



UNIVERSITY OF
CAMBRIDGE

Good Practices using Machine Learning

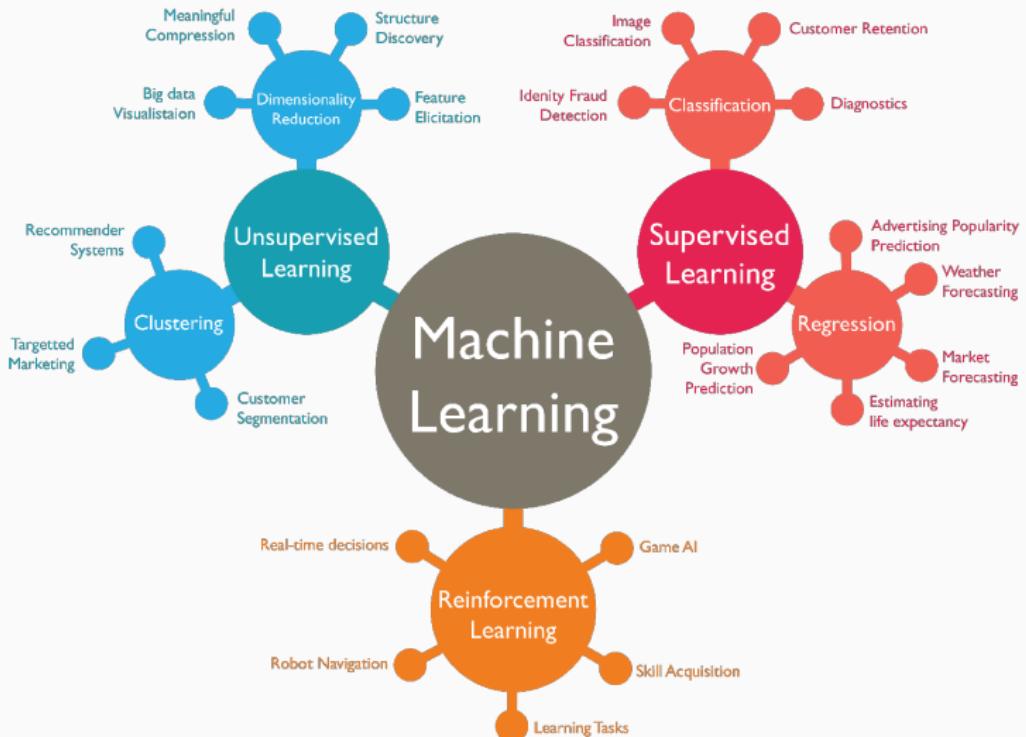
Carl Henrik Ek - che29@cam.ac.uk

September 29, 2021

<http://carlhenrik.com>

Introduction

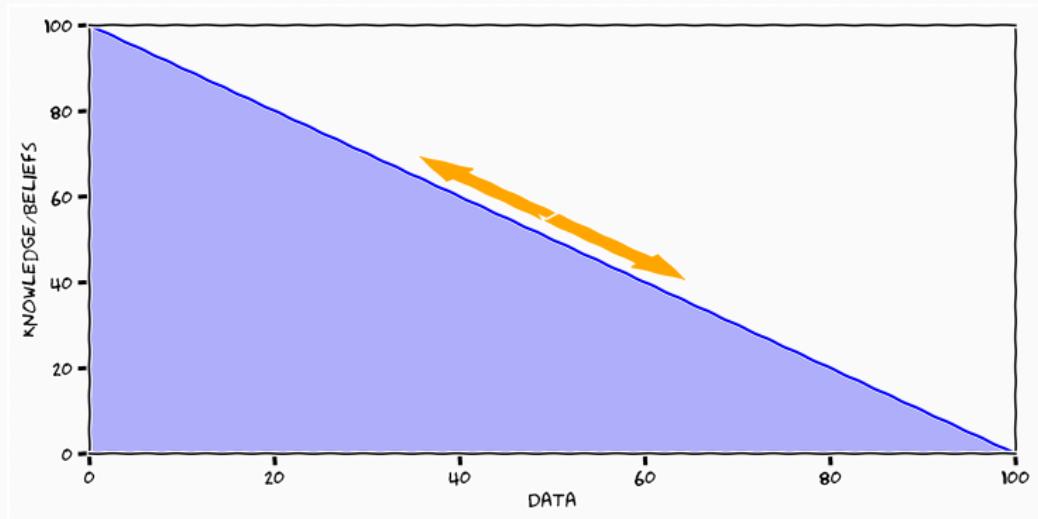
Map of ML



Handle



Pareto



Paradigms

- Supervised Learning
- Unsupervised Learning
- Reinforcement Learning
- Active Learning
- Meta Learning

Reinforcement Learning

Machine Learning

Supervised Learning predict output from input

$$\mathcal{D} = \{y_i, x_i\}_{i=1}^N$$

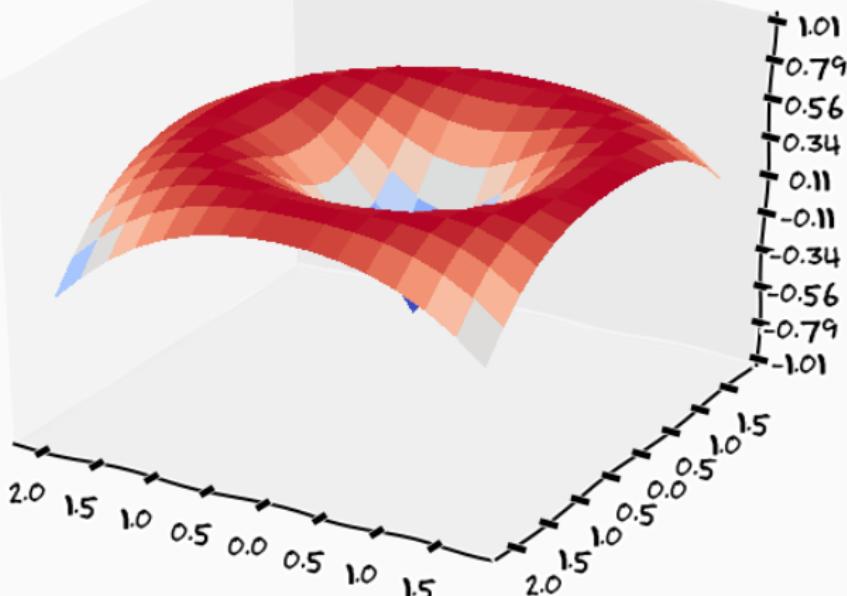
$$p(\mathbf{y}|\mathbf{x}, \theta)$$

Unsupervised Learning model the data

$$\mathcal{D} = \{y_i\}_{i=1}^N$$

$$p(\mathbf{y}|\theta)$$

Structure



Reinforcement



Reinforcement Learning



Can we learn without specifying how the task should be achieved by providing, rewards (positive) and punishment (negative)?

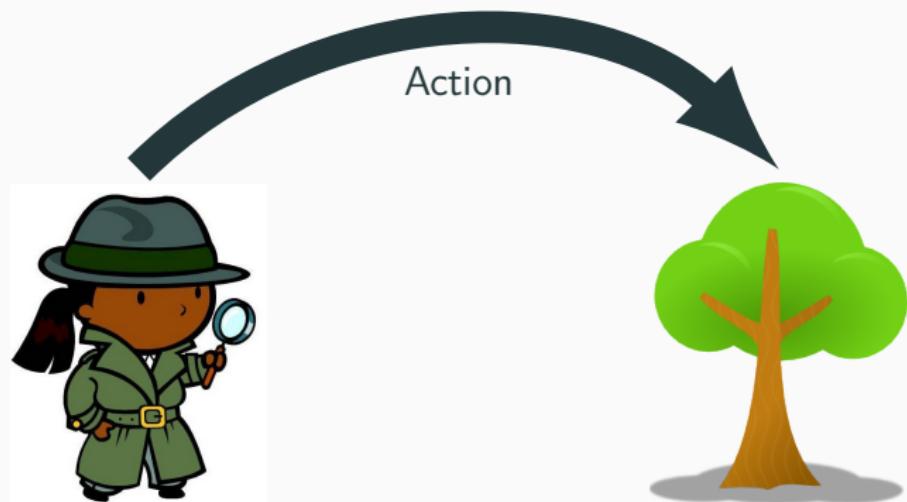
Formalism



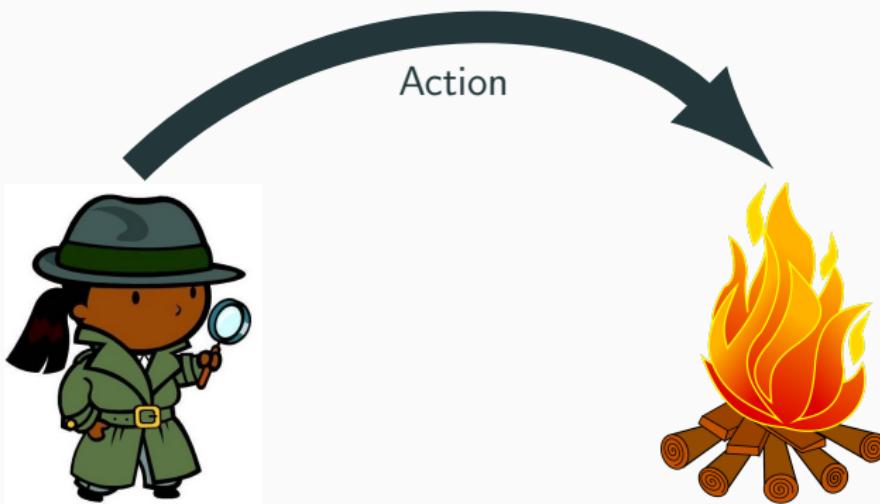
Formalism



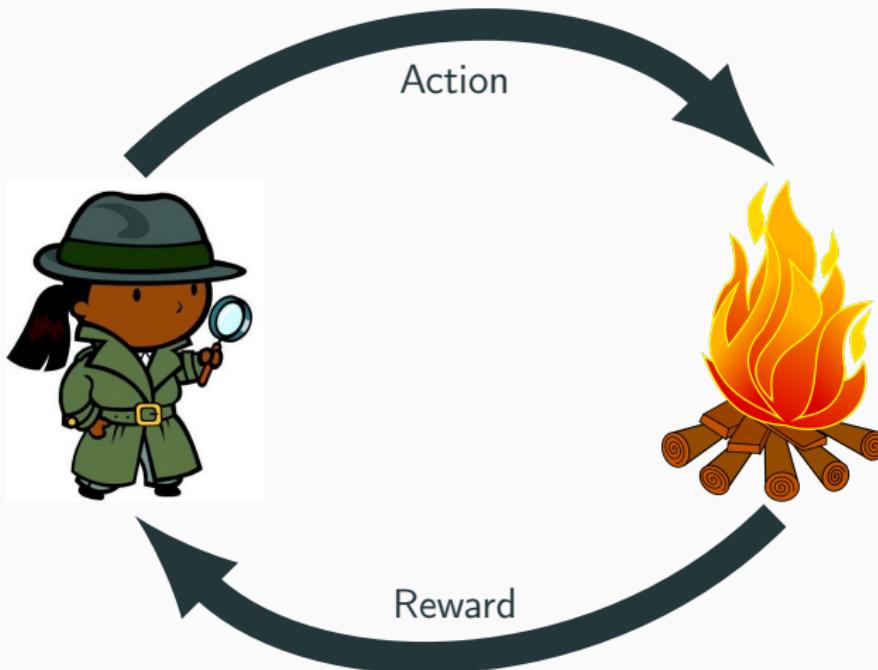
Formalism



Formalism



Formalism



Optimal Behaviour

- Finite time horizon

$$E \left[\sum_{t=0}^h r_t \right]$$

Optimal Behaviour

- Finite time horizon

$$E \left[\sum_{t=0}^h r_t \right]$$

- Average reward

$$\lim_{h \rightarrow \infty} E \left[\frac{1}{h} \sum_{t=0}^h r_t \right]$$

Optimal Behaviour

- Finite time horizon

$$E \left[\sum_{t=0}^h r_t \right]$$

- Average reward

$$\lim_{h \rightarrow \infty} E \left[\frac{1}{h} \sum_{t=0}^h r_t \right]$$

- Infinite horizon (discounted reward)

$$E \left[\sum_{t=0}^{\infty} \gamma^t r_t \right]$$

$$0 \leq \gamma \leq 1$$

Reinforcement Learning

- Dynamic model

$$p(\mathbf{s}_t | \mathbf{a}_{t-1}, \mathbf{s}_{t-1}, f)$$

- Policy

$$p(\mathbf{a}_t | \mathbf{s}_t, \pi)$$

- Reward

$$p(r_t | \mathbf{s}_t)$$

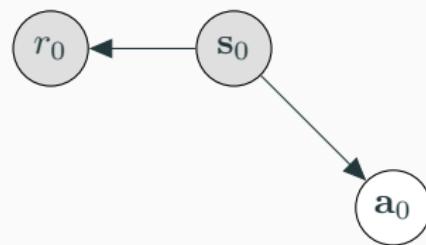
Reinforcement Learning

s_0

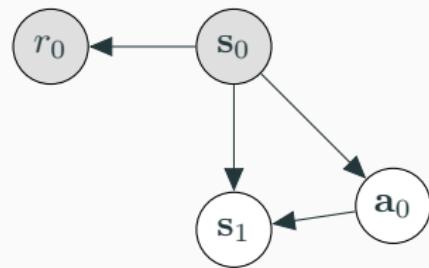
Reinforcement Learning



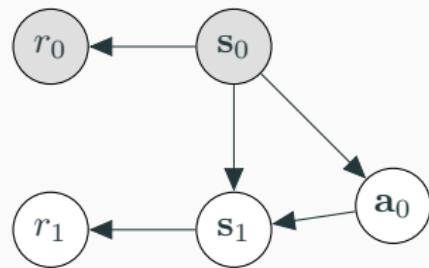
Reinforcement Learning



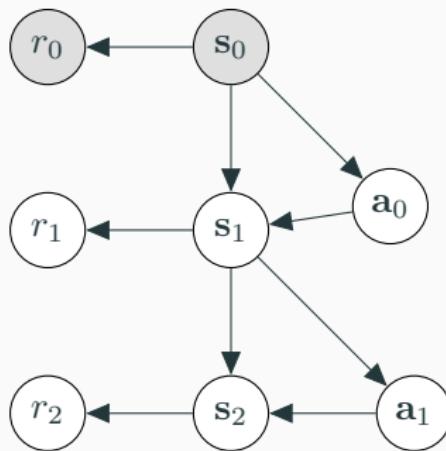
Reinforcement Learning



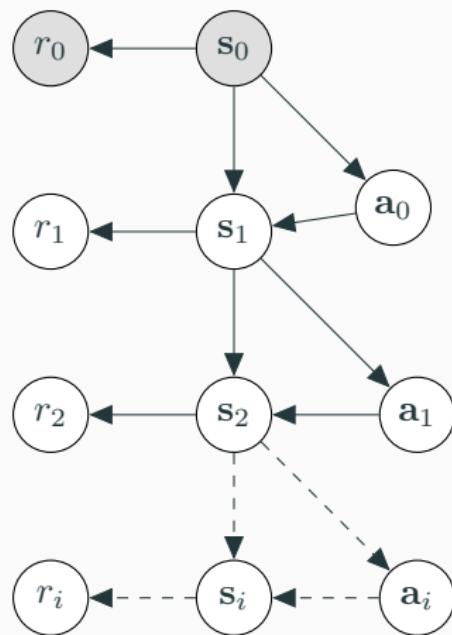
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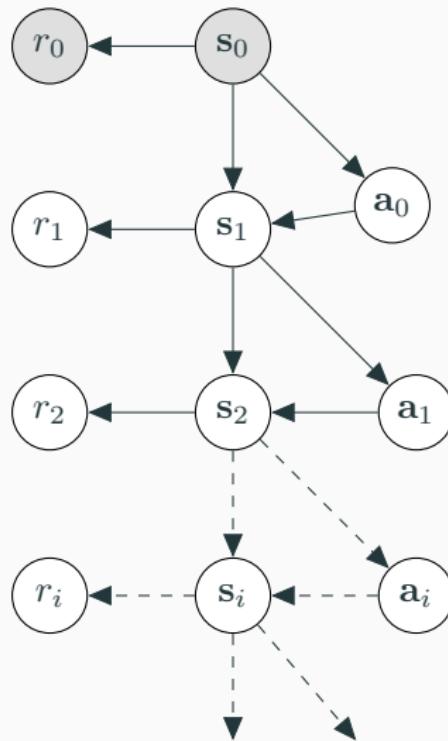
Reinforcement Learning



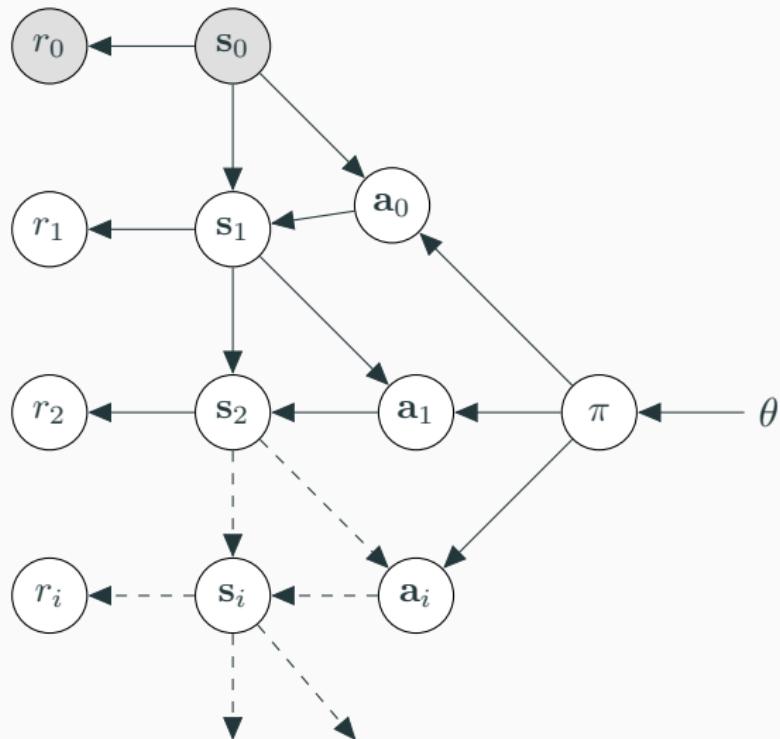
Reinforcement Learning



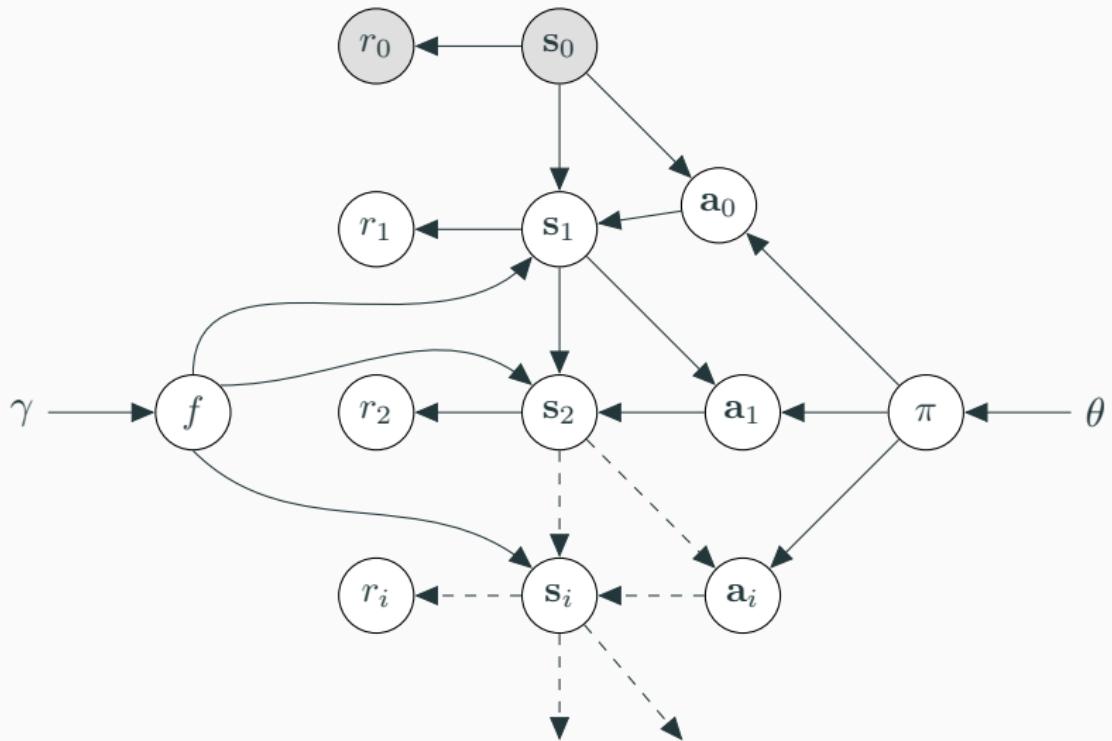
Reinforcement Learning



Reinforcement Learning



Reinforcement Learning



Reinforcement Learning : Rollout

$$\begin{aligned} p(\mathbf{s}_{0,\dots,T}, \mathbf{a}_{0,\dots,T-1}, r_{0,\dots,T}, f, \pi, \gamma, \theta | \mathbf{s}_0) = \\ p(\mathbf{s}_T | a_{T-1}, s_{T-1}, f) p(a_{T-1} | s_{T-1}, \pi) \dots \\ \vdots \\ p(\mathbf{s}_2 | a_1, s_1, f) p(a_1 | s_1, \pi) \\ p(\mathbf{s}_1 | a_0, s_0, f) p(a_0 | s_0, \pi) \\ p(f | \gamma) p(\pi | \theta) p(\gamma) p(\theta) \end{aligned}$$

- if we want to learn dynamics and policy we need to marginalise out f, π, θ and γ
- very very hard problem as uncertainty propagated a long way

Reinforcement Learning

- Is not a model but an inference structure

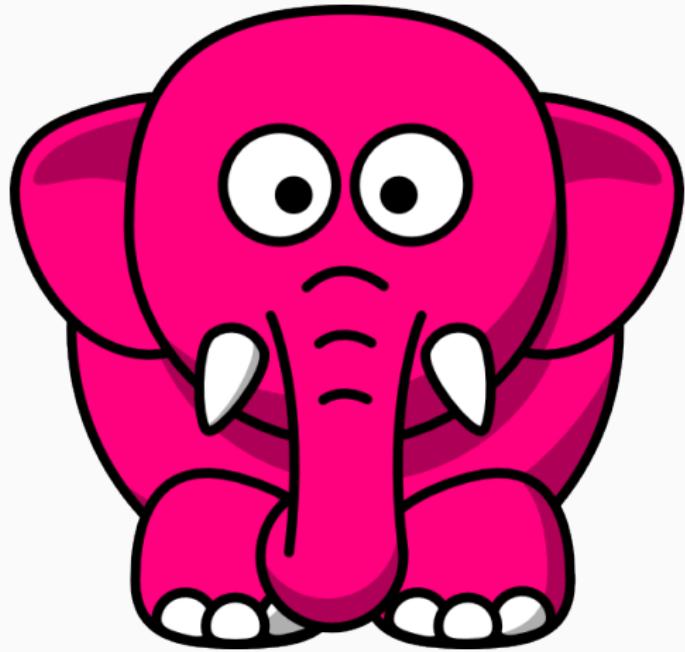
Reinforcement Learning

- Is not a model but an inference structure
- The data capture should be part of the model

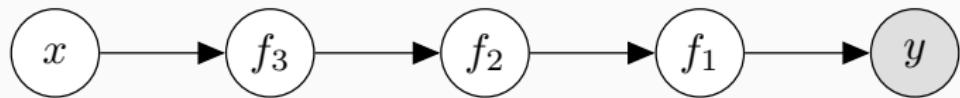
Reinforcement Learning

- Is not a model but an inference structure
- The data capture should be part of the model
 - Exploration vs. Exploitation

Neural Networks

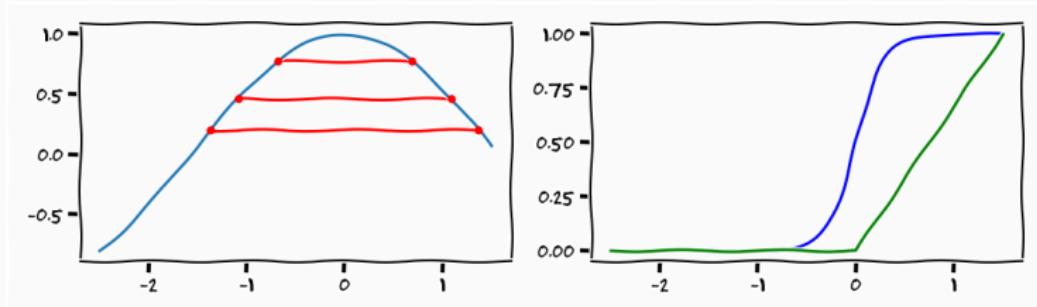


Neural Networks



$$y = f_k(f_{k-1}(\dots f_0(x))) = f_k \circ f_{k-1} \circ \dots \circ f_1(x)$$

Composite Functions



$$y = f_k(f_{k-1}(\dots f_0(x))) = f_k \circ f_{k-1} \circ \dots \circ f_1(x)$$

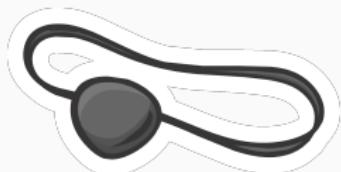
$$\text{Kern}(f_1) \subseteq \text{Kern}(f_{k-1} \circ \dots \circ f_2 \circ f_1) \subseteq \text{Kern}(f_k \circ f_{k-1} \circ \dots \circ f_2 \circ f_1)$$

$$\text{Im}(f_k \circ f_{k-1} \circ \dots \circ f_2 \circ f_1) \subseteq \text{Im}(f_k \circ f_{k-1} \circ \dots \circ f_2) \subseteq \dots \subseteq \text{Im}(f_k)$$

A piece of string



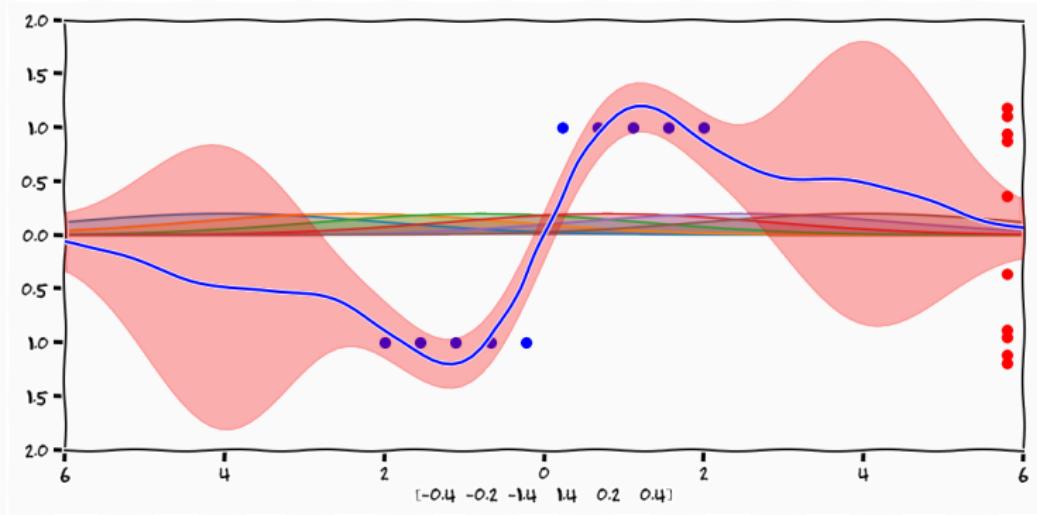
Assumptions: Algorithms



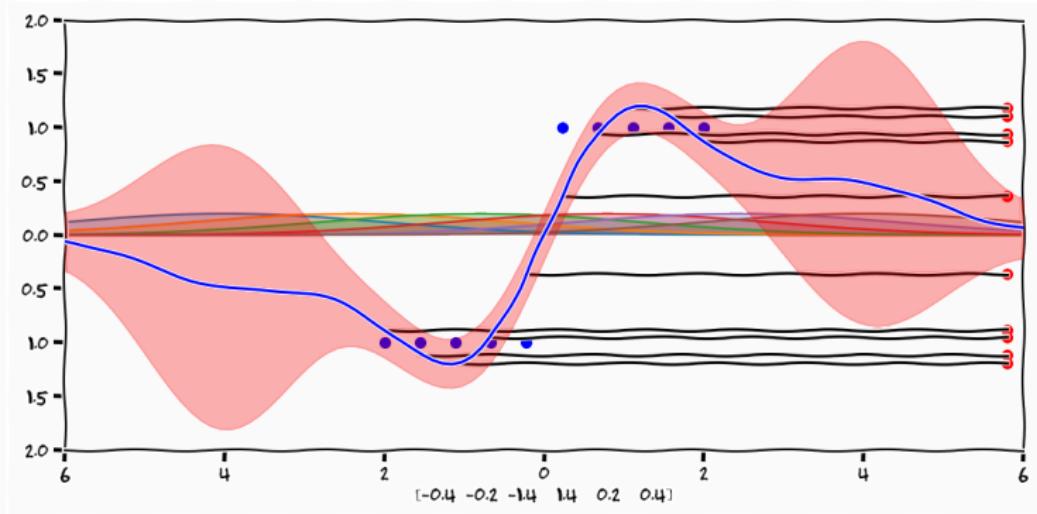
Statistical Learning

$$\mathcal{A}_{\mathcal{F}}(\mathcal{S})$$

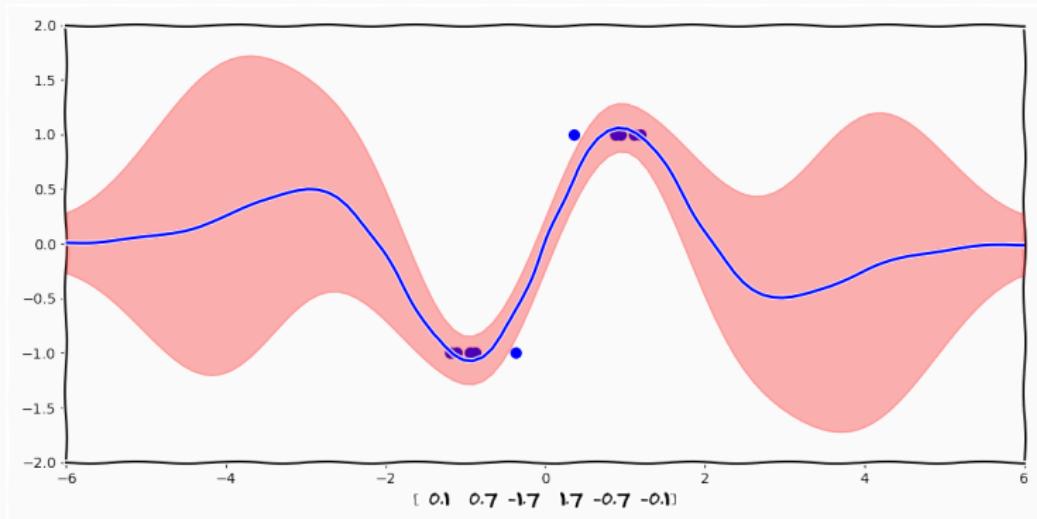
Composite Functions



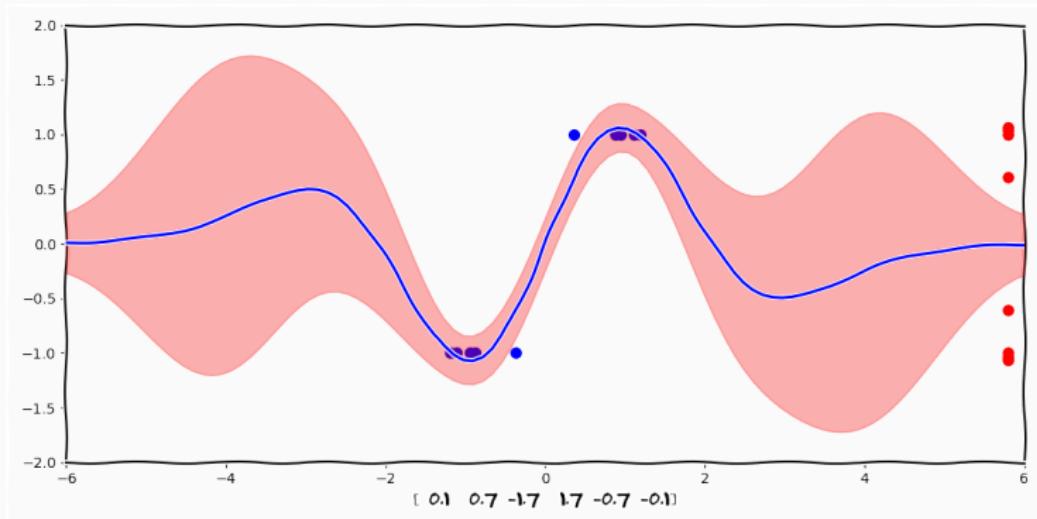
Composite Functions



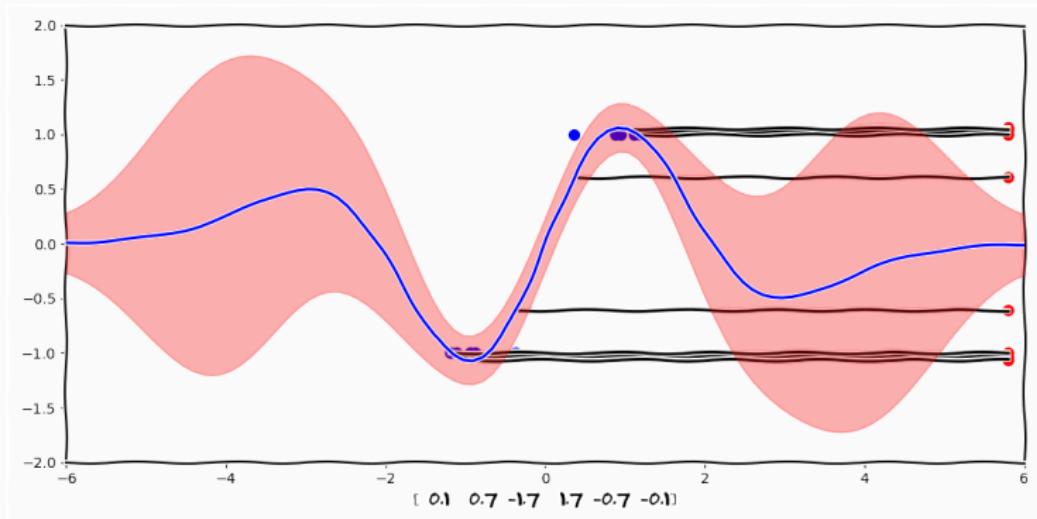
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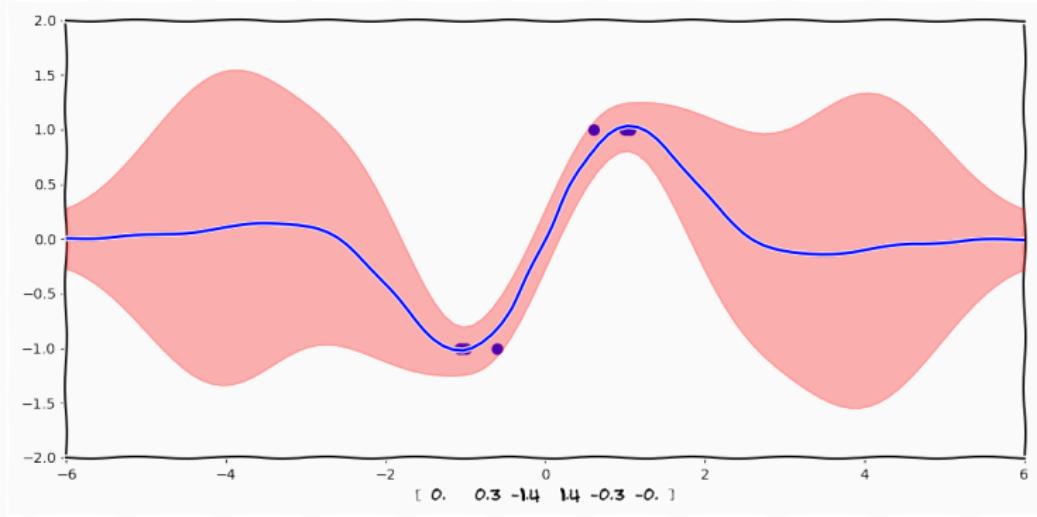
Composite Functions



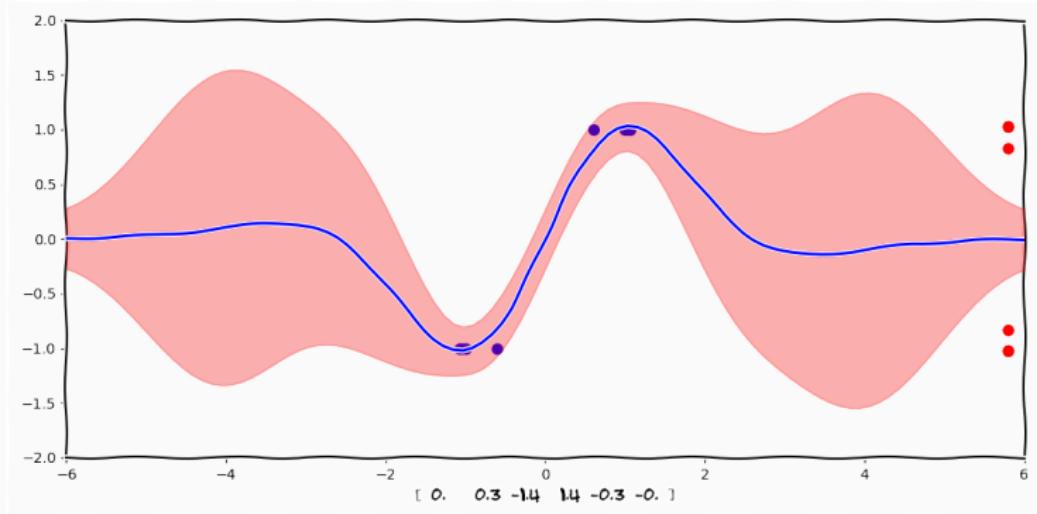
Composite Functions



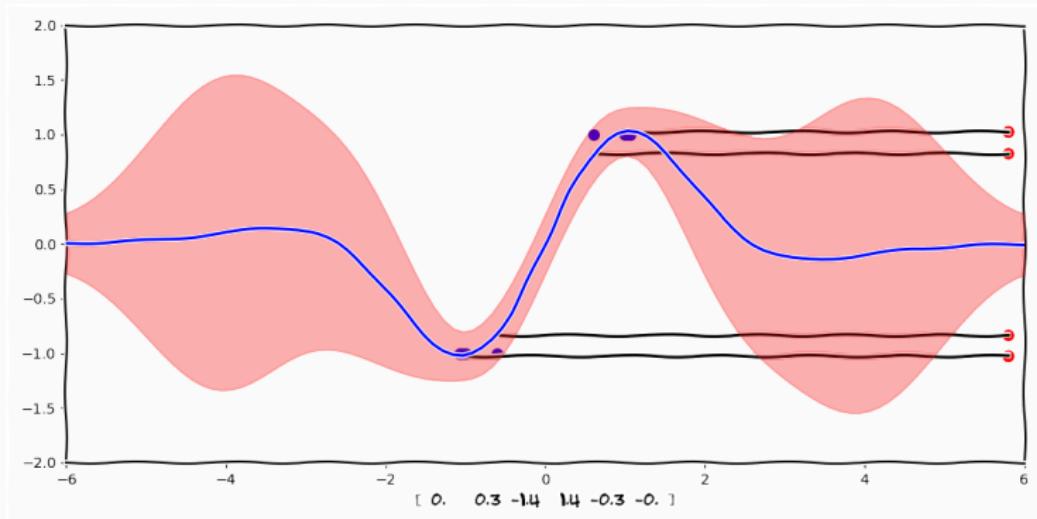
Composite Functions



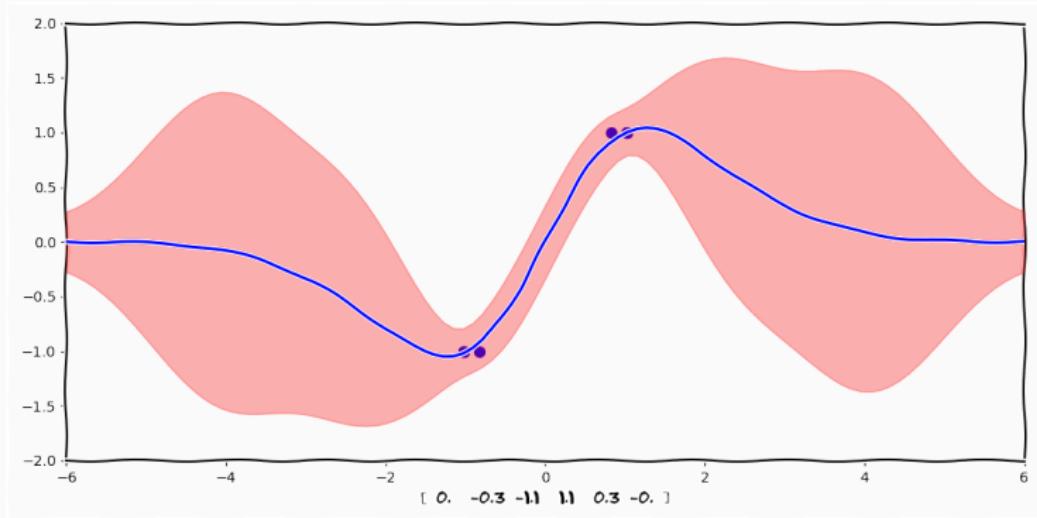
Composite Functions



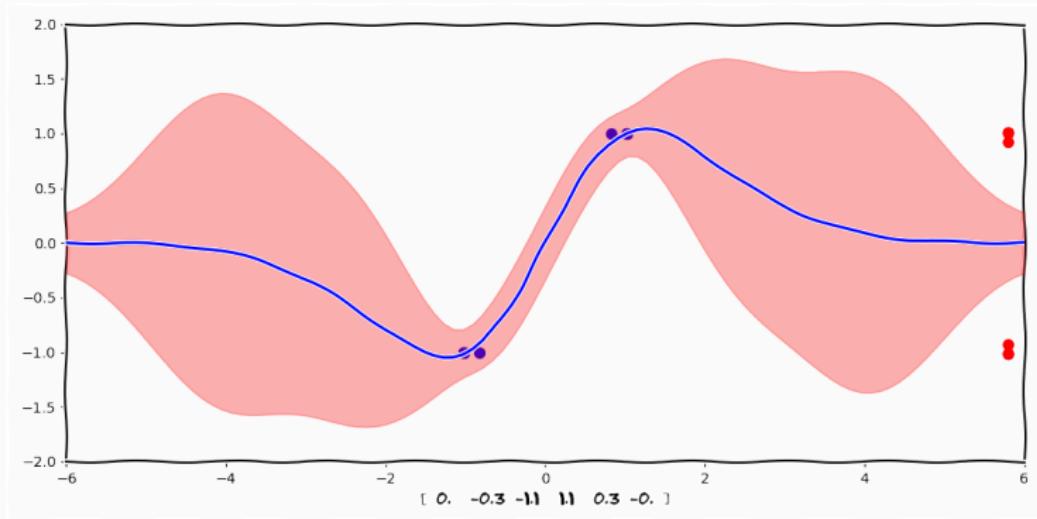
Composite Functions



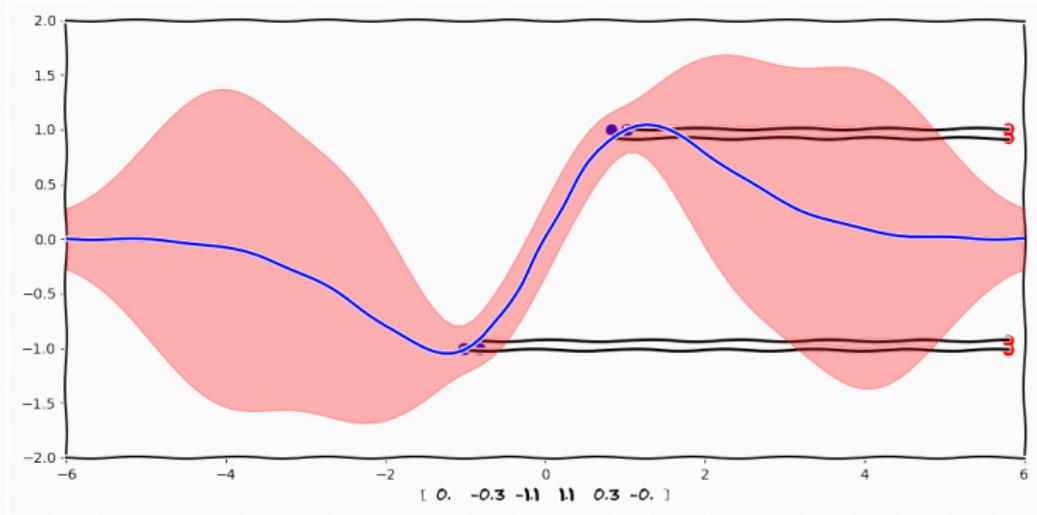
Composite Functions



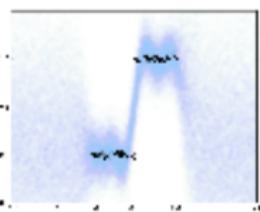
Composite Functions



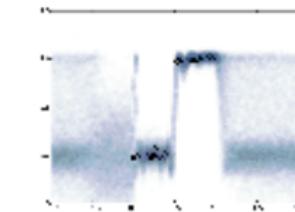
Composite Functions



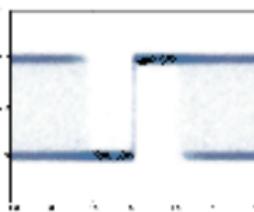
The Final Composition



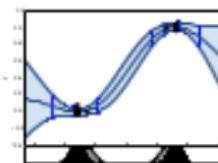
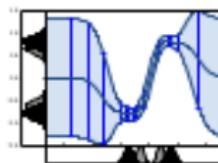
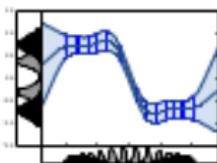
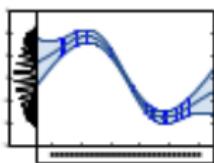
(a) GP



(b) 2 layers

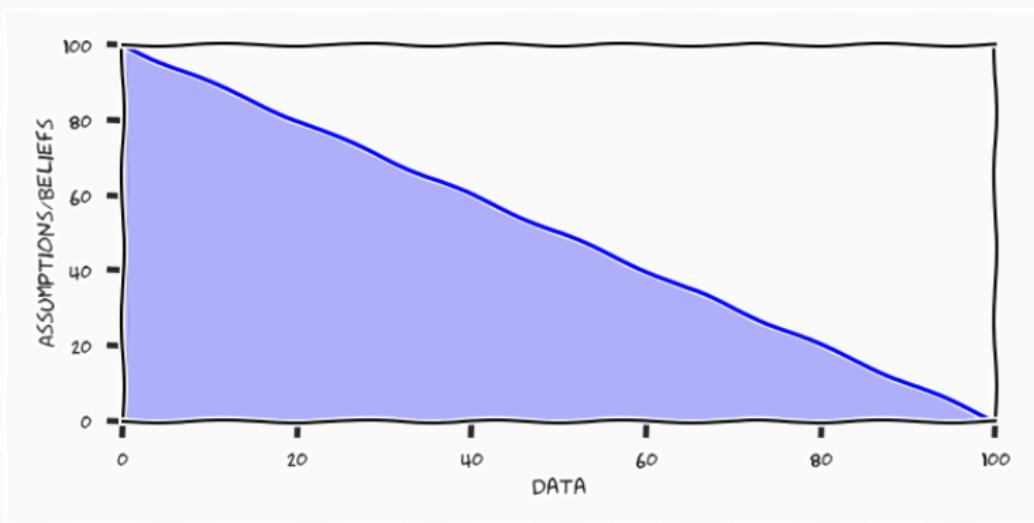


(c) 4 layers



(d) Hidden spaces for 4 layer model

Data and Beliefs



Active Learning

Black-box Optimisation

$$x^{(*)} = \underset{x \in \mathcal{X}}{\operatorname{argmin}} f(x)$$

- \mathcal{X} is a bounded domain

Black-box Optimisation

$$x^{(*)} = \operatorname{argmin}_{x \in \mathcal{X}} f(x)$$

- \mathcal{X} is a bounded domain
- f is explicitly unknown

Black-box Optimisation

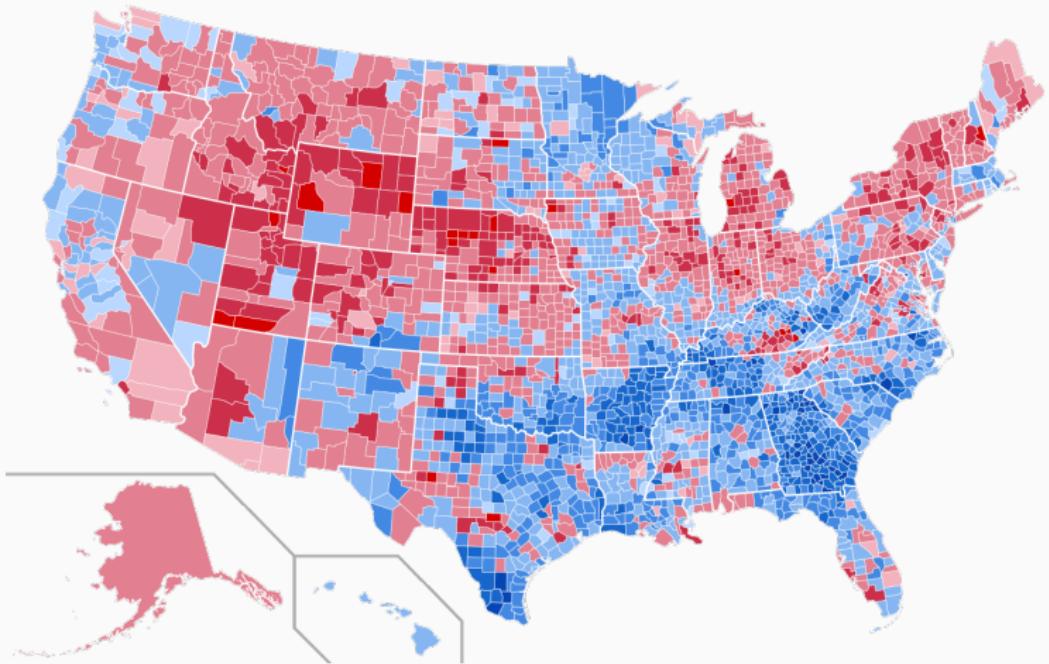
$$x^{(*)} = \operatorname{argmin}_{x \in \mathcal{X}} f(x)$$

- \mathcal{X} is a bounded domain
- f is explicitly unknown
- Evaluations of f may be noisy

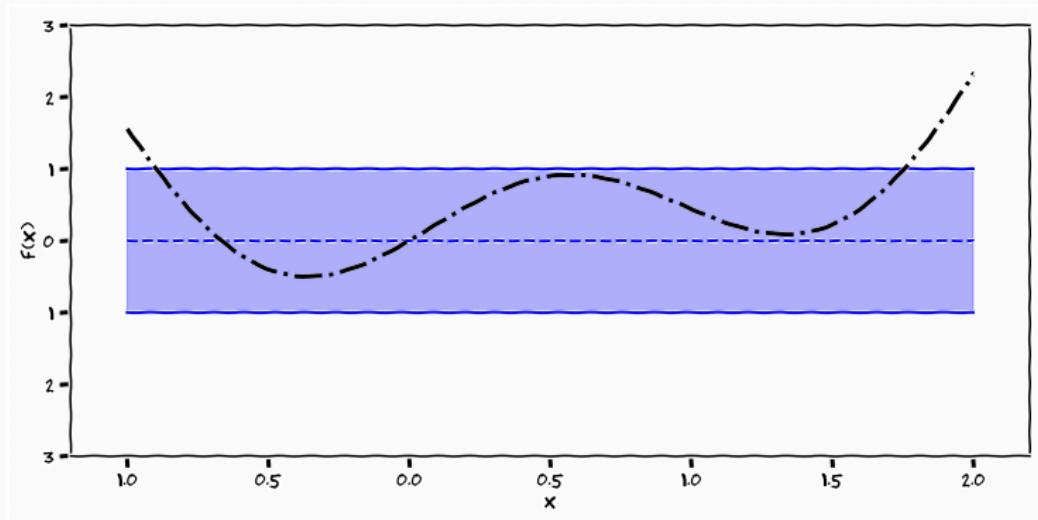
Black-box Optimisation

$$x^{(*)} = \operatorname{argmin}_{x \in \mathcal{X}} f(x)$$

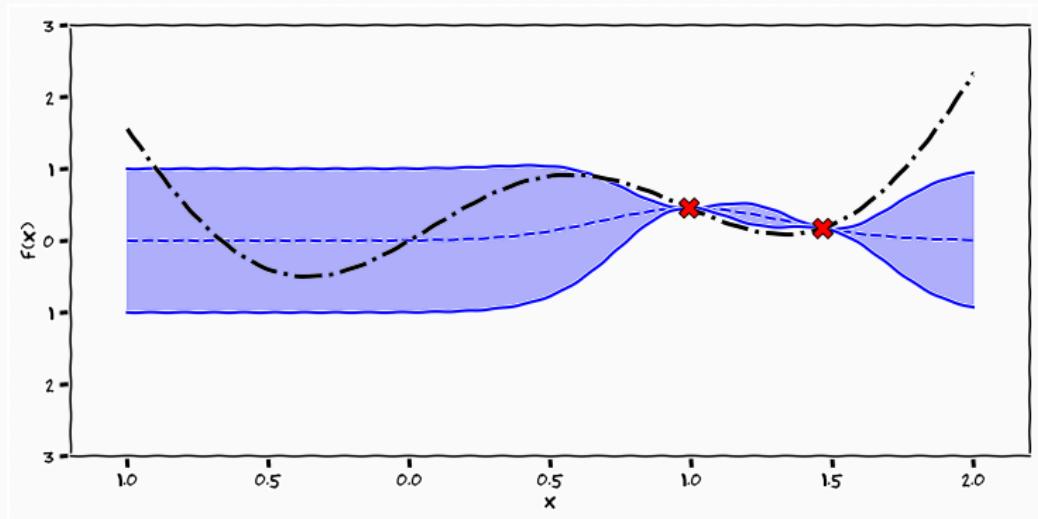
- \mathcal{X} is a bounded domain
- f is explicitly unknown
- Evaluations of f may be noisy
- Evaluations of f is expensive



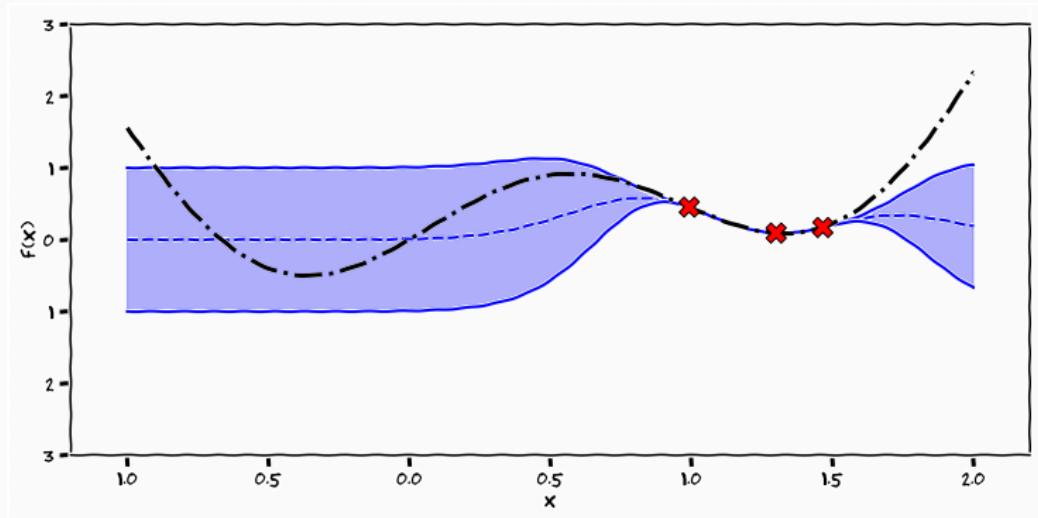
Posterior Search



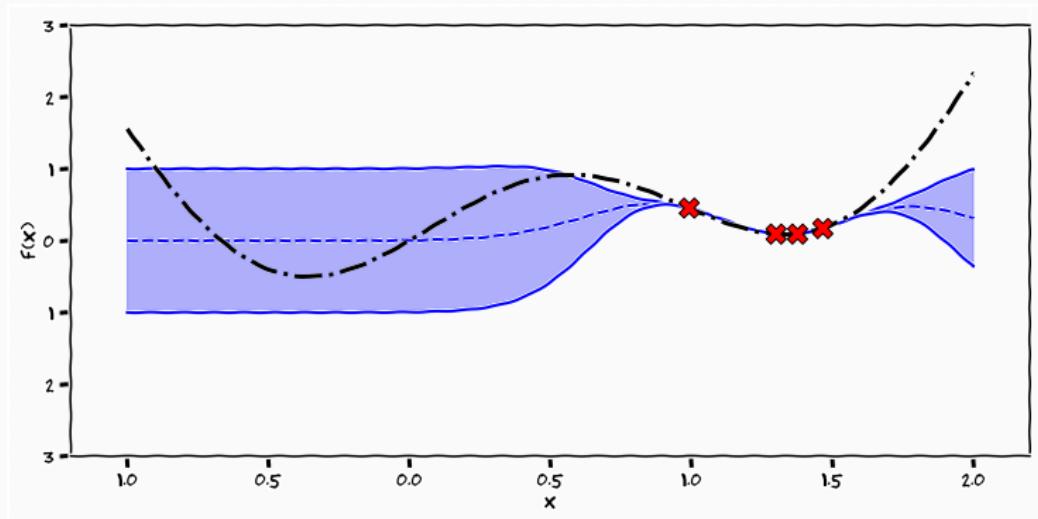
Posterior Search



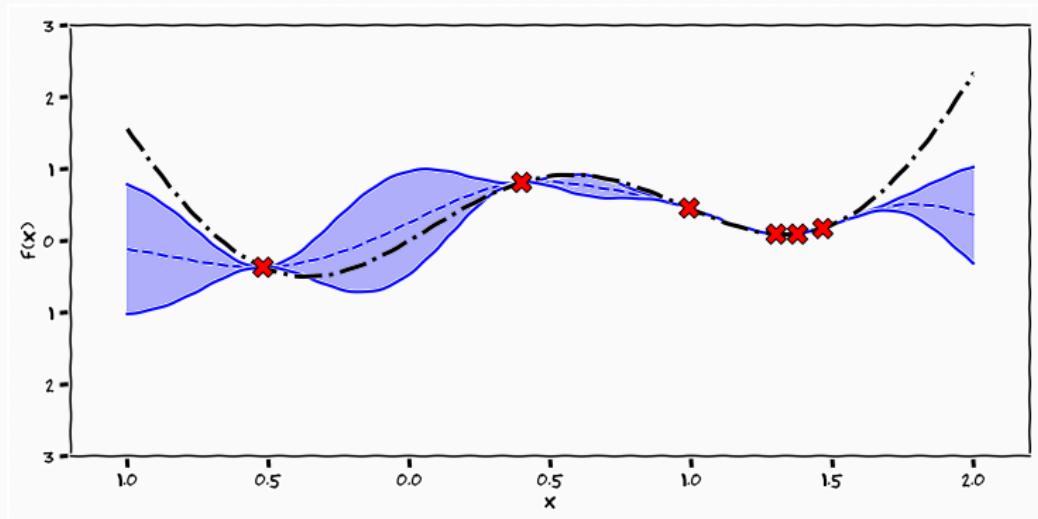
Posterior Search



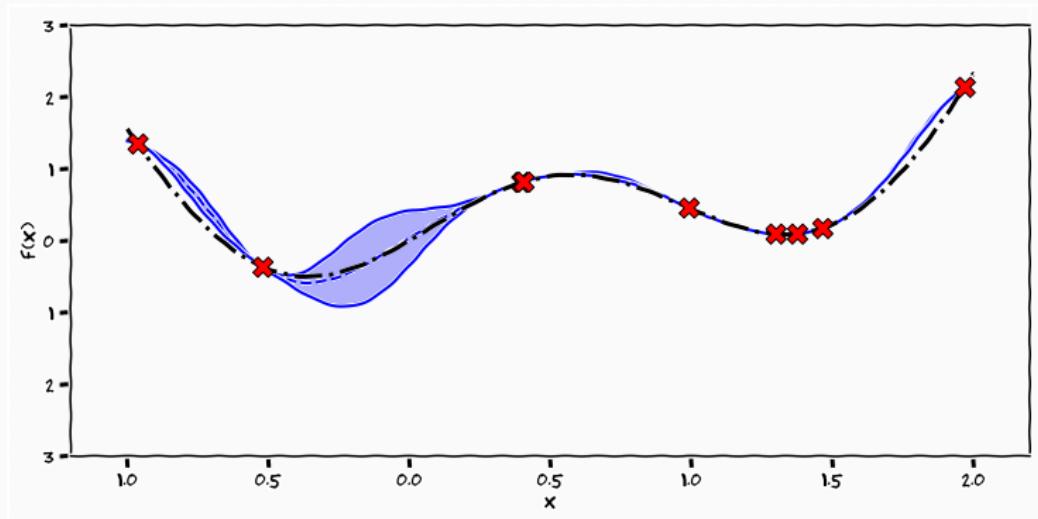
Posterior Search



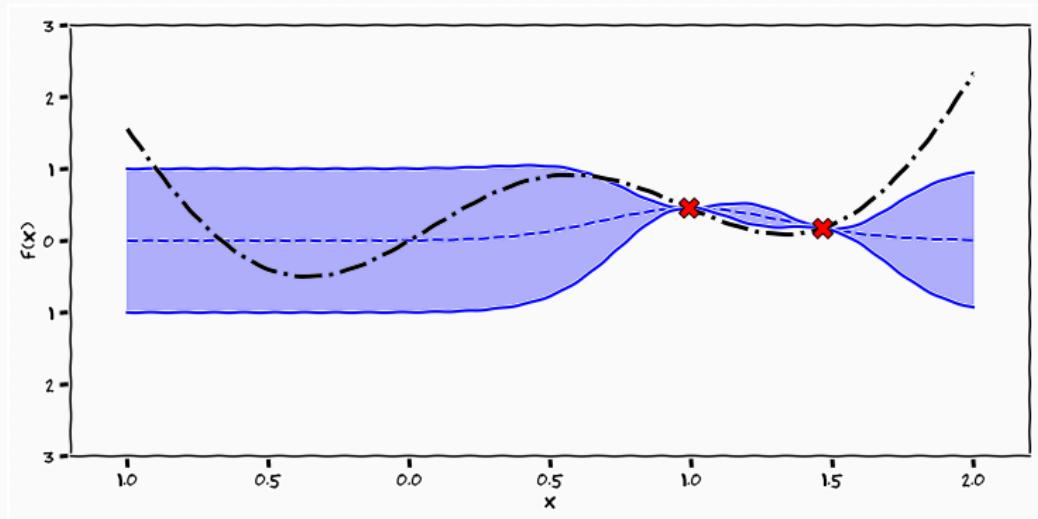
Posterior Search



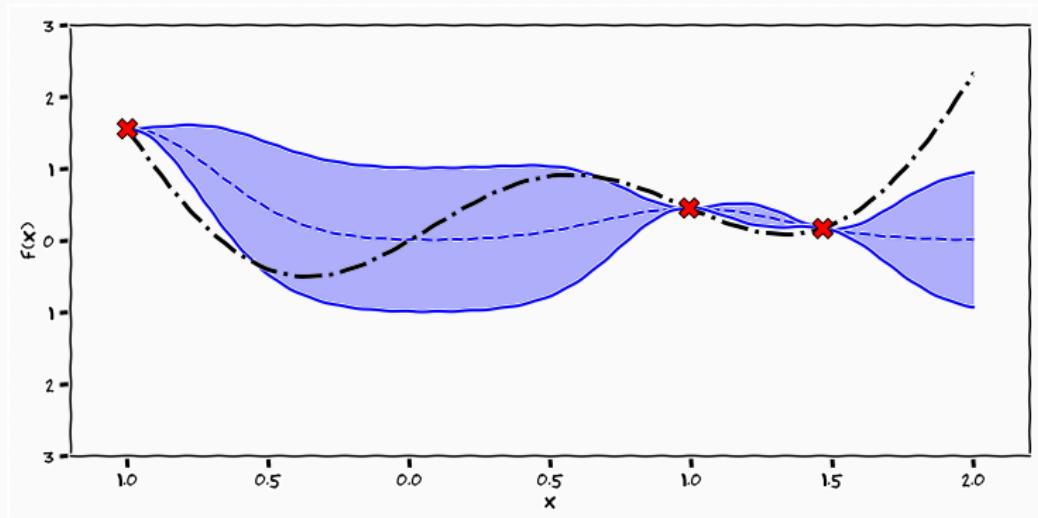
Posterior Search



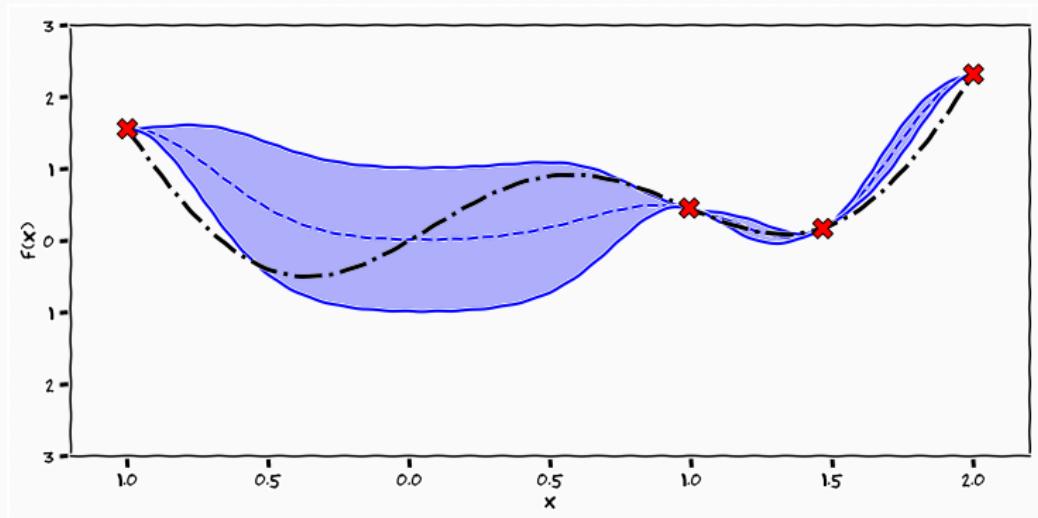
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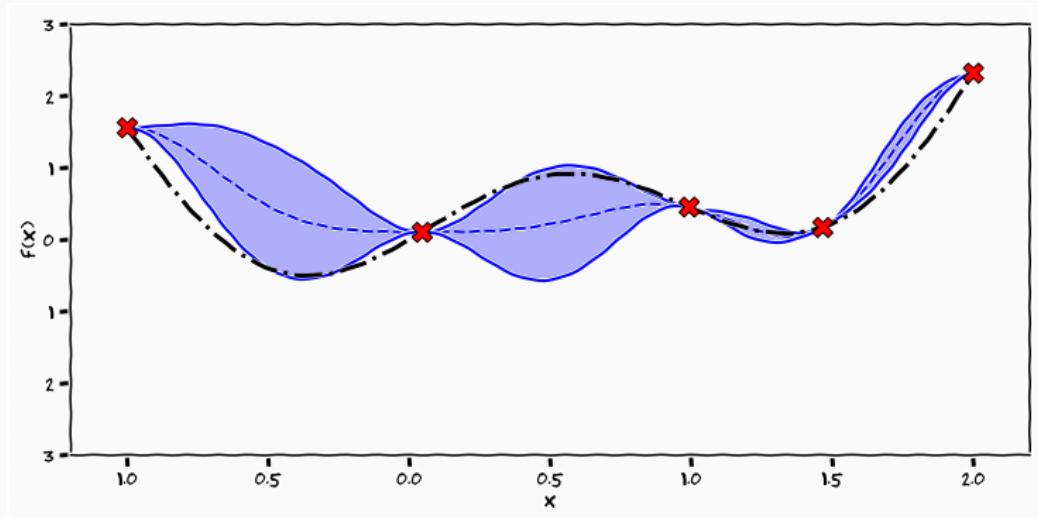
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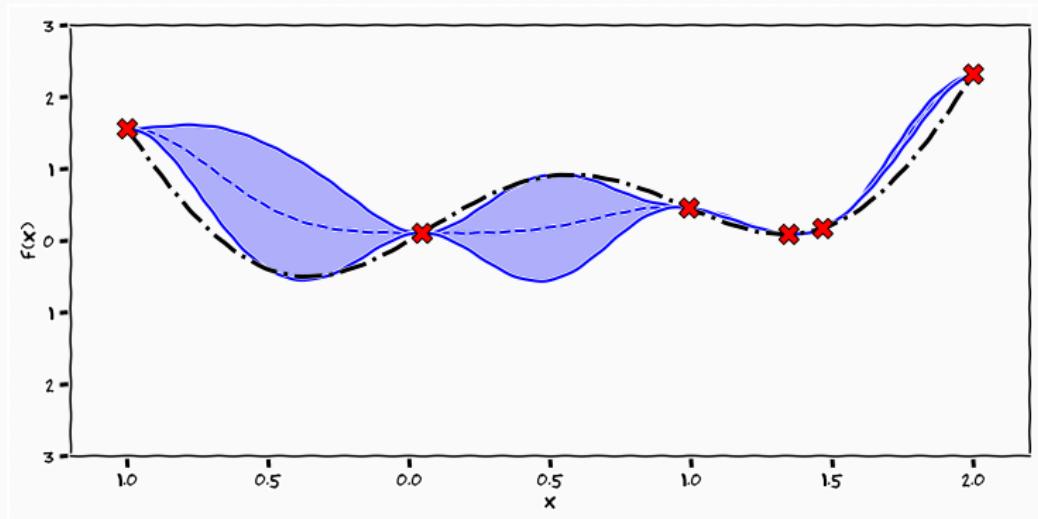
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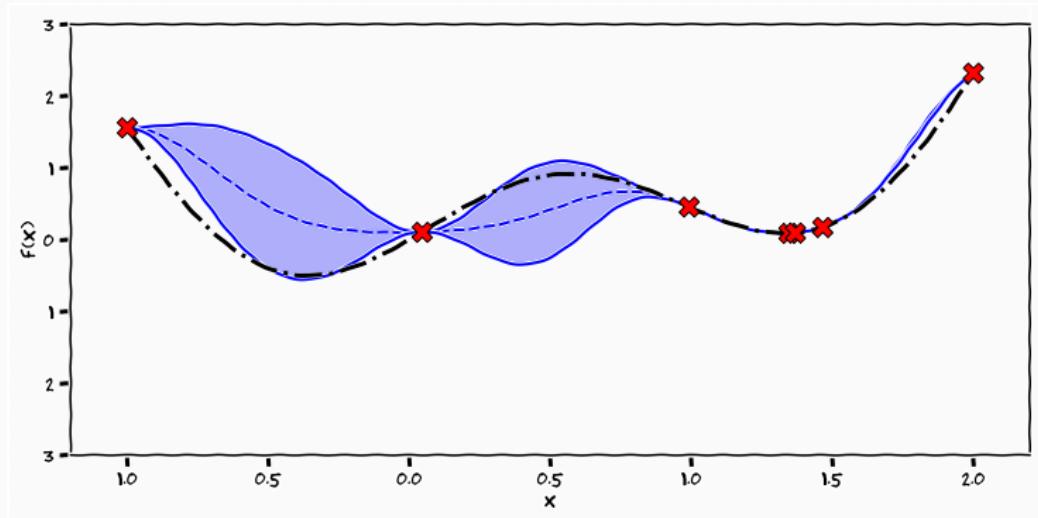
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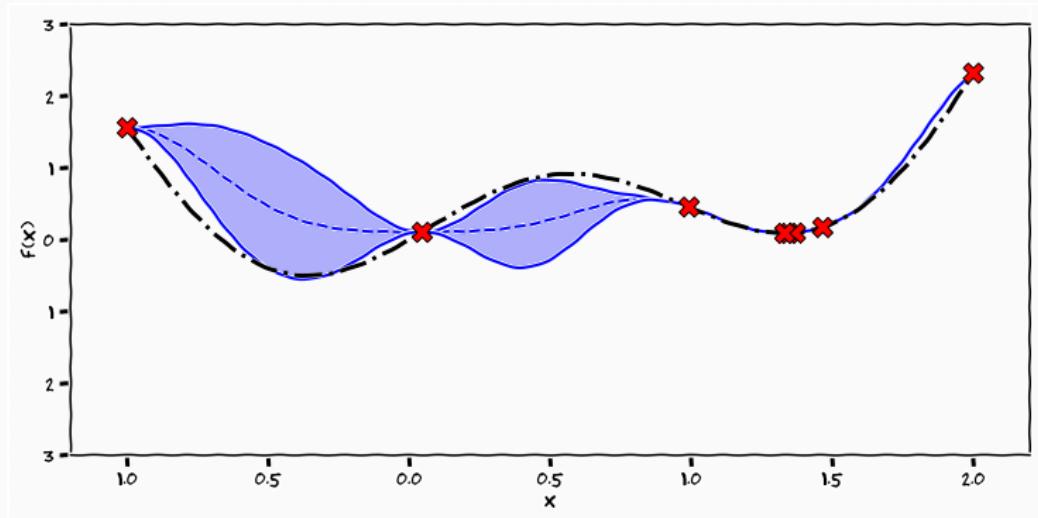
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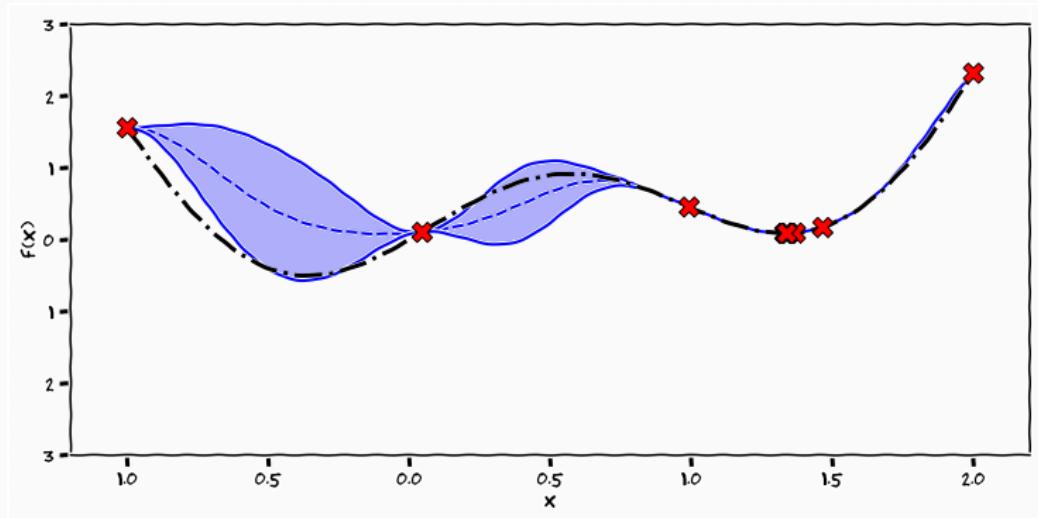
Posterior Search



Posterior Search



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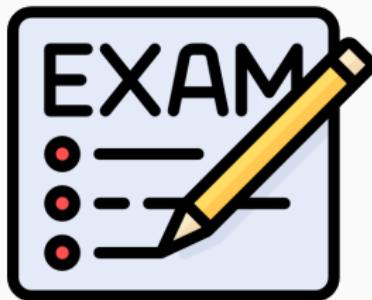








Exploration and Exploitation



Exploitation use the knowledge that we currently have

Exploration try to gain new knowledge by trying new things

Surrogate Uncertainty

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MLPW/bin/lectures/06./bin/ass/bo-aq-int-gp-1.png"

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Surrogate Uncertainty

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Surrogate Uncertainty

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Surrogate Uncertainty

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Surrogate Uncertainty

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Surrogate Uncertainty

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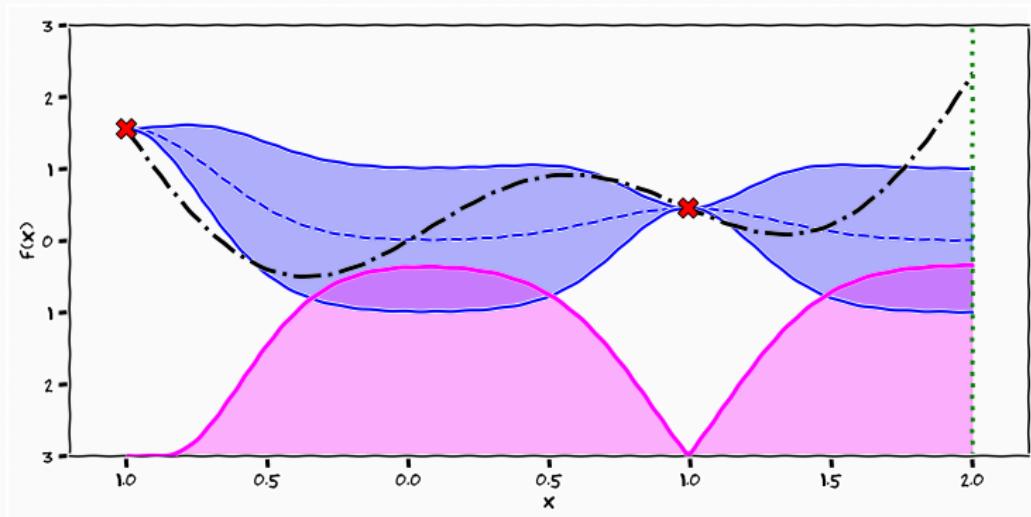
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Acquisition Function

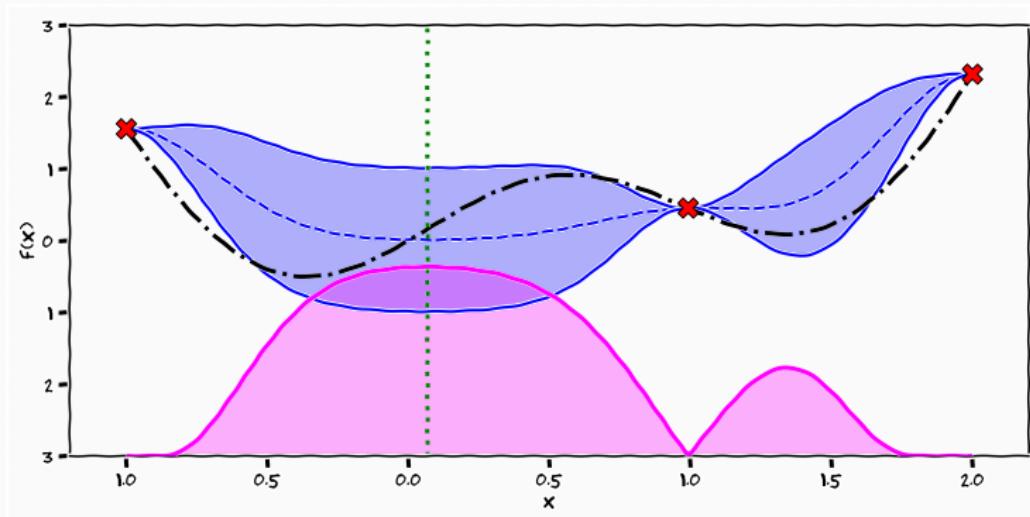
$$x_{n+1} = \operatorname{argmax}_{x \in \mathcal{X}} \alpha(x; \{x_i, y_i\}_{i=1}^n, \mathcal{M}_n)$$

- Formulate a sequential decision problem
- This will work well if $\alpha(x)$
 - is cheap to compute
 - balances *exploration* and *exploitation*

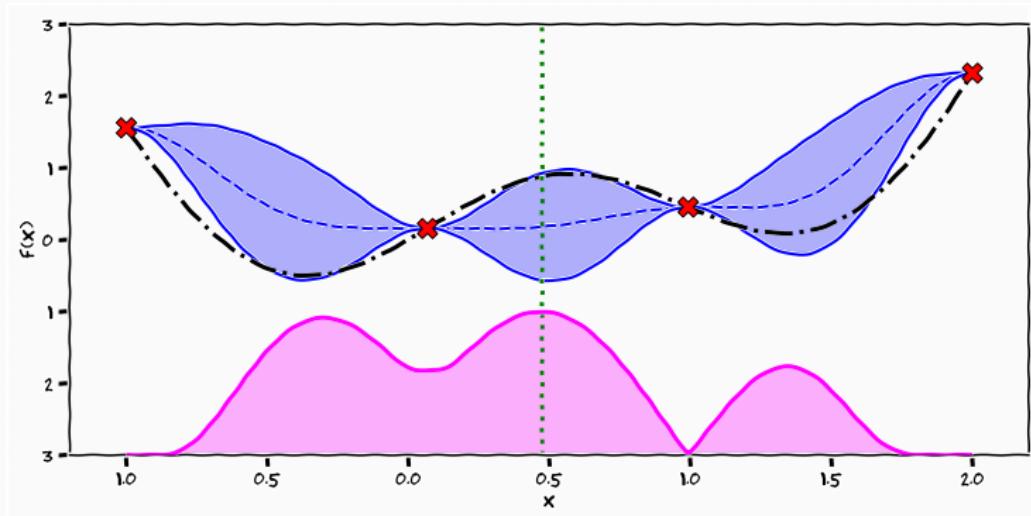
Expected Improvement



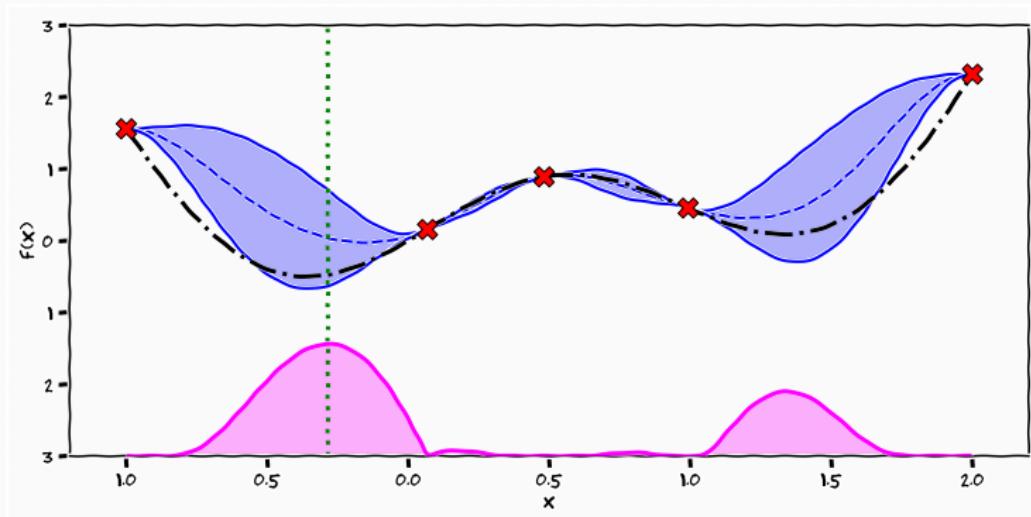
Expected Improvement



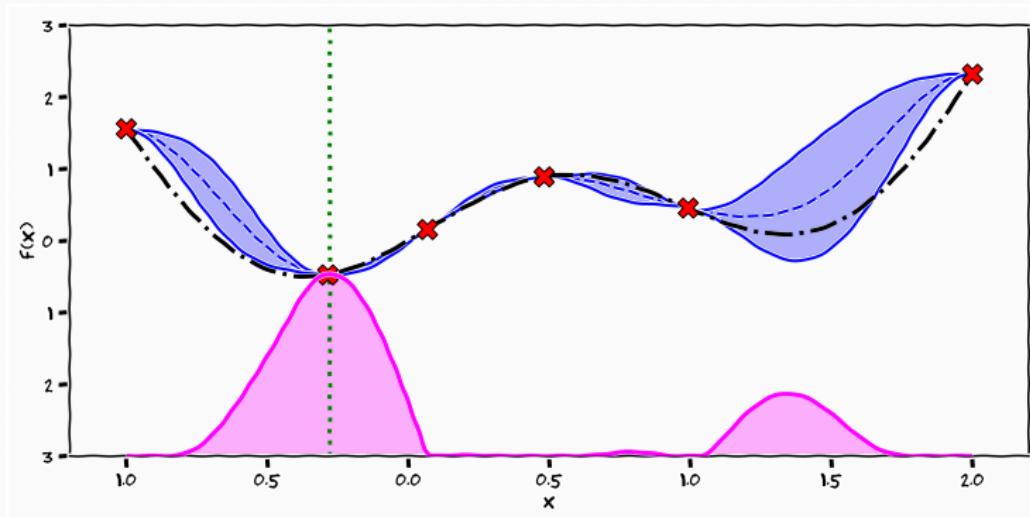
Expected Improvement



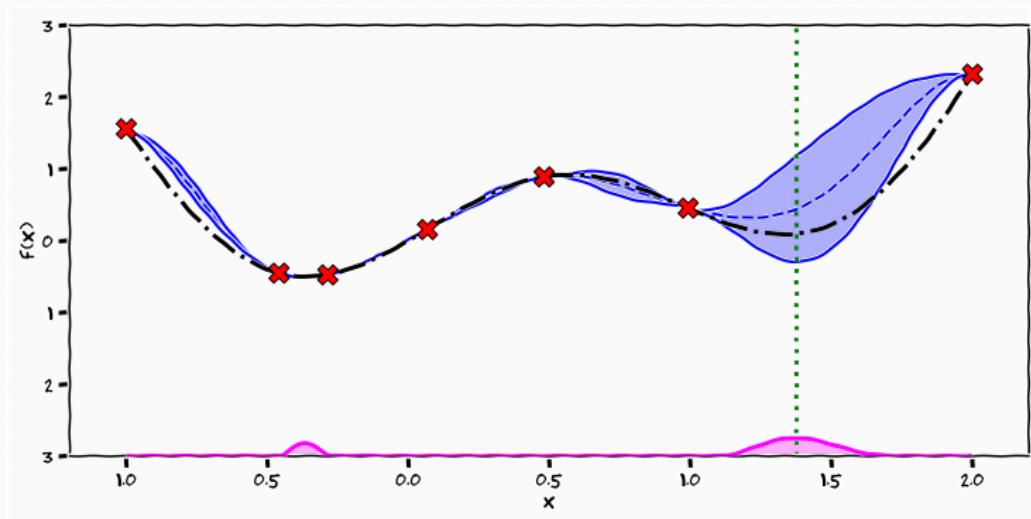
Expected Improvement



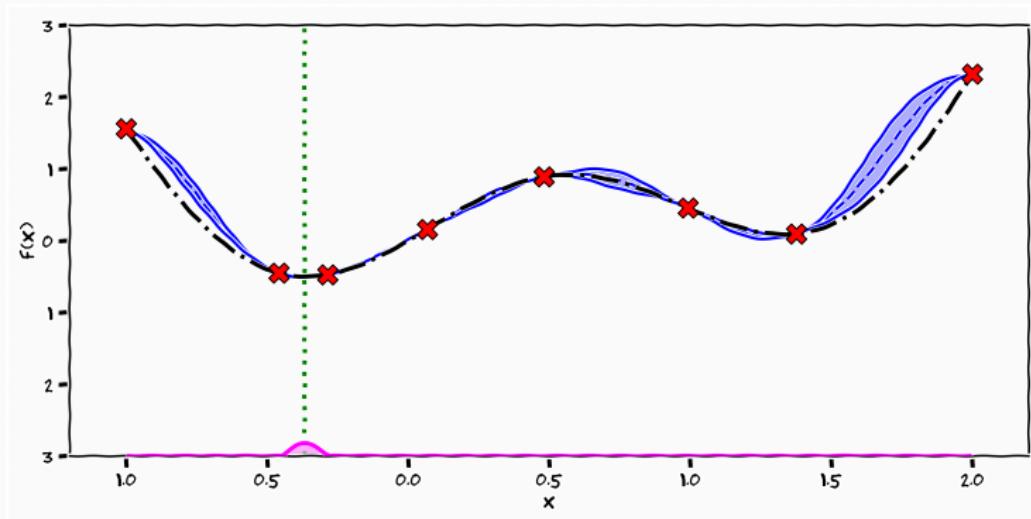
Expected Improvement



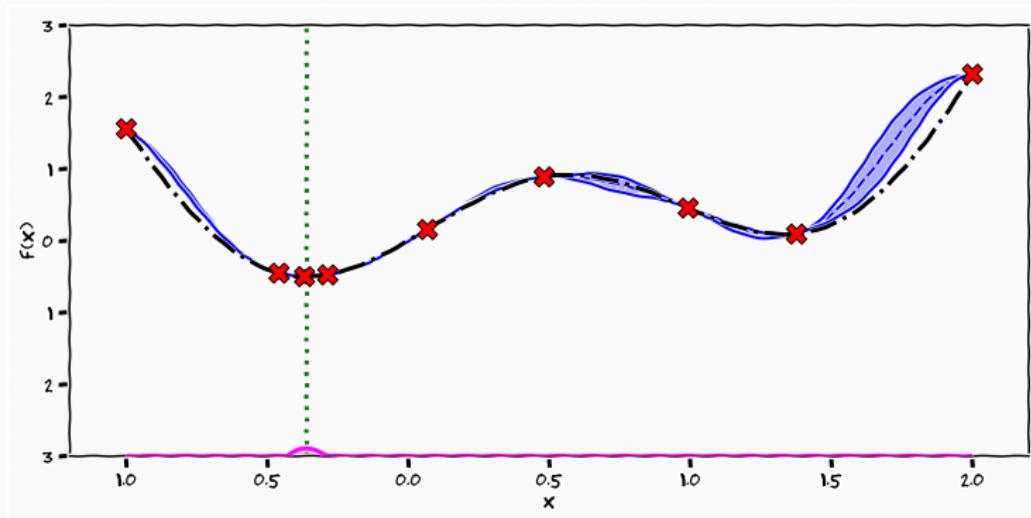
Expected Improvement



Expected Improvement



Expected Improvement



Task 1 encode your knowledge about **the function** in the GP prior

¹till they open the door to the exam.

Task 1 encode your knowledge about **the function** in the GP prior

Task 2 randomly sample some data

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Task 2 randomly sample some data

Task 3 specify your acquisition function

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Task 1 encode your knowledge about **the function** in the GP prior

Task 2 randomly sample some data

Task 3 specify your acquisition function

Task 4 evaluate and maximise the acquisition function

¹till they open the door to the exam.

Task 1 encode your knowledge about **the function** in the GP prior

Task 2 randomly sample some data

Task 3 specify your acquisition function

Task 4 evaluate and maximise the acquisition function

Task 5 add new data to model and **re-estimate** hyperparameters

¹till they open the door to the exam.

Task 1 encode your knowledge about **the function** in the GP prior

Task 2 randomly sample some data

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Task 4 evaluate and maximise the acquisition function

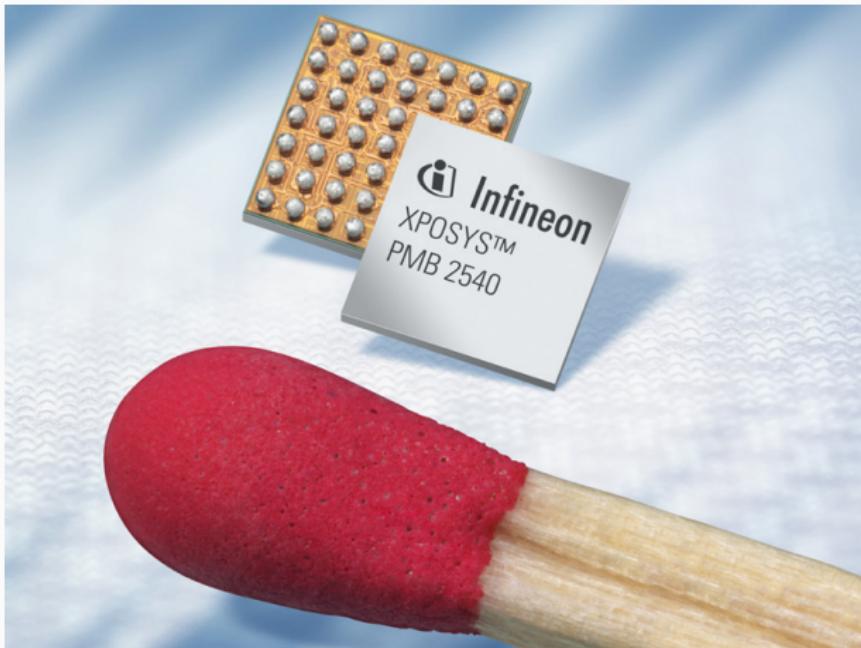
Task 5 add new data to model and **re-estimate** hyperparameters

Loop 4-5 till budget is gone¹

¹till they open the door to the exam.

How to Work on ML Problems





AI Replacement

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MLPW/bin/lectures/06/bin/ass/ai-replacement.jpeg"

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ML Colonialism



The Three Asses²



²Neil Lawrence and Andrei Paleyes coined the term

- **A*cce*ss** - how to acquire data?
- ***Ass*ess** - what can you do with the data before you have a question?
- **A*ddre*ss** - what can you do when you have a question?

Infrastructure

- Tensorflow
- PyTorch
- Torch
- NumpyTorch
- Edward
- Numpy
- Gluon
- Theano
- Tensorflow Probabilities
- JAX
- R
-

Benefits

$$p(f_* | \mathbf{x}_*, \mathbf{x}, \mathbf{y}, \boldsymbol{\theta}) = \mathcal{N}(k(\mathbf{x}_*, \mathbf{x})^T(K(\mathbf{x}, \mathbf{x}) + \sigma^2 \mathbf{I}))^{-1} \mathbf{y},$$
$$k(\mathbf{x}_*, \mathbf{x}_*) - k(\mathbf{x}_*, \mathbf{x})^T(K(\mathbf{x}, \mathbf{x}) + \sigma^2 \mathbf{I})^{-1} K(\mathbf{x}, \mathbf{x}_*))$$

$$k(x_1, x_2) = \alpha e^{-\frac{1(x_1 - x_2)^2}{2 \cdot \ell}}$$

AutoDiff Compute $\frac{\partial}{\partial x_1} p(f_* | \mathbf{x}_*, \mathbf{x}, \mathbf{y}, \boldsymbol{\theta})$

Operations $\log(e^x) = x$

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- L is a triangular matrix $|A| = \prod \text{diag}(A)$

$$\log(|K|) = 2 \cdot \log \left(\prod \text{diag}(L) \right) = 2 \sum \log(\text{diag}(L))$$

Gaussian processes

Code

```
from scipy.spatial.distance import cdist
x = np.linspace(-3,3,300).reshape(-1,1)
K = scip.exp(-cdist(X, X, 'squeclidean') / s**2)
y_samp = np.random.multivariate_normal(mu,K,Nsamp)
```

Summary

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How to **not** do a PhD



Courses

- Foundations of Data Science
- Advanced Data Science
- Probabilistic Machine Learning
- Machine Learning and the Physical World
- Deep Learning

eof