4. **Define:** UNUSED_{TM} as the set of $\langle$M,q $\rangle$such that M is a tm and q is a state of M and there does not exist a string w such that M enters q while processing w.

   WTS: whether UNUSED_{TM} is decidable, r.e. but not decidable, co-r.e., but not r.e. or none of the above.

   Intuition: UNUSED_{TM} isn't r.e. or decideable, but co is r.e.

   Proof by contradiction: We can show that UNUSED_{TM} is decidable and that TM R decides it. Therefore since R solves UNUSED_{TM}, we can leverage R to check if q_{accept} is unused in E_{TM}. Intuitively we know that q_{accept} is only unused iff $L(M) = \emptyset$. We create a TM S that decides E_{TM} using R as a subroutine:

   S = on input $\langle$M $\rangle$, where M is a TM

   1. Run TM R on $\langle$M, q_{accept} $\rangle$
   2. If R accepts, accept, if R rejects, reject

   Therefore we have built a decider for E_{TM}, however this is a contradiction! We know that E_{TM} is undecidable, therefore there cannot not exist a TM that decides UNUSED_{TM}

   We can prove that the complement of UNUSED_{TM} is r.e.:

   WTS: USED_{TM} is r.e.

   Proof by construction:

   Build E an enumerator: (must output in canonical order)

   1. Simulate USED_{TM} on tape 1
   2. For each string s in canonical order
      a. Run USED_{TM} on s
      b. If we pass through state q accept.
      c. else, continue

   Since we know we can find q in a finite amount of time or it keeps looping we know that this is USED_{TM} r.e. We know then that since UNUSED_{TM} is undecidable and USED_{TM} is r.e. that UNUSED_{TM} must not be r.e.