CS 254: Computability and Complexity	**Anonymous submission**
Problem Set #01	*October 17, 2019*

1. **A.** Show that if L is regular, then scramble(L) is context free Proof by description of a PDA: We will call the DFA for L M

   PDA for scramble(L) is P

   P has a very similar set of states to M. The difference is that P will non-deterministically work through to find which permutation of the scramble that is in L

   PDA P starts by putting an ϵ on the stack

   Then begins reading inputs:

   you non-deterministacally choose where to go:
   if we read in symbol x:
   put x into the DFA and push x onto the stack
   If the height of the stack is ≥ 1: There are two cases:

   Case 1: The input symbol matches the symbol on top of the stack. In this case we read the symbol into the DFA

   Case 2: The input symbol does not match the top of the stack. In this case we push the complement of the input symbol onto the stack

   At the end of the input, if the stack is empty and you are in an accept state then P has read the correct permutation such that scramble(L) $\in$ L

   Argument: This is correct because with the PDA we are given the ability to use non-determinism. Meaning we will create lots of different branches in the PDA but all that matters is that if one is successful we have found the correct permutation.

   B. With three or more symbols Scramble(L) is not context free.

   If we have regular language(L) = $(123)^*$ intersect with the regular language 1^* 2^* 3^* the language contains $1^n 2^n$ 3^n which is not context free.

   Proof by contradiction:

   Assume $1^n 2^n$ 3^n is C.F. we choose the string s = $1^p 2^p$ 3^p Since this string is in L and has length $\geq$ p we know we can pump string s.

   To pass the pumping lemma we must follow the three conditions:
   i. $xy^i z$ in L for every $i \geq 0$

   ii.$|y| > 0$

   iii. $|xy| \leq$ p

   if we set p = 3 our string is aaa bbb ccc

   Case 1: vxy contains all a's or all b's or all c's
   If we pump $uv^i xyuv^i z$ where i = 2, condition 1 states that this must exist in L
   As we can see if we pump where vxy = aaa then string s will be aaa aaa bbb ccc which $\notin$ L which violates the first condition.
   Note: this is the same if vxy were set to a single a or if they were set to b's or c's.

   Case 2: vxy contains a's and b's or b's and c's If we pump $uv^i xyuv^i z$ where i = 2, condition 1 states that this must exist in L
   As we can see if we pump where vxy = a b then string s would be

aa ab ab bbb ccc which $\notin$ L which violates the first condition.
Note: this is the same if vxy were set to a b's and c's.

Case 3: Due to the propery that $|xy| \leq p$ we know vxy cannot contains a's b's and c's.

This is a contradiction. Therefore L cannot be context free as proven by the pumping lemma.