CS 254: Computability and Complexity	**Anonymous submission**
Problem Set #10	*November 19, 2019*

1. Let DOUBLE-SAT = $\{\langle\emptyset\ \rangle|\ \emptyset$ has at least two satisfying assignments$\}$ Show DOUBLE-SAT is NP-complete

   Proof:

   Show that DOUBLE-SAT is NP:

   On input $\emptyset$ a nondeterministic polynomial machine can guess two different assignments, and accept if $\emptyset$ returns true for both.

   Show that DOUBLE-SAT is NP-complete by mapping SAT $\leq_p$ DOUBLE-SAT.

   TM M = "On input $\langle\emptyset\ \rangle$where $\langle\emptyset\ \rangle$is the encoding of a boolean formula.

   1. Create a new variable x.
   2. Let $\emptyset_2 = \emptyset \wedge$ (x $\vee$ $\bar{x}$)
   3. Output $\emptyset_2$

   Validation: If $\emptyset \in$ SAT, then $\emptyset$ has a least 1 satisfying assignment. Due to this we know that $\emptyset_2$ has two satisfying assignments:

   The assignment that satisfied $\emptyset \wedge$ x=True

   The assignment that satisfied $\emptyset \wedge$ x=False

   Therefore If $\emptyset \in$ SAT, then $\emptyset_2 \in$ DOUBLE-SAT.

   Otherwise $\emptyset \notin$ SAT, then $\emptyset$ has no satisfying assignment, therefore $\emptyset \wedge$ (x $\vee$ $\bar{x}$) will always evaluate to false, because false and (anything) is always false.

   Therefore If $\emptyset \notin$ SAT, then $\emptyset_2 \notin$ DOUBLE-SAT.

   Thus, SAT $\leq_p$ DOUBLE-SAT and DOUBLE-SAT is NP-complete.