

5. Want to show that  $\sum = a, b, c$  where  $L$  has a length of  $w$  that is 3 times the number of  $a$ 's in  $w$ .

Proof: by contradiction

Assume  $L$  is a regular language with a pumping length  $p$ . Our string  $s = a^p b^p c^p$ . Because  $|s| \geq p$  we know  $s$  can be pumped. To prove that  $s$  is regular there exists a way to write  $s = xy^iz$  s.t. it follows the 3 conditions of pumping lemma:

i.  $xy^iz$  in  $L$  for every  $i \geq 0$

ii.  $|y| > 0$

iii.  $|xy| \leq p$

Let  $p=2$ ,  $x = a$ ,  $y = a$ ,  $z = bbcc$

$s = aabbcc$  and since  $1/3$  of the letters are  $a$  this  $\in L$

However, if we let  $i = 0$  meaning  $y$  becomes  $\epsilon$  the string becomes  $xy^0z = abbcc$ . Therefore the ratio of  $a$ 's is only  $1/5 \neq 1/3$ .

This is a contradiction as it does not follow case i that state  $xy^iz \in L$  for every  $i \geq 0$ .