5. Claim: $(x^R)^R = x$

   Base cases:

   $x = \epsilon$. This holds true as the reverse of nothing is still nothing, and the reverse of that is still nothing

   x = ax' where x' = ϵ. This holds true as the reverse of one item is still that one and the reverse of that is again that one item $a = a^R = (a^R)^R$

   Proof to help inductive hypothesis (proof provided by Anna Rafferty)

   $(xy)^R = (ax'y)^R$ becasue x = ax'

   LHS = $(a(x'y))^R$ associative prop. of string concatenation

   LHS = $(x'y)^R a$ definiton of reverse of a(x'y)

   LHS = $y^R(x')^R a)$

   LHS = $y^R(ax')^R$ associative prop. of string concatenation

   LHS = $y^R x^R$

   Inductive hypothesis:

   $(x^R)^R = (ax')^{R\ R}$

   Since we know that $(xy)^R = y^R x^R$ we can assume:

   $((ax')^R)^R = ((x')^R(a)^R)^R$

   $((x')^R(a)^R)^R = ((a)^R)^R((x')^R)^R$

   $((a)^R)^R((x')^R)^R = (a)^R\ ((x')^R)^R$

   $(a)^R\ ((x')^R)^R = a\ ((x')^R)^R$

   Thus, we have proven for all strings $(x^R)^R = x$ when x = ax'