

5. Let  $EQ_{DFA} = \{(A, B) \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$  describe a TM that decides  $EQ_{DFA}$  by testing A and B on all strings up to a certain length.

Algorithm:

Start by calculating the pumping length P for A and B. We can do this by counting the number of states in their DFA. Since we set our pumping length  $p =$  to the number of states, we know any string with a length longer than that has a repeated substring according to the pigeon hole theorem.

Choose the longer of the two pumping lengths p. Use a for loop  $i = 0$  to p. Go through every possible combination using i steps in the two DFAs. If we find a path that ends in an accept state write the path to the tape.

Since p is finite and the amount of states in the two DFAs is finite our TM is capable of performing this task.

Finally, simulate all the accept strings that were found on both DFA A and DFA B. If every single string is accepted, then accept. If any string is rejected, reject.

Proof: The pumping length p is all we need as it generates all strings that don't have repeated substrings, as shown by the pigeonhole theorem. We know that  $p =$  the number of states of which ever DFA has more states. We then can enumerate to find all the strings with the TM as p is finite and the two DFA's have a finite amount of states. Lastly since A and B are inputs we we can simulate A and B on any string that we wrote down on the tape.