A Non-Parametric Elicitation of Probability Weights

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November 9, 2019

In this note, we propose a method to elicit probability weights as in Prospect Theory (Kahneman, 1979), based on the assumptions of time separability and symmetric weighting function. This method has two features: first, it does not require functional form assumptions on the utility function, nor its elicitation; second, it is compatible with both monetary and non-monetary incentives. Requiring only few measurements, it is also easy to implement.

Set up the Preferences

Denote with (x, p, t) a lottery that pays out a positive reward x with probability p at period t, and 0 in any other period.[1] Throughout this paper we assume a preference relation $\succsim$ over elementary lotteries (x, p, t) that obeys Prospect Theory (Kahneman, 1979) and time-separability. Specifically, $\succsim$ can be represented by three functions: a strictly increasing *weighting function* $w : [0, 1] \to [0, 1]$ which satisfies symmetry $w(1 - p) = 1 - w(p)$; a continuous value function $u : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$; and a discounting function $d : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$. We normalize $d(0) = 1$, $w(1) = 1$ and

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[1] A time period here can have any unit such as day, week, month etc.

$u(0) = 0$.[2] Then

$$\begin{aligned} (x, p, t) &\succsim (a, q, s) \\ \Leftrightarrow d(t)w(p)u(x) &\geq d(s)w(q)u(a). \end{aligned} \tag{1}$$

The Elicitation Method

We propose a two-step method to elicit the weighting function $w(p)$ without eliciting the utility function $u(x)$ nor the discounting function $d(t)$.

Step 1 For each subject, first we elicit the period $\bar{t}$ at which the subject is indifferent between a lottery that pays off a positive $\bar{x}$ with probability $\frac{1}{2}$ at each period from time 0 to $\bar{t}$ and $\bar{x}$ with certainty at each period from period $\bar{t}+1$ to T.[3] That is,

$$\begin{aligned} \sum_{k=0}^{\bar{t}} (\bar{x}, \frac{1}{2}, k) &\simeq \sum_{i=\bar{t}+1}^{T} (\bar{x}, 1, i) \\ \Leftrightarrow \quad \sum_{k=0}^{\bar{t}} d(k)w(\frac{1}{2})u(\bar{x}) &= \sum_{i=\bar{t}+1}^{T} d(i)w(1)u(\bar{x}) \\ \Leftrightarrow \quad \frac{1}{2} &= \frac{\sum_{i=\bar{t}+1}^{T} d(i)}{\sum_{k=0}^{\bar{t}} d(k)} \end{aligned} \tag{2}$$

, where we used that $d(0) = 1$ and $w(1) = 1$ given the normalization, and that $w(\frac{1}{2}) = \frac{1}{2}$ given symmetric w. Define

$$D(t) = \frac{\sum_{i=t+1}^{T} d(i)}{\sum_{k=0}^{t} d(k)} \tag{3}$$

The purpose of step 1 is to pin down period $\bar{t}$, for which the subject's $D(\bar{t})$ as defined in equation (3) is $\frac{1}{2}$.

[2] Note that the normalizations are without loss of generality. In particular, we allow for positive background consumption utility $\tilde{u}(0) > 0$: for any $\tilde{u}$ with $\tilde{u} > 0$, the value function $u(x) = \tilde{u}(x) - \tilde{u}(0)$ represents the same preferences $\succsim$ and satisfies $u(0) = 0$.

[3] Note that $\bar{x}$ can be non-monetary rewards. The only requirement is that for each individual subject the same reward is used for all lotteries in the elicitation. The valuation of this reward does not need to be homogeneous for all subjects.

Step 2 Let $p_0 = \frac{1}{2}$. Now we fix $\bar{t}$ elicited in step 1 and elicit a sequence of points on the weighting function w by iteration: $p_1, \ldots, p_n$ such that for all $j = 1, \ldots, n$, the subject is indifferent between receiving $\bar{x}$ with probability p_j from time 0 to period $\bar{t}$, and receiving $\bar{x}$ with probability p_{j-1} from period $\bar{t}+1$ to the end period T. That is,

$$\begin{aligned}
\sum_{k=0}^{\bar{t}} (\bar{x}, p_j, k) &\simeq \sum_{i=\bar{t}+1}^{T} (\bar{x}, p_{j-1}, i) \\
\sum_{k=0}^{\bar{t}} d(k) w(p_j) u(\bar{x}) &= \sum_{i=\bar{t}+1}^{T} d(i) w(p_{j-1}) u(\bar{x}) \\
\Rightarrow \quad w(p_j) &= \frac{\sum_{i=\bar{t}+1}^{T} d(i)}{\sum_{k=0}^{\bar{t}} d(k)} \cdot w(p_{j-1}) \\
&= D(\bar{t}) \cdot w(p_{j-1}) \\
&= \frac{1}{2} w(p_{j-1}).
\end{aligned}$$

By induction and $w(p_0) = w(\frac{1}{2}) = \frac{1}{2}$, the weights of $p_1, \ldots, p_n$ must satisfy

$$w(p_i) = \frac{1}{2^{i+1}}.$$

Now we have obtained $n+1$ points on the weighting function $w(p)$. By symmetry, we can obtain another n points: $w(1-p_i) = 1 - \frac{1}{2^{i+1}}$ for $i = 1, \ldots, n$.

In practice, depending on the need of the research question, the user of this method can extrapolate the weighting function $w(p)$, by fitting a polynomial to these $2n+1$ points elicited in step 2.

References

Kahneman, Daniel. 1979. "Prospect theory: An analysis of decisions under risk." *Econometrica* 47:278.