

1. Prove the operation star is closed under recognizable languages:

For a recognizable language L , we can create a nondeterministic turing machine M that recognizes L^* .

We will non-deterministically cut input string w into parts w_1, w_2, \dots, w_n and run w_i on M . Since the input is finite we can try every possible cut. If M halts or rejects for any i , then reject. Otherwise, if for every i , M accepts w_i then accept.

If there is a way to partition w into strings such that w_1, w_2, \dots, w_n where $w_i \in L$ then there will be a branch that M will accept.

Therefore L^* is closed under recognizable languages.