in ω, i.e. the likelihood that a random type t votes A conditional on $x(t) > 0$ in ω is given by

$$\begin{aligned} & \int_t \mu(t,a)(\frac{1}{2} + x(t)) + \mu(t,b)(\frac{1}{2} - x(t))dH(t|x(t) > 0) \\ = \ & \frac{1}{2} + \int_{t\in L} x(t)dH(t) - \int_{t\in C} x(t)dH(t), \end{aligned} \tag{45}$$

and the vote share of the uninformed in β is

$$\frac{1}{2} - \int_{t\in L} x(t)dH(t) + \int_{t\in C} x(t)dH(t). \tag{46}$$

Therefore, Observation 2 implies that the vote share of the informed voters is larger than 50% in α and smaller than 50% in β if and only if the power index of the aligned types is larger. Given Observation 1, the uninformed votes split almost $50 - 50$ such that both observations together suggest that the order of the vote shares in any informative equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ is, for any n large enough, given by

$$\begin{aligned} & q(\alpha;\sigma_n) < \frac{1}{2} < q(\beta;\sigma_n) \\ & \Leftrightarrow W(L,\hat{p}) < W(C,\hat{p}), \end{aligned}$$

where $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*)$.[30] Therefore, the outcome preferred by the group with the higher power index is more likely to be elected in each state. Intuitively, when information is sufficiently "cheap," there is sufficient information acquisition and outcomes are determinate in each state, implying that the outcome preferred by the group with the higher power index is elected with

[30] Note that $W(L,\hat{p}) \neq W(C,\hat{p})$ in any informative equilibrium sequence; this follows from the genericity condition (28) and since $\hat{p} \in \Phi^{-1}(\frac{1}{2})$ by (37).

probability converging to 1 as $n \to \infty$. Now, suppose that $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$ and that the outcomes were not determinate in a state $\omega \in \{\alpha, \beta\}$. Using the central limit theorem, I show in the Appendix (Lemma 14) that this is equivalent to the vote shares being only a finite number of standard deviations away from the majority threshold $\frac{1}{2}$ as $n \to \infty$, i.e. $\delta_\omega \in \mathbb{R}$. But then, Observation 3 implies that the outcomes must be determinate, which yields a contradiction. In conclusion, the observations suggest that any informative equilibrium sequence implements the outcome prefered by the group with the higher power index, i.e., it satisfies condition (29) of Theorem 1.

3.4 Sketch of Proof: Uniform Types

3.4.1 Setting

Each voter is of an aligned type with probability $\lambda > \frac{1}{2}$ and of a contrarian type with a probability $(1-\lambda)$ independently. The cost is polynomial $c(x) = x^d$ and the total intensity is the same for all aligned and contrarian voters respectively, but might depend on the voter group, i.e., $k(t) = k_L$ for all $t \in L$ and $k(t) = k_C$ for all $t \in C$. The prior belief about the state is uniform and the conditional distribution of the threshold of doubt is the same for the aligned and contrarian types respectively, denoted as F, and uniform.

3.4.2 Symmetric Candidate Equilibrium

I claim that there is a *symmetric equilibrium*, meaning that the expected margin of victory is the same in both states, i.e. $q(\alpha; \sigma_n^*) - \frac{1}{2} = \frac{1}{2} - q(\beta; \sigma_n^*)$. This implies that the election is equally close to being tied in both states and that voters do not learn anything from the pivotal event, i.e.,

$$\Pr(\alpha|\text{piv}; \sigma_n^*, n) = \frac{1}{2}, \tag{47}$$

given (13). The expected utility of an aligned type who chooses precision $x > 0$ is given by the expected utility, K, from all the events when her vote does not affect the outcome, by the cost $c(x)$, and by the expected utility from the pivotal event. The expected utility from the pivotal event is

$$\Pr(\text{piv};\sigma_n^*,n)\Big[t_\alpha \Pr(\alpha|\text{piv};;\sigma_n^*,n)(\frac{1}{2}+x)+t_\beta(\Pr(\alpha|\text{piv};;\sigma_n^*,n)(\frac{1}{2}-x)\Big]. \quad (48)$$

To see why, note that when the citizen is pivotal and votes B, she receives a utility of zero. When she is pivotal and votes A, she receives utility of t_ω, depending on the state. Given (17), she only votes A after a; and she receives a in α with probability $\frac{1}{2}+x$ and in β with probability $\frac{1}{2}-x$.

3.4.3 Equilibrium Conditions

Take any aligned type. Using that $\Pr(\alpha|\text{piv};\sigma_n^*,n) = \frac{1}{2}$ in any symmetric candidate equilibrium, and using $t_\alpha = k(1-y)$ and $t_\beta = -ky$ (see (22)) and (48), the first-order condition, which equates marginal cost and marginal benefits, is

$$\Pr(\text{piv}\sigma_n^*,n)\frac{k_L}{2} = c'(x). \quad (49)$$

The first-order condition pins down the precision $x_L^* > 0$ chosen by the aligned types that get partially informed; compare to Lemma 2 and (19). Who are these types? A simple calculation shows that the expected utility from an informed choice, with precision $x_L^* > 0$, is lower than the expected utility from an uninformed vote for B if[31]

$$y > \frac{1}{2} + x_L^{**} \quad (50)$$

[31] Using (22) and (48), the expected utility from an informed choice, with precision $x_L^* >$ 0, is lower than the expected utility from an uninformed vote for B if $\Pr(\text{piv})\frac{1}{2}\Big[(\frac{1}{2}+x)k(1-y)-(\frac{1}{2}-x)k_g y\Big]-c(x)\geq 0$. This is equivalent to $\Pr(\text{piv})\frac{k}{2}\Big[\frac{1}{2}(1-y)-\frac{1}{2}y\Big]+\Pr(\text{piv})\frac{k}{2}x\Big[(1-y)+y)\Big]-c(x)<0$. Using the first-order condition, this is equivalent to $c'(x_L^*)(\frac{1}{2}-y)+c'(x_L^*)x_L^*-c(x_L^*)<0$. Dividing by $c(x_L^*)$ yields $x^*(1-\frac{c(x_L^*)}{x_L^*c'(x_L^*)})<(y-\frac{1}{2})$. Rearranging gives (50).

where $x_L^{**} = x_L^*(1 - \frac{c(x_L^*)}{x_L^* c'(x_L^*)})$. Analogously, all types with

$$y < \frac{1}{2} - x_L^{**} \tag{51}$$

prefer to vote for A uninformed over the informed vote with precision x_L^*. A similar analysis shows that there is a precision $x_C > 0$ chosen by all the contrarian types that get partially informed. This precision is given by the first-order condition

$$\Pr(\text{piv})\frac{k_C}{2} = c'(x), \tag{52}$$

and pins down the contrarian types that get partially informed. These are the types with a threshold of doubt

$$\frac{1}{2} - x_C^{**} \leq y \leq \frac{1}{2} + x_C^{**}, \tag{53}$$

where $x_C^{**} = x_C(1 - \frac{c(x_C)}{x_C c'(x_C)})$.

3.4.4 Fixed Point Argument

Using the symmetry of the signals, and that the distribution of the threshold of doubt y is uniform, (50), (51) and (53) together imply that the best response $\hat{\sigma}_n$ to any symmetric candidate equilibrium is symmetric, i.e., the vote shares satisfy $q(\alpha; \hat{\sigma}_n) - \frac{1}{2} = \frac{1}{2} - q(\beta; \hat{\sigma}_n)$. An application of Kakutani's fixed point theorem implies that, for any n, there is a symmetric equilibrium σ_n^*. In this equilibrium, only the types $y \in [\phi_g^-(k), \phi_g^+(k)]$ get partially informed, where $\phi_g^+(k) = \frac{1}{2} - x_g^{**}$ and $\phi_g^-(k) = \frac{1}{2} + x_g^{**}$.

3.4.5 Limit Outcomes

I will now conclude the proof of Theorem 1 for the uniform types case. For this, I will first establish Observation 2 for the sequence $(\sigma_n^*)_{n \in \mathbb{N}}$ of the symmetric equilibria.

Observation 2. The types that get partially informed are those for which

$y(t) \in [\frac{1}{2} - x_g^*(1 - \frac{c(x_g^*)}{x_g^* c'(x_g^*)}), \frac{1}{2} + x_g^*(1 - \frac{c(x_g^*)}{x_g^* c'(x_g^*)})]$, and all these types choose the precision $x_g^* > 0$. I conclude, therefore, that

$$\begin{aligned} \lim_{n\to\infty} \frac{\int_{t\in L} x(t)dH(t)}{\int_{t\in C} x(t)dH(t)} &= \lim_{n\to\infty} \frac{\Pr(t\in L)}{\Pr(t\in C)} \frac{(x_L^*)^2(1-\frac{c(x_L^*)}{x_L^* c'(x_L^*)})}{(x_C)^2(1-\frac{c(x^*)}{x_g^* c'(x^*)})} \\ &= \frac{\Pr(t\in L)}{\Pr(t\in C)} \frac{(x_L^*)^2}{(x_C)^2}, \end{aligned} \tag{54}$$

where, for the last line, I used $\frac{c(x)}{xc'(x)} = \frac{1}{d}$, given that $c(x) = x^d$. The first-order condition (49) pins down the ratio of the precision of aligned and contrarians, $\frac{x_L^*}{x_C^*} = (\frac{k_L}{k_C})^{\frac{1}{d-1}}$. Finally, the definition of the power indices $W(g)$, i.e., (27), together with (54) implies (39) for the sequence of symmetric equilibria.

Given the symmetry of the equilibrium, the margin of victory in each state is the same. Thus, (38) and (39) imply that the outcome preferred by the group with the higher power index has the larger vote share in expectation in each state. Intuitively, when information is sufficiently "cheap," outcomes of the election will be determinate in each state. This will be the case when $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$, which is the condition in Theorem 1. If we assume that the outcomes were not determinate, then using the central limit theorem, I show in the Appendix (Lemma 14) that this is equivalent to the vote shares being only a finite number of standard deviations away from the majority threshold $\frac{1}{2}$, i.e. $\delta_\omega \in \mathbb{R}$. But then, Observation 3 implies that the outcomes must be determinate, which yields a contradiction. I conclude that under the sequence of symmetric equilibria, the outcome preferred by the group with the higher power index is elected with probability converging to 1 in each state, as $n \to \infty$. This concludes the sketch of the proof of Theorem 1 for the special case.

3.5 Sketch of Proof: General Case

In this Section, I will sketch the proof of Theorem 1 for the general case.

3.5.1 Observation 2

In this section, I sketch the proof of Observation 2 for the general case. The analysis follows along similar lines as in the special case with uniform types. However, it must be shown that the order of several types of asymmetric behaviour is sufficiently small such that the results from the example carry over asymptotically as $n \to \infty$. Before I do this, I make preliminary remark: the boundaries $\phi_g^+(k)$ and $\phi_g^-(k)$ of the information acquisition regions are implicitly determined by

$$\begin{aligned}\frac{1}{2} + x^{**}(\phi_g^-(k), k) &= \frac{\Pr(\beta|\text{piv})\phi_g^-(k)}{\Pr(\alpha|\text{piv})(1-\phi_g^-(k)) + \Pr(\beta|\text{piv})\phi_g^-(k)}, &(55)\\ \frac{1}{2} - x^{**}(\phi_g^+(k), k) &= \frac{\Pr(\beta|\text{piv})\phi_g^+(k)}{\Pr(\alpha|\text{piv})(1-\phi_g^+(k)) + \Pr(\beta|\text{piv})\phi_g^+(k)}, &(56)\end{aligned}$$

where $x^{**}(y,k;\sigma,n) = x^*(y,k;\sigma,n)(1-\frac{c(x^*(y,k;\sigma,n))}{x^*(y,k;\sigma,n)c'(x^*(y,k;\sigma,n))})$ is defined as in the uniform types case and $x^*(y,k;\sigma,n)$ is the solution to the first-order condition (19); the proof is in Section A of the Appendix.[32]

Approximation 1. First of all, even when fixing the total intensity, different types choose different precisions in general. I show that this heterogeneity is sufficiently small and the information precision of all types that get partially informed is well approximated by the precision of the critical types, which satisfy $y = \Pr(\alpha|\text{piv};\sigma_n,n)$. Using a Taylor approximation, for all k,

$$x(y,k) > 0 \Rightarrow \frac{x(y,k)}{x(\Pr(\alpha|\text{piv};\sigma_n,n),k)} \approx 1. \tag{57}$$

The basic intuition is that only types arbitrarily close to the critical types acquire information when n is growing large. More intuition comes from a close inspection of the first-order condition (19) that equates marginal benefits

[32] Note that the conditions (55) and (56) reduce to the conditions (50), (51) and (53) from the uniform types example when $\Pr(\alpha|\text{piv};\sigma_n^*) = \frac{1}{2}$ and $\Phi_g^-(k) = 1 - \Phi_g^+(k)$.

and marginal cost. Implicit differentiation shows

$$\frac{\partial \text{Marginal Cost}}{\partial x^*(y,k;\sigma_n,n))}\frac{\partial x^*(y,k;\sigma_n,n))}{\partial y}=\frac{\partial \text{Marginal Benefit}}{\partial y}. \tag{58}$$

Changes in the marginal benefit on the right-hand side of (19) due to changes in the threshold of doubt y are of an order of the marginal cost $c'(x(y,k))$. These changes relate to infinitely smaller changes in the equilibrium precision $x^*(y,k;\sigma_n,n)$ on the left-hand side since the second derivative $c''(x(y,k))$ of the cost function is much larger than the first derivative $c'(x(y,k))$ when the precision $x(y,k)$ is small.[33]

Approximation 2. Second, the information acquisition region might be asymmetric around the critical types with $y=\Pr(\alpha|\text{piv};\sigma_n,n)$. I show that this asymmetry is of a sufficiently small order. Similar to the uniform types example (see (50), (51) and (53)),

$$\frac{x(\hat{y},k)(1-\frac{c(x(\hat{y},k))}{c'(x(\hat{y}_n,k))x(\hat{y}_n,k)})}{\Big[\phi_g^-(k)-\Pr(\alpha|\text{piv};\sigma_n,n)\Big]}\approx\frac{x(\hat{y}_n,k)(1-\frac{c(x(\hat{y}_n,k))}{c'(x(\hat{y}_n,k))x(\hat{y}_n,k)})}{\Big[\Pr(\alpha|\text{piv};\sigma_n,n)-\phi_g^+(k)\Big]}\approx K, \tag{59}$$

where K is a constant that only depends on $\lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma_n,n)=\hat{p}$. To show this, I consider the linear (Taylor) approximations at $\hat{y}_n=\Pr(\alpha|\text{piv};\sigma_n^*)$ of the left-hand side, $\chi(y)$, of the equations (55) and (56) that define the boundaries $\phi_g^+(k)$ and $\phi_g^-(k)$. Since the left-hand side takes the value $\frac{1}{2}$ at $\hat{y}_n$, this yields[34]

$$\chi'(\hat{y}_n)\Big[\phi_g^+(k)-\hat{y}_n)\Big]\approx x^{**}(\phi_g^+(k)), \tag{60}$$

$$\chi'(\hat{y}_n)\Big[\hat{y}_n-\phi_g^-(k)\Big]\approx x^{**}(\phi_g^-(k)). \tag{61}$$

[33]Consider the example of polynomial cost $c(x)=x^d$. Then $\lim_{x\to 0}\frac{c''(x)}{c'(x)}=\lim_{x\to\infty}\frac{d-1}{x}=\infty$.

[34]The approximation errors are of an order $\Big[\phi_g^-(k)-\hat{y}_n\Big]^2$ and $\Big[\phi_g^+(k)-\hat{y}_n\Big]^2$ respectively. Since only types that are arbitrarily close to the critical types acquire information as $n\to\infty$, it holds$\phi_g^-(k)\overset{n\to\infty}{\to}\phi_g^+(k)$ for all $k>0$. Therefore, the error terms are negligible.

The intuition here is again that only types that are arbitrarily close to the critical types acquire information when n is large such that the boundaries are arbitrarily close to $\hat{y}$ and the linear approximation is asymptotically precise. Finally, (59) follows from (57), the continuity of c and the Taylor approximations (60) and (61).

Approximation 3. Third, when the distribution of the thresholds is not uniform, the linear (Taylor) approximation of the distribution around the critical types is asymptotically precise in describing the mass of types that acquire information,

$$\frac{f(\hat{y},k|k'=k,g)\Big[\phi^g(k)-\psi^g(k)\Big]}{F(\psi^g(k)|k'=k,g)-F(\phi^g(k)|k'=k,g)} \quad\approx\quad 1. \tag{62}$$

where $F(-|k'=k,g)$ is the conditional distribution of the thresholds and $f(-|k'=k,g)$ its density.

Finally, given (57), (62), and (59),

$$\begin{aligned}
& \int_{t\in g} x(t)dH(t) \\
\approx \quad & \int_k f(\hat{y}_n,k|k'=k,g)2Kx(\hat{y}_n,k)^2(1-\frac{c(x(\hat{y}_n,k))}{c'(x(\hat{y}_n,k))x(\hat{y}_n,k)})dJ(k|g) \\
= \quad & \Pr(g)f(\hat{y}_n|g)\mathrm{E}_k(x(y,k)^2(1-\frac{c(x(y,k))}{c'(x(y,k))x(y,k)})|y=\hat{y}_n,g).
\end{aligned} \tag{63}$$

where $J(\cdot|g)$ is the conditional distribution of the total intensity of the types of group g. Thus,

$$\lim_{n\to\infty}\frac{\int_{t\in g}x(t)dH(t)}{\Pr(g)f(\hat{p}|g)\mathrm{E}_k\Big[x(y,k)^2|y=\hat{p},g\Big]\frac{2K(d-1)}{d}} \quad=\quad 1 \tag{64}$$

where I used the notation $\hat{p}=\lim_{n\to\infty}\Pr(\alpha|\text{piv},\sigma_n^*)$ and, second, that $(1-\frac{c(x(y,k))}{c'(x(y,k))x(y,k)})\to\frac{d-1}{d}$ as $n\to\infty$, which follows from Lemma 5 in the Appendix and since $x(y,k)\to 0$ for all (y,k). The first-order condition (19) together with $\lim_{x\to 0}\frac{c'(x)}{x^{d-1}} \in \mathbb{R}$ implies $x(\hat{p},k) \approx k^{\frac{1}{d-1}} x(\hat{p},1)$ for all $k>0$. Thus, since $x(\hat{p},1)$ does not depend on the group $g \in \{L, C\}$, it follows from (64) that the ratio of the power indices $W(g)$ defined by (27) is asymptotically equivalent to the ratio of the likelihood-weighted average precision $\int_{t\in g} x(t)dH(t)$, that is (39) holds.

In the next two sections, I sketch the argument for the existence of informative equilibrium sequences.

3.5.2 Inference in Large Elections

This section prepares the final fixed point argument used to construct the equilibrium sequences of Theorem 1. I record the intuitive fact that voters update toward the substate in which the vote share is closer to 1/2, that is, in which the election is closer to being tied in expectation.

Lemma 6 *Take any strategy σ for which* $\Pr(\text{piv}|\beta;\sigma,n) \in (0,1)$. *If*

$$\left|q(\alpha;\sigma)-\frac{1}{2}\right| < (\leq) \left|q(\beta;\sigma)-\frac{1}{2}\right|, \tag{65}$$

then

$$\frac{\Pr(\text{piv}|\alpha;\sigma,n)}{\Pr(\text{piv}|\beta;\sigma,n)} > (\geq) 1. \tag{66}$$

Proof. The function $q(1-q)$ has an inverse u-shape on $[0,1]$ and is symmetric around its peak at $q=\frac{1}{2}$, as is illustrated in Figure 3. So, $\left|q-\frac{1}{2}\right| < (\leq) \left|q'-\frac{1}{2}\right|$ implies that $q(1-q) > (\geq) q'(1-q')$. Thus, it follows from (13) that (65) implies (66). ■

Moreover, Lemma 6 extends in an extreme form as the electorate grows large ($n\to\infty$): the event that the election is tied is infinitely more likely in the state in which the election is closer to being tied in expectation. In fact, the likelihood ratio of the pivotal event diverges exponentially fast.

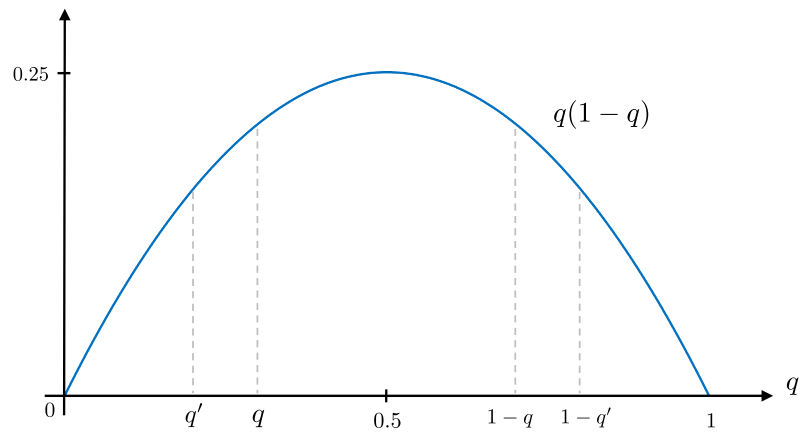


Figure 3: The function $q(1-q)$ for $q \in [0,1]$. If $|q - \frac{1}{2}| < |q' - \frac{1}{2}|$, then $q(1-q) > q'(1-q')$.

Lemma 7 *Consider any sequence of strategies* $(\sigma_n)_{n \in \mathbb{N}}$. *If,*

$$\lim_{n\to\infty} \left| q(\alpha;\sigma_n) - \frac{1}{2} \right| < (>) \lim_{n\to\infty} \left| q(\beta;\sigma_n) - \frac{1}{2} \right|, \tag{67}$$

then, for any $\kappa \geq 0$,

$$\lim_{n\to\infty} \frac{\Pr(\text{piv}|\alpha;\sigma_n,n)}{\Pr(\text{piv}|\beta;\sigma_n,n)} n^{-\kappa} = \infty(0). \tag{68}$$

Proof. Let

$$k_n = \frac{q(\alpha;\sigma_n)(1-q(\alpha;\sigma_n))}{q(\beta;\sigma_n)(1-q(\beta;\sigma_n))}.$$

From (13), the left-hand side of (68) is $\frac{(k_n)^n}{n^\kappa}$. The function $q(1-q)$ has an inverse u-shape on $[0,1]$ and is symmetric around its peak at $q = \frac{1}{2}$, as is illustrated in Figure 3. Therefore, (67) implies that $\lim_{n\to\infty} k_n > 1$. So, $\lim_{n\to\infty} (k_n)^n = \infty$. Moreover, $(k_n)^n$ diverges exponentially fast and, hence, dominates the denominator n^κ, which is polynomial. ∎

3.5.3 Fixed Point Argument

This section sketches the formal fixed point argument used to prove Theorem 1. First, I provide a useful compact representation of equilibrium.

Equilibrium Vote Shares. It follows from the analysis of the best response in Section 2 that, for n large enough, an equilibrium is a (non-degenerate) strategy $\sigma = (x, \mu)$ that satisfies (5)-(8), with $\sigma' = \sigma$, (19) for all types t with $x(t) > 0$, and (24).
I claim that equilibrium can be alternatively characterized in terms of the vector of the expected vote shares of outcome A in state α and β, i.e.,

$$\mathbf{q}(\sigma) = (q(\alpha; \sigma), q(\beta; \sigma)). \tag{69}$$

Note that for any σ and any $\omega \in \{\alpha, \beta\}$, the vote share $q(\omega; \sigma)$ pins down the likelihood of the pivotal event conditional on ω, given (13). Given (5)-(8), (19), and (55)-(56), the vector of the pivotal likelihoods is a sufficient statistic for the best response, and therefore $\mathbf{q}(\sigma)$ as well. Given some vector of expected vote shares $\mathbf{q} = (q(\alpha), q(\beta)) \in (0, 1)$, let $\sigma^{\mathbf{q}}$ be the best response to $\mathbf{q}$. Therefore, σ^* is an equilibrium, if and only if, $\sigma^* = \sigma^{\mathbf{q}(\sigma^*)}$. Conversely, an equilibrium can be described by a vector of vote shares $\mathbf{q}^*$ that is a fixed point of $\mathbf{q}(\sigma^-)$, i.e.,[35]

$$\mathbf{q}^* = \mathbf{q}\left(\sigma^{\mathbf{q}^*}\right). \tag{70}$$

In the following, I use the notation $\Pr(\alpha|\text{piv}; \mathbf{q})$ to denote the posterior consistent with (13) and the vote shares $\mathbf{q}$, and also similar notation analogous to the previous notation.

Now, I am going to sketch a fixed point argument showing that there is a sequence of equilibrium vote shares $(\mathbf{q}_n^*)_{n\in\mathbb{N}}$ such that the corresponding

[35] The ability to write an equilibrium as a finite-dimensional fixed point via (70) is a significant advantage. Similarly, a reduction to finite dimensional equilibrium beliefs has been useful in other settings; see Bhattacharya (2013b), Ahn & Oliveros (2012) and Heese & Lauermann (2017).

sequence of equilibrium strategies implements the power index rule of Theorem 1. I sketch the argument for the case when Φ is strictly increasing such that there is a unique $\hat{p} \in \Phi^{-1}(\frac{1}{2})$. The argument for the non-monotone case follows along the same lines. In addition, I consider the case when $\Phi(\Pr(\alpha)) < \frac{1}{2}$ and when the contrarians have the higher power index, i.e. $W(L, \hat{p}) < W(C, \hat{p})$. In the first step, I use the results from Section 3.5.2 regarding the inference from the pivotal event, and show: for any vote share $q(\alpha)$ in α close to $\frac{1}{2}$, I find a vote share in $q(\beta)$ such that the vote share in α under the best response is again $q(\alpha)$.

Step 1 *Let Φ be strictly increasing, $\Phi(\Pr(\alpha)) < \frac{1}{2}$ and $W(L, \hat{p}) < W(C, \hat{p})$ for $\hat{p} \in \Phi^{-1}(\frac{1}{2})$. For any $\epsilon > 0$ small enough, any $\frac{1}{2} - \frac{\epsilon}{2} \le q(\alpha) \le \frac{1}{2}$, and any n large enough, there is $q^*_n(\beta) \ge \frac{1}{2}$ such that*

$$q(\alpha) \quad = \quad q(\alpha; \sigma^{(q(\alpha), q^*_n(\beta))}). \tag{71}$$

and $q^*(\beta)$ is continuous in $q(\alpha)$.

As we will see momentarily, the first step reduces the problem of finding an equilibrium vote share to a problem in one-dimension (see Step 2).

Proof.

Step 1.1 *If $q(\beta) = \frac{1}{2} + \epsilon$, then, for ϵ small enough and n large enough,*

$$\hat{q}(\alpha; \sigma^{\mathbf{q}}) > q(\alpha). \tag{72}$$

The election is more close to being tied in α, and, by Lemma 7, voters become convinced that the state is α, i.e., $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \mathbf{q}, n) = 1$. It follows from Lemma 3 that $\lim_{n\to\infty} q(\alpha; \sigma^{\mathbf{q}}) = \Phi(1)$. Finally, (148) follows when ϵ is small enough, since $\Phi(1) > \frac{1}{2}$.

Step 1.2 *If $q(\beta) = \frac{1}{2}$, then for ϵ small enough and any n,*

$$\hat{q}(\alpha; \sigma^{\mathbf{q}}) < q(\alpha). \tag{73}$$

The election is more close to being tied in β, and, by Lemma 6, voters update towards β, that is $\Pr(\alpha|\text{piv};\mathbf{q},n) \leq \Pr(\alpha)$. Since $\Phi(\Pr(\alpha)) < \frac{1}{2}$, Lemma 3 implies that $\lim_{n\to\infty} q(\alpha;\sigma^{\mathbf{q}}) < \frac{1}{2}$. Finally, (148) follows when ϵ is small enough.

Since $q(\alpha;\sigma^{\mathbf{q}})$ is continuous in $q(\beta)$, it follows from Step 1.1, Step 1.2, and the intermediate value theorem that, for n large enough, there is $q^*_n(\beta)$ such that (71) holds. It follows from the implicit function theorem that $q^*(\beta)$ is continuous in $q(\alpha)$. ■

In what follows, I show that, for any n large enough, there is a vote share $q^*_n(\alpha)$ such that $\mathbf{q}^*_n = (q^*_n(\alpha), q^*_n(\beta))$ is a fixed point of $\mathbf{q}(\sigma^-)$, thereby constructing equilibria $\sigma^{\mathbf{q}^*_n}$ of the voting game. Given (71), it is sufficient to show the following.

Step 2 *For any n large enough, there is $q^*_n(\alpha)$ such that*

$$q^*_n(\beta) = q(\beta;\sigma^{(q^*_n(\alpha),q^*_n(\beta))}). \tag{74}$$

For this, again, I provide an intermediate value theorem argument, leveraging that information cost are sufficiently low, i.e. $\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$. Roughly speaking, when $q(\alpha) = \frac{1}{2}$, then the likelihood of being pivotal is relatively high and, when the information cost are sufficiently low, there is relatively much information acquisition under the best response. Using (38) and (39), the vote share in β under the best response, i.e. $q(\beta;\sigma^{(\frac{1}{2},q^*_n(\beta))})$, is, as a consequence, relatively much larger than $q(\alpha) = \frac{1}{2}$. One can show that, therefore,

$$q(\beta;\sigma^{(\frac{1}{2},q^*_n(\beta))}) > q^*_n(\beta), \tag{75}$$

when n is large. Essentially, this is because , given that Φ is strictly increasing, (71) together with Lemma 3 implies

$$\lim_{n\to\infty} \Pr(\alpha|\text{piv};\mathbf{q}_n,n) = \Phi^{-1}(\frac{1}{2}), \tag{76}$$

for $\mathbf{q}_n = (\frac{1}{2}, q^*_n(\beta))$, so the inference from the pivotal event is bounded such that $q^*(\beta)$ is relatively close to $q(\alpha) = \frac{1}{2}$ as opposed to $q(\beta;\sigma^{(\frac{1}{2},q^*(\beta))})$. Conversely, for $q(\alpha) = \frac{1}{2} - \epsilon$, (71) together with Lemma 3 implies that $\lim_{n\to\infty} q(\beta; \sigma^{(q(\alpha), q^*(\beta))}) = \frac{1}{2} - \epsilon$, so, for n large enough,

$$q(\beta; \sigma^{(\frac{1}{2}-\epsilon, q^*(\beta))}) < q_n^*(\beta), \tag{77}$$

since $q_n^*(\beta) > \frac{1}{2}$ by construction. Finally, using (75) and (77) and that $q(\beta; \sigma^{(q(\alpha), q_n^*(\beta))})$ is continuous in $q(\alpha)$, the intermediate value theorem implies Step 2.

Take equilibrium vote shares $(\mathbf{q}_n^*)_{n\in\mathbb{N}}$ as constructed by Step 2 and recall that $q_n^*(\alpha) \leq \frac{1}{2} \leq q_n^*(\beta)$ for n large enough, by construction. Suppose that the outcomes in the two states were not determinate: using the central limit theorem, I show in the Appendix (Lemma 14) that this is equivalent to the vote shares being only a finite number of standard deviations away from the majority threshold $\frac{1}{2}$, i.e. $\delta_\omega \in \mathbb{R}$. But then, Observation 3 implies that the outcomes must be determinate, which is a contradiction. Consequently, the outcome prefered by the group of contrarians is elected with probability converging to 1 in both states. This concludes the sketch of proof for Theorem 1.

4 Determinants of Elections

4.1 The Degree of Political Conflict

In this section, consider the settings without partisan voters,

$$\Pr(t \in L) = 1 - \Pr(t \in C) > 0, \tag{78}$$

mainly for the simplicity of exposition. Theorem 2 shows that information cost can screen intensities perfectly, and this implies that the contrarians, which represent a minority in expectation, might win the election. Conversely, without information costs, all voters become perfectly informed, and elections choose the median-voter preferred outcome in each state.

When the welfare at stake for both groups is very similar, screening becomes more difficult. Intuitively, this benefits the group of aligned voters, which is larger in expectation. For any preference distribution H satisfying (30) - (80) and $\Pr(t \in C) > 0$, the *degree of conflict* is

$$C(H) = \Big[\big| \frac{\Pr(t \in L)}{\Pr(t \in C)} \frac{\mathrm{E}(k(t)|t \in L)}{\mathrm{E}(k(t)|t \in C)} - 1 \big| \Big]^{-1}. \tag{79}$$

The degree of conflict is larger, when the welfare at stake for the voter groups, aligned and contrarians, is more similar in expectation, thereby capturing how contested the election is.

The next result verifies the intuition and shows that the election being more contested is typically in the interests of the group that is larger in expectation. For this result, I focus on the informative equilibrium sequence of Lemma 1[36] and preference distributions such that all voters of the same group g share the same (total) intensity $k_g > 0$, i.e., for all $g \in \{C, L\}$ there is a $k_g > 0$ such that

$$t \in g \Rightarrow k(t) = k_g. \tag{80}$$

I show that, when the degree of conflict is arbitrarily large, the outcome preferred by the contrarians, which are a minority in expectation, is not elected, except for an arbitrarily small set of cost functions among those that permit informative equilibrium sequences, i.e. satisfy $\lim_{x\to\infty} \frac{c'(x)x}{c(x)} \geq 3$.

Theorem 3 *There is a function $\epsilon : \mathbb{R}^{>0} \to \mathbb{R}^{>0}$ with $\lim_{\kappa\to\infty} \epsilon(\kappa) = 0$ such that the following holds. Take $\kappa > 0$ and any preference distribution H with $C(H) > \kappa$, and satisfying (30), (78) (80), and (28). Then, for any cost function with $\lim_{x\to\infty} \frac{c'(x)x}{c(x)} > 3+\epsilon(\kappa)$, there is an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ such that*

$$\lim_{n\to\infty} \Pr(A|\alpha; \sigma_n^*) = \lim_{n\to\infty} \Pr(B|\beta; \sigma_n^*) = 1, \tag{81}$$

[36] The later characterization of all equilibrium sequences in Section 5 shows that, generically, there are no other informative equilibrium sequences when $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$.

Proof. Fix the elastictiy $d = \lim_{x\to\infty} \frac{c'(x)x}{c(x)} > 3$ and the preference distribution. Take any equilibrium sequence $(\sigma_n^*)_{n\in\ltimes}$ and let $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n)$. Suppose that $\frac{W(L,\hat{p})}{W(C,\hat{p})} < 1$. Given (30) and (80),

$$\frac{W(L,\hat{p})}{W(C,\hat{p})} < 1 \Leftrightarrow \left[\frac{k_L}{k_C}\right]^{\frac{2}{d-1}} < \frac{\Pr(t \in C)}{\Pr(t \in L)} \tag{82}$$

Given the definition (79), $\kappa \to \infty$ implies $\left[\frac{k_L}{k_C}\right] \to \frac{\Pr(t\in C)}{\Pr(t\in L)}$. Since $0 < \frac{\Pr(t\in C)}{\Pr(t\in L)} < 1$, for any κ large enough, the ratio $\left[\frac{k_L}{k_C}\right]^{\frac{2}{d-1}}$ is strictly increasing in d and the derivative has a positive lower bound.[37] Thus, considering the cost functions with elasticities $d \geq 3$ that permit informative equilibrium sequences, for κ arbitrarily large, $\frac{W(L,\hat{p})}{W(C,\hat{p})} < 1$ can only hold when d arbitrarily close to $d = 3$. Consequently, Theorem 1 implies the theorem. ■

4.2 Polarization of Preferences

This section argues that groups of voters that share common interests are less likely to win an election when the preference intensities vary more strongly across the voters in the group.

For any preference distribution H and $g \in \{L, C\}$, let F_H^g be the conditional distribution of the threshold of doubt $y(t)$ of the types $t \in g$, and let J_H^g be the conditional distribution of the (total) intensities $k(t)$ of the types $t \in g$. A distribution H is a *g-intensity spread of H* if

$$H(-|t \in g') = H'(-|t \in g') \text{ for } g' \neq g \in \{C, L\}, \tag{83}$$

$$F_H^g = F_{H'}^g, \tag{84}$$

$$J_H^g <_{\text{mps}} J_{H'}^g, \tag{85}$$

where (85) means that $J_{H'}^g$ is a mean-preserving spread of J_H^g.

[37]A calculation shows $\frac{\partial a^{\frac{2}{d-1}}}{\partial d} = -\frac{2a^{\frac{2}{d-1}} log(a)}{(d-1)^2}$ which is strictly bounded above zero when $0 < a < 1$ since $\log(a) < 0$ for $0 < a < 1$.

The following result shows that a voter group acquires less information relative to other groups when the preference intensities are more dispersed within the group. In other words, when the intensities are more polarized, this aggravates the free-rider problem of the group relative to the free-rider problem of the other groups. Recall Observation 2: the ratio $\frac{W_H(L,\hat{p})}{W_H(C,\hat{p})}$ of the power indices of the voter groups pins down the ratio of the equilibrium amount of information acquired by the voter groups, where for clarity I added the preference distribution as a subscript. More precisely, the next result shows that the ratio $\frac{W_H(L,\hat{p})}{W_H(C,\hat{p})}$ is smaller for any L-intensity spread and larger for any C-intensity spread.

Lemma 8 *Let* $\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$. *Let* $g \in \{C, L\}$. *Take any preference distributions* H, H' *satisfying (30) - (32). Take any equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$ *given* H *and any equilibrium sequence* $(\hat{\sigma}_n)_{n\in\mathbb{N}}$ *given* H' *such that* $\lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma_n^*,n) = \lim_{n\to\infty}\Pr(\alpha|\text{piv};\hat{\sigma_n},n)$.

1. *If* H' *is an* L*-intensity spread of* H*,*

$$\frac{W_{H'}(L,\hat{p})}{W_{H'}(C,\hat{p})} < \frac{W_H(L,\hat{p})}{W_H(C,\hat{p})}. \tag{86}$$

2. *If* H' *is a* C*-intensity spread of* H*,*

$$\frac{W_{H'}(L,\hat{p})}{W_{H'}(C,\hat{p})} > \frac{W_H(L,\hat{p})}{W_H(C,\hat{p})}. \tag{87}$$

The proof for this can be found in Section B of the Appendix.

I use the previous results of Lemma 2 and Lemma 1, which characterize equilibrium outcomes depending on the order of the power indices, and show that when the intensities within a voter group are sufficiently dispersed, there is no equilibrium sequence where the outcome preferred by the voter group is more likely to be elected in both states. Second, there is an equilibrium sequence where the outcome that is preferred by the group is *never* elected as $n \to \infty$.

Theorem 4 *Let* $\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$*. Let* $g = L$ $(g = C)$*. Take any preference distribution* H *satisfying (30) - (32). There is a g-intensity spread* H' *of* H *such that:*

1. *For all equilibrium sequences* $(\sigma_n^*)_{n\in\mathbb{N}}$ *given* H'*,*

$$\lim_{n\to\infty}\Pr(A|\alpha;\sigma_n^*,n) \leq (\geq)\frac{1}{2} \quad \textit{or} \quad \lim_{n\to\infty}\Pr(B|\beta;\sigma_n^*,n) \leq (\geq)\frac{1}{2}.$$

2. *There is an equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$ *given* H' *such that*

$$\lim_{n\to\infty}\Pr(A|\alpha;\sigma_n^*,n) = \lim_{n\to\infty}\Pr(B|\beta;\sigma_n^*,n) = 0(1).$$

The proof is in Section B of the Appendix.

5 All Equilibria

5.1 Tyranny of the Uninformed Majority

In this section, I will show that, generically, there is an equilibrium sequence where the outcomes are as if all voters would have no access to further information about the state when the electorate is large. This means that, given this equilibrium sequence, the election chooses the outcome preferred by the median voter given the prior belief. The theorem implies that when signals of low precision are sufficiently cheap, i.e. when $\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$, there are, generally multiple types of equilibrium sequences: an informative equilibrium sequence (see Lemma 1) and an equilibrium sequence that is not informative.

Theorem 5 *Let* $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$*. There exists an equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$ *for which*

$$\lim_{n\to\infty}\Pr(A|\alpha;\sigma_n^*,n) = \lim_{n\to\infty}\Pr(A|\beta;\sigma_n^*,n) = \begin{cases} 1 \;\; \textit{if} \;\; \Phi(\Pr(\alpha)) > \frac{1}{2}, \\ 0 \;\; \textit{if} \;\; \Phi(\Pr(\alpha)) < \frac{1}{2}, \end{cases} \tag{88}$$

Proof. Recall that equilibrium can be alternatively characterized in terms of the vector of the expected vote shares of outcome A in state α and β, i.e. (69). Let

$$\mathrm{B}_\delta = \{\mathbf{q} = (q(\alpha), q(\beta)) : |\mathbf{q} - (\Phi(\Pr(\alpha)), \Phi(\Pr(\alpha)))| < \delta. \tag{89}$$

Given $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$, there is $\delta > 0$ small enough such that the vote shares in B_δ are all larger than $\frac{1}{2}$ or all smaller than $\frac{1}{2}$. Hence, it follows from (14) that the likelihood of the pivotal event is exponentially small, given any such $\mathbf{q}$. The verbatim argument of the proof of (37) implies that voters cannot learn anything from the pivotal event, as $n \to$, i.e.

$$\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \mathbf{q}, n) = \Pr(\alpha), \tag{90}$$

compare to (120). The intuition is that the precision of each voter is exponentially small since the value of information is proportional to the likelihood of the pivotal event. Consequently, the difference of the vote shares in the two states is exponentially small, which implies that that the pivotal event contains no information about the relative probability of α and β as the electorate grows large. Consequently, voting according to the prior belief is optimal as the electorate grows large.
Now, take any $\mathbf{q} \in \mathrm{B}_\delta$. Lemma 3, together with (90) implies that the vote shares of $\sigma^{\mathbf{q}}$ are again in $\mathbf{B}_\delta$ when n is large enough, i.e. $\mathbf{q}(\sigma^{\mathbf{q}}) \in \mathrm{B}_\delta$. An application of Kakutani's fixed point theorem shows that there is a sequence of equilibrium vote shares $(\mathbf{q}^*_n)_{n\in\mathbb{N}}$, i.e. vote shares satisfying (69), and for all $\omega \in \{\alpha, \beta\}$,

$$\lim_{n\to\infty} q^*_n(\omega) = \Phi(\Pr(\alpha)). \tag{91}$$

The theorem follows from the weak law of large numbers, given $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$.
■

I state a corollary of the proof of Theorem 5 that will be useful for the characterization of all equilibrium sequences in Section 5.2.

Corollary 2 *Take any equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$. *Either* $\lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma_n^*,n) = \Pr(\alpha)$ *or* $\hat{p} = \lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma_n^*,n) \in \Phi^{-1}(\frac{1}{2})$.

5.2 Characterization Result

In this section, I finish the characterization of equilibrium sequences.

Theorem 6 *Take any preference distribution* H *satisfying (28).*

1. *If* $\lim_{x\to 0}\frac{c'(x)x}{c(x)} < 3$, *all equilibrium sequences satisfy (88).*
2. *If* $\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$, *there are three types of equilibrium sequences. There is an equilibrium sequence satisfying (29). There is an equilibrium satisfying (88), and there is an equilibrium sequence* $(\sigma_n^*)_{n\in\mathbb{N}}$ *satisfying*

$$\lim_{n\to\infty}\Pr(A|\alpha;\sigma_n^*,n) = \lim_{n\to\infty}\Pr(A|\beta;\sigma_n^*,n)$$
$$= \begin{cases} 1 & \textit{if } \operatorname{sgn}(W(L,\hat{p}) - W(C,\hat{p})) \neq \operatorname{sgn}(\hat{p} - \Pr(\alpha)) \\ 0 & \textit{if } \operatorname{sgn}(W(L,\hat{p}) - W(C,\hat{p})) = \operatorname{sgn}(\hat{p} - \Pr(\alpha)), \end{cases} \tag{92}$$

and $\hat{p} \in \Phi^{-1}(\frac{1}{2})$, *where* $\hat{p} = \lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma_n^*,n)$. *Any equilibrium sequence satisfies either (29), (88), or (92).*

Let $\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$. Then, Theorem 1 establishes the existence of an equilibrium sequence satisfying (29). Theorem 5 establishes the existence of an equilibrium sequence satisfying (88). I prove another theorem in the Appendix, Theorem 9, that establishes the existence of an equilibrium sequence satisfying (92), $W(L,\hat{p}) \neq W(C,\hat{p})$, and $\hat{p} \neq \Pr(\alpha)$, where $\hat{p} = \lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma_n^*,n)$. The construction of this equilibrium sequence follows the same lines of argument as the construction of the equilibrium sequence satisfying (29). The remaining parts of the proof of Theorem 6 are in Section D of the Appendix. Using the characterization of equilibrium sequences, I prove Observation 1 in Section D of the Appendix.

5.2.1 Illustrations of the Equilibrium Sequences

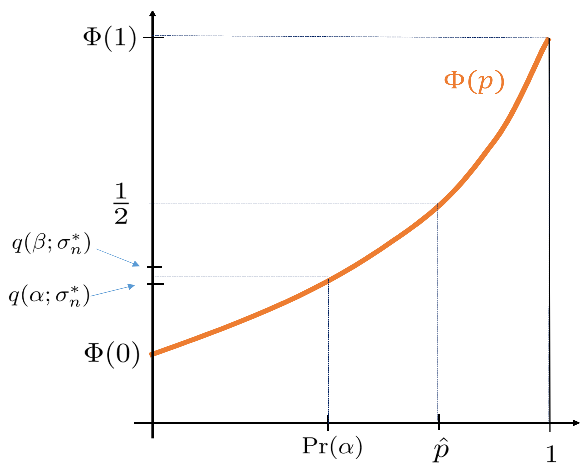
Figures 4- 6 illustrate the equilibrium sequences satisfying (92), (29), and (88) in the following case: Φ is strictly increasing such that $\hat{p} \in \Phi^{-1}(\frac{1}{2})$ is unique,

$$\hat{p} < \Pr(\alpha), \tag{93}$$

$$W(L, \hat{p}) < W(C, \hat{p}), \tag{94}$$

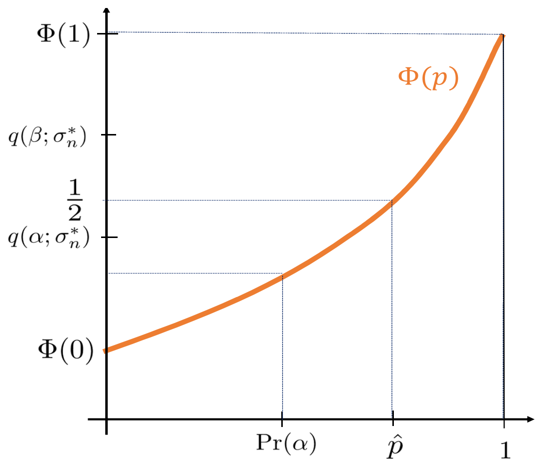
meaning that the power index of the contrarian voters is larger. Hence, the vote shares are ordered as $q(\alpha; \sigma_n^*) < q(\beta; \sigma_n^*)$ in all equilibria when n is large (see Observation 2).

Figure 4: **The equilibrium sequence(s)** $(\sigma_n^*)_{n \in \mathbb{N}}$**.** satisfying (88).



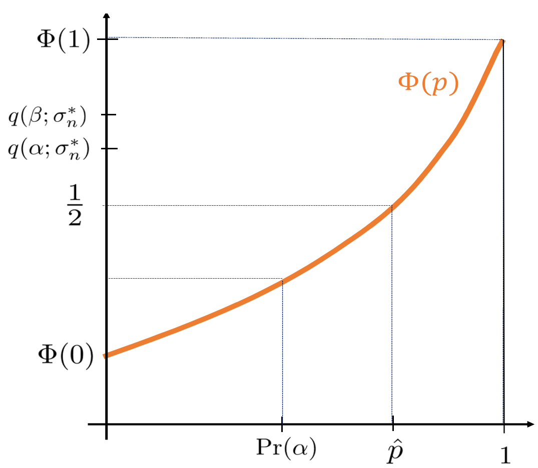
The vote shares are far away from $\frac{1}{2}$ and exponentially close to each other. The voters cannot learn anything from the pivotal event as $n \to \infty$, hence $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n) = \Pr(\alpha)$, and $\lim_{n\to\infty} q(\omega; \sigma_n^*) = \Phi(\Pr(\alpha))$ (see Lemma 3).

Figure 5: **The equilibrium sequence(s)** $(\sigma_n^*)_{n\in\mathbb{N}}$. satisfying (29).



The vote share of policy A in α, $q(\alpha;\sigma_n^*)$ is closer to $\frac{1}{2}$ than the vote share in β, $q(\alpha;\sigma_n^*)$. As a consequence, the voters update towards $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*, n)$ when conditioning on the pivotal event. As $n \to \infty$, the election is endogenously close in expectation, $\lim_{n\to\infty} q(\omega;\sigma_n^*) = \frac{1}{2}$ for $\omega \in \{\alpha,\beta\}$.

Figure 6: **The equilibrium sequence(s)** $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfying (92).



The vote share of policy A in α, $q(\alpha;\sigma_n^*)$ is closer to $\frac{1}{2}$ than the vote share in β, $q(\alpha;\sigma_n^*)$. As a consequence, the voters update towards $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*, n)$ when conditioning on the pivotal event.As $n \to \infty$, the election is endogenously close in expectation, $\lim_{n\to\infty} q(\omega;\sigma_n^*) = \frac{1}{2}$ for $\omega \in \{\alpha,\beta\}$.

6 Extensions

6.1 Heterogeneous Cost

In this section, I show that cost of information and preference intensities are strategically equivalent such that the previous results extend to a setting where the cost functions of the voters are heterogeneous. I show, in particular, that there are equilibria with first-best outcomes when the cost function of a given voter is independent of the preference type of the voters.

Let the information cost of the voters depend on a private type γ. For a given cost function c, a voter of *effort type* γ pays $c(\gamma, x) = \gamma c(x)$ for a signal of precision x. The effort type γ is distributed independently and identically across voters according to some distribution G, with density g, the *effort type distribution*, and independently of the the signals of the voters and the preference types of the other voters. The support of G is bounded below by a strictly positive constant.

The model with heterogeneous information cost is equivalent to a model with homogenous cost and a suitable distribution of preference types since the best response of an aligned or contrarian voter with effort type γ, total intensity k and threshold of doubt y is the same as that of the voter with effort type $\gamma' = 1$, total intensity $\frac{k}{\gamma}$ and threshold of doubt y, given the characterization of the best response, (5)-(8), (19), (55) and (56).

Consider the settings with heterogeneous cost where the effort type is independent of the voter's own preference type, i.e. G is independent of H, and where the distribution H of the preference types satisfies (30) - (32). Then, the corresponding preference distribution with homogenous cost types also satisfies (30) - (32). As a consequence, Theorem 2 extends to settings with heterogeneous information cost. With the appropriate extension of the definitions to the heterogeneous cost model,

Theorem 7 *Take any preference distribution H that satisfies (30)- (32) and (28). Take any effort type distribution G that is independent of H. There is an open set of cost functions $c(\cdot)$ and an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ given c for* *which the welfare-maximizing outcome is elected with probability converging to* 1 *as* $n \to \infty$.

I conjecture that Theorem 7 extends to settings where the distribution of effort types γ has a support with lower end $\gamma = 0$, and more generally, that the equilibrium representation through power indices (as in Lemma 2) extends, given effort types that are independent of the preference types t. Indeed, for any fixed γ, the ratio of the power indices captures the ratio of the information acquired. However, then, this remains true when integrating over γ.

6.2 Rich vs Coarse Choice of Information Quality

In this section, I illustrate that the "richness" of the informational choice set of the voters matters greatly. For this, I compare the previous results of the model where voters choose the quality of their information from a continuous set with a setting where the informational choice of the citizens is coarse. Interestingly, in the setting with a coarse information choice, there are *always* equilibrium sequences where, in each state, the welfare-maximizing outcome is elected, whereas in the continuous setting this depends on the degree of conflict κ, that is how similar the welfare at stake is for voters that gain from policy A and those that are harmed by it (see Section 4.1). This finding suggests that the richness of the informational choice present in modern societies might sometimes be harmful to social welfare. This is particularly surprising since a richer choice set for the voters should intuitively facilitate the screening of intensities.

Binary Precision Setting. A random voter is of an aligned type with probability $\lambda = \Pr(L) > \frac{1}{2}$, and of a contrarian type with probability $1 - \lambda = \Pr(C)$. All types share a common threshold of doubt $y(t) = \frac{1}{2}$. Each voter can choose an uninformative signal at no cost or a binary, symmetric signal of precision $\frac{1}{2} + x$ with $x > 0$ at a cost $c \geq 0$, $\Pr(a|\alpha) = \Pr(b|\beta) = \frac{1}{2} + x$. The cost c is drawn independently and identically across voters from the uniform distribution and are the private information of the voter. The cost of a voter is independent of the (preference) types and the signals of the voters. The common prior of the voters is uniform.

Now, take any symmetric strategy σ_n, meaning that $|q(\alpha;\sigma_n)-\frac{1}{2}| = |q(\beta;\sigma_n)-\frac{1}{2}|$. Given (13), this implies that $\Pr(\alpha|\text{piv};\sigma_n,n) = \frac{1}{2}$. The expected utility from the pivotal event of an aligned voter who chooses the informative signal is

$$\begin{aligned} & \Pr(\text{piv})\Big[k_L \Pr(\alpha|\text{piv};\sigma)(\frac{1}{2}+x) - k_L(\Pr(\alpha|\text{piv};\sigma)(\frac{1}{2}-x)\Big] - c \quad (95) \\ = \; & \Pr(\text{piv})\frac{k_L}{2}x - c. \quad (96) \end{aligned}$$

To see why, note that when the citizen is pivotal and votes B, she receives a utility of zero. When she is pivotal and votes A, she receives a utility of k_L or $-k_L$, depending on the state. Note that Lemma 17 extends to this setting. Given Lemma 4, she only votes A after a; and she receives a in α with probability $\frac{1}{2}+x$ and in β with probability $\frac{1}{2}-x$.

Thus, it is optimal for an aligned voter to choose the informative signal, if and only if, $c \leq \Pr(\text{piv})\frac{k}{2}x$. Given that the cost is uniformly distributed, the likelihood-weighted mean precision of the aligned types is

$$\begin{aligned} \int_{t\in L} x(t)dH(t) &= \int_{t\in L} \frac{k(t)}{2}\Pr(\text{piv})xdH(t) \\ &= \Pr(\text{piv})x\lambda \mathrm{E}_H(k(t)|t\in L). \quad (97) \end{aligned}$$

Similarly, the likelihood-weighted mean precision of the contrarian types is

$$\int_{t\in C} x(t)dH(t) = \Pr(\text{piv})x(1-\lambda)\mathrm{E}_H(k(t)|t\in C). \quad (98)$$

When the types that choose the uninformative signal vote $50-50$ for policy A and B, the vote shares of the best response are symmetric to $\frac{1}{2}$. An application of Kakutani's fixed point theorem implies that, for all n, there is a symmetric equilibrium σ_n^*. In this equilibrium, the vote shares are ordered as follows,

$$q(\alpha;\sigma_n^*) \quad > \quad q(\beta;\sigma_n^*) \tag{99}$$

$$\Leftrightarrow \lambda \mathrm{E}_H(k(t)|t\in L) \quad > \quad (1-\lambda)\mathrm{E}_H(k(t)|t\in C). \tag{100}$$

Note that the preference distribution satisfies (31) and (30) such that the welfare-maximizing outcome is given by (33). Thus, (99) implies that the welfare-maximizing policy is more likely to be elected in each state. The next result shows that the welfare-maximizing outcome is elected with probability converging to 1 in each state as the electorate grows large.

Theorem 8 *Consider the setting with a binary choice of the signal precision. For any preference distribution H with $(1-\lambda)\mathrm{E}_H(k(t)|t\in C)\neq\lambda\mathrm{E}_H(k(t)|t\in L)$, there is an equilibrium sequence $(\sigma_n)_{n\in\mathbb{N}}$, for which the welfare-maximizing outcome is elected with probability converging to 1.*

Proof. Consider a sequence $(\sigma_n)_{n\in\mathbb{N}}$ of symmetric equilibria, as I have just constructed. Clearly, Lemma 14 is valid in this setting such that it remains to show that $\delta_\omega\in\{\infty,-\infty\}$ for all $\omega\in\{\alpha,\beta\}$. By symmetry of the equilibria, $\delta_\alpha=-\delta_\beta$. Suppose that $\delta_\alpha\in\mathbb{R}$. Given (97) and (98),

$$\begin{aligned}&(q(\alpha)-\frac{1}{2})s(\alpha;\sigma_n^*)^{-1}\\ =\ &\frac{1}{2}\Pr(\mathrm{piv})x(\Big[\mathrm{E}_H(k(t)|t\in L)-(1-\lambda)\mathrm{E}_H(k(t)|t\in C)\Big]\end{aligned}$$

Multiplication of both sides with the inverse of the standard deviation $s(\alpha;\sigma_n^*)^{-1}=\frac{q(\omega;\sigma_n^*)(1-q(\omega;\sigma_n^*)}{2n+1}$ and taking limits $n\to\infty$ and using Lemma 12 yields

$$\delta_\alpha=\lim_{n\to\infty}s(\alpha;\sigma_n^*)^{-1}\phi(\delta_\alpha)x(\Big[\mathrm{E}_H(k(t)|t\in L)-(1-\lambda)\mathrm{E}_H(k(t)|t\in C)\Big] \tag{101}$$

The right-hand side diverges, and, by assumption, the left-hand side does not diverge, $\delta_\alpha\in\mathbb{R}$. This yields a contradiction. Hence, $\delta_\alpha=-\delta_\beta\in\{\infty,-\infty\}$, and the theorem follows from Lemma 14 and (99). ■

7 Remarks

7.1 Median-Voter Outcomes

Given Observation 2, whenever the contrarians have a larger power index, policy A is more likely to be elected in β than in α. Clearly, this implies that the median voter-preferred outcome is less likely to be elected in one of the states since the median voter prefers A only in α.

Corollary 3 *Let $W(L,p) < W(C,p)$ for any $p \in [0,1]$ with $\Phi(p) = \frac{1}{2}$. For any equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$, there is a state $\omega \in \{\alpha, \beta\}$ where the median-voter preferred outcome is less likely to be elected as $n \to \infty$, i.e.,*

$$\lim_{n\to\infty} \Pr(A|\alpha;\sigma_n^*, n) \leq \frac{1}{2} \quad \text{or} \quad \lim_{n\to\infty} \Pr(B|\beta;\sigma_n^*, n) \leq \frac{1}{2}. \tag{102}$$

Proof. In the Appendix, Lemma 10 (a corollary of the proof of (37) shows that any equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfies $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*, n) \in \{\Pr(\alpha), \Phi^{-1}(\frac{1}{2})\}$. Since $\Phi(0) \neq \frac{1}{2}$, $\Phi(1) \neq \frac{1}{2}$, and $\Pr(\alpha) \in (0,1)$ by assumption, the corollary follows from Lemma 2. ■

7.2 Aggregate Cost

I show that the sum of the voters' cost converges to zero in all equilibrium sequences when $\lim_{x\to 0} \frac{c'(x)x}{c(x)} \neq 3$. This shows that the equilibrium sequences with first-best outcomes imply first-best results, even when taking into account the costs of voters.

Lemma 9 *Let $\lim_{x\to 0} \frac{c'(x)x}{c(x)} \neq 3$. Take any equilibrium sequence $(\sigma_n)_{n\in\mathbb{N}}$ and let x_i be the realisation of the precision of voter $i \in \{1, \ldots, 2n+1\}$. Then,*

$$\lim_{n\to\infty} \Big[\sum_{i=1,\ldots,2n+1} c(x_i) \Big] = 0. \tag{103}$$

The proof is provided in Section E of the Appendix.

7.3 Bergsonian Welfare

In this section, I return to the welfare analysis of Section 3.2, but consider more general welfare rules than utilitarian welfare. A well-known class of welfare functions are the Bergsonian welfare functions, parametrized by a parameter $\rho \in [0, \infty]$.[38] The Bersonian welfare function with parameter ρ chooses, in each state ω, the outcome that maximizes the ρ-weighted sum of intensities, i.e. $\sum_{i=1,\ldots,2n+1}(t^i_{\omega'})^\rho$.[39][40] Theorem 1 implies the following corollary.

Corollary 4 *Take any preference distribution H satisfying (28) and (30)-(32). For any cost function c with limit elastiticty $d = \lim_{x\to 0}\frac{c(x)x}{c(x)} > 3$, there is an equilibrium sequence $(\sigma^*_n)_{n\in\mathbb{N}}$ that implements the Bergson welfare function with parameter $\rho = \frac{2}{d-1} > 1$ as $n \to \infty$, that is for all $\omega \in \{\alpha, \beta\}$:*

$$\lim_{n\to\infty} \Pr(A|\omega; \sigma^*_n, n) = 1 \Leftrightarrow \mathrm{E}(t_\omega^{\frac{2}{d-1}}|\omega = \omega) > 0. \tag{104}$$

$$\lim_{n\to\infty} \Pr(A|\omega; \sigma^*_n, n) = 0 \Leftrightarrow \mathrm{E}(t_\omega^{\frac{2}{d-1}}|\omega = \omega) < 0. \tag{105}$$

Proof. Recall (34). Thus, the order of the power indices of the groups of the aligned and the contrarian voter types is the same as the order of the means of the ρ-weighted intensities of the types. Note that it follows from the independence of the preference types of the voters and the law of large numbers that $\frac{1}{2n+1}\sum_{i=1,\ldots,2n+1}(t^i_{\omega'})^\rho \to \mathrm{E}_G((t_\omega)^\rho|\omega')$ almost surely. Hence, the corollary follows from (34) and Theorem 1. ∎

Pigou-Dalton Transfer Principle. Note that $\rho > 1$, as in the corollary, is equivalent to the weighting function $g(t^i_\omega) = t^i_\omega$ of the Bergson welfare function being concave. Furthermore, $\rho > 1$ is equivalent to the Bergson welfare function satisfying the Pigou-Dalton transfer principle, a principle of fairness, which expresses an aversion for pure inequality (see Moulin (2004) for the definition and the proof of this statement).

[38]See Burk (1936).

[39]The superscript i is added to the notation for the intensities t_ω of voter i for clarity.

[40]It is well-known that these welfare rules describe the class of symmetric, homothetic and additively separable welfare functions, see Theorem 6 of Roberts (1980).

Appendices

A Information Acquisition

A.1 Proof of Lemma 2

The benefits of information acquisition depend on how often the voters' ballot decides the election, that is the likelihood of the pivotal event. Consider an aligned type. The expected utility of an aligned type who chooses precision $x > 0$ is given by the expected utility, K, from all the events when her vote does not affect the outcome, by the cost, and by the expected utility from the pivotal event. The expected utility from the pivotal event is

$$\Pr(\text{piv})\Big[t_\alpha \Pr(\alpha|\text{piv};\sigma)(\frac{1}{2}+x)+t_\beta(\Pr(\alpha|\text{piv};\sigma)(\frac{1}{2}-x)\Big]. \tag{106}$$

To see why, note that when the citizen is pivotal and votes B, she receives utility of zero. When she is pivotal and votes A, she receives utility of t_ω, depending on the state. Given (17), it is optimal to vote A only after a; and she receives a in α with probability $\frac{1}{2}+x$ and in β with probability $\frac{1}{2}-x$. Equating marginal cost and marginal benefits gives

$$c'(x)=\Pr(\text{piv}|\sigma')\Big[\Pr(\alpha|\text{piv};\sigma')t_\alpha-\Pr(\beta|\text{piv};\sigma')t_\beta\Big]. \tag{107}$$

I claim that there is a unique solution $x^*(t;\sigma',n)$ to (107) when n is large enough. First, the marginal cost are strictly increasing since c is strictly convex. So, any solution to (107) is unique. Since $\lim_{x\to 0}\frac{c'(x)}{x^{d-1}}\in\mathbb{R}$ for some $d>1$ and since c is continuously differentiable, marginal cost at zero are zero, i.e. $c'(0)=0$. Given $c'(0)=0$, $c'(1)>0$ and since $0\le\Pr(\text{piv}|\sigma',n)<c'(1)$ for any n large enough, given (14), it follows from the intermediate value theorem that there is a solution to (107). It follows from the implicit function theorem that $x^*(t;\sigma',n)$ is continuously differentiable. The argument for the contrarian types is analogous. This finishes the proof of the lemma.

A.2 Proof of Lemma 4.

Step 1 *There is $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$: take any strategy σ' and any type t. It is a best response for a type t to acquire information $x(t) > 0$ if and only if the following two conditions hold,*

$$\frac{1}{2} + x^{**}(t) \geq \frac{\Pr(\beta|\text{piv};\sigma',n)\phi_g^-(k)}{\Pr(\alpha|\text{piv};\sigma',n)(1-\phi_g^-(k)) + \Pr(\beta|\text{piv};\sigma',n)\phi_g^-(k)}, \quad (108)$$

$$\frac{1}{2} - x^{**}(t) \leq \frac{\Pr(\beta|\text{piv};\sigma',n)\phi_g^+(k)}{\Pr(\alpha|\text{piv};\sigma',n)(1-\phi_g^+(k)) + \Pr(\beta|\text{piv};\sigma',n)\phi_g^+(k)}, \quad (109)$$

where $x^{**}(t;\sigma',n) = x^*(t;\sigma',n)(1 - \frac{c(x^*(t;\sigma',n))}{x^*(t;\sigma',n)c'(x^*(t;\sigma',n))})$ and $x^*(t;\sigma',n)$ is the unique solution to the first-order condition (19).

Consider an aligned type. Recall Lemma 2 which says that if an aligned type acquires information, then $x(t) = x^*(t;\sigma',n)$. Recall (17) which says that if an aligned type acquires information, the type votes A after a and B after b. So, the comparison of the expected utility from voting A or B without further information and the expected utility from choosing $x = x^*(t;\sigma',n)$ and voting A after a and B after b minus the cost $c(x^*(t;\sigma',n))$, pins down the aligned types that acquire information. Using (106), an aligned type prefers choosing precision $x = x^*(t;\sigma',n)$ over voting A without further information if

$$\begin{aligned} & \Pr(\text{piv}|\sigma',n)\Big[\Pr(\alpha|\text{piv};\sigma',n)(\frac{1}{2}+x)t_\alpha + \Pr(\beta|\text{piv};\sigma',n)(\frac{1}{2}-x)t_\beta\Big] - c(x) \\ \geq\ & \Pr(\text{piv}|\sigma',n)\Big[\Pr(\alpha|\text{piv};x,\sigma',n)t_\alpha + \Pr(\beta|\text{piv};\sigma',n)t_\beta\Big]. \end{aligned} \quad (110)$$

I rewrite (110) as

$$\begin{aligned} & \Pr(\text{piv}|\sigma',n)\Big[(\frac{1}{2}+x)\Big[\Pr(\alpha|\text{piv};\sigma',n)t_\alpha - \Pr(\beta|\text{piv};\sigma',n)t_\beta\Big] + \Pr(\beta|\text{piv};\sigma',n)t_\beta\Big] - c(x) \\ \geq\ & \Pr(\text{piv}|\sigma',n)\Big[\Pr(\alpha|\text{piv};\sigma',n)t_\alpha - \Pr(\beta|\text{piv};\sigma',n)t_\beta + 2\Pr(\beta|\text{piv};\sigma',n)t_\beta\Big] \end{aligned} \quad (111)$$

Let $x = x^*(t;\sigma', n)$ in the following. Plugging (107) into (111) gives

$$\begin{aligned} & (\frac{1}{2}+x)c'(x) - c(x) + \Pr(\text{piv}|\sigma', n)\Pr(\beta|\text{piv};\sigma')t_\beta \\ \geq \quad & c'(x) + 2\Pr(\text{piv}|\sigma', n)\Pr(\beta|\text{piv};\sigma', n)t_\beta. \end{aligned} \tag{112}$$

I divide by $c'(x)$ rearrange, and use (107) again,

$$(\frac{1}{2}+x) - \frac{c(x)}{c'(x)} \geq 1 + \frac{\Pr(\beta|\text{piv};\sigma', n)(t_\beta)}{\Pr(\alpha|\text{piv};\sigma', n)t_\alpha + \Pr(\beta|\text{piv};\sigma', n)(-t_\beta)}. \tag{113}$$

I use (22) and (23),

$$(\frac{1}{2}+x) - \frac{c(x)}{c'(x)} \geq 1 + \frac{-\Pr(\beta|\text{piv};\sigma', n)y(t)}{\Pr(\alpha|\text{piv};\sigma', n)(1-y(t)) + \Pr(\beta|\text{piv};\sigma', n)y(t)}. \tag{114}$$

Rearranging gives (109). In the same way one shows that an aligned type prefers choosing precision $x = x^*(t;\sigma', n)$ over voting B without further information only if (108) holds. The argument for the contrarian types is analogous. This finishes the proof of the first step.

Fix a group of voter types, aligned or contrarians. Recall that any type of the group is uniquely determined by the pair $(y(t), k(t))$, see (22) and (23).

Step 2 *For any n large enough: for any $g \in \{C, L\}$ and for any $k > 0$,*

$$\frac{1}{2} + x^{**}(y,k) - \chi(y), \quad \textit{and} \tag{115}$$

$$\frac{1}{2} - x^{**}(y,k) - \chi(y). \tag{116}$$

are strictly decreasing in $y \in (0,1)$, where $\chi(y) = \frac{\Pr(\beta|\text{piv};\sigma', n)y}{\Pr(\beta|\text{piv};\sigma', n)y + \Pr(\alpha|\text{piv};\sigma', n)(1-y)}$. For any $\epsilon > 0$, there is $\delta > 0$ such that the derivatives of (115) and (116) are bounded below by δ.

Consider the derivatives of the summands individually: recall that $x^*(y, k;\sigma', n)$ is implicitly defined by the first-order condition 19. First, it follows from im-

plicit differentiation[41] that the derivative of $x^*(y, k; \sigma', n)$ with respect to y converges to zero uniformly as $n \to \infty$. Intuitively, changes in the marginal benefit on the right hand side of (19) due to changes in the threshold of doubt y are of an order of the marginal cost $c'(x(y,k))$. These changes relate to infinitely smaller changes in the equilibrium precision $x^*(y, k; \sigma', n)$ on the left hand side since the derivative of the marginal cost, $c''(x(y,k))$, is much larger than the marginal cost $c'(x(y,k))$ when the precision is small. Lemma 5 together with the definition of $x^{**}(y, k; \sigma', n)$ implies that the derivative of $x^{**}(y, k; \sigma', n)$ with respect to y converges to zero uniformly as $n \to \infty$.
Second, note that χ is continuously differentiable in y; moreover, for any $\epsilon > 0$, there is $\delta > 0$ such that $\chi'(y) > \delta$ for any $y \in (\epsilon, 1-\epsilon)$ and any n.[42] I conclude that for n large enough, $\frac{1}{2} + x^{**}(y,k) - \chi(y)$ and $\frac{1}{2} - x^{**}(y,k) - \chi(y)$ are strictly decreasing.

Step 3 *For any n large enough, there are $\phi_g^-(k), \phi_g^+(k)$ with $\phi_g^-(k) < \Pr(\alpha|\text{piv}; \sigma_n, n) < \phi_g^+(k)$ such that*

$$\chi(y) \leq \frac{1}{2} + x^{**}(y,k) \quad \Leftrightarrow \quad y \leq \phi_g^+(k), \tag{117}$$

$$\chi(y) \geq \frac{1}{2} - x^{**}(y,k) \quad \Leftrightarrow \quad y \geq \phi_g^-(k). \tag{118}$$

Note that $\chi(\hat{y}_N) = \frac{1}{2}$ and $x^{**}(\hat{y}, k) > 0$ for $\hat{y}_n = \Pr(\alpha|\text{piv}; \sigma', n)$. Thus, $\chi(\hat{y}_n) \leq \frac{1}{2} + x^{**}(\hat{y}_n, k)$ and $\chi(\hat{y}_n) \geq \frac{1}{2} - x^{**}(\hat{y}_n, k)$. The claim of the step follows from Step 2 and since $\lim_{n\to\infty} x^{**}(\hat{y}_n, k) = 0$.

Finally, Lemma 4 follows from Step 1 and Step 3. Note that it follows from the implicit function theorem that the functions $\phi_g^-(k)$ and $\phi_g^+(k)$ are continuously differentiable in k.

[41] For any continuously differentiable $F : [0,1] \times [0, \frac{1}{2}] \to \mathbb{R}$ and an (implicit) function $g : [0,1] \to [0, \frac{1}{2}]$ with $F(a, g(a)) = 0$, $g'(a) = \frac{\frac{\partial F(x,y)}{\partial x}|_{(x,y)=(a,g(a))}}{\frac{\partial F(x,y)}{\partial y}|_{(x,y)=(a,g(a))}}$, which is an implication of the chain rule of differentiation.

[42] For any $p \in (0,1)$, $\frac{\partial}{\partial y}(\frac{py}{py+(1-p)(1-y)}) = \frac{(1-p)p}{(p(2y-1)-y+1)^2}$. Thus, for any $\epsilon > 0$, there is $\delta > 0$ such that for all $p \in (\epsilon, 1-\epsilon)$, $\frac{\partial}{\partial y}(\frac{py}{py+(1-p)(1-y)}) > \delta$. The assumption $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma', n) \in (0,1)$ implies that, moreover, there is $\delta > 0$ such that $\chi'(y)$ is uniformly bounded below by a positive constant for any n large enough.

B Proof of Theorem 1

This section proves Theorem 1. Section B.1 proves (37) and Observation 2. Section 3.5.2 and Section B.2 prepare a fixed point argument. Section 3.5.2 analyzes the inference of the voters from the pivotal event. Section B.2 proves the results from Section 3.3.3 about which cost functions allow for sufficient information acquisition by the voters, as needed for election outcomes to be *informative*, $\lim_{n\to\infty} \Pr(A|\alpha;\sigma_n,n) \neq \lim_{n\to\infty} \Pr(B|\alpha;\sigma_n,n)$ and determinate in both states, $\lim_{n\to\infty} \Pr(B|\omega;\sigma_n,n) \in \{0,1\}$ for all $\omega \in \{\alpha,\beta\}$.

B.1 Critical Observations

B.1.1 Proof of (37)

Suppose that $\lim_{n\to\infty} \Phi(\Pr(\alpha|\text{piv};\sigma_n^*,n)) \neq \frac{1}{2}$. Then, the pivotal likelihood is exponentially small when n is large, given (14). Thus, the first-order condition (19) implies that the precision $x(t)$ of each voter is exponentially small. It follows from (38) that the difference in the vote shares is exponentially small,

$$q(\alpha;\sigma_n^*) - q(\beta;\sigma_n^*) = \lim_{n\to\infty} 2\Big[\int_{t\in L} x(t)dH(t) - \int_{t\in C} x(t)dH(t)\Big] < z^n c_3 \quad (119)$$

for some constant $c_3 \neq 0$ and n large enough. Consider the voter's inference about the relative likelihood of α and β. Intuitively, given (119), the pivotal event contains no information about the relative probability of α and β as the electorate grows large. I claim that (119) implies

$$\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*,n) = \Pr(\alpha). \quad (120)$$

To see why, consider the likelihood ratio

$$\frac{\Pr(\text{piv}|\alpha;\sigma_n^*,n)}{\Pr(\text{piv}|\beta;\sigma_n^*,n)} = \Big[\frac{q(\alpha;\sigma_n^*)(1-q(\alpha;\sigma_n^*)}{q(\beta;\sigma_n^*)(1-q(\beta;\sigma_n^*)}\Big]^n. \quad (121)$$

The inequality (119) states that the difference in vote shares is exponentially small such that $(1-\frac{1}{n^2})^n \leq \frac{\Pr(\alpha|\text{piv};\sigma_n^*,n)}{\Pr(\beta|\text{piv};\sigma_n^*,n)} \leq (1+\frac{1}{n^2})^n$ for all n large enough.

Then, the description $\lim_{n\to\infty}(1-\frac{x}{n})^n = e^x$ of the e-function implies that the likelihood ratio converges to 1 and in turn (120).

If $\Phi(\Pr(\alpha)) = \frac{1}{2}$, (37) follows from (120) since Φ is continuous. Suppose that $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$. Note that (120) implies that $\lim_{n\to\infty} q(\omega;\sigma_n^*) = \Phi(\Pr(\alpha))$ for $\omega \in \{\alpha,\beta\}$, given Lemma 3 and since Φ is continuous. Then, the weak law of large numbers implies $\lim_{n\to\infty}\Pr(A|\alpha;\sigma_n^*,n) = \lim_{n\to\infty}\Pr(B|\beta;\sigma_n^*,n) \in \{0,1\}$. This contradicts with the equilibrium sequence being informative; see the definition (25). I conclude, that any informative equilibrium sequence must satisfy (37). This finishes the proof of (37).

I record the following corollary of the proof of (37).

Lemma 10 *Take any preference distribution H satisfying (28). For any equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$, $\lim_{n\to\infty}\Pr(\alpha|\mathrm{piv};\sigma_n^*) \in \{\Pr(\alpha)\} \cup \Phi^{-1}(\frac{1}{2})$.*

Proof. Suppose that $\lim_{n\to\infty}\Pr(\alpha|\mathrm{piv};\sigma_n^*) \notin \Phi^{-1}(\frac{1}{2})$. Then, Lemma 3 implies that $\lim_{n\to\infty} q(\omega;\sigma_n^*) \neq \frac{1}{2}$ for any $\omega \in \{\alpha,\beta\}$. Then, the proof of (37) implies (120), and hence, $\lim_{n\to\infty}\Pr(\alpha|\mathrm{piv};\sigma_n^*) = \Pr(\alpha)$. This finishes the proof of Lemma 10. ■

B.1.2 Proof of Observation 2

I show a more general lemma, which implies Observation 2.

Lemma 11 *Take any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$. For all n, let $\hat{\sigma}_n$ be any best response to σ_n. If $\hat{p} = \lim_{n\to\infty}\Pr(\alpha|\mathrm{piv};\sigma_n,n) \in (0,1)$, for any $g \in \{L,C\}$, the sequence of best responses satisfies*

$$\int_{t\in g} x(t)dH(t) \;\approx\; c_4\Big[\Pr(\mathrm{piv}|\sigma_n^*,n)\Big]^{\frac{2}{d-1}} W(g,\hat{p})$$

for some constant $c_4 > 0$ that does not depend on g.

Proof. For the simplicity of the exposition, in the following, we consider cost functions $c(x) = \gamma x^d$ for some $d > 1$. More generally, recall that any cost function considered in this paper satisfies $\lim_{x\to 0}\frac{c'(x)}{x^{d-1}} \in \mathbb{R}$ for some $d > 1$ such that l'Hospital's rule implies that $\lim \frac{c(x)}{x^d} \in \mathbb{R}$. The proof for a general cost function is verbatim except that one has to replace the equality $c(x) = \gamma x^d$ with the approximation $c(x) \approx \gamma x^d$.

Notation. Recall that, when fixing the voter group $g \in \{L, C\}$, we can view the conditional distribution $H(-|t \in g)$ of the preference types $t \in g$ as a distribution of the threshold of doubt $y(t)$ and the total intensity $k(t)$ of the types $t \in g$, see (22) and (23). The marginal distribution $F(-|t \in g)$ of the threshold of doubt of the types $t \in g$ has the density $f^g(y) = \int_{k\in[0,2]} h^g(y,k)dk$ where h^g is the density of $H(-|t \in g)$. The marginal distribution J of the total intensity of the voter types $t \in g$ has the density $j^g(k) = \int_{y\in[0,1]} h^g(y,k)dy$. Similarly, let the densities of the conditional distributions $F(y'|k = k', t \in g) = \int_{y\le y'} \frac{h^g(y,k)}{j^g(k')}$ and $J(k'|y = y', t \in g) = \int_{k\le k'} \frac{h^g(y,k)}{f^g(y')}$ be $f^g(y|k = k')$ and $j^g(k|y = y')$, for any $k' \in [0, 2]$ and any $y' \in [0, 1]$. In the following, I consider $x(t)$, $x^*(t; \sigma_n, n)$ and $x^{**}(t; \sigma_n, n)$ as functions of (y, k). Recall from 2 that $x^*(t; \sigma_n, n)$ and $x^{**}(t; \sigma_n, n)$ only depend on the treshold of doubt and the total intensity, and are independent of the group that the type t belongs to.

Take any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$ with $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n, n) \in (0, 1)$. The first step shows that the region $[\phi_g^-(k), \phi_g^+(k)]$ of types that acquire information, i.e. for which $x(y, k) > 0$, is sufficiently symmetric around the critical types with $\hat{y}_n = \Pr(\alpha|\text{piv}; \sigma_n)$. This first step implies (59) from the sketch of proof in the main text.

Step 1 *For all $k \in (0, 2)$,*

$$\lim_{n\to\infty} \frac{\phi_g^+(k) - \hat{y}_n}{x^{**}(\hat{y}, k; \sigma_n, n)} = -\lim_{n\to\infty} \frac{\phi_g^-(k) - \hat{y}_n}{x^{**}(\hat{y}_n, k; \sigma_n, n)} = \lim_{n\to\infty} \frac{1}{\chi'(\hat{y}_n)}. \tag{122}$$

where $\hat{y}_n = \Pr(\alpha|\text{piv}; \sigma_n)$.

Let $\chi(y) = \frac{\Pr(\beta|\text{piv};\sigma_n,n)y}{\Pr(\beta|\text{piv};\sigma',n)y+\Pr(\alpha|\text{piv};\sigma_n,n)(1-y)}$, which is the left hand side of (55) and

(56). Recall that $\chi(\hat{y}_n) = \frac{1}{2}$. A Taylor expansion of χ at $\hat{y}_n = \Pr(\alpha|\text{piv};\sigma_n)$ implies

$$\phi_g^-(k) - \hat{y}_n = -\frac{1}{\chi'(\epsilon_{1,n})}x^{**}(\phi_g^-(k), k) \tag{123}$$

for some $\epsilon_{1,k,n} \in [\phi_g^-(k), \hat{y}_n]$. I plug in the definition of $x^{**}(y,k)$,

$$\phi_g^-(k) - \hat{y}_n = -\frac{1}{\chi'(\epsilon_{1,k,n})}x^*(\phi_g^-(k), k;\sigma_n, n)\Big[1 - \frac{x^*(\phi_g^-(k), k;\sigma_n, n)c'(x^*(\phi_g^-(k), k;\sigma_n, n))}{c(x^*(\phi_g^-(k), k;\sigma_n, n))}\Big] \tag{124}$$

A Taylor expansion of $x^*(y,k;\sigma_n,n)$ at $\hat{y}_n = \Pr(\alpha|\text{piv};\sigma_n,n)$ yields

$$x^*(\phi_g^-(k), k;\sigma_n, n) = x^*(\hat{y}_n, k;\sigma_n, n) + (x^*(\epsilon_{2,k,n}, k;\sigma_n, n))'\Big[\phi_g^-(k) - \hat{y}_n\Big]. \tag{125}$$

for some $\epsilon_{2,n} \in [\phi_g^-(k), \hat{y}_n)]$. As in the proof of Lemma 4, implicit differentiation shows that the derivative of the function $x^*(y,k;\sigma_n,n)$ with respect to y converges to zero uniformly. Therefore, (124), (125) and together imply

$$\lim_{n\to\infty}\frac{\phi_g^-(k) - \hat{y}_n}{x^{**}(\hat{y}, k;\sigma_n, n)} = -\lim_{n\to\infty}\frac{1}{\chi'(\epsilon_{1,n})} = -\lim_{n\to\infty}\frac{1}{\chi'(\hat{y}_n)}. \tag{126}$$

where the last equality follows from the continuity of χ' and since $\epsilon_{1,k,n} \to \hat{y}_n$ as $n \to \infty$. The analogous argument for $\phi_g^+(k)$ shows that

$$\lim_{n\to\infty}\frac{\phi_g^+(k) - \hat{y}_n}{x^{**}(\hat{y}_n, k;\sigma_n, n)} = \lim_{n\to\infty}\frac{1}{\chi'(\hat{y}_n)}, \tag{127}$$

which finishes the proof of (122).

The next step shows (57) from the sketch of the proof in the main text.

Step 2 *For all $k \in (0,2)$,*

$$x(y,k) > 0 \Rightarrow \frac{x(y,k)}{x(y^*,k)} \approx 1. \tag{128}$$

where the convergence is uniform across (y,k).

Take any (y, k) such that $x(y, k) > 0$. Lemma 4 implies that $y \in [\phi_g^-(k), \phi_g^+(k)]$. A Taylor expansion of $x^*(y, k; \sigma_n, n)$ at $\hat{y}_n = \Pr(\alpha|\text{piv}; \sigma_n, n)$ implies

$$x(y, k) = x^*(\hat{y}_n, k; \sigma_n, n) + (x^*(\epsilon_{y,k,n}, k; \sigma_n, n))'(y - \hat{y}_n) \tag{129}$$

for some $\epsilon_{y,k,n} \in [y, \hat{y}_n]$. Since $\lim_{n\to\infty} x^*(\hat{y}, k; \sigma_n, n) = 0$, Lemma 5 implies that

$$\lim_{n\to\infty} \frac{x^*(\hat{y}_n, k; \sigma_n, n)}{x^{**}(\hat{y}_n, k; \sigma_n, n)} = \frac{d}{d-1}. \tag{130}$$

Hence, Step 1 implies $\lim_{n\to\infty} \frac{x^*(\hat{y}_n,k;\sigma_n,n)}{\phi_g^+(k)-\hat{y}_n} \in \mathbb{R}$, and, thus $\lim_{n\to\infty} \frac{x^*(\hat{y}_n,k;\sigma_n,n)}{y-\hat{y}_n} \in \mathbb{R}$ Recall that the derivative of $x^*(y, k; \sigma_n, n)$ with respect to y converges to zero uniformly. Therefore, I conclude that (129) yields (128).

The next step shows (62) from the sketch of the proof in the main text.

Step 3 *For all $k' \in (0, 2)$ and any $g \in \{L, C\}$,*

$$\frac{f^g(\hat{y}_n, k; \sigma_n, n|k = k')\left[\phi_g^-(k') - \phi_g^+(k')\right]}{F(\phi_g^+(k)|k(t) = k, t \in g) - F(\phi_g^-(k)|k = k', t \in g)} \approx 1. \tag{131}$$

A Taylor expansion of $F(-|k = k', t \in g)$ at $\hat{y} = \Pr(\alpha|\text{piv}; \sigma_n, n)$ implies

$$F(\phi(k)|k = k', t \in g) = F(\hat{y}|k = k', t \in g) + f^g(\epsilon_{3,k',n}|k = k')\left[\phi_g^-(k') - \hat{y}_n\right], \tag{132}$$
$$F(\psi(k)|k = k', t \in g) = F(\hat{y}|k = k', t \in g) + f^g(\epsilon_{4,k',n}|k = k')\left[\phi_g^+(k') - \hat{y}_n\right] \tag{133}$$

for some $\epsilon_{3,k,n} \in [\phi_g^-(k'), \hat{y}_n]$ and some $\epsilon_{4,k,n} \in [\hat{y}_n, \phi_g^+(k')]$. Since $\lim_{n\to\infty} \phi_g^-(k') - \phi_g^-(k) = 0$, the continuity of $f^g(-|k = k')$ implies

$$\lim_{n\to\infty} \frac{f^g(\epsilon_{3,k',n}|k = k')}{f^g(\hat{y}_n|k = k')} = 1, \tag{134}$$
$$\lim_{n\to\infty} \frac{f^g(\epsilon_{4,k',n}|k = k')}{f^g(\hat{y}_n|k = k')} = 1. \tag{135}$$

Finally, (134) and (135) together with (132) and (133) imply (131).

The last two steps finish the proof of Lemma 2.

Step 4 *For any $g \in \{C, L\}$ and any $k' \in [0,2]$,*

$$\lim_{n\to\infty} \frac{\int_y x(y,k')dF(y|k=k',t\in g)}{x^*(\hat{y}_n,k';\sigma_n,n)^2} = \lim_{n\to\infty} \frac{2f^g(\hat{y}_n|k=k')}{\chi'(\hat{y}_n)}\frac{d-1}{d}. \tag{136}$$

I use the first three steps,

$$\begin{aligned}
& \lim_{n\to\infty}\int_y x(y,k')dF(y|k=k',t\in g) \\
\approx\ & \Big[F(\psi^g(k')|k=k') - F(\phi^g(k')|k=k')\Big]x^*(\hat{y}_n,k';\sigma_n,n) \\
\approx\ & f^g(\hat{y}_n,k'|k=k')\Big[\phi^g(k') - \psi^g(k')\Big]x^*(\hat{y}_n,k';\sigma_n,n) \\
\approx\ & 2\frac{f^g(\hat{y}_n,k'|k=k')}{\chi'(\hat{y}_n)}x^{**}(\hat{y}_n,k';\sigma_n,n)x^*(\hat{y}_n,k';\sigma_n,n) \\
\approx\ & 2\frac{f^g(\hat{y}_n,k'|k=k')}{\chi'(\hat{y}_n)}x^*(\hat{y}_n,k';\sigma_n,n)^2\frac{d-1}{d},
\end{aligned}$$

where I used Step 2 for the second line, Step 3 for the third line, Step 1 for the fourth line and (130) for the last line.

Step 5 *For all $g \in \{L, C\}$,*

$$\int_{t\in g} x(t)dH(t) \approx \frac{2(d-1)}{d}\frac{x^*(\hat{p},1;\sigma_n,n)^2}{\chi'(\hat{p})}\Pr(g)f^g(\hat{p})\mathrm{E}(k(t)^{\frac{2}{d-1}}|t\in g, y(t)=\hat{p}).$$

for $\hat{p} = \lim_{n\to\infty}\Pr(\alpha|\mathrm{piv};\sigma_n,n)$.

First, I change coordinates,

$$\int_{t\in g} x(t)dG(t) = \Pr(g)\int_{k'\in[0,2]}\int_{y\in[0,1]} x(y,k')f^g(y,k')dydk' \tag{137}$$

and, then, apply (136) to obtain

$$\begin{aligned}
&\int_{t\in g} x(t)dG(t) \\
\approx\ & \Pr(g)\int_{k'\in[0,2]} j^g(k')x^*(\hat{y}_n,k')^2\frac{f^g(\hat{y}_n|k=k')}{\chi'(\hat{y}_n)}\frac{2(d-1)}{d}dk'. \qquad (138)
\end{aligned}$$

It follows from the first-order condition (19) and since $c(x)=\gamma x^d$ that for all $k'\in[0,2]$,

$$x^*(\hat{y}_n,k';\sigma_n,n) = (k')^{\frac{1}{d-1}}x^*(\hat{y}_n,1;\sigma_n,n). \qquad (139)$$

Combining (138) and (139),

$$\int_{t\in g} x(t)dG(t) \approx \Pr(g)\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2\int_{k'\in[0,2]}(k')^{\frac{2}{d-1}}j^g(k')f^g(\hat{y}_n|k=k')dk'.$$

I rewrite,

$$\begin{aligned}
\int_{t\in g} x(t)dG(t) &\approx \Pr(g)\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2 f^g(\hat{y}_n)\int_{k'\in[0,2]}(k')^{\frac{2}{d-1}}\frac{h^g(\hat{y}_n,k')}{f^g(\hat{y}_n)}dk' \\
&= \Pr(g)\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2 f^g(\hat{y}_n)\int_{k'\in[0,2]}(k')^{\frac{2}{d-1}}j^g(k'|y=\hat{y}_n)dk' \\
&= \Pr(g)\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2 f^g(\hat{y}_n)\mathrm{E}_H((k')^{\frac{2}{d-1}}|y=\hat{y}_n) \\
&\approx c_{4,n}\Pr(g)f^g(\hat{p})\mathrm{E}_H((k')^{\frac{2}{d-1}}|y=\hat{p}) \qquad (140)
\end{aligned}$$

where $c_{4,n}=\frac{2(d-1)}{d\chi'(\hat{y}_n)}x^*(\hat{y}_n,1;\sigma_n,n)^2$. I used (37) and the continuity of f^g, and $f^g(k'|y=y')$ in y' for the statement on the last line. The continuity of χ' and $x^*(\cdot,1;\sigma_n,n)$ in y implies $\lim_{n\to\infty}\frac{c_4}{c_{4,n}}=1$ for $c_4=\frac{2(d-1)}{d}\frac{x^*(\hat{p},1;\sigma_n,n)^2}{\chi'(\hat{p})}>0$. This finishes the proof of Step 4. ■

B.2 Preliminaries: Value and Cost of Information

This section analyzes which cost functions allow for sufficient information acquisition by the voters, as needed for election outcomes to be *informa-*

tive, $\lim_{n\to\infty} \Pr(A|\alpha;\sigma_n,n) \neq \lim_{n\to\infty} \Pr(B|\alpha;\sigma_n,n)$ and determinate in both states, $\lim_{n\to\infty} \Pr(B|\omega;\sigma_n,n) \in \{0,1\}$. What matters is that cost go to zero fast enough when a voter chooses an arbitrarily uninformative signal. The critical condition is that that the first three derivatives of the cost function are zero at the precision of the uninformative signal. Or equivalently, that $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$.[43]

Value of Information. The marginal value of information is proportional to the likelihood of the pivotal event, given (19). How large is this likelihood? Since the vote share in a state is the empirical mean of $2n+1$ i.i.d. Bernoulli variables which take the value 1 with probability $q(\omega;\sigma_n)$, an application of the local central limit[44] tells how likely events close to the expected vote share are. In particular, the next result shows that when the expected vote share in a state is sufficiently close to the majority threshold $\frac{1}{2}$, the pivotal likelihood is proportional to the standard deviation of the vote share, which is of order $n^{-\frac{1}{2}}$.[45] For any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$ and any n, let $s(\omega;\sigma_n)$ be the standard deviation of the vote share. Let

$$\delta_\omega = \lim_{n\to\infty} \frac{1}{s(\omega;\sigma_n)}\Big[q(\omega;\sigma_n) - \frac{1}{2}\Big] \tag{141}$$

be the normalized distance of the expected vote share to the majority threshold as $n \to \infty$.

Lemma 12 *For any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$ and for all $\omega \in \{\alpha,\beta\}$,*

$$\lim_{n\to\infty} \Pr(\text{piv}|\omega;\sigma_n) s(\omega;\sigma_n)^{-1} \;=\; \phi(\delta_\omega), \tag{142}$$

where ϕ the probability density function of the standard normal distribution.

[43] This condition also appears in Martinelli (2006).

[44] See Gnedenko (1948), and McDonald (1980) for the local limit theorem for triangular arrays of integer-valued variables.

[45] The number of A-votes is distributed according to a binomial distribution with parameters n and $q_{\omega,n}$. Hence, its variance is $nq(\omega;\sigma_n)(1-q(\omega;\sigma_n))$, and the standard deviation $(nq(\omega;\sigma_n)(1-q(\omega;\sigma_n))^{\frac{1}{2}}$. Consequently, the standard deviation of the vote share is distributed according to $\frac{1}{n}\mathcal{B}(n,q(\omega;\sigma_n))$, so its standard deviation is $(\frac{q(\omega;\sigma_n)(1-q(\omega;\sigma_n))}{n})^{\frac{1}{2}}$.

The proof is omitted since the result directly follows from the local central limit theorem.

Figure 7 provides intuition for Lemma 14.

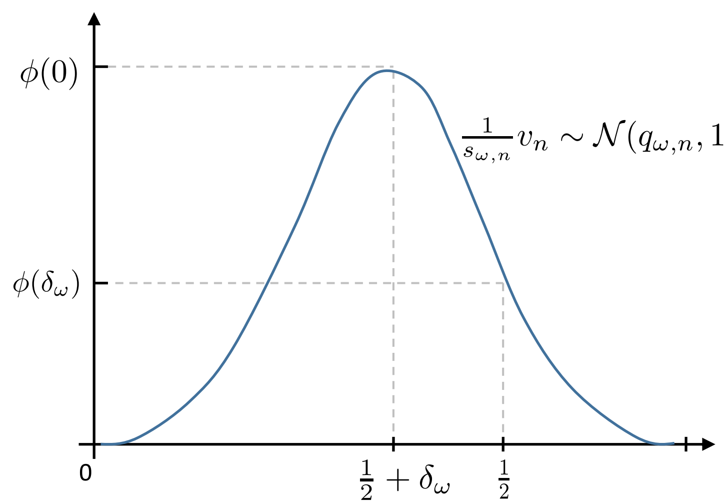


Figure 7: Conditional on ω, the realized vote share v_n follows a binomial distribution with parameters $q(\omega; \sigma_n^*)$ and $2n+1$. It follows from the Central Limit Theorem for triangular arrays that the distribution of the normalized vote share converges a normal distribution.

Cost of Information. It depends on the cost of information, how the value of information translates into information acquisition and election outcomes. In what follows, I describe a condition on the cost function that allows for informative and determinate outcomes under the best response to any strategies with vote shares sufficiently close to $\frac{1}{2}$. Conversely, I give a condition when this is not the case; then, intuitively, determinate and informative outcomes are not possible under the best response to *any* strategy since the value of information will just be lower if the election is less close to being tied. It turns out that the key statistic is the elasticity of the cost function for low levels of precision $x \approx 0$. To see why, in the first step, I show the following result.

Lemma 13 *Take a sequence of strategies* $(\sigma_n)_{n\in\mathbb{N}}$ *with* $\delta_\alpha \in \mathbb{R}$ *and suppose that* $W(L,\hat{p}) \neq W(C,\hat{p})$ *for* $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n,n)$*. Let* $\hat{\sigma_n}$ *be a best reponse to* σ_n*.*

1. *If* $\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$*, then*

$$\lim_{n\to\infty}\frac{q(\alpha;\hat{\sigma_n}) - q(\beta;\hat{\sigma_n})}{s(\alpha;\sigma_n)} = \infty. \tag{143}$$

2. *If* $\lim_{x\to 0}\frac{c'(x)x}{c(x)} < 3$*, then*

$$\lim_{n\to\infty}\frac{q(\alpha;\hat{\sigma_n}) - q(\beta;\hat{\sigma_n})}{s(\alpha;\sigma_n)} = 0. \tag{144}$$

The proof is at the end of this section.

How do the information acquisition and expected vote shares translate into election outcomes? Take a strategy $(\sigma_n)_{n\in\mathbb{N}}$ with $\delta_\alpha \in \mathbb{R}$ and let $d > 3$. Given Lemma 13, it might be that the expected vote shares are arbitrarily many standard deviations away from the majority threshold under the best response, i.e. $\delta_\omega((\hat{\sigma}_n)_{n\in\mathbb{N}}) \in \{\infty, -\infty\}$ for all $\omega \in \{\alpha,\beta\}$. The following result characterizes the distribution of the election outcomes in state ω as $n\to\infty$ for any given sequence of strategies as a function of $\delta_\omega \in \mathbb{R}\cup\{\infty,-\infty\}$. The result implies that outcomes are determinate in ω as $n\to\infty$ when $\delta_\omega \in \{\infty,-\infty\}$.

Lemma 14 *Consider any sequence of strategies* $(\sigma_n)_{n\in\mathbb{N}}$ *and any state* $\omega \in \{\alpha,\beta\}$*. The probability that* A *gets elected in* ω *converges to*

$$\lim_{n\to\infty}\Pr(A|\omega_i;\sigma_n) = \Phi(\delta_\omega),$$

where $\Phi(\cdot)$ *is the cumulative distribution of the standard normal distribution.*

The proof relies on an application of the central limit theorem and is provided Section B.2.2 of the Appendix. Figure 7 illustrates Lemma 14.

B.2.1 Proof of Lemma 13

Recall that $d = \lim_{x \to 0} \frac{c'(x)x}{c(x)}$ (see Lemma 11). What will matter is if the mean square of the precision of the critical types, i.e. $\mathrm{E}_k\left[x(y,k)^2 | y = \hat{p}\right]$, is of an order larger or smaller than the pivotal likelihood. Consider the first-order condition, (19),

$$\begin{aligned} c'(x^*(t;\sigma_n,n)) &= \Pr(\text{piv}|\sigma_n,n)\mathrm{E}_\omega(|t_\omega||\text{piv};\sigma_n,n). \\ \Rightarrow x^*(t;\sigma_n,n)^2 &\approx \left[\Pr(\text{piv}|\sigma_n,n)\frac{\mathrm{E}_\omega(|t_\omega||\text{piv};\sigma_n,n)}{d}\right]^{\frac{2}{d-1}}. \end{aligned} \tag{145}$$

We see that the critical elasticity is $d = 3$. For $d > 3$, the squared precision is of an order larger than the likelihood of the pivotal event, $\lim_{n\to\infty} \frac{x^*(t;\sigma_n,n)^2}{\Pr(\text{piv}|\sigma_n,n)} = \infty$. Let $d > 3$ in the following. Then, given (64) and the observation just made, the likelihood-weighted average precision of a voter group is of an order larger than the likelihood of the pivotal event, $\lim_{n\to\infty} \frac{\int_{t\in g} x(t)dH(t)}{\Pr(\text{piv}|\sigma_n,n)} = \infty$. Take any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$ with $\delta_\alpha \in \mathbb{R}$. Then, the likelihood of the pivotal event in α is of the order of the standard deviation of the vote share, $s(\alpha;\sigma_n)$, given Lemma 12. I conclude that the likelihood-weighted average precision of each voter group is as large as arbitrarily many standard deviations of the vote share in α, as $n \to \infty$, i.e. $\lim_{n\to\infty} \frac{\int_{t\in g} x(t)dH(t)}{s(\alpha;\sigma_n)} = \infty$. Hence, the first item of the lemma follows from (38), Lemma 2 and the assumption that $W(L,\hat{p}) \neq W(C,\hat{p})$. The proof of the second item is analogous.

B.2.2 Proof of Lemma 14

Proof. Let $q_n = q(\omega,\sigma_n)$. By using the normal approximation[46]

[46] For this normal approximation, we cannot rely on the standard central limit theorem, because q_n varies with n. Recall that for any undominated strategy, types t with $t_\alpha > 0, t_\beta > 0$ vote A and types t with $t_\alpha < 0, t_\beta < 0$ vote B. Hence, since the type distribution has a strictly positive density, there exists $\epsilon > 0$ such that $\epsilon < q_n < 1 - \epsilon$ for all $n \in \mathbb{N}$. As a consequence, we can apply the Lindeberg-Feller central limit theorem (see Billingsley (2008), Theorem 27.2). To see why, one checks that a sufficient condition for the the Lindeberg condition is that $(2n+1)q_n(1-q_n) \to \infty$ as $n \to \infty$ since this implies that for n sufficiently large the indicator function in the condition takes the value zero.

$$\mathcal{B}(2n+1, q_n) \simeq \mathcal{N}((2n+1)q_n, (2n+1)q_n(1-q_n)),$$

we see that the probability that A wins the election in ω converges to

$$\Phi(\frac{\frac{1}{2}(2n+1) - (2n+1)\cdot q_n}{((2n+1)q_n(1-q_n))^{\frac{1}{2}}}).$$

Taking limits $n \to \infty$, gives

$$\begin{aligned}
& \lim_{n\to\infty} \Phi(\frac{\frac{1}{2}(2n+1) - (2n+1)\cdot q_n}{(2n+1)q_n(1-q_n))^{\frac{1}{2}}}) \\
= & \lim_{n\to\infty} \Phi(\frac{(2n+1)\frac{1}{2} - (2n+1)(\frac{1}{2} + (q_n - \frac{1}{2}))}{((2n+1)^{\frac{1}{2}}(q_n(1-q_n))^{\frac{1}{2}}}) \\
= & \lim_{n\to\infty} \Phi((q_n - \frac{1}{2})\Big[\frac{(2n+1)}{q_n(1-q_n)}\Big]^{\frac{1}{2}}) \\
= & \Phi(\delta_\omega),
\end{aligned}$$

where the equalities on the last two lines hold both when $\delta_\omega \in \{\infty, -\infty\}$ and when $\delta_\omega \in \mathbb{R}$. For the equality on the last line, I used that the standard deviation of the vote share is given by $s(\omega;\sigma_n) = (\frac{q_n(1-q_n)}{2n+1}^{\frac{1}{2}})$. ■

In the next section, I prove Lemma 1, using the previous lemmas 13 and 14. Thereby, I show the existence of informative equilibrium sequences with both determinate and different outcomes in the states, given $\lim_{n\to\infty} \frac{c'(x)x}{c(x)} > 3$.

B.3 Fixed Point Argument

Proof of Theorem 1. First, I simplify the problem of finding an informative equilibrium sequence further to a problem in one dimension.

For this, I define a constrained best response $\hat{q}(\omega, \sigma^{\mathbf{q}})$ as follows. Let $\hat{p} \in [0,1]$ be the minimal belief for which $\Phi(\hat{p}) = \frac{1}{2}$ and $\Phi'(p^*) > 0$.[47] Consider the case when $\Pr(\alpha) < \hat{p}$ and $W(L,\hat{p}) < W(C,\hat{p})$. Take $\delta > 0$ small enough

[47] Recall that $\Phi(0) < \frac{1}{2}$ and $\Phi(1) > \frac{1}{2}$ by assumption and that Φ is continuously differentiable and not constant on any open interval. This implies the existence of $\hat{p}$.

such that Φ is strictly increasing on $[\hat{p}-\delta, \hat{p}+\delta]$ and $\Pr(\alpha) < \hat{p}-\delta$. Now, if $\Pr(\alpha|\text{piv}; \mathbf{q}) \in [\hat{p}-\delta, \hat{p}+\delta]$, then let $\hat{q}(\omega, \sigma^{\mathbf{q}}) = q(\omega, \sigma^{\mathbf{q}})$. If $\Pr(\alpha|\text{piv}; \mathbf{q}) < \hat{p}-\delta$, then let $\hat{q}(\omega, \sigma^{\mathbf{q}})$ be the vote share of the best response to any strategy σ' with induced prior $\Pr(\alpha|\text{piv}; \sigma', n) = \hat{p}-\delta$ and for which the likelihood of the pivotal event is $\Pr(\text{piv}|\mathbf{q}, n)$. Conversely, if $\Pr(\alpha|\text{piv}; \mathbf{q}) > \hat{p}+\delta$, then let $\hat{q}(\omega, \sigma^{\mathbf{q}})$ be the vote share of the best response to any strategy σ' with induced prior $\Pr(\alpha|\text{piv}; \sigma', n) = \Phi(\hat{p}+\delta)$ and for which the likelihood of the pivotal event is $\Pr(\text{piv}|\mathbf{q}, n)$. The constrained best response is a 'truncation' of $q(\omega; \sigma^{\mathbf{q}})$ and therefore continuous in $\mathbf{q}$.

Step 1 *For any $\epsilon > 0$ small enough, any $\frac{1}{2} - \frac{\epsilon}{2} \leq q(\alpha) \leq \frac{1}{2}$, and any n large enough, there is $q_n^*(\beta) \geq \frac{1}{2}$ such that*

$$q(\alpha) = \hat{q}(\alpha; \sigma^{(q(\alpha), q_n^*(\beta))}) \tag{146}$$

$$= q(\alpha; \sigma^{(q(\alpha), q_n^*(\beta))}). \tag{147}$$

and $q_n^*(\beta)$ is continuous in $q(\alpha)$.

Take any $q(\alpha) \in [\frac{1}{2}, \frac{1}{2} - \frac{\epsilon}{2}]$. Let $\mathbf{q} = (q(\alpha), q(\beta))$ in the following.

Step 1.1 *If $q(\beta) = \frac{1}{2} + \epsilon$, then, for ϵ small enough and n large enough,*

$$\hat{q}(\alpha; \sigma^{\mathbf{q}}) > q(\alpha). \tag{148}$$

The election is more close to being tied in α, and, by Lemma 7, voters become convinced that the state is α, that is $\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \mathbf{q}, n) = 1$. It follows from the definition of $\hat{q}(\omega; \sigma^{\mathbf{q}})$ and Lemma 3 that $\lim_{n\to\infty} \hat{q}(\alpha; \sigma^{\mathbf{q}}) = \Phi(\hat{p}+\delta)$. Finally, (148) follows when ϵ is small enough, since $\Phi(\hat{p}+\delta) > \frac{1}{2}$.

Step 1.2 *If $q(\beta) = \frac{1}{2}$, then for ϵ small enough and any n,*

$$\hat{q}(\alpha; \sigma^{\mathbf{q}}) < q(\alpha). \tag{149}$$

The election is more close to being tied in β, and, by Lemma 6, voters update towards β, that is $\Pr(\alpha|\text{piv}; \mathbf{q}, n) \leq \Pr(\alpha)$. Since $\Pr(\alpha) < \hat{p}-\delta$, it follows from the definition of $\hat{q}(\alpha;\sigma^{\mathbf{q}})$ and Lemma 3 that $\lim_{n\to\infty}\hat{q}(\alpha;\sigma^{\mathbf{q}}) = \Phi(\hat{p}-\delta)$. Finally, (149) follows for $\epsilon > 0$ small enough since $\Phi(\hat{p}-\delta) < \frac{1}{2}$.

Since $\hat{q}(\alpha;\sigma^{\mathbf{q}})$ is continuous in $q(\beta)$, it follows from Step 1.1, Step 1.2, and the intermediate value theorem that, for n large enough, there is $q^*(\beta)$ such that (146) holds. It follows from the implicit function theorem that $q^*(\beta)$ is continuous. Now, suppose that $\Pr(\alpha|\text{piv};\sigma^{(q(\alpha),q^*_n(\beta))},n) \notin [\hat{p}-\delta,\hat{p}+\delta]$. Then, Lemma 3 together with the definition of $\hat{q}(\alpha;\sigma^{\mathbf{q}})$ implies that $\lim_{n\to\infty}\hat{q}(\alpha;\sigma^{\mathbf{q}}) \in \{\Phi(\hat{p}-\delta),\Phi(\hat{p}+\delta)\}$. This contradicts with (146) since $\Phi(\hat{p}-\delta) < q(\alpha) < \Phi(\hat{p}+\delta)$ for $\epsilon > 0$ small enough. Hence, $\lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma^{(q(\alpha),q^*(\beta))},n) \in [\hat{p}-\delta,\hat{p}+\delta]$ and therefore, given the definition of the "truncation" $\hat{q}$, (146) implies (147).

In what follows, I show that, for any n large enough, there is a vote share $q^*_n(\alpha)$ such that $\mathbf{q}^*_n = (q^*_n(\alpha), q^*(\beta))$ is a fixed point of $\mathbf{q}(\sigma^\cdot)$, thereby constructing equilibria $\sigma^{\mathbf{q}^*_n}$ of the voting game. Given (146), it is sufficient to show the following.

Step 2 *For any n large enough, there is $q^*_n(\alpha)$ such that*

$$q^*_n(\beta) = q(\beta;\sigma^{(q^*_n(\alpha),q^*_n(\beta))}). \tag{150}$$

I consider sequences of vote shares $(\mathbf{q}_n)_{n\in\mathbb{N}}$ with $\mathbf{q}_n = (q(\alpha)_n, q^*(\beta)_n)$.

Step 2.1 *If $\lim_{n\to\infty}(\frac{1}{2}-q(\alpha)_n)s(\alpha;\mathbf{q}_n) \in \mathbb{R}^{\geq 0}$, then, for n large enough,*

$$q^*(\beta)_n < q(\beta;\sigma^{\mathbf{q}_n}). \tag{151}$$

Recall that $\lim_{x\to 0}\frac{c'(x)x}{c(x)} = d$ (see Lemma 11). Now, (147) together with the assumption $\lim_{n\to\infty}(\frac{1}{2}-q(\alpha)_n)s(\alpha;\mathbf{q}_n) \in \mathbb{R}^{\geq 0}$ implies

$$\delta(\alpha)((\sigma^{\mathbf{q}_n})_{n\in\mathbb{N}}) = \lim_{n\to\infty}(\frac{1}{2}-q(\alpha;\sigma^{\mathbf{q}_n}))s(\alpha;\mathbf{q}_n) \in \mathbb{R}^{\geq 0}. \tag{152}$$

Then, Lemma 3 implies $\lim_{n\to\infty}\Pr(\alpha|\text{piv};\mathbf{q}_n) \in \Phi^{-1}(\frac{1}{2})$. Given the definition of the "truncated" best response $\hat{\sigma}$,

$$\lim_{n\to\infty} \Pr(\alpha|\text{piv};\mathbf{q}_n) = \hat{p}. \tag{153}$$

Since I consider the case where $W(L,\hat{p}) < W(C,\hat{p})$, it follows from Lemma 2 and (38) that $q(\alpha;\sigma^{\mathbf{q}_n}) < q(\beta;\sigma^{\mathbf{q}_n})$ for n large enough. Given (152), Lemma 13 implies that

$$\lim_{n\to\infty} \left[q(\beta;\sigma^{\mathbf{q}_n}) - \frac{1}{2}\right]|s(\alpha;\mathbf{q}_n) = \infty. \tag{154}$$

Given (153) and since $\hat{p} \in (0,1)$, the inference from the pivotal event is bounded as $n \to \infty$, given $(\mathbf{q}_n)_{n\in\mathbb{N}} = (q(\alpha)_n, q^*_n(\beta))_{n\in\mathbb{N}}$. Therefore, Lemma 12 implies that

$$(q^*(\beta) - \frac{1}{2})s(\beta;\mathbf{q}_n) \in \mathbb{R} \tag{155}$$

since otherwise $\lim_{n\to\infty} \frac{\Pr(\alpha|\text{piv};\mathbf{q})}{\Pr(\alpha|\text{piv};\mathbf{q})} = \frac{\Phi(\delta_\alpha((\sigma^{\mathbf{q}_n})_{n\in\mathbb{N}}))}{\Phi(\delta_\beta((\sigma^{\mathbf{q}_n})_{n\in\mathbb{N}}))} = 0$. Note that $\lim_{n\to\infty} \frac{s(\alpha;\mathbf{q}_n)}{s(\beta;\mathbf{q}_n)} = 1$.[48] Therefore, (154) and (155) imply (151) for n large enough.

Step 2.2 *If $q_n(\alpha) = \frac{1}{2} - \frac{\epsilon}{2}$, then, for n large enough,*

$$q^*_n(\beta) > q(\beta;\sigma^{\mathbf{q}_n}). \tag{156}$$

Lemma 3 together with (146) implies that $\lim_{n\to\infty} q(\beta;\sigma^{\mathbf{q}_n}) = \frac{1}{2} - \epsilon$. Since $q^*_n(\beta) \geq \frac{1}{2}$, clearly, (156) holds for n large enough.

Finally, since $q(\beta;\sigma^{\mathbf{q}_n})$ is continuous in $q(\alpha)$, an application of the intermediate value theorem shows that, for all n large enough, there is $q^*(\alpha) < \frac{1}{2}$ for which (150) holds. This finishes the proof of Step 2.2. The corresponding strategies $\sigma^*_n = \sigma^{(q^*_n(\alpha),q^*(\beta))}$ form an equilibrium sequence .

It remains to show that the election chooses the outcome preferred by the contrarians in each state with probability converging to 1, given the equilibrium

[48] The standard deviation of the vote share in ω is $(\frac{q(\omega))(1-q(\omega)_n}{n})^{\frac{1}{2}}$. Note that $\lim_{n\to\infty} q(\alpha)_n - q(\beta)_n = 0$, which implies that the ratio of the standard deviations converges to 1 as $n \to \infty$.

sequence $(\sigma_n^*)_{n\in\mathbb{N}}$. Since $q(\alpha;\sigma_n^*) \leq \frac{1}{2} \leq q(\beta;\sigma_n^*)$ by construction, Lemma 12 implies that it remains to show that $|\delta_\omega| = \infty$ for all states ω.

Suppose that $\delta_\omega \in \mathbb{R}$ for some $\omega \in \{\alpha, \beta\}$. I claim that this implies that $\delta_\omega \in \mathbb{R}$ for all states. Otherwise, Lemma 14 implies that the inference from the pivotal event is not bounded, that is $\lim_{n\to\infty} \frac{\Pr(\alpha|\text{piv};\sigma_n^*)}{\Pr(\alpha|\text{piv};\sigma_n^*)} = \frac{\Phi(\delta_\alpha)}{\Phi(\delta_\beta)} \in \{0, 1\}$. But then $\lim_{n\to\infty} q(\omega;\sigma_n^*) \in \{\Phi(0), \Phi(1)\}$ and $\delta_\omega = \infty$ for all states since $\Phi(0) < \frac{1}{2}$ and $\Phi(1) > \frac{1}{2}$. Second, when $\delta_\alpha \in \mathbb{R}$, Lemma 13 implies that there is a state ω for which $|\delta_\omega| = \infty$. However, this contradicts with the observation that $\delta_\omega \in \mathbb{R}$ for all states. Consequently, it must be that $|\delta_\omega| = \infty$ for all states ω. This finishes the proof of Theorem 1 for the case when $\Pr(\alpha) < \hat{p}$ and $W(L,\hat{p}) < W(C,\hat{p})$. The proof of the other cases is analogous.

Note that in the same way that I constructed an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ with $q(\alpha;\sigma_n^*) \leq \frac{1}{2} \leq q(\beta;\sigma_n^*)$ in the case when $\hat{p} > \Pr(\alpha)$ and $W(L,\hat{p}) < W(C,\hat{p})$, I can construct an equilibrium sequence with $\frac{1}{2} \leq q(\alpha;\sigma_n^*) \leq q(\beta;\sigma_n^*)$ for n large enough in this case; essentially since the likelihood of the pivotal event as a function of the vote shares $\mathbf{q}$ is symmetric around $\frac{1}{2}$. The proof is verbatim and only the relevant inequality signs need to be flipped. Conversely, if $\hat{p} > \Pr(\alpha)$ and $W(L,\hat{p}) > W(C,\hat{p})$, I can construct an equilibrium sequence with $q(\alpha;\sigma_n^*) \leq q(\beta;\sigma_n^*) \leq \frac{1}{2}$ for n large enough. By symmetry of the other cases, I conclude:

Theorem 9 *Let* $\lim_{x\to 0} \frac{c'(x)x}{c(x)} > 3$. *Take any preference distribution H satisfying (28). There is an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfying (92).*

C Determinants of Elections

C.1 Proof of Lemma 8

Proof. Recall that $\lim_{x\to 0} \frac{c'(x)x}{c(x)} = d$ (Lemma 5). The basic intuition for Lemma 8 is that the that power index $W(g,\hat{p})$ of the group g is proportional to $\mathrm{E}_H(k^{\frac{2}{d-1}})$. Thus, since $k^{\frac{2}{d-1}}$ is strictly concave, given $d > 3$, an application of Jensen's inequality shows that for any g-intensity spread H' of H and $g \neq$

$g' \in \{L, C\}$,

$$\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g) < \mathrm{E}_H(k^{\frac{2}{d-1}}|t \in g), \tag{157}$$

$$\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g') = \mathrm{E}_H(k^{\frac{2}{d-1}}|t \in g'). \tag{158}$$

First, note that by the definition of a g-intensity spread, $\Pr_{H'}(L) = \Pr_H(L)$ and $\Pr_{H'}(C) = \Pr_H(C)$; see (83)-(84). Second, $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n) = \lim_{n\to\infty} \Pr(\alpha|\text{piv}; \hat{\sigma_n}, n)$ and (84) together imply $f_H^g(\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n)) = f_{H'}^g(\lim_{n\to\infty} \Pr(\alpha|\text{piv}; \hat{\sigma_n}, n))$. Third, since H and H' satisfy (31), $\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g) = \mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g, y(t) = \hat{p})$ and $\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g') = \mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g', y(t) = \hat{p})$. These observations together with (157) and (158) imply (86), given the definition (27) of the power indices. ■

C.2 Proof of Theorem 4

Consider the case $g = L$. Given Lemma 2 and Lemma 1, it remains to show that for any H there is a L-intensity spread such that $\frac{W_{H'}(L,\hat{p})}{W_{H'}(C,\hat{p})} < 1$ across all equilibrium sequences. Note that $W_H(C\hat{p}) = W_{H'}(C, \hat{p})$ for any L-intensity spread H' of H. Given that H satsfies (28), it holds $\Pr(t \in C) > 0$, and the continuity of $f(\cdot|t \in g)$ and $\mathrm{E}(k(t)^{\frac{2}{d-1}|t \in g, y(t)=p}$ in p implies that $f(\cdot|t \in g)$ and $\mathrm{E}(k(t)^{\frac{2}{d-1}|t \in g, y(t)=p}$ in p are bounded below by a strictly positive constant. In particular, $W(C, p)$ is bounded below by a positive constant. Thus, it suffices to show that for any H and for any $\epsilon > 0$, there is an L-intensity spread H' of H such that for all equilibrium sequences $(\sigma_n^*)_{n\in\mathbb{N}}$ given H',

$$\mathrm{E}_{H'}(k^{\frac{2}{d-1}}|t \in g) < \epsilon. \tag{159}$$

For this, consider L-intensity spreads $H'(\kappa)$ of H such that the conditional distributions $J_{H'(\kappa)}^L$ of the intensities k of the types of group g are concentrated in neighbourhoods around 0 and $K > 0$, that is $\Pr_{J_{H'(\kappa)}^L}(\kappa \leq k \leq \kappa + \delta) + \Pr_{J_{H'(\kappa)}^L}(0 \leq k \leq \epsilon) \geq 1 - \delta$. Since the mean of the intensities is the same under the L-intensity spread, the iterated law of expectation implies

$\lim_{\delta\to 0}\Pr_{J^L_{H'(\kappa)}}(\kappa \le k \le \kappa+\delta)\kappa = E_{J^L_H}(k)$. Hence,

$$\begin{aligned}\lim_{\delta\to 0}\mathrm{E}_{J^L_{H'(\kappa)}}(k^{\frac{2}{d-1}}) &= \lim_{\delta\to 0}\Pr_{J^L_{H'(\kappa)}}(\kappa \le k \le \kappa+\delta)\kappa^{\frac{2}{d-1}} \\ &= \frac{E_{J^L_H}(k)}{\kappa}\kappa^{\frac{2}{d-1}} \overset{\kappa\to\infty}{\to} 0, \qquad (160)\end{aligned}$$

where I used that $d = \lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$ and hence $\frac{2}{d-1} < 1$. This finishes the proof of the theorem for the case $g = L$. The case for $g = C$ is analogous.

D Characterization of All Equilibria

D.1 Proof of Theorem 6

Case $\lim_{x\to 0}\frac{c'(x)x}{c(x)} > 3$**.**

I show that any equilibrium sequence satisfies either (29), (92) or (88).

Take any equilibrium sequence $(\sigma^*_n)_{n\in\mathbb{N}}$. It follows from Lemma 10 that $\hat{p} = \lim_{n\to\infty}\Pr(\alpha|\text{piv};\sigma^*_n) \in \Phi^{-1}(\frac{1}{2}) \cup \{\Pr(\alpha)\}$. Given Lemma 2, the order of the vote shares is given by the order of $W(L,\hat{p})$ and $W(C,\hat{p})$. Consider the case when

$$W(L,\hat{p}) < W(C,\hat{p}) \qquad (161)$$

such that there are the following two cases:

Case 1 *For any n large enough, $\frac{1}{2} \le q(\alpha;\sigma^*_n) \le q(\beta;\sigma^*_n)$.*

Suppose that $\delta(\alpha) \in \mathbb{R}$. Then, Lemma 13 implies that $\delta(\alpha) = \infty$, a contradiction. Hence $\delta(\alpha) = \delta(\beta) = \infty$. Lemma 14 implies that the equilibrium sequence satisfies $\lim_{n\to\infty}\Pr(A|\alpha;\sigma^*_n,n) = \lim_{n\to\infty}\Pr(B|\alpha;\sigma^*_n,n) = 1$. Suppose that $\Phi(\Pr(\alpha|\text{piv};\sigma_n,n)) < \frac{1}{2}$. Then, the equilibrium sequence satisfies (92). Suppose that $\Phi(\Pr(\alpha|\text{piv};\sigma_n,n)) > \frac{1}{2}$. Then, the equilibrium sequence satisfies (88).

Case 2 *For any n large enough, $q(\alpha;\sigma^*_n) \le \frac{1}{2} \le q(\beta;\sigma^*_n)$.*

Suppose that $\delta(\alpha) \in \mathbb{R}$. First, note that this implies $\delta(\beta) \in \mathbb{R}$. To see why, note that otherwise Lemma 12 implies that $\lim_{n\to\infty} \frac{\Pr(\alpha|\text{piv};\sigma_n^*,n)}{\Pr(\alpha|\text{piv};\sigma_n^*,n)} = \frac{\Phi(\delta_\alpha)}{\Phi(\delta_\beta)} = 1$. Then, Lemma 3 implies that $\lim_{n\to\infty} q(\alpha;\sigma_n^*) = \Phi(1)$. This yields a contradiction to $q(\alpha;\sigma_n^*) \leq \frac{1}{2}$ since $\Phi(1) > \frac{1}{2}$ by assumption. Second, given $\delta(\alpha) \in \mathbb{R}$, Lemma 13 implies that $\delta(\alpha) \in \{\infty, -\infty\}$ or $\delta(\beta) \in \{\infty, -\infty\}$. Clearly, this contradicts the first observation that $\delta(\omega) \in \mathbb{R}$ for any $\omega \in \{\alpha, \beta\}$. In the same way, the assumption $\delta(\beta) \in \mathbb{R}$ leads to a contradiction. Consequently, $-\delta(\alpha) = \delta(\beta) = \infty$ and Lemma 14 implies that the equilibrium sequence satisfies (29).

This finishes the proof for the case when $W(L,\hat{p}) < W(C,\hat{p})$. The proof for the case when $W(L,\hat{p}) > W(C,\hat{p})$ is analogous.

Case $\lim_{x\to 0} \frac{c'(x)x}{c(x)} < 3$**.**

Take any equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$. First, recall Lemma 10, which implies that $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*,n) \in \Phi^{-1}(\frac{1}{2}) \cup \{\Pr(\alpha)\}$.

Case 1 $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*,n) = \Pr(\alpha)$

Lemma 3 together with $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$ and the law of large numbers imply that $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfies (88).

Case 2 $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*,n) \in \Phi^{-1}(\frac{1}{2})$

Step 1 *Let* $\delta_{\omega,n} = (q(\omega;\sigma_n^*) - \frac{1}{2})s(\omega;\sigma_n^*)^{-1}$ *for* $\omega \in \{\alpha,\beta\}$*. Then,*

$$\lim_{n\to\infty} \delta_{\alpha,n} - \delta_{\beta,n} = 0. \tag{162}$$

Note that $s(\omega;\sigma_n^*) = (\frac{q(\omega;\sigma_n^*)(1-q(\omega;\sigma_n^*)}{2n+1})^{\frac{1}{2}}$ such that $\hat{p} \in \Phi^{-1}(\frac{1}{2})$ together with Lemma 3 implies $\lim_{n\to\infty} \frac{s(\alpha;\sigma_n^*)}{s(\beta;\sigma_n^*)} = 1$. Note that Lemma 13 provides an upper bound for the limit of the difference of the vote shares measured in standard deviations since it considers the case when the vote share in one or both of the states is arbitrarily close to $\frac{1}{2}$. Consequently, Lemma 13 implies (162).

Step 2 $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) = \Pr(\alpha)$.

Consider the ratio of the likelihoods of the pivotal event in the two states,

$$
\begin{aligned}
& \frac{\Pr(\text{piv}|\alpha;\sigma_n^*,n)}{\Pr(\text{piv}|\beta;\sigma_n^*,n)} \\
= \ & \Big[\frac{q(\alpha;\sigma_n^*)(1-q(\alpha;\sigma_n^*)}{q(\beta;\sigma_n^*)(1-q(\beta;\sigma_n^*)}\Big]^n. \\
= \ & \Big[1-\frac{(q(\alpha;\sigma_n^*)-\frac{1}{2})^2-(q(\beta;\sigma_n^*)-\frac{1}{2})^2}{(q(\beta;\sigma_n^*)-\frac{1}{2})^2-\frac{1}{4}}\Big]^n \\
= \ & \Big[1-\frac{1}{2n+1}(\frac{q(\alpha;\sigma_n^*)(1-q(\alpha;\sigma_n^*))}{(q(\beta;\sigma_n^*)-\frac{1}{2})^2-\frac{1}{4}}\delta_{\alpha,n}^2-\frac{q(\beta;\sigma_n^*)(1-q(\beta;\sigma_n^*))}{(q(\beta;\sigma_n^*)-\frac{1}{2})^2-\frac{1}{4}}\delta_{\beta,n}^2)\Big]^n.
\end{aligned}
$$

Let

$$
x_n=(\frac{q(\alpha;\sigma_n^*)(1-q(\alpha;\sigma_n^*))}{(q(\beta;\sigma_n^*)-\frac{1}{2})^2-\frac{1}{4}}\delta_{\alpha,n}^2-\frac{q(\beta;\sigma_n^*)(1-q(\beta;\sigma_n^*))}{(q(\beta;\sigma_n^*)-\frac{1}{2})^2-\frac{1}{4}}\delta_{\beta,n}^2. \tag{163}
$$

The likelihood ratio simplifies to

$$
\frac{\Pr(\text{piv}|\alpha;\sigma_n^*,n)}{\Pr(\text{piv}|\beta;\sigma_n^*,n)}=\Big[1-\frac{1}{2n+1}x_n\Big]^n. \tag{164}
$$

I rewrite,

$$
\frac{\Pr(\text{piv}|\alpha;\sigma_n^*,n)}{\Pr(\text{piv}|\beta;\sigma_n^*,n)}=(\Big[1-\frac{1}{2n+1}x_n\Big]^n-e^{-\frac{1}{2}x_n})+e^{-\frac{1}{2}x_n} \tag{165}
$$

and analyse the two summands separately in the following. First, note that Lemma 3 together with $\hat{p}\in\Phi^{-1}(\frac{1}{2})$ implies $\lim_{n\to\infty}q(\omega;\sigma_n^*)=\frac{1}{2}$ for $\omega\in\{\alpha,\beta\}$. Thus, Step 1 and (163) together imply

$$
\lim_{n\to\infty}x_n=0. \tag{166}
$$

This yields

$$
\lim_{n\to\infty}e^{-\frac{1}{2}x_n}=1, \tag{167}
$$

Second, using the Lemmas 4.3 and 4.3 in Durrett (1991) [p.94], for all $n \in \mathbb{N}$,

$$|(1 - \frac{x_n}{(2n+1)})^n - e^{-x_n}| \leq \frac{x_n^2}{(2n+1)^3} \tag{168}$$

Finally, it follows from (165) - (168) that

$$\lim_{n\to\infty} \frac{\Pr(\text{piv}|\alpha;\sigma_n^*, n)}{\Pr(\text{piv}|\beta;\sigma_n^*, n)} = 1, \tag{169}$$

which was to be shown.

Now, the result of Step 2 is a contradiction to $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) \in \Phi^{-1}(\frac{1}{2})$ since $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$ by assumption. Consequently, for all equilibrium sequences $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) = \Pr(\alpha)$ (Case 1). It follows from the law of large numbers that all equilibrium sequences satisfy (88).

D.2 Proof of Observation 1

Take any informative equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$. Let $\hat{q}(\omega;\sigma_n^*)$ be the likelihood that a randomly drawn uninformed type votes A in ω; I call $\hat{q}(\omega;\sigma_n^*)$ the *vote share of the uninformed*, for $\omega \in \{\alpha, \beta\}$,

$$\hat{q}(\omega;\sigma_n^*) = \int_t \mu(t,a)dH(t|x(t) = 0). \tag{170}$$

Let $\tilde{q}(\omega;\sigma_n^*)$ be the likelihood that a randomly drawn uninformed type votes A in ω; $\tilde{q}(\omega;\sigma_n^*)$ is the *vote share of the informed*,

$$\begin{aligned}
\tilde{q}(\alpha;\sigma_n^*) &= \int_t (\frac{1}{2} + x(t))\mu(t,a) + (\frac{1}{2} - x(t))\mu(t,b)dH(t|x(t) > 0) \\
&= \frac{1}{2} + \int_{t\in L} x(t)dH(t) - \int_{t\in C} x(t)dH(t), \qquad (171) \\
\tilde{q}(\beta;\sigma_n^*) &= \int_t (\frac{1}{2} - x(t))\mu(t,a) + (\frac{1}{2} + x(t))\mu(t,b)dH(t|x(t) > 0) \\
&= \frac{1}{2} - \int_{t\in L} x(t)dH(t) + \int_{t\in C} x(t)dH(t). \qquad (172)
\end{aligned}$$

Now, first, Theorem 6 implies that $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfies (29). Therefore, $q(\alpha;\sigma_n^*) < \frac{1}{2} < q(\beta;\sigma_n^*)$ for all n large enough or $q(\beta;\sigma_n^*) < \frac{1}{2} < q(\alpha;\sigma_n^*)$ for all n large enough. Second, since the precision of the signals of the voters is symmetric across the states, Claim 17 implies that the vote shares in the two states α and β of the informed voters are symmetric to $\frac{1}{2}$. Third, suppose that $\lim_{n\to\infty} |\hat{q}(\omega;\sigma_n^*) - \frac{1}{2})|n^{-\frac{1}{2}} = \infty$, and note that $\hat{q}(\alpha;\sigma_n^*) = \hat{q}(\beta;\sigma_n^*)$ for all n, given (170). Calculations similar to (163) - (168) show that $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) \in \{0,1\}$, given the symmetry of the vote share of the informed. Hence, Lemma 3) implies that $\lim_{n\to\infty} q(\omega;\sigma_n^*) \in \{\Phi(0),\Phi(1)\}$. This yields a contradiction with the earlier observation that $q(\alpha;\sigma_n^*) < \frac{1}{2} < q(\beta;\sigma_n^*)$ for all n large enough or $q(\beta;\sigma_n^*) < \frac{1}{2} < q(\alpha;\sigma_n^*)$ for all n large enough. I conclude that $\lim_{n\to\infty} |\hat{q}(\omega;\sigma_n^*) - \frac{1}{2})|n^{-\frac{1}{2}} \in \mathbb{R}$. Finally, Observation 1 follows from Lemma 14 and $s(\omega;\sigma_n^*) = (\frac{q(\omega;\sigma_n^*)(1-q(\omega;\sigma_n^*)}{2n+1})^{\frac{1}{2}}$.

E Extensions

E.1 Proof of Lemma 9

Given Lemma 10, it is sufficient to consider two cases.

Case 1 $\lim_{n\to\infty} \Pr(\alpha|\text{piv};\sigma_n^*) = \Pr(\alpha)$.

Since $\Phi(\Pr(\alpha)) \neq \frac{1}{2}$, (14) implies that the likelhood of the pivotal event is exponentially small,

$$\Pr(\text{piv}|\alpha;\sigma_n^*) < z^n c_1 \tag{173}$$

for some constant $c_1 \in \mathbb{R}$, some $0 < z < 1$ and for n large enough. This translates into the information acquisition of the voters. For all t,

$$x^*(t) \approx \Pr(\text{piv}|\sigma_n^*,n)^{\frac{1}{d-1}} \Big[\Pr(\alpha|\text{piv};\sigma_n^*,n)t_\alpha + \Pr(\beta|\text{piv};\sigma_n^*,n)t_\beta \Big] < z^n c_2 \tag{174}$$

for some constant $c_2 \in \mathbb{R}$, where I used the first-order condition (107) and that $\lim_{x\to\infty} \frac{c'(x)}{x^{d-1}} \in \mathbb{R}$ for the approximation. The approximation (174) implies that the sum of the voters cost converges to zero since the cost function is approximately polynomial when $x \approx 0$. This finishes the proof of Case 1.

Case 1 $\hat{p} = \lim_{n\to\infty} \Pr(\alpha|\text{piv}; \sigma_n^*, n) \in \Phi^{-1}(\frac{1}{2})$.

Fix a group $g \in \{\ell, s\}$. Any type $t \in g$ can be uniquely described by the total intensity and the threshold of doubt; see Section 2.1.7. For the ease of exposition, let $\alpha = \arg\max\left(|q(\alpha;\sigma_n^*) - \frac{1}{2}|, |q(\alpha;\sigma_n^*) - \frac{1}{2}|\right)$. It follows from Bayes' rule that

$$\lim_{n\to\infty} \frac{\Pr(\text{piv}|\alpha;\sigma_n^*)}{\Pr(\text{piv}|\beta;\sigma_n^*)} = \frac{\Pr(\beta)}{\Pr(\alpha)} \frac{\hat{p}}{1-\hat{p}}. \tag{175}$$

Multiplication of the first-order condition (107) by $n^{\frac{1}{2}}$ and using (175) yields

$$n^{\frac{1}{2}} c'(x^*(\Pr(\alpha|\text{piv};\sigma_n^*, n), 1)) = n^{\frac{1}{2}} \Pr(\alpha) \Pr(\text{piv}|\alpha;\sigma_n^*) \left[t_\alpha - t_\beta \frac{\Pr(\text{piv}|\alpha;\sigma_n^*)}{\Pr(\text{piv}|\beta;\sigma_n^*)}\right] \tag{176}$$

Note that

$$\begin{aligned} \forall q \in (0,1) : 4^n (q(1-q))^n &= 4^n \left[(\frac{1}{2} - (\frac{1}{2} - q))(\frac{1}{2} + (\frac{1}{2} - q)\right]^n \\ &= 4^n (\frac{1}{4} - (\frac{1}{2} - q)^2)^n \\ &= (1 - 4\frac{(n^{\frac{1}{2}}(\frac{1}{2} - q))^2}{n})^n. \end{aligned} \tag{177}$$

Combining (14) with (176) and (177) gives

$$n^{\frac{1}{2}} c'(x^*(\Pr(\alpha|\text{piv};\sigma_n^*, n), 1)) \approx c_2 (1 - 4\frac{(n^{\frac{1}{2}}(\frac{1}{2} - q(\alpha;\sigma_n^*))^2}{n})^n$$

for some constant $c_2 > 0$. Multiplication of both sides with $\delta_n = n^{\frac{1}{2}} |q(\alpha;\sigma_n^*) - \frac{1}{2}|$ yields

$$\delta_n n^{\frac{1}{2}} c'(x^*(\Pr(\alpha|\text{piv};\sigma_n^*, n), 1)) \approx c_3 \delta_n e^{-4\delta_n^2} + \delta_n \left[(1 - 4\frac{\delta_n^2}{n})^n - e^{-4\delta_n^2}\right]. \tag{178}$$

for some constant $c_3 > 0$. Using Lemmas 4.3 and 4.3 in Durrett (1991),

$$(1 - 4\frac{\delta_n^2}{n})^n - e^{-4\delta_n^2} \leq \frac{16\delta_n^4}{n^3}. \tag{179}$$

Therefore, $\lim_{n\to\infty} \delta_n \Big[(1 - 4\frac{\delta_n^2}{n})^n - e^{4\delta_n}\Big] = 0$. It follows from the characterization of all equilibrium sequences in Theorem 6 and (28) that $\hat{p} \in \Phi^{-1}(\frac{1}{2})$ implies $\lim_{n\to\infty} \delta_n = \infty$, which in turn implies $\lim_{n\to\infty} \delta_n e^{-4\delta_n} = 0$. I conclude that

$$\lim_{n\to\infty} \delta_n n^{\frac{1}{2}} c'(x^*(\Pr(\alpha|\text{piv}; \sigma_n^*, n), 1)) = 0. \tag{180}$$

Recall (38) which states that

$$q(\alpha; \sigma_n^*) - q(\beta; \sigma_n^*) = 2\Big[\int_{t\in\ell} x(t)dH(t) - \int_{t\in s} x(t)dH(t)\Big]. \tag{181}$$

The genericity assumption (28) together with $\hat{p} \in \Phi^{-1}(\frac{1}{2})$ implies $W(L, \hat{p}) \neq W(C, \hat{p})$. I combine Step 5, (181) and $W(L, \hat{p}) \neq W(C, \hat{p})$ to obtain

$$x^*(p^*, 1)^2 \approx c_4\Big[q(\alpha; \sigma_n^*) - q(\beta; \sigma_n^*)\Big]$$

for some constant $c_4 \neq 0$. Then,

$$\begin{aligned} x^*(p^*, 1)^2 &\leq c_4\Big[|q(\alpha; \sigma_n^*) - \frac{1}{2}| + |q(\beta; \sigma_n^*) - \frac{1}{2}|\Big] \\ &\leq 2c_4 \frac{\delta_n}{n^{-\frac{1}{2}}}, \end{aligned} \tag{182}$$

where I used the triangle equality on the first line. Hence, (180) implies

$$\lim_{n\to\infty} n x^*(\Pr(\alpha|\text{piv}; \sigma_n^*, n), 1)^2 c'(x^*(\Pr(\alpha|\text{piv}; \sigma_n^*, n), 1)) = 0 \tag{183}$$

Using Lemma 5,

$$\lim_{n\to 0} \frac{x^2 c'(x)}{x c(x)} = d. \tag{184}$$

Recall that $x^*(\Pr(\alpha|\text{piv};\sigma_n^*,n),1)$ converges to zero as $n\to\infty$, such that combining (183) and (184) gives

$$\lim_{n\to\infty} nx^*(\Pr(\alpha|\text{piv};\sigma_n^*,n),1)c(x^*(\Pr(\alpha|\text{piv};\sigma_n^*,n),1)) = 0. \tag{185}$$

I claim that any equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ satisfies

$$\int_{t\in g} c(x(t))dH(t) \approx \frac{2(d-1)}{d}\frac{c(x^*(p^*,1))x^*(p^*,1)}{\chi'(p^*)}\Pr(g)f(p^*|t\in g)\mathrm{E}(k(t)^{\frac{2}{d-1}}|t\in g, y(t)=\hat{p}) \tag{186}$$

The proof of (186) follows from previous arguments, analogous to the ones in Step 5 of the Proof of Observation 2. The proof is verbatim with the required changes in notation. Finally, the lemma follows from (185) and (186).

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