CS 254: Computability and Complexity **Anonymous submission**
Problem Set #04 *October 8, 2019*

1. A.
We want to find a compliment to language L where L $a^n b^n$ | n $\geq$ 0
Notice the complement of this is simply where n $\neq$ m or (a $\cup$ b)*ba(a $\cup$ b)*
To do this we will construct two separate CFGs then combine them to create one:

n $\neq$ m
S $\rightarrow$ aSb | X | Y
X $\rightarrow$ aX | a
Y $\rightarrow$ Yb | b

(a $\cup$ b)*ba(a $\cup$ b)*
S $\rightarrow$ ZbaZ | Z
Z $\rightarrow$ aZ | bZ | ϵ

CFG complement of L:
S $\rightarrow$ ZbaZ | aSb | X | Y | Z
X $\rightarrow$ aX | a
Y $\rightarrow$ Yb | b
Z $\rightarrow$ aZ | bZ | ϵ
Therefore there is a CFL

B.
language L x_1#x_2#...#x_k | k $\geq$ 1, for each $x_i \in$ a, b^*, and some i and j, $x_i = x_j^i$
For a string s $\in$ L it must have contain some $x_i = x_j^i$ or have some palindrome where $x_i = x_j^i$ where it is the same string.
To do this we will construct two separate CFGs then combine them to create one:

$x_i = x_j^i$
S $\rightarrow$ X#Y | Y#X | Y
Y $\rightarrow$ aYa | bYb | a | b | ϵ
X $\rightarrow$ a | b | # | ϵ

palindrome
S $\rightarrow$ Z | X#Z | Z#X
Z $\rightarrow$ aZa | bZb | #X#
X $\rightarrow$ a | b | # | ϵ

Now we must combine the two:
S $\rightarrow$ S_1 | S_2
S_1 $\rightarrow$ X#Y | Y#X | Y
Y $\rightarrow$ aYa | bYb | a | b | ϵ
S_2 $\rightarrow$ Z | X#Z | Z#X
Z $\rightarrow$ aZa | bZb | #X#
X $\rightarrow$ a | b | # | ϵ
Therefore there is a CFL