

2.  $ISO = \{\langle G, H \rangle \mid G \text{ and } H \text{ are graphs isomorphic to one another.}\}$  Show that  $ISO \in NP$ .

Lets assume there is a TM  $M$  for  $ISO$

$M =$  "on input  $\langle G, H \rangle$  where  $G$  and  $H$  are undirected graphs.

1. Nondeterministically select a permutation of nodes
2. for each pair of nodes  $x$  and  $y$  check to see if  $(x,y)$  is an edge in  $G$
3. If there is an edge, check for an edge  $(x, y)$  in  $H$ . If not reject.
4. accept.

step 1 All we are doing is selecting elements which runs in  $NP$ .

Step 2 in  $NP$  because we will do  $O(n^2)$  work checking every node against every other node.

Step 3 is based on step two, and step 3 runs in  $O(n^2)$  so it would be  $c * O(n^2) * O(n^2)$  which is still polynomial therefore  $\in NP$ .

Step 4 accepts in  $O(1)$ .

Therefore our highest order is  $\in NP$  so this construction must be  $\in NP$ .

This construction is correct because we will check that every single edge from  $G$  is in  $H$  ensuring that every node leads to the proper node in both of the graphs, and through the above proof we know we can do this is polynomial time with a nondeterminism.