5. NECESSARY$_{CFG}$ is a CFG where A is a variable that is necessary

   Show that NECESSARY$_{CFG}$ is recognizable:

   Proof by Construction:

   TM D on input G,A where G is a CFG and A is necessary

   1. create grammar X where x = G/A

   for i=1,2,3,...

   generate every string of length i from G

   If any string from set $\notin$ X, accept

   else, loop

   Since all we need to do is find a string that is in G but not in X we just keep iterating until we eventually find a string that is not in X but is in G and accept, or we keep looping. Therefore NECESSARY$_{CFG}$ is recognizable.

   Prove that NECESSARY$_{CFG}$ is not decidable:

   Intuition: This can't be decidable because we can never really know if something is necessary because what if it isn't in the next string?

   Using Rice's theorem

   NECESSARY$_{CFG}$ is a language of TM descriptions. It satisfies the two conditions of Rice's Theorem. One, it is non-trivial, as some TMs will have a necessary component A, while others will not. Two, it depends only on the language. If two TM's were to recognize the same language, they would both have descriptions in NECESSARY$_{CFG}$ or neither would. Therefore, by Rice's theorem, we know NECESSARY$_{CFG}$ is un-decidable.