

3. Show that CFLs are closed under union, concatenation, and star:

CFLs are closed under union as  $\forall L_1$  and  $L_2 \in \text{a CFL}, L_1 \cup L_2$

If we take and combine these two languages we can create a new CFG such that

$S \rightarrow S_1 \mid S_2$

This will create a string from  $S_1$ ,  $S_2$ , or both  $S_1$  and  $S_2$  which is the definition of union

CFLs are closed under concatenation as  $\forall L_1$  and  $L_2 \in \text{a CFL}, \{s_1 s_2 \mid s_1 \in L_1 \text{ and } s_2 \in L_2\}$

$S \rightarrow S_1 S_2$

which will create a string from  $L_1$  and then  $L_2$  which is the definition of concatenation

CFLs are closed under star as  $\forall L \in \text{a CFL}, L^* \in \text{a CFL}$ :

$S \rightarrow S_1 S \mid \epsilon$

This will generate any string from  $L$  but also can create no string which is the definition of star.