CS 254: Computability and Complexity	**Anonymous submission**
Problem Set #08	*November 8, 2019*

5. Definition for O($2^{f(n)}$) and $2^{O(f(n))}$.

O($2^{f(n)}$):
b(x) $\in$ O($2^{f(n)}$) = b(x) $\leq$ c * $2^{f(n)}$ for all n > n_0

$2^{O(f(n))}$:
d(x) $\in$ $2^{O(f(n))}$
$2^{k(x)}$ where k(x) $\in$ O(f(n)).
So k(x) $\leq$ c * f(n) Therefore $2^{k(x)} \leq 2^{c*f(n)}$
Thus d(x) $\leq 2^{c*f(n)}$ for all n > n_0
.
Prove that that is O($2^{f(n)}$) is a proper subset of $2^{O(f(n))}$:
First we will prove that it is a subset.
we know: $a^{bc} = (a^b)^c$
$2^{2x} = (2^2)^x = 4^x$
We know that O(2^x) $\neq$ O(3^x) $\neq$ O(4^x).
c * $2^{f(n)} \leq 2^c$ * $2^{(f(n)}$ We know that we can cross out $2^{(f(n))}$ on both sides.
this leaves us with c $\leq 2^c$, and we know this is true for any c. Therefore O($2^{f(n)}$) $\subset 2^{O(f(n))}$

Now we need to prove that it is a proper subset:
Intuition: c * $2^{f(n)} \leq$ g $\leq 2^c * f(n)$
c * $2^x \leq$ g $\leq 2^{c*x}$
so c * $2^x \leq 2.01^x \leq 2^{c*x}$
as x $\rightarrow \infty$ then there's a c that makes the above true.
Therefore c * $2^{f(n)} \leq 2.01^{f(n)} \leq 2^c * f(n)$
Since g $\in$ O(2.01^x) but g $\notin$ O(2^x)
Since g $\notin$ O($2^{f(n)}$) meaning there are elements in $2^{O(f(n))}$ but not in O($2^{f(n)}$) makes it a proper subset.
Therefore we have proven that O($2^{f(n)}$) $\subseteq 2^{O(f(n))}$