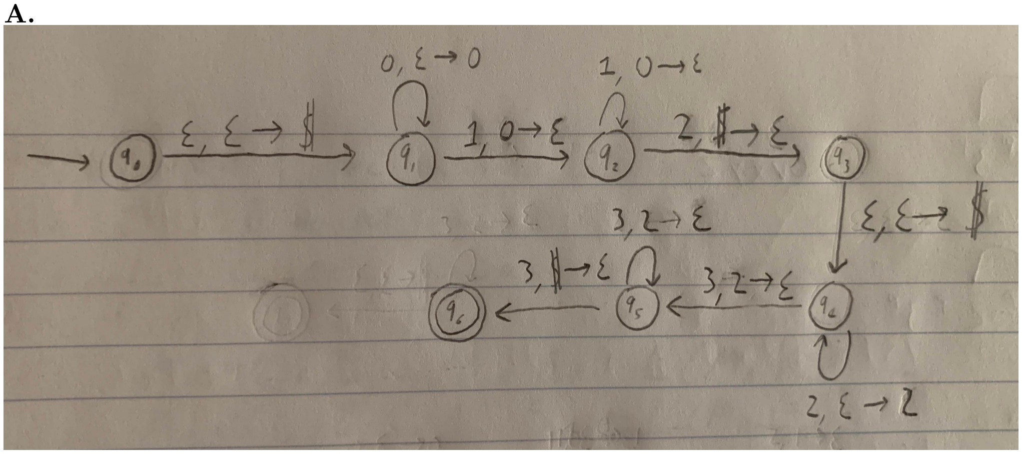
CS 254: Computability and Complexity | **Anonymous submission**
Problem Set #04 | *October 8, 2019*

2. **A.**



B.

Proof by contradiction. Assume that language L $0^n1^m2^p3^q$ is a context free language. By the pumping lemma we should be able to chose and string s ∈ L and split it into s = uvxyz and satisfy these three conditions:
i. for each i ≥ 0, uv^{i}xy^{i}z ∈ L,
ii. |vy| > 0, and
iii. |vxy| ≤ p
To prove that there is a contradiction I will set string s equal to $0^p1^p2^p3^p$.
There are a few cases:

The case where u = ϵ: then vxy = 0^p or some fraction of $0^{p/x}$ but it cannot be greater than that due to the iii condition. We clearly see that this will not work as if we set i = 2 the number of 0s ≠ the number of 2's.

The case where u = 0^p: then vxy = 1^p or some fraction of $1^{p/x}$ but it cannot be greater than that due to the iii condition. We also know that |vy| > 0 Therefore they must be some portion of 1^p. Again this case is trivial when pumping and setting i = 2 we see the number of 1s ≠ the number of 3s. Note, we could set u = 0^p 1^p and then vxy = 2^p and we could keep extending u but we would result in the same conclusion.

The case where u = $0^{p/2}$: then vxy = $0^{p/2}1^{p/2}$ This violates case 1 as it states for each i ≥ 0, uv^{i}xy^{i}z ∈ L, if we set i to two then we would have 0101 which ∉ L.

Since any placement of uvxyz does not follow the pumping lemma conditions this is a contradiction and have proven that this is not a CFL.

C.

Proof by contradiction. Assume that language L $0^n1^m2^p3^q$ is a context free language. By the pumping lemma we should be able to chose and string s ∈ L and split it into s = uvxyz and satisfy these three conditions:
i. for each i *geq* 0, uv^{i}xy^{i}z ∈ L,
ii. |vy| > 0, and
iii. |vxy| ≤ p
To prove that there is a contradiction I will set string s equal to $0^p1^p2^p3$.
There are a few cases:

vxy = the same number for example vxy = 0^p then when we set i = 0 we will have a greater number of 0s than 1's, therefore we break condition one. The same argument works for if vxy = 1^p or vxy = 2^p. It will still also work if v or y is set to a variable and the other is *epsilon* i.e. v=0^p and y = ϵ as there will still be an inequality.

We could split the numbers across two different ones i.e. v = $0^{p/3} x = 1^{p/3} y = 1^{p/3}$ and u = $0^{p/3}$ z = $1^{p/3} 2^p 3$ In this situation if we pump with i = 2 we will get more 1s than 2s therefore be break condition 1. This same argument holds if we set u =0^p and shift vxy over any amount.

Since any placement of uvxyz does not follow the pumping lemma conditions this is a contradiction and have proven that this is not a CFL.

D.