

1. Let  $C_{CFG} = \{(G,k) \mid G \text{ is a CFG and } L(G) \text{ contains exactly } k \text{ strings where } k \geq 0 \text{ or } k = \infty\}$  show that  $C_{CFG}$  is decidable.

We can use a machine  $M$  to check whether the language  $L(G)$  is an infinite or finite set:

- $L(G)$  is infinite and  $k = \infty$ , accept
- $L(G)$  is infinite and  $k \neq \infty$ , reject
- $L(G)$  is finite and  $k = \infty$ , reject
- $L(G)$  is finite and  $k \neq \infty$ , continue

calculate the pumping length  $p$  for grammar  $G$

set count = 0

use for loop  $i = 0$  to  $p$

Use the loop to find all the strings whose length =  $i$  for every  $i$ . Increment count for every string found.

After for loop, check, if count =  $k$ , accept. Else reject.

$M$  discovers if  $L(G)$  is an infinite set or not and matches  $k$ . After that if  $L(G)$  is not infinite and  $k \neq \infty$ . Then to be able to prove that  $C_{CFG}$  is decidable we need to prove the size of  $L(G)$  is  $k$ . We do this by looping and keeping track of all the strings that can be generated by grammar  $G$ . Since the grammar is finite, we cannot generate strings longer than  $p$ . Lastly we check the value of  $k$  against our increment variable count. Thus  $C_{CFG}$  is decidable.