4. Firstly, to show that book-shelf $\in$ NP we can show that we would non-deterministically choose books to include or not include on the shelf and then check the edges between them. Obviously this would be polynomial and checking every node against every other is O(n^2 which is in poly time.

   Show that book-shelf is NP-complete I will show IndSet $\leq_p$ Book-shelf. Since IndSet is np-complete showing the reductions would show that Book-shelf is also np-complete. If we do not connect any of the books together but only connect the tags together and the books to their respective tags, then if we just ran the IndSet on that graph we should get the correct solution.

   Validation: Since we have proven Book-shelf $\in$ NP and $\leq_p$ Book-shelf then Book-shelf must be NP-complete. The reason this mapping works is because we correctly find a subset of k books where no books belong to two different tags.