

5. A. There are two cases $B = \epsilon$ and $B = y$. y being some set of strings

The quotient rule for strings states that you take the original string and remove the portion that is on the denominator side. Therefore

Case one: $B = \epsilon$ $A = xy$

This is quite simple as removing ϵ would leave xy therefore $A/B = xy$

Case two: $B = y$ $A = xy$

$A/B = x$ as that is the only part not in B .

B. To illustrate the case where $(A/B)B = A$ Let $A = x^+y^+$ and $B = y^+$

$$(A/B) = x^+y^+ / y^+ = x^+$$

Now we must concatenate y^+ back on

$$(x^+)y^+ = x^+y^+ \text{ By string concatenation}$$

Therefore $(A/B)B = A$ in this example

To illustrate the case where $(A/B)B \neq A$ Let $A = x^+$ and $B = y^+$

$$(A/B) = x^+ \text{ as we cannot remove anything with } B$$

Now we must concatenate y^+ onto A leaving us with:

$$x^+y^+ \text{ Therefore } (A/B)B \neq A$$

C. We know that there must be a DFA for language A . By the quotient rule, we know that $A/B = w$ where $A = wx$ and $B = x$. This will take off a portion of the strings from the back end of them. Knowing this, this problem is reverse to problem 3. In this problem we will simply remove states that dealt with the x portion of A . To do this start at q_0 and see if a string from language B works. If so, we need to remove the states involved. Try this on every state from q_0 to q_n

DFA for A/B :

$Q = Q \subseteq Q_a$ excluding states that accept B

$\Sigma = \text{Alphabet from } A$

$\delta = \text{Transitions will be the same except for to and from states that were removed}$

$q_0 = q_0$ from A

$F = \text{Any state that was a start state when testing language } B$