

2. Consider the "prefix" operation over languages A and B:

A prefix B = { w | w  $\in$  A and some prefix of w  $\in$  B }

A string x is a prefix of string w if there exists a string y = w. Show that B is regular.

Let DFA<sub>A</sub> be the DFA recognizing A and DFA<sub>B</sub> the DFA recognizing B.

Proof: Let DFA<sub>A</sub> = (Q,  $\Sigma$ ,  $\delta$ , q<sub>0</sub>, F<sub>A</sub>)

Notice: The only difference between A and B should be the accept states so DFA<sub>B</sub> = (Q,  $\Sigma$ ,  $\delta$ , q<sub>0</sub>, F<sub>b</sub>)

The accept states for machine DFA<sub>B</sub> should be any state along a path to an accept state in DFA<sub>A</sub>. so q  $\in$  F<sub>b</sub> if and only if there's a path from q to an accepting state in DFA<sub>A</sub>

Therefore since we can construct a DFA for B we know it is regular