

6. Want to show that the language:

$$D = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w\}$$

Proof: by contradiction

Assume D is a regular language with a pumping length p . To prove that D is a regular language under the pumping lemma we will use string s to prove that it holds for three cases at the same time:

i. xy^iz in L for every $i \geq 0$

ii. $|y| > 0$

iii. $|xy| \leq p$

$$s = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^p \begin{bmatrix} 0 \\ 1 \end{bmatrix}^p$$

Under the definition of the pumping lemma we know we can split s into xy^iz and because $|s| \geq p$ we know s can be pumped. To prove that s is regular there exists a way to write $s = xy^iz$ s.t. it follows the 3 conditions of pumping lemma:

since s can be broken up:

$$\text{We can set } x = \epsilon \quad y = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^p \quad \text{and } z = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^p$$

We consider the case where xy^2z

we get: $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \notin D$ but this is a contradiction of case i which states that i. xy^iz in L for every $i \geq 0$. Therefore by the pumping lemma, D is not a regular language