

5. The language $L = \{ \text{cat enabler}^n \text{ fancier}^m \text{ o cat enabler}^p \text{ fancier}^q \mid n, m, p, q \geq 1 \text{ and } n = p \text{ and } m = q \}$

Proof by contradiction: Assume that L is a CFL we will use the pumping lemma. Since this is greater than p and string $S \in L$ we know we can pump and follow these conditions: i. for each $i \geq 0$, $uv^i xy^i z \in L$,
ii. $|vy| > 0$, and
iii. $|vxy| \leq p$

There are a few conditions for the layout of $s = uv^i xy^i z$

Case one: $u = \epsilon$, $vxy = \text{cat enabler}^p \text{ fancier}^p$, $z = \text{o cat enabler}^p \text{ fancier}^p$. If we pump S where $i = 2$ we will end up with more enablers than fanciers which breaks the condition that $n = p$ and is not in L which means we break the first case of the pumping lemma.

Case two: $u = \text{cat enabler}$, $vxy = \text{fancier}^p$, $z = \text{o cat enabler}^p \text{ fancier}^p$. Again if we pump S where $i = 2$ we will end up with more fanciers than enablers and given the condition that $m = n$ we know this is not in L and therefore breaks condition 1 of the pumping lemma.

Case three: $u = \text{cat enabler}^{p/2}$, $v = \text{enabler}^{p/2}$, $xy = \text{fancier}^{p/2}$, $z = \text{o cat enabler}^p \text{ fancier}^p$. If we pump S where $i = 2$ we will end up with $n \neq p$ and $m \neq q$ therefore it is not in L for any i and we have broken the first condition of the pumping lemma.

Through all of these cases we have shown any layout for $uvxyz$ will break a condition for the pumping lemma therefore L is not a CFL by our proof by contradiction.