

3. Base Case: $x = \epsilon$ then $xy = \epsilon y = y$ by the identity property. Since $y \in A$ it is regular
inductive step: $x = ax'$

Assume that $x' \circ A$ is regular

want to prove that $ax' \circ A$ is regular

Since $x'y$ is a regular expression meaning there is an NFA that accepts $x'y$ and only $x'y$. We can add a stage(s) to the front of the preexisting NFA to accept a and only a . We know that this is possible since $a \in \Sigma$

To do this we add a stage that would accept the a and only a . If a is accepted, let it lead into q_0 of the NFA for $x'A$.

This will work because we can make a machine that will accept a and only a and then connect that machine to our preexisting machine. If a is not accepted there is no where for it to go therefore it will die resulting in a reject.

Since $x = ax'$ and we have proven to be able to concatenate x with y where y is any string in A we know $x \circ A$ is regular