

5. Assume the L is a CFL. $L = \{ ww \mid w \in \{0,1\}^* \}$. By the pumping lemma we should be able to choose and string $s \in L$ and split it into $s = uvxyz$ and satisfy these three conditions:
- for each $i \geq 0$, $uv^i xy^i z \in L$,
 - $|vy| > 0$, and
 - $|vxy| \leq p$

To prove this I choose string $0^p 1 0^p 1$. It is a member of L and the length is greater than p so we know we can pump.

$$u = 0^{2p/3}, v = 0^{p/3}, x = 1, y = 0^{p/3}, z = 0^{2p/3} 1$$

If we choose $i = 3$ then we get $00010001 \in L$.

Therefore this can be pumped as it follows all the conditions and acts as a CFL.

However, if we try pumping $0^p 1^p 0^p 1^p$ we notice that it cannot be pumped:

Case one:

If the substring occurs only in the first half of s , when we pump by setting $i = 2$ we move a 1 into the first position of the second half therefore it cannot be in the form ww since w must start with a 0. The same occurs if vxy occurs in the second half of s moving a 0 into the last position of the first half so it also cannot be in the form ww .

Case two: vxy straddles the midpoint.

If vxy is straddling the midpoint and we pump down where $i = 0$ then uxz has the form $0^p 1^i 0^j 1^p$ where i and j both cannot equal p . Thus this string is not in the form ww .

Since there is no positioning of $uvxyz$ to pump the string we know L is not a CFL.