

1. Prove that there is a decider for N for A/B

Proof by contradiction:

Lets assume we can build a decider N

We learned from problem set 2 that if $B = \emptyset$ then $A/B = \emptyset$

Therefore our decider N will be able to decide E_{TM} where $E_{TM} = \{M \mid M \text{ is a TM and } L(M) = \emptyset\}$.

N on input D, M :

1. output C
2. run C on any string
 - i. If C accepts, reject M
 - ii. If C rejects, accept M

If $M \notin E_{TM}$ then N can decide M as $L(M) = \Sigma^*$ because it can halt on $M \neq \emptyset$

However if $M \in E_{TM}$ then N can decide E_{TM} which we know is undecidable by theorem 5.2. This means our assumption about N must be wrong and that there is a contradiction! Therefore A/B is undecidable