CS 254: Computability and Complexity	**Anonymous submission**
Problem Set #01	*October 29, 2019*

2. **Intuition**: L_1 is r.e. and L_2 is not r.e. because there are obviously languages we can recognize that have more than 254 elements, but we cannot recognize a language to have less than 254 elements, because what if there is one we just haven't tried yet?

   WTS: $L_1 = \{M \mid |\text{L(M)}| \geq 254\}$. is r.e.

   Proof by construction:

   Build E an enumerator : (must output in canonical order)

```
1. Simulate M on tape 1
2. count = 0
3. For each string s in {canonical order}
    a. Run M on s
    b. If accept
        i. count++
        ii. if count $\geq$ 254, accept
    c. else, continue
```

   Therefore we have shown that L_1 is r.e. as we will accept if there are 254 elements or loop until we do find 254 elements.

   WTS: $L_2 = \{M \mid |\text{L(M)}| \leq 254\}$. is not r.e.

   Proof:

   First we will prove that L_2 is not decideable using Rice's Thm.

   L_2 is a language of TM descriptions. It satisfies the two conditions of Rice's Theorem. First, it is non-trivial as some TMs more than 254 elements in their languages, however not every language has more than 254 elements. Second, it is not implementation dependent, meaning it only depends on the language. If two turing machines were to recognize the same language, then both should have descriptions in L_2 or neither will. Therefore Rice's theorem implies that L_2 is un-decideable.

   We can also prove that co-L_2 is r.e. We use a similar proof to L_1's Since the complement is simply co-$L_2 = \{M \mid |\text{L(M)}| \geq 255\}$.

   proof by construction:

   Build E an enumerator : (must output in canonical order)

```
1. Simulate M on tape 1
2. count = 0
3. For each string s in {canonical order}
    a. Run M on s
    b. If accept
        i. count++
        ii. if count $\geq$ 255, accept
    c. else, continue
```

   Therefore we have shown that co-L_2 is r.e. as we will accept if there are 255 elements or loop until we do find 254 elements.

   Now, since we have proved that L_2 is both undecidable and co-L_2 is r.e., we know that L_2 must not be r.e.