CS 254: Computability and Complexity	**Anonymous submission**
Problem Set #06	*October 24, 2019*

1. Show that a 2-stack PDA is equivalent in power to a Turing Machine.

   Big idea: We can have the 2-stack PDA act similar to the TM by using the first stack to represent the portion of the tape to the left of the TM head and the second stack to represent the tape to the right of the head.

   Proof:

   A PDA with two stacks is as powerful as a Turing Machine

   The PDA will start by pushing $ onto each stack. Then read through the entire input and push it to the left stack.

   The PDA can simulate the TM by popping symbols from the left stack and pushing them onto the right stack to simulate moving to the right, or popping from the right stack and pushing to the left stack to simulate moving to the left. Otherwise we can have the same states that tell us what to change the current input to and which way to move. Since The two stack PDA can fully simulate the TM then we know it is as powerful as the TM

   A turing machine is as powerful as a PDA with two stacks:

   a PDA with two stacks is equivalent to a TM with three tapes. The first tape reads input, the second tape and third tape keep track of the two stacks. As we move along the input we ”pop” the input from one of second or third tapes and ”push” it onto the opposite. Therefore a 3-tape TM can fully simulate a 2-stack PDA.

   Since we know a 3-tape TM is equivalent to a standard TM we know that the two stack PDA is not move powerful than a standard TM.

   Since we have shown that a two stack is no less or more powerful than a TM it must be the case that it is as powerful as a standard TM.