

6. Claim: $\#1(vw) = \#1(v) + \#1(w)$ proved by structural induction

Base Cases: $v = \epsilon$ and $w = \epsilon$. From identity property of strings we know $\epsilon + \epsilon = \epsilon$ and $\#1(\epsilon) = \#1(\epsilon) + \#1(\epsilon)$ which is the same as $0=0$

v has elements and $w = \epsilon$. From the additive property of strings we know $\epsilon + v = v$ which is the same as ϵv and $\#1(\epsilon v)$ is the same as $\#1(v)$

vis versa w has elements and $v = \epsilon$. From the additive property of strings we know $\epsilon + w = w$ which is the same as ϵw which is the same as ϵw and $\#1(\epsilon w)$ is the same as $\#1(w)$

Inductive hypothesis: $v = 1 + \epsilon$

From string additive property we know $w + 1 + \epsilon = 1w\epsilon$

Therefore a recursively defined function counting ones will treat vw and $v + w$ the same.